Student ID: ______________________________

Student Name: ______________________________

Signature: ______________________________

The University of New South Wales
Session 2, 2018

GSOE9210 Engineering Decisions

sample mid-term test

Instructions:

- Time allowed: 1 hour
- Reading time: 5 minutes
- This paper has 17 pages
- Total number of questions: 23 (multiple choice)
- Total marks available: 40 (not all questions are of equal value)
- Allowed materials: UNSW approved calculator, pencil (2B), pen, ruler, graph paper (1 blank page), working out paper (1 blank page)

  This exam is closed-book. No books, study notes, or other study materials may be used.
- Answers should be marked in pencil (2B) on the accompanying multiple choice answer sheet
- The exam paper may not be retained by the candidate

Start of exam

1. (*1 mark*) In a decision tree a leaf node represents:

   a) a strategy
   b) a condition
   c) an outcome
   d) a random variable
   e) none of the above

   Solution
   c)—Leaf nodes represent outcomes.

2. (*2 marks*) A decision tree with n ($n \geqslant 1$) nodes has how many branches/edges:

   a) $\frac{n}{2}$
   b) $n!$
   c) n
   d) $n - 1$
   e) none of the above

   Solution
   d)—Each node in a tree, except the root, has a unique parent to which it is connected by a single branch. Therefore, if there are n nodes, there must be $n - 1$ branches.

3. (*2 marks*) Which of the following decision rules will always eliminate (*i.e.*, will never select) weakly dominated strategies:

   a) *MaxiMax*
   b) *Maximin*
   c) *miniMax Regret*
   d) Laplace's
   e) none of the above

   Solution
   d)—Laplace's rule is the only one that will always eliminate weakly dominated strategies. All others may admit some weakly dominated strategies.

Questions 4 to 8 refer to the decision table below.

	s_1	s_2
A	v	3
B	1	4

Solution
Note, first of all, that if $v < 1$, then B dominates A. So A will never be chosen unless $v \geqslant 1$.

4. (*1 mark*) Which is the full range of values of v ($v \in \mathbb{R}$) for which the *MaxiMax* decision rule would choose A?

   a) $v \geqslant 1$
   b) $v \geqslant 3$
   c) $v \geqslant 4$
   d) for all values of v
   e) for no value of v

   Solution
   c)—$v \geqslant 4$.

   $V_M(\mathrm{A}) = \max\{v, 3\}$. $V_M(\mathrm{B}) = \max\{1, 4\} = 4$. $V_M(\mathrm{A}) \geqslant V_M(\mathrm{B})$ iff $\max\{v, 3\} \geqslant 4$.

   If $v \leqslant 3$ then $\max\{v, 3\} = 3 < 4$, in which case B is preferred.

   If $v \geqslant 3$ then $\max\{v, 3\} = v$, in which case A is preferred if $v \geqslant 4$.

   Combining: A is preferred if $v \geqslant 4$.

5. (*1 mark*) Which is the maximum range of values of v ($v \in \mathbb{R}$) for which the *Maximin* decision rule would choose A?

   a) $v \geqslant 1$
   b) $v \geqslant 3$
   c) $v \geqslant 4$
   d) for all values of v
   e) for no value of v

Solution
a)—$v \geqslant 1$.

$V_m(\mathrm{A}) = \min\{v, 3\}$. $V_m(\mathrm{B}) = \min\{1, 4\} = 1$. $V_m(\mathrm{A}) \geqslant V_m(\mathrm{B})$ iff $\min\{v, 3\} \geqslant 1$.

If $v \geqslant 3$ then $\min\{v, 3\} = 3 \geqslant 1$, in which case A is preferred.

If $v \leqslant 3$ then $\min\{v, 3\} = v$, in which case A is preferred if $v \geqslant 1$.

Therefore, A is preferred if $v \geqslant 3$ or $1 \leqslant v \leqslant 3$. Combining: A is preferred if $v \geqslant 1$.

6. (*2 marks*) What is the maximum range of values of v ($v \in \mathbb{R}$) for which Laplace's decision rule would choose A?

    a) $v \geqslant 1$
    b) $v \geqslant 2$
    c) $v \geqslant 3$
    d) for all values of v
    e) for no value of v

Solution
b)—$v \geqslant 2$.

$V_L(\mathrm{A}) = v + 3$. $V_L(\mathrm{B}) = 1 + 4 = 5$. $V_L(\mathrm{A}) \geqslant V_L(\mathrm{B})$ iff $v + 3 \geqslant 5$.

Therefore, $v \geqslant 2$.

7. (*2 marks*) For which range of values of v ($v \in \mathbb{R}$) below would Savage's *miniMax Regret* decision rule choose A?

    a) $v \leqslant 1$
    b) $1 \leqslant v \leqslant 2$
    c) $v \geqslant 2$
    d) for all values of v
    e) for no value of v

Solution
c)—$v \geqslant 2$.

The regret matrix is:

	s_1	s_2
A	$M - v$	1
B	$M - 1$	0

where $M = \max\{v, 1\}$.

That is:

$$M = \begin{cases} v & \text{if } v \geqslant 1 \\ 1 & \text{if } v \leqslant 1 \end{cases}$$

The two cases are shown below:

	s_1	s_2
A	0	1
B	$v-1$	0

$(v \geqslant 1)$

	s_1	s_2
A	$1-v$	1
B	0	0

$(v \leqslant 1)$

For *miniMax Regret* action A is preferred if $V_{MR}(\text{A}) \leqslant V_{MR}(\text{B})$.

If $v \geqslant 1$, then:

$$\begin{aligned} V_{MR}(\text{A}) &\leqslant V_{MR}(\text{B}) \\ 1 &\leqslant v - 1 \\ v &\geqslant 2 \end{aligned}$$

So A would be preferred for $v \geqslant 2$.

Alternatively, note that if $v \leqslant 1$, then A is dominated by B. In this case A would be chosen for no value of v.

Therefore, the combined range is $v \geqslant 2$.

8. (*1 mark*) For which range of values of v ($v \in \mathbb{R}$) shown below would B be weakly dominated by A?

   a) $v \leqslant 1$
   b) $1 \leqslant v \leqslant 3$
   c) $v \geqslant 4$
   d) for all values of v
   e) for no value of v

   Solution
   e) For no values of v.

   Because the value of action A in state s_2 is strictly less than that of B, it follows that A can never dominate B weakly or strictly, regardless of the value of A in state s_1.

Questions 9 to 11 refer to the decision table below.

	s_1	s_2
A	10	2
B	3	3

9. (*2 marks*) Suppose an agent was indifferent between A and B. What would be the value of the agent's optimism index α?

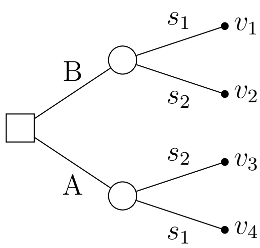
   a) $\frac{1}{3}$
   b) $\frac{2}{7}$
   c) $\frac{3}{4}$
   d) $\frac{1}{8}$
   e) none of the above

   Solution
   d)—$\frac{1}{8}$.

   Set:

$$
\begin{aligned}
V_H(\text{A}) &= V_H(\text{B}) \\
\alpha(10) + (1-\alpha)(2) &= \alpha(3) + (1-\alpha)(3) \\
8\alpha + 2 &= 3 \\
\therefore \qquad \alpha &= \tfrac{1}{8}
\end{aligned}
$$

10. (*2 marks*) For which values does the following tree best represent the table above:



a) $v_1 = 3$, $v_2 = 10$, $v_3 = 3$, $v_4 = 2$
b) $v_1 = 2$, $v_2 = 3$, $v_3 = 10$, $v_4 = 3$
c) $v_1 = 10$, $v_2 = 3$, $v_3 = 2$, $v_4 = 3$
d) $v_1 = 3$, $v_2 = 3$, $v_3 = 2$, $v_4 = 10$
e) none of the above

Solution
d)

11. (*2 marks*) Which action would be chosen under *miniMax Regret*?

a) both A and B
b) neither A nor B
c) A only
d) B only
e) none of the above

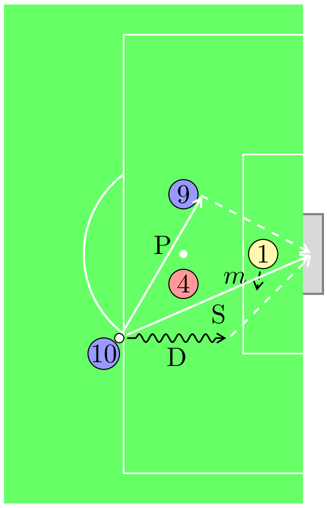
Solution
c)—A.

The regret table is shown below:

	s_1	s_2	M
A	0	1	1
B	7	0	7

The maximum regret of A is less than that of B. Hence the *miniMax Regret* solution is A.

Questions 12 to 19 refer to the diagram below.



Alice plays football and finds herself in the situation shown above. Alice (blue #10), who has the ball, and a teammate (blue #9), are trying to score against an opposition defender (red #4) and goal-keeper (yellow #1). Suppose Alice has three actions to choose from:

- P pass to her team-mate (blue #9) to shoot;
- D dribble towards goal then shoot; or
- S shoot from where she is.

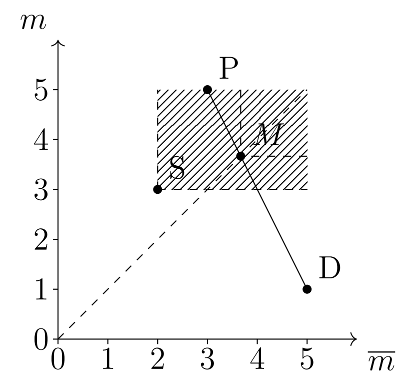
Alice believes that her team's chances of scoring if she passes to her team-mate are 3 in 10. The chances of scoring if she dribbles toward goal before shooting are 5 in 10. Her chances of scoring by shooting from where she is are 2 in 10.

There is the possibility that the goal-keeper (yellow #1) might move (m) toward the ball as shown, in which case the chances of scoring by passing and shooting would improve respectively to 5, 3, and the chances of scoring if she dribbles would be reduced to 1.

Solution
The corresponding decision and regret tables for her team's chances of scoring are shown below:

	$\overline{m}$	m	min
P	3	5	3
D	5	1	1
S	2	3	2

	$\overline{m}$	m	max
P	2	0	2
D	0	4	4
S	3	2	3



12. (*1 mark*) Which, if any, pure actions above are strictly dominated?

   a) P only
   b) D only
   c) S only
   d) D and S
   e) none of the above

   Solution
   c)—S is strictly dominated. Alice can disregard shooting as a viable option.

13. (*1 mark*) Which is the *Maximin* pure action?

   a) P only
   b) D only
   c) S only
   d) D and S
   e) none of the above

   Solution
   a)—P.

14. (*2 marks*) The *Maximin* mixed action is:

   a) passing twice as often as dribbling
   b) dribbling twice as often as passing
   c) shooting twice as often as dribbling
   d) passing as often as shooting
   e) none of the above

   Solution
   a)—Passing twice as often as dribbling.

   Let μ be the amount of P in the mixture of P and D. Let the mixtures of P and D be represented by points $P = (x, y) = \mu(3, 5) + (1 - \mu)(5, 1) = (5 - 2\mu, 1 + 4\mu)$.

   Setting $x = y$:

$$5 - 2\mu = 1 + 4\mu$$
$$4 = 6\mu$$
$$\mu = \tfrac{2}{3}$$

   So $M = \frac{2}{3}\mathrm{P}\frac{1}{3}\mathrm{D}$. That is, Alice should pass twice as often as she dribbles.

15. (*2 marks*) Alice could guarantee that, in the worst case, her chances of scoring were no worse than:

   a) 1 in 10
   b) 2 in 10
   c) 3 in 10
   d) 4 in 10
   e) 5 in 10

   Solution
   c)—no worse than 3 in 10.

   Setting $\mu = \frac{2}{3}$ gives $V_m(M) = 1 + 4\mu = 1 + \frac{8}{3} = \frac{11}{3} > 3$. By using the *Maximin* mixed action, she can guarantee that her chances of scoring are at least 3 in 10.

16. (*2 marks*) Which mixtures of passing and dribbling would be at least as preferred as shooting in all possible cases (states)?

    a) dribbling at least twice as often as passing
    b) passing at least three times as often as dribbling
    c) dribbling no more than three times as often as passing
    d) passing at least as often as dribbling
    e) none of the above

    Solution
    d)—passing at least as often as dribbling.

    The condition that an action be at least as preferred in all states amounts to weak dominance.

    We know that $M = (5-2\mu, 1+4\mu)$. In order for M to be at least as preferred to S in all states it must dominate S; *i.e.*, $5 - 2\mu > 2$ and $1 + 4\mu > 3$.

    Hence $\mu < \frac{3}{2}$ and $\mu > \frac{1}{2}$; *i.e.*, $\frac{1}{2} < \mu < \frac{3}{2}$.

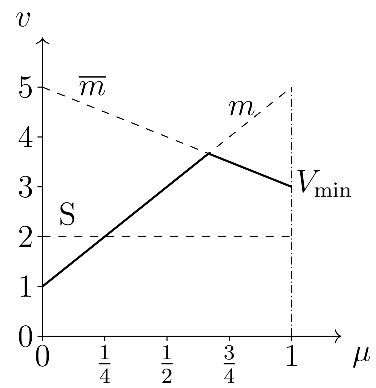
    Because $0 \leqslant \mu \leqslant 1$, this reduces to $\frac{1}{2} < \mu \leqslant 1$. That is, passing at least as often as dribbling.

17. (*2 marks*) Which mixtures of passing and dribbling would be preferred under *Maximin* to the strategy "always shoot"?

    a) dribbling at least twice as often as passing
    b) passing at least three times as often as dribbling
    c) dribbling no more than three times as often as passing
    d) passing at least as often as dribbling
    e) none of the above

    Solution
    c)—dribbling no more than three times as often as passing.

    The graph below shows the *Maximin* value of mixtures of P and D (μ is the amount of P in the P–D mixture). The *Maximin* value of S is 2.



From the graph we can see that for $\mu \geqslant \frac{1}{4}$, the *Maximin* value of the mixture is above that of S; *i.e.*, passing at least as often as dribbling. Analytically, because in state $\overline{m}$ the value of any mixture of P and D is greater than that of S, the only value that matters is the value in state m. Therefore we need:

$$1 + 4\mu > 2$$
$$\mu > \tfrac{1}{4}$$

So the mixture with the least amount of passing is $\frac{1}{4}$P$\frac{3}{4}$D; *i.e.*, dribbling no more than three times as often as passing.

Let $p = P(m)$ be the probability that the goal-keeper will move as shown.

18. (*2 marks*) For what range of values of p would it be better for Alice to dribble than to shoot?

    a) $p < \frac{2}{3}$
    b) $p > \frac{3}{5}$
    c) $p > \frac{2}{5}$
    d) $p < \frac{3}{5}$
    e) none of the above

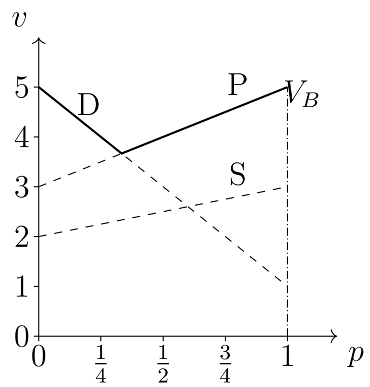
    Solution
    d)—$p < \frac{3}{5}$.

The *Bayes* values of the respective actions are given below:

$$
\begin{aligned}
V_B(\mathrm{P}) &= (1-p)(3) + p(5) \\
&= 3 + 2p \\
V_B(\mathrm{D}) &= (1-p)(5) + p(1) \\
&= 5 - 4p \\
V_B(\mathrm{S}) &= (1-p)(2) + p(3) \\
&= 2 + p
\end{aligned}
$$

Setting:

$$
\begin{aligned}
V_B(\mathrm{D}) &> V_B(\mathrm{S}) \\
5 - 4p &> 2 + p \\
3 &> 5p \\
p &< \tfrac{3}{5}
\end{aligned}
$$



19. (*2 marks*) Which percentage below gives the proportion of occasions in which, if the goal-keeper were to move, he could minimise Alice's chances of scoring despite her best efforts to score?

    a) 80%
    b) 70%
    c) 60%
    d) 50%
    e) 40%

Solution
e)—40%

Setting:

$$
\begin{aligned}
V_B(\mathrm{P}) &= V_B(\mathrm{D}) \\
3 + 2p &= 5 - 4p \\
6p &= 2 \\
p &= \tfrac{1}{3}
\end{aligned}
$$

The desired proportion corresponds to the least favourable probability distribution; *i.e.*, move 33% of the time. Notice from the graph above that for $p > \frac{1}{3}$ Alice's best option is to pass, which gives increasingly better chances of scoring as p increases, so in order to keep Alice's scoring chances as low as possible the goal-keeper should make his move as close to $p = \frac{1}{3}$ as possible. Of the options above, this is achieved by picking the lowest; *i.e.*, 40%.

---

20. (*2 marks*) Suppose Bob strictly prefers A to B. Which of the following is *false*:

a) if $p \neq 0$ and $q \leqslant p$, then $[q : A \,|\, (1-q) : B] \sim [\frac{q}{p} : [p : A \,|\, (1-p) : B] \,|\, (1 - \frac{q}{p}) : B]$
b) if $0 < p < 1$, then $A \succ [p : A \,|\, (1-p) : B] \succ B$
c) for all $0 \leqslant p \leqslant 1$ and all P, $[p : A \,|\, (1-p) : P] \succ [p : B \,|\, (1-p) : P]$
d) $A \sim [p : A \,|\, (1-p) : A]$, for any $0 \leqslant p \leqslant 1$
e) none of the above

Solution
c) is false for $p = 0$ (*i.e.*, A and B are impossible in their respective lotteries) $[p : A \,|\, (1-p) : P] \sim [p : B \,|\, (1-p) : P]$.

Questions 21 to 23 refer to the problem below.

Carla is a contractor bidding on either of two government contracts, A and B, each of which could be extended if her bid is successful. Contract B is more lucrative (has a higher profit) but would be harder to win than A.

She estimates that putting together a bid for A will cost $\$2.5K$, and the profit would be $\$25K$ (including costs) if the contract is extended, and $\$15K$ otherwise.

Similarly, B would cost $\$5K$ and return $\$35K$ if the contract were extended, and $\$25K$ otherwise.

Carla believes there would be a 50% chance of her bid for B being successful, and that bidding for A would have a slightly greater (60%) chance of success.

Government statistics show that 80% of contracts are extended, but the decision to extend is only known after the contracts have been awarded.

21. (*2 marks*) If Carla is risk-neutral, her expected monetary value (EMV) for A, in units of thousands (K), is closest to:

    a) \$9
    b) \$11
    c) \$13
    d) \$15
    e) \$17

    Solution
    c)—\$13 (\$12.8)

    If her bids for A and B were successful she would value them according to their *Bayes* values (evaluating outcomes in \$) follows:

$$\begin{aligned} V_{\$}(s_{\mathrm{A}}) &= P(e)v_{\$}(25K) + P(\bar{e})v_{\$}(15K) \\ &= \frac{8}{10} \times 25K + \frac{2}{10} \times 15K \\ &= 20K + 3K = \$23K \end{aligned}$$

$$\begin{aligned} V_{\$}(s_{\mathrm{B}}) &= P(e)v_{\$}(35K) + P(\bar{e})v_{\$}(25K) \\ &= \frac{8}{10} \times 35K + \frac{2}{10} \times 25K \\ &= 28K + 5K = \$33K \end{aligned}$$

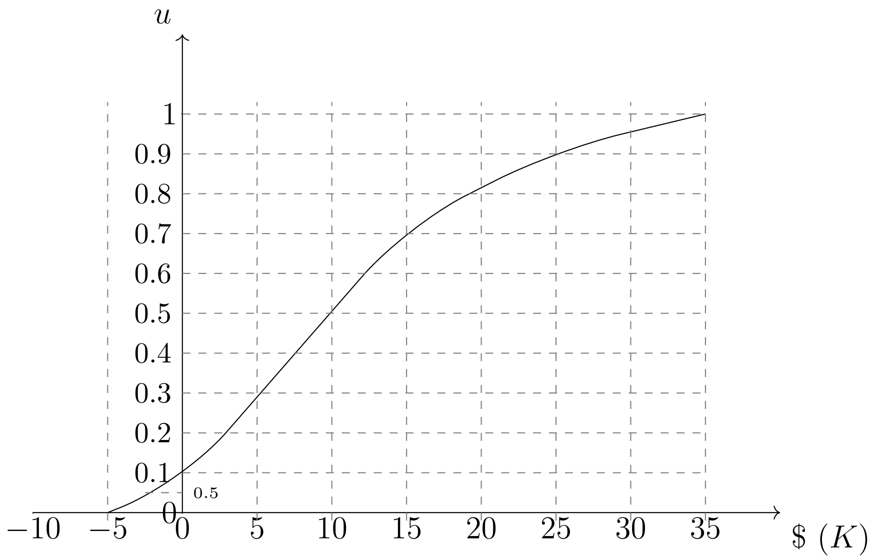
    To measure the value of bidding for each she must also consider the chances of success for each:

$$\begin{aligned} V_{\$}(\mathrm{A}) &= P(s_{\mathrm{A}})V_{\$}(s_{\mathrm{A}}) + P(\overline{s_{\mathrm{A}}})v_{\$}(-2.5K) \\ &= \tfrac{6}{10} \times 23K + \tfrac{4}{10} \times -2.5K \\ &= \tfrac{1}{5}(3 \times 23K + 2 \times -2.5K) \\ &= \tfrac{1}{5}(69K - 5K) = \tfrac{1}{5} \times 64K \\ &= \$12.8K \end{aligned}$$

$$\begin{aligned} V_{\$}(\mathrm{B}) &= P(s_{\mathrm{B}})V_{\$}(s_{\mathrm{B}}) + P(\overline{s_{\mathrm{B}}})v_{\$}(-5K) \\ &= \tfrac{1}{2} \times 33K + \tfrac{1}{2} \times -5K \\ &= \tfrac{1}{2}(33K - 5K) \\ &= \tfrac{1}{2}(28K) = \$14K \end{aligned}$$

This value is $\$12.8K$ for A.

22. (*2 marks*) Suppose Carla's utility for money is as shown below:



Her utility value (given utilities in the interval $[0, 1]$) for B would be closest to:

a) 0.50
b) 0.55
c) 0.60
d) 0.65
e) 0.70

Solution
a)—0.50 (0.49)

Setting $U(\mathrm{B}) = V_B(\mathrm{B})$, gives:

$$\begin{aligned}
V_B(\mathrm{B}) &= P(s_\mathrm{B})V_B(s_\mathrm{B}) + P(\overline{s_\mathrm{B}})u(-\$5K)\\
&= \tfrac{1}{2} \times [\tfrac{8}{10}u(\$35K) + \tfrac{2}{10}u(\$25K)] + \tfrac{1}{2} \times u(-\$5K)\\
&= \tfrac{1}{2} \times [\tfrac{8}{10} \times 1 + \tfrac{2}{10} \times \tfrac{9}{10}] + \tfrac{1}{2} \times 0\\
&= \tfrac{1}{20}[8 \times 1 + 2 \times \tfrac{9}{10}] + 0\\
&= \tfrac{1}{200}[8 \times 10 + 2 \times 9] = \tfrac{1}{200} \times 98\\
&= \tfrac{1}{100} \times 49 = 0.49
\end{aligned}$$

23. (*2 marks*) Using the utility function for Question 22, if Carla were offered a lottery with a probability p of payoff $\$bK$, otherwise payoff $\$aK$, which of the following statement(s) is most precise?

a) if $a = -5$ and $b = 5$, her risk premium would be negative
b) if $a = -5$ and $b = 35$, her risk premium would be positive for some probabilities and negative for others
c) if $a = 15$ and $b = 25$, and $p = \frac{1}{2}$, her certainty equivalent would be less than $\$20K$
d) all of the first three above
e) none of the first three above

Solution
d)—all three

The utility curve is concave down for values greater than $\$10K$, which means for any binary lottery whose values are both in this range she would be risk averse (case c).

For values less than $\$5K$ the curve is concave up, so she would be risk seeking for lotteries for which both outcomes are in this range (case a).

For lotteries whose outcomes respectively have values less than and greater than $\$10K$ (case b) the risk premium could be either negative or positive depending on the outcomes and their respective chances.

Therefore, all three properties hold.

End of exam