Tööleht nr 4 aines „Matemaatiline analüüs”

I. Lähtudes tuletise definitsioonist, leida järgmiste funktsioonide tuletised

1. $y = x^3$
2. $y = \frac{1}{x}$
3. $y = \sqrt{x}$
4. $y = \frac{1}{\sqrt{x}}$

II. Leida tuletised (*a, b, c, m, n* tähistavad konstante)

5. $y = x^4 + 3x^2 - 6$
6. $y = 6x^{7/2} + 4x^{5/2} + 2x$
7. $y = \frac{a - x}{a + x}$
8. $y = \sqrt{3x} + \sqrt[3]{x} + \frac{1}{x}$
9. $y = \frac{x}{m} + \frac{m}{x} + \frac{x^2}{n^2} + \frac{n^2}{x^2}$
10. $y = (2x - 1)(x^2 - 6x + 3)$
11. $y = \frac{2x^4}{b^2 - x^2}$
12. $y = \sqrt{x^2 + a^2}$
13. $y = \sqrt{\frac{1+x}{1-x}}$
14. $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$
15. $y = 2\sin x + \cos 3x$
16. $y = \sin 2x \cos 3x$
17. $y = \sin\sqrt{1 - x^2}$
18. $y = a\sqrt{\cos 2x}$
19. $y = \ln \tan x$
20. $y = \log_3(x^2 + x)$
21. $y = \log_a(x^2 + 1)$
22. $y = 2^{-x}$
23. $y = a^{x^2}$
24. $y = c^{a^2 - x^2}$
25. $y = x^{\frac{1}{x}}$
26. $y = (x/n)^{nx}$
27. $y = (\sin x)^{\tan x}$
28. $y = \frac{(x-2)^2\sqrt[3]{x+1}}{(x-5)^3}$
29. $y = \frac{\sqrt[3]{(4x-1)^2}}{(x-1)\sqrt[5]{(3-x)^7}}$
30. $y = \sqrt{1-x} \cdot \sqrt[3]{\frac{x}{x^2+3}}$
31. $y = \arctan(x^2 + 1)$
32. $y = \arccos(x^2)$
33. $y = x^{\arcsin x}$

III. Leida ilmutamata kujul antud funktsioonide tuletised muutuja *x* järgi

34. $x^{100} - y^{100} = x^2 y^2$
35. $b^2x^2 + a^2y^2 = a^2b^2$
36. $x^{2/3} + y^{2/3} = a^{2/3}$
37. $\cos(xy) = x$

IV. Leida parameetriliselt antud funktsioonide tuletised

38. $\begin{cases} x = a\cos t \\ y = b\sin t \end{cases}$
39. $\begin{cases} x = a\cos^3 t \\ y = b\sin^3 t \end{cases}$
40. $\begin{cases} x = \frac{3at}{1+t^2} \\ y = \frac{3at^2}{1+t^2} \end{cases}$

V. Leida kõrgemat järku tuletised

41. $y = 3x^3 - 2x^2 + 5x - 1$, leida y''
42. $y = \sqrt[5]{x^3}$, leida y'''
43. $y = \sqrt{\sin 2x}$, leida y''
44. $y = \ln(x+1)$, leida $y^{(4)}$
45. $y = x^2 \ln x$, leida $y^{(5)}$
46. $y = \frac{(x-1)^3}{(x+1)^2}$, leida y''
47. $y = \frac{x^3}{2(x+1)^2}$, leida y''
48. $y = \frac{(x+1)^2}{x-2}$, leida y''

Vastused:

1. $3x^2$
2. $-\frac{1}{x^2}$
3. $\frac{1}{2\sqrt{x}}$
4. $-\frac{1}{2x\sqrt{x}}$
5. $4x^3+6x$
6. $21x^{5/2}+10x^{3/2}+2$
7. $-\frac{2a}{(a+x)^2}$
8. $\frac{\sqrt{3}}{2\sqrt{x}}+\frac{1}{3\sqrt[3]{x^2}}-\frac{1}{x^2}$
9. $\frac{1}{m}-\frac{m}{x^2}+\frac{2x}{n^2}-\frac{2n^2}{x^3}$
10. $6x^2-26x+12$
11. $\frac{4x^3(2b^2-x^2)}{(b^2-x^2)^2}$
12. $\frac{x}{\sqrt{x^2+a^2}}$
13. $\frac{1}{(1-x)\sqrt{1-x^2}}$
14. $\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(1+\frac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\frac{1}{2\sqrt{x}}\right)\right)$
15. $2\cos x-3\sin 3x$
16. $2\cos 2x\cos 3x-3\sin 2x\sin 3x$
17. $-\frac{x\cos\sqrt{1-x^2}}{\sqrt{1-x^2}}$
18. $-\frac{a\sin 2x}{\sqrt{\cos 2x}}$
19. $\frac{2}{\sin 2x}$
20. $\frac{2x+1}{(x^2+x)\ln 3}$
21. $\frac{2x}{(x^2+1)\ln a}$
22. $-2^{-x}\ln 2$
23. $2xa^{x^2}\ln a$
24. $-2xc^{a^2-x^2}\ln c$
25. $x^{\frac{1}{x}}\left(\frac{1-\ln x}{x^2}\right)$
26. $n(x/n)^{nx}(1+\ln\frac{x}{n})$
27. $(\sin x)^{\tan x}\left(1+\frac{\ln\sin x}{\cos^2 x}\right)$
28. $\frac{(x-2)^2\sqrt[3]{x+1}}{(x-5)^3}\left(\frac{2}{x-2}+\frac{1}{3(x+1)}-\frac{3}{x-5}\right)$
29. $\frac{\sqrt[3]{(4x-1)^2}}{(x-1)\sqrt[5]{(3-x)^7}}\left(\frac{8}{3(4x-1)}-\frac{1}{x-1}+\frac{7}{5(3-x)}\right)$
30. $\sqrt{1-x}\cdot\sqrt[3]{\frac{x}{x^2+3}}\left(-\frac{1}{2(1-x)}+\frac{1}{3x}-\frac{2x}{3(x^2+3)}\right)$
31. $\frac{2x}{1+(x^2+1)^2}$
32. $\frac{-2x}{\sqrt{1-x^4}}$
33. $x^{\arcsin x}\left(\frac{\arcsin x}{x}+\frac{\ln x}{\sqrt{1-x^2}}\right)$
34. $\frac{50x^{99}-xy^2}{50y^{99}+x^2y}$
35. $-\frac{b^2x}{a^2y}$
36. $-\sqrt[3]{\frac{y}{x}}$
37. $-\frac{1+y\sin(xy)}{x\sin(xy)}$
38. $-\frac{b}{a}\cot t$
39. $-\frac{b}{a}\tan t$
40. $\frac{2t}{1-t^2}$
41. $y''=18x-4$
42. $y'''=\frac{42}{125}x^{-12/5}$
43. $y''=-\frac{\sin^2 2x+1}{\sin 2x\sqrt{\sin 2x}}$
44. $y^{(4)}=-\frac{6}{(x+1)^4}$
45. $y^{(5)}=4/x^3$
46. $y''=\frac{24(x-1)}{(x+1)^4}$
47. $y''=\frac{3x}{(x+1)^4}$
48. $y''=\frac{18}{(x-2)^3}$