Objective - Find basis $\{a_1, a_2, a_3\}$ for R^3 such that P is the change-of-coordinates matrix $\{a_1, a_2, a_3\}$ to the basis $\{b_1, b_2, b_3\}$

$$P = \begin{bmatrix} 4 & -9 & 5 \\ -3 & -1 & 6 \\ 4 & -2 & -6 \end{bmatrix}, b_1 = \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix}, b_2 = \begin{bmatrix} 4 \\ 5 \\ -4 \end{bmatrix} \text{ \& } b_3 = \begin{bmatrix} 3 \\ 3 \\ -6 \end{bmatrix}$$

lets calc the product of 1R of P & 1st col of B

$$P = \qquad b = \begin{bmatrix} 0 & 4 & 3 \\ -1 & 5 & 3 \\ 3 & -4 & -6 \end{bmatrix}$$

$4 \cdot 0 + -9 \cdot -1 + 5 \cdot 3 = 18$

Do the same w/ 1st R P & second col

$4 \cdot 4 + -9 \cdot 5 + 5 \cdot -4 = -29$

Now third col

$4 \cdot 3 - 9 \cdot 3 \; 5 \cdot -6 = 3$

Now 2nd R 1st col

$-3 \cdot 0 + -1 \cdot -1 + 6 \cdot 3 = 7$

2nd R P 2nd col B

$-3 \cdot 4 + -1 \cdot 5 + 6 \cdot -4 = 19$

2nd R P 3rd col B

$-3 \cdot 3 + -1 \cdot 3 + 6 \cdot -6 = -3$

3rd R P 1st col b

$4 \cdot 0 + -2 \cdot -2 + -6 \cdot 3 = -7$

3rd R P 2nd col b

$4 \cdot 4 + -2 \cdot 5 + -6 \cdot -4 = 36$

3rd R P 3rd col B

$4 \cdot 3 + -2 \cdot 3 + -6 \cdot -6 = 0$

so P·b

$$P \cdot \vec{b} = \begin{bmatrix} 18 & -29 & 3 \\ 7 & 19 & -3 \\ -7 & 38 & 0 \end{bmatrix}$$

$a_1 \quad a_2 \quad a_3$

Probably Not thought to hard & did it in my head

Thers this is the basis for $\{a_1, a_2, a_3\}$ for R^3

Double Check

$$P = \begin{bmatrix} 4 & -9 & 5 \\ -3 & -1 & 6 \\ 9 & -2 & -6 \end{bmatrix} \quad b_1 = \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix}, b_2 = \begin{bmatrix} 4 \\ 5 \\ -4 \end{bmatrix} \quad b_3 = \begin{bmatrix} 3 \\ 3 \\ -6 \end{bmatrix}$$

$$\underset{A \leftarrow B}{P} \left[[\bar{b}_1]_A \; [\bar{b}_2]_A \; [\bar{b}_3]_A \right]$$

$$\underset{B \leftarrow A}{P} = \left[[\bar{a}_1]_B \; [\bar{a}_2]_B \; [\bar{a}_3]_B \right]$$

$$A = BP$$

$$= \begin{bmatrix} 0 & 4 & 3 \\ -1 & 5 & 3 \\ 3 & -4 & -6 \end{bmatrix} \begin{bmatrix} 4 & -9 & 5 \\ -3 & -1 & 6 \\ 4 & -2 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 15 & -10 & 6 \\ 8 & -2 & 7 \\ -30 & -11 & 27 \end{bmatrix}$$