GSOE9210 Engineering Decisions

Problem Set 02

1. For the software project budgeting example in lectures, how would the decision be affected if the discount rate was 10%?

   Solution
   With a *discount rate* for each period of 10%:

$$
\begin{aligned}
NPV(\mathrm{A}) &= -10 + \frac{5}{1.1} + \frac{25}{1.1^2} &&= 15.2 \\
NPV(\mathrm{B}) & &&= 13.0 \\
NPV(\mathrm{C}) &= -5 + \frac{10}{1.1} + \frac{12}{1.1^2} &&= 14.0
\end{aligned}
$$

   In this case, project A would be most preferred and B would become least favourable.

2. For the travelling problem in lectures, the values of outcomes based on walking distance (km) are given below:

	b_L	b_P
Tr	2	2
Bu	1	4

   (a) Describe each action as a lottery.
   (b) What is the *MaxiMax* (MM) action for this problem?
   (c) How would this be affected if the traveller had to visit the hospital's clinic, a further kilometre south of the hospital, afterwards?
   (d) Draw the decision table for the same problem using walking time instead, assuming a person walks at an average speed of 3km/h.
   (e) Which action is chosen under the *MaxiMax* rule when considering walking time?

   Solution

   (a) Below the lottery associated with an action 'A' is denoted ℓ_{A}:

$$
\ell_{\mathrm{Tr}} = [b_L : \mathrm{E} | b_P : \mathrm{E}] = [\mathrm{E}] \qquad \ell_{\mathrm{Bu}} = [b_L : \mathrm{D} | b_P : \mathrm{C}]
$$

   (b) Take the bus (Bu), with least distance 1km. Note that, here, more preferred outcomes have lower values. Equivalently, preferences values could be associated with negative distances.
   (c) Adding the 1km walk to the clinic would just add 1 to every entry. This would not affect the outcomes' relative desirability, and hence would not affect the choice under *MaxiMax*.

(d) Assuming no disruptions to the journey (*e.g.*, traffic lights at road crossings, *etc.*), the decision table expressed in minutes is:

	b_L	b_P
Tr	40	40
Bu	20	80

(e) Since walking time is a scaling factor, which preserves the relative values of the outcomes, this also has no effect on *MaxiMax*.

3. Repeat the above exercise for the *Maximin* (Mm) rule.

Solution

(a) Take the train (Tr), with worst possible case a walk of 2km.
(b) Adding the 1km walk to the clinic would just add 1 to every entry, which, again, would not affect the outcomes relative desirability, and hence would not affect *Maximin*'s choice.
(c) As above.
(d) Since walking time is a scaling factor, which preserves the relative values of the outcomes, this has no effect on *Maximin* either.

4. Suppose Alice is the principal in the school fund-raising problem discussed in lectures:

	d	w
S	120	85
F	150	75

d day is dry
w day is wet

(a) Represent each action as a lottery.
(b) Which action is preferred under *MaxiMax* and *Maximin*?
(c) What optimism level (*i.e.*, value of index α under *Hurwicz*'s rule) would Alice have if she were 'indifferent' between (*i.e.*, have equal preference for) the two options?
(d) Derive a general expression for the value of the optimism index α^* for which Alice would be indifferent between actions A_1 and A_2, with best and worst outcomes M_1 and m_1, and M_2 and m_2, respectively.
(e) Suppose there was a third option, an *indoor trivia night* (T), which generates profit \$100 regardless of the weather. How optimistic would Alice have to be to prefer the sports day over the trivia night?

Solution

(a) The corresponding lotteries are shown below:

$$\ell_{\mathrm{S}} = [d : \$120 | w : \$85] \qquad \ell_{\mathrm{F}} = [d : \$150 | w : \$75]$$

(b) Consider the decision table below:

	d	w	m	M
S	120	85	85	120
F	150	75	75	150

It follows that *Maximin* would prefer S and *MaxiMax* F.

(c) The *Hurwicz* values are given by:

$$\begin{aligned} V_H(\mathrm{S}) &= 120\alpha + 85(1-\alpha) \\ V_H(\mathrm{F}) &= 150\alpha + 75(1-\alpha) \end{aligned}$$

The two are equivalent when:

$$\begin{aligned} V_H(\mathrm{S}) &= V_H(\mathrm{F}) \\ 120\alpha + 85(1-\alpha) &= 150\alpha + 75(1-\alpha) \\ 10(1-\alpha) &= 30\alpha \\ 40\alpha &= 10 \\ \therefore\ \alpha &= \tfrac{1}{4} \end{aligned}$$

(d) In general, two actions A_1 and A_2 are equivalent when:

$$\begin{aligned} V_H(\mathrm{A}_1) &= V_H(\mathrm{A}_2) \\ M_1\alpha + m_1(1-\alpha) &= M_2\alpha + m_2(1-\alpha) \\ (M_1 - M_2 + m_2 - m_1)\,\alpha &= m_2 - m_1 \\ \alpha &= \frac{m_2 - m_1}{(m_2 - m_1) + (M_1 - M_2)} \\ \text{rearranging} \qquad \alpha &= \frac{1}{1 - \frac{(M_1 - M_2)}{(m_1 - m_2)}} \end{aligned}$$

(e) Action S is preferred when:

$$\begin{aligned} V_H(\mathrm{S}) &> V_H(\mathrm{T}) \\ 120\alpha + 85(1-\alpha) &> 100 \\ 35\alpha &> 15 \\ \therefore\ \alpha &> \tfrac{3}{7} \end{aligned}$$

5. How could you simplify Laplace's decision rule of insufficient reason? That is, can you give an equivalent, but simpler, criterion for choosing between actions?

   Solution
   Provided all states are exhaustive and mutually exclusive, Laplace's rule amounts to choosing the action which maximises the sum of the values of its outcomes.

6. Alice has a choice of buying an investment property in either of two suburbs: A and B. In five years, house prices are likely to go up by $\$2K$ in B, and by $\$1K$ in A. However, there is an existing proposal to build a shopping centre in A in the next year. If the shopping centre is approved

(a), house prices in A will increase in value over the next five years by \$6K.

For the problem described above:

(a) Which is the *Maximin* action?
(b) Which is the best action if approval from the shopping centre is granted? If approval is not granted?
(c) Which is the *miniMax Regret* action?
(d) Which of the two decision rules above would be most relevant for a property investor?

Solution
The decision table looks as follows, where the entries represent the value in five years:

	a	$\overline{a}$	m
A	6	1	1
B	2	2	2

(a) The *Maximin* action is B.
(b) In state a, the best action is A. In state $\overline{a}$, the best action is B.
(c) The regret matrix/table is given by:

	a	$\overline{a}$	M
A	0	1	1
B	4	0	4

It follows that the *miniMax Regret* action is A.
(d) Speculative investors tend to be risk takers, looking for high returns even where there's risk. Such a person would probably choose according to *miniMax Regret* rather than *Maximin*.

7. Consider the following decision table:

	s_1	s_2	s_3	s_4	V
A_1	2	2	0	1	
A_2	1	1	1	1	
A_3	0	4	0	0	
A_4	1	3	0	0	

(a) Evaluate each action under the following decision rules, and determine which action will be chosen under each rule: i. *MaxiMax* (MM) ii. *Maximin* (Mm) iii. *Hurwicz*'s rule for values of $\alpha = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$.
(b) Which decision rules above *agree* on this problem; *i.e.*, choose the same actions?
(c) Two decision rules are said to be *equivalent* if they choose the same action for every possible decision problem. Which of the rules above are equivalent?

Solution

(a) For the case $\alpha = \frac{1}{4}$:

	s_1	s_2	s_3	s_4	M	m	H
A_1	2	2	0	1	2	0	$\frac{1}{2}$
A_2	1	1	1	1	1	1	1
A_3	0	4	0	0	4	0	1
A_4	1	3	0	0	3	0	$\frac{3}{4}$

i. A_3
ii. A_2
iii. A_2 and A_3

(b) *MaxiMax* and *Maximin* don't agree on this problem. *Hurwicz*'s rule will agree with the other rules for some values of α: *e.g.*, with *MaxiMax* for $\alpha = 1$; with *Maximin* for $\alpha = 0$.
For other values, such as $\alpha = \frac{1}{4}$, *Hurwicz*'s rule agrees with neither.

(c) None. The problem above is a counterexample showing that none of the rules are equivalent, as all the rules produce different choices for at least one (*e.g.*, this) decision problem.

8. For the problem above, which is the *miniMax Regret* action?

Solution

The regret matrix/table is given by:

	s_1	s_2	s_3	s_4	M
A_1	0	2	1	0	2
A_2	1	3	0	0	3
A_3	2	0	1	1	2
A_4	1	1	1	1	1

Action A_4 has the least maximum regret (1).

9. For the raffle problem discussed in lectures:

(a) Draw the decision tree and table
(b) Should you draw a ticket in the raffle?
(c) What if you knew there were three blue tickets? Four? None?
(d) How many blue tickets would there have to be to make it worth entering?
(e) If there were n blue tickets ($0 \leqslant n \leqslant 4$), how would the the value of the prize which makes it worthwhile entering depend on n?

Solution

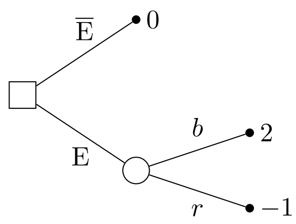
The raffle corresponds to the lottery:

$$\ell = [b : \$2 | r : -\$1]$$

where:

b blue ticket drawn
r red ticket drawn

(a) A concise decision tree (there may be others) and table are shown below:



	b	r
$\overline{\mathrm{E}}$	0	0
E	2	−1

where E stands for 'Enter the raffle'.

(b) To enter is to risk losing your entry fee (−1) for the chance to win 2. I probably would enter, due to the small amounts involved.

(c) If I knew there were three blue tickets (*i.e.*, chances of winning are 3 to 1 in favour) I would definitely enter. I would also enter if there were four blue tickets. This would represent a certain win.
If there were no blue tickets that would mean certain loss; I wouldn't enter.

(d) For me, probably two or more. Note that, based on expected values:

$$\begin{aligned} E(\mathrm{E}) &= P(b)(2) + P(r)(-1) \\ &= \tfrac{1}{2}(2) + \tfrac{1}{2}(-1) = \tfrac{1}{2} \\ E(\overline{\mathrm{E}}) &= 0. \end{aligned}$$

(e) In general, let \$$w$ be value of winning and \$$l$ that of losing:

$$\begin{aligned} E(\mathrm{E}) &= P(b)w + P(r)l \\ &= \tfrac{n}{4}w + \tfrac{4-n}{4}(-1) \qquad \text{(setting } l = -1\text{)} \\ &= \tfrac{1}{4}(wn - 4 + n) \\ &= \tfrac{1}{4}((w+1)n - 4) \\ E(\overline{\mathrm{E}}) &= 0. \end{aligned}$$

We would require (for $n > 0$):

$$\begin{aligned} E(\mathrm{E}) &> E(\overline{\mathrm{E}}) \\ (w+1)n - 4 &> 0 \\ w &> \tfrac{4}{n} - 1 \end{aligned}$$

For $n = 0$ there would be no finite value that would make entering worthwhile.