CSE 3380: Linear Algebra for CSE
University of Texas at Arlington
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Assignment 5

You must show your work to receive credit.

Topics Covered

- Gram-Schmidt
- Least Squares
- Linear Models
- Eigenvectors and Eigenvalues
- Symmetric Matrices
- Singular Value Decomposition

1. Find the best approximation to $\mathbf{z}$ by vectors of the form $c_1\mathbf{v}_1 + c_2\mathbf{v}_2$, where

$$\mathbf{v}_1 = \begin{bmatrix} 3 \\ -1 \\ 2 \\ -4 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ -5 \\ -2 \\ -5 \end{bmatrix}, \text{ and } \mathbf{z} = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 2 \end{bmatrix}$$

2. Compute the QR factorization of the given matrix A using `scipy.linalg.qr`. Verify R by hand using the Q matrix that was computed. Save your script as `problem3.py`.

$$A = \begin{bmatrix} 1 & 0 & 4 \\ -2 & 3 & -2 \\ -2 & 0 & 6 \end{bmatrix}$$

3. Find the least squares solution given A and $\mathbf{b}$.

$$A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \\ 1 & 1 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}$$

4. Using the dataset `dataset1.txt`, available through Canvas, find the least squares solution using `np.linalg.lstsq`. You can load the data using `np.loadtxt`. After finding the least squares solution, plot the data and the solution using `matplotlib`.

5. Using the dataset `dataset2.txt`, available through Canvas, find the least squares solution using `np.linalg.lstsq`. You can load the data using `np.loadtxt`. After finding the least squares solution, plot the data and the solution using `matplotlib`.

6. By hand, find a basis for the eigenspace corresponding to each listed eigenvalue.

$$A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}, \lambda = 1, 2, 3$$

7. Find an orthonormal eigenbasis for the following matrix.

$$A = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 4 & -4 \\ 2 & -4 & 4 \end{bmatrix}$$

8. Compute the Singular Value Decomposition for the given matrix

$$A = \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$$

9. Let $X \in \mathbb{R}^{N \times M}, P = X^T X,$ and $C = XX^T$. Show that if $\mathbf{v}_i$ is an eigenvector of P then $X\mathbf{v}_i$ is an eigenvector of C.

10. Use `np.linalg.eig` to calculate the eigenvalues and eigenvectors of the given matrix. Using `matplotlib`, plot the standard basis vectors, the vectors defined by the columns of A, and the calculated eigenvectors. Save your script as `problem2.py`.

$$A = \begin{bmatrix} 1 & -2 \\ -4 & 1 \end{bmatrix}$$

You may submit your work as either a scanned PDF OR you may take pictures of your homework solutions and combine them into a PDF. Compress the written part with the programming files into a single zip file. **Do not submit individual images. Rename your submission as LASTNAME_ID_A5.zip**.