Student ID: ____________________________

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The University of New South Wales
Session 2, 2018

GSOE9210 Engineering Decisions

sample final exam

Instructions:

- Time allowed: 2 hours
- Reading time: 10 minutes
- This paper has 19 pages
- Total number of questions: 53 (multiple choice)
- Total marks available: 60 (not all questions are of equal value)
- Allowed materials: UNSW approved calculator, pencil (2B), pen, ruler, graph paper (1 blank page), working out paper (1 blank page)

  This exam is closed-book. No books, study notes, or other study materials may be used.
- Provided materials: graph paper (1 page), exam booklet, generalised multiple choice answer sheet
- Answers should be marked in pencil (2B) on the accompanying multiple choice answer sheet
- The exam paper may not be retained by the candidate

Start of exam

Questions 1 to 7 refer to the problem below.

Recall the school fund-raiser example from lectures in which Alice, the principal of a local school, is planning to hold a single-day fund-raiser. There are two options for the fund-raising activity: a fête (F) or a sports day (S). Proceeds of each activity depend on the (unpredictable) weather: on a dry day (d) a fête will make a profit of \$150 and a sports day only \$120; however, on a wet day (w) the sports day will net \$85 and the fête only \$75.

Suppose Alice has no information about the likelihood of whether any given day will be dry or wet.

1. (*1 mark*) On any given day, which of the two activities (S or F) will ensure the greatest lower bound on profit? [1]

   a) S only
   b) F only
   c) both S and F
   d) neither S nor F
   e) a mixture of S and F

2. (*1 mark*) Suppose Alice is more concerned about limiting the maximum regret—she doesn't like to miss out on opportunities. Which activity would Alice prefer? [2]

   a) S only
   b) F only
   c) both S and F
   d) neither S nor F
   e) a mixture of S and F

For the following questions assume the following:

Imagine Alice works for the local branch of the Government's education department. She is in charge of twelve local schools, and is planning to hold a single-day fund-raiser in each school on the same day. She can hold different activities in different schools, if she wishes.

3. (*1 mark*) In how many schools should Alice hold a sports day if she wants to ensure the greatest minimum profit? [3]

   a) in none of them
   b) in four of them
   c) in six of them
   d) in eight of them
   e) in all twelve of them

4. (*1 mark*) In how many schools should a sports day be hosted if limiting the maximum regret is the main consideration? [4]

   a) in none of them
   b) in three of them
   c) in four of them
   d) in six of them
   e) in all twelve of them

For the following question, suppose that fund-raising events are held in one day of each week of every month.

5. (*1 mark*) Let $p = P(d)$ be the probability that any given day is dry. Which is the *Bayes* action for probability $p = \frac{1}{2}$? [5]

   a) S only
   b) F only
   c) both S and F
   d) neither S nor F
   e) a mixture of S and F

Records kept over the last ten years indicate that, on average, the number of dry days per month in Alice's geographic area are as follows:[1]

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Dry days	15	13	10	8	6	5	5	7	11	13	14	16

[1]Note that Alice lives in a very wet area; perhaps a mountain valley.

6. (*2 marks*) Alice holds her fund-raisers every month except the one month in which she takes her annual holidays. If Alice is concerned with limiting the maximum regret, which of the options below would be best time for Alice to take her holidays? [7]

   a) Jan or Feb
   b) Feb or Sep
   c) June or July
   d) Apr or Aug
   e) Jan or Dec

7. (*1 mark*) If Alice were concerned with securing the greatest minimum profit, in which months should she schedule her holidays? [8]

   a) Jan or Feb
   b) Feb or Sep
   c) June or July
   d) Apr or Aug
   e) Jan or Dec

Questions 8 to 22 refer to decision table below.

Consider the following decision table for a problem in which the outcomes are measured in dollars ($).

	s_1	s_2
a_1	10	50
a_2	40	20

There are two agents, A and B, who are making independent decisions on which of the possible actions (a_1 and a_2) to take—note that this is *not* a game: both agents are choosing separate decisions at different times.

Consider agent A first. Agent A's utility function for money is logarithmic (with base 2); *i.e.*, $u(x) = \log_2(x - a)$, where $a \in \mathbb{R}$ is a parameter to be determined.

8. (*1 mark*) If $u(10) = 0$, which alternative below best describes the utility function $u(x)$? [9]

a) $\log(x)$
b) $\log(x-1)$
c) $\log(x+9)$
d) $\log(x-9)$
e) none of the above

9. (*1 mark*) Let $p = P(s_1)$. If $p = \frac{1}{2}$, which of the following statements is correct? [10]

a) a_1 has greater expected dollar value than a_2
b) a_2 has greater expected dollar value than a_1
c) both actions have the same expected dollar value
d) a_1 is dominated
e) none of the above

10. (*2 marks*) For $p = \frac{1}{2}$, which of the following statements is true? [12]

a) A prefers a_1 to a_2
b) A prefers a_2 to a_1
c) A is indifferent between the two actions
d) A prefers neither action
e) none of the above

11. (*1 mark*) For which value(s) of p would A be indifferent between the two actions? [13]

a) $p = 0$
b) $0 < p \leqslant \frac{1}{4}$
c) $\frac{1}{4} < p \leqslant \frac{1}{2}$
d) $\frac{1}{2} < p < \frac{3}{4}$
e) $\frac{3}{4} \leqslant p$

12. (*1 mark*) For $p = \frac{1}{2}$, the certainty equivalent of a_1 is closest to … [14]

a) \$0
b) \$10
c) \$15
d) \$25
e) \$45

13. (*1 mark*) For $p = \frac{1}{2}$, the certainty equivalent of a_2 is closest to … [15]

a) \$0
b) \$10
c) \$15
d) \$25
e) \$45

14. (*1 mark*) For $p = \frac{1}{2}$, what is the approximate value of the risk premium of a_1? [16]

a) \$0
b) −\$10
c) −\$6
d) \$15
e) \$20

15. (*1 mark*) For $p = \frac{1}{2}$, what is the approximate value of the risk premium of a_2? [17]

a) \$0
b) −\$10
c) −\$3
d) \$3
e) \$10

For agent B all we know is that she is indifferent between a certain \$20 and 10% chance of \$50 and 90% of \$10. She is also indifferent between \$40 and the lottery $[\frac{6}{10} : \$50 | \frac{4}{10} : \$10]$.

Assume in the following questions that $p = P(s_1) = \frac{1}{2}$.

16. (*1 mark*) Which of the following statements is true? [18]

   a) B prefers a_1 to a_2
   b) B prefers a_2 to a_1
   c) B is indifferent between the two actions
   d) B prefers neither action
   e) none of the above

17. (*2 marks*) Assume that utilities for dollar values other than those given can be linearly interpolated. For a utility scale in the range $[0, 10]$, which expression below best represents $u(x)$ for $\$20 \leqslant x \leqslant \$40$? [20]

   a) $x - 10$
   b) $\frac{1}{10}x - 1$
   c) $\frac{2}{5}x - 10$
   d) $4 - 4x$
   e) $\frac{1}{4}x - 4$

18. (*1 mark*) The certainty equivalent of a_1 is closest to . . . [21]

   a) \$20
   b) \$25
   c) \$30
   d) \$35
   e) \$40

19. (*1 mark*) The certainty equivalent of a_2 is closest to . . . [22]

   a) \$0
   b) \$10
   c) \$15
   d) \$25
   e) \$45

20. (*1 mark*) What is the approximate value of the risk premium of a_1? [23]

   a) \$0
   b) $-\$10$
   c) $-\$6$
   d) \$15
   e) \$20

21. (*1 mark*) What is the approximate value of the risk premium of a_2? [24]

   a) \$0
   b) $-\$10$
   c) $-\$3$
   d) \$3
   e) \$10

22. (*1 mark*) For which value of $p = P(s_1)$ would B be indifferent between the two actions? [25]

   a) $\frac{1}{10}$
   b) $\frac{1}{5}$
   c) $\frac{2}{5}$
   d) $\frac{3}{5}$
   e) $\frac{7}{10}$

Questions 23 to 26 refer to the problem below.

Two friends agree to "meet at *the* park", but subsequently each realises that there are two identical parks (A and B) nearby, to which the other might've referred. Each friend has to decide, independently, to which park to go to meet their friend. The game is modelled by the matrix below.

	A	B
A	$1, 1$	$0, 0$
B	$0, 0$	$1, 1$

23. (*1 mark*) How many plays survive simplification by elimination of dominated strategies? [26]

   a) none
   b) one
   c) two
   d) three
   e) four

24. (*1 mark*) How many equilibrium points does this game have? [27]

   a) none
   b) one
   c) two
   d) three
   e) four

25. (*1 mark*) How many Pareto optimal plays are there in this game? [28]

   a) none
   b) one
   c) two
   d) three
   e) four

26. (*1 mark*) Suppose Alice believes that the probability of Bob going to park A is $p = P_B(\text{A})$. Which value of p would leave Alice indifferent between going to either park? [29]

   a) $p = 0$
   b) $p = \frac{1}{4}$
   c) $p = \frac{1}{3}$
   d) $p = \frac{1}{2}$
   e) for any $p \in [0, 1]$

Questions 27 to 30 refer to problem below.

Alice and Bob have agreed to meet for lunch. Alice prefers restaurant A and Bob prefers restaurant B. Unfortunately, they didn't specify at which restaurant they were to meet. This ‘game’ is modelled by the following game matrix.

	a	b
A	$2,1$	$0,0$
B	$0,0$	$1,2$

27. (*1 mark*) How many plays survive simplification by elimination of dominated strategies? [30]

    a) none
    b) one
    c) two
    d) three
    e) four

28. (*1 mark*) How many equilibrium points does this game have? [31]

    a) none
    b) one
    c) two
    d) three
    e) four

29. (*1 mark*) How many Pareto optimal plays are there in this game? [32]

    a) none
    b) one
    c) two
    d) three
    e) four

30. (*1 mark*) Suppose Alice believes that the probability of Bob going to restaurant A is $p = P_B(\mathrm{a})$. Which value of p would leave Alice indifferent between going to either restaurant? [33]

a) $p = 0$
b) $p = \frac{1}{4}$
c) $p = \frac{1}{3}$
d) $p = \frac{1}{2}$
e) for any $p \in [0, 1]$

Questions 31 to 33 refer to the problem below.

Alice and Bob, who are tennis partners, agreed to play this weekend. There are two tennis courts near them, A and B, but they didn't specify at which court they would play. Court A is closer to both. This 'game' is modelled by the following game matrix.

	a	b
A	2, 2	0, 0
B	0, 0	1, 1

31. (*1 mark*) How many equilibrium points does this game have? [34]

a) none
b) one
c) two
d) three
e) four

32. (*1 mark*) How many Pareto optimal plays are there in this game? [35]

a) none
b) one
c) two
d) three
e) four

33. (*1 mark*) Suppose Alice believes that the probability of Bob going to court A is $p = P_B(\mathrm{a})$. Which value of p would leave Alice indifferent between going to either court? [36]

a) $p = 0$
b) $p = \frac{1}{4}$
c) $p = \frac{1}{3}$
d) $p = \frac{1}{2}$
e) for any $p \in [0, 1]$

Questions 34 to 36 refer to problem below.

Alice sells magazines. She advertises her business by sending out promotional leaflets to her customers. She has printed three types of leaflet (A, B, or C), but she can only afford to send one leaflet per customer. Her market—the customers to which she sells her magazines—is segmented into two categories, s_1 and s_2.

Her average sales, per 100 leaflets sent, are shown in the table below.

	s_1	s_2
A	0	19
B	15	5
C	10	12

34. (*1 mark*) For the decision problem described by the table above, Alice's guaranteed minimum average sales per hundred leaflets, if she didn't know to which segment her customers belong when she sent out her leaflets, is: [37]

a) $\frac{65}{12}$
b) $\frac{75}{12}$
c) $\frac{85}{12}$
d) $\frac{95}{12}$
e) none of the above

35. (*1 mark*) Let $p = P(s_1)$ be the probability that a customer belongs to segment s_1. If $p = \frac{7}{10}$, which leaflet would be most profitable? [38]

a) A
b) B
c) C
d) a non-pure mixture of A and C
e) none of the above

36. (*2 marks*) Assume $p = \frac{7}{10}$, as in the previous question. Suppose Alice could hire an oracle who could predict to which segment each customer belonged with complete accuracy. If each unit sold makes a profit of $10, what is the highest rate, in dollars per 100 leaflets/customers, which Alice should pay for the oracle's service? [40]

a) $29
b) $42
c) $23
d) $37
e) none of the above

Questions 37 to 43 refer to zero-sum game matrix below.

	b_1	b_2	b_3	b_4
a_1	4	2	5	2
a_2	2	1	-1	-2
a_3	3	2	4	2
a_4	-6	0	6	1

37. (*1 mark*) Which plays by the row player are best responses to column player's b_3? [41]

a) a_1 only
b) a_2 only
c) a_3 only
d) a_4 only
e) there are multiple best responses

38. (*1 mark*) Which plays by the row player are best responses to column player's b_2? [42]

   a) a_1 only
   b) a_2 only
   c) a_3 only
   d) a_4 only
   e) there are multiple best responses

39. (*1 mark*) Which plays by the row player are best responses to column player's b_1? [43]

   a) a_1 only
   b) a_2 only
   c) a_3 only
   d) a_4 only
   e) there are multiple best responses

40. (*1 mark*) Which plays by the column player are best responses to row player's a_2? [44]

   a) b_1 only
   b) b_2 only
   c) b_3 only
   d) b_4 only
   e) there are multiple best responses

41. (*1 mark*) How many saddle points does this game have? [45]

   a) none
   b) one
   c) two
   d) three
   e) four

42. (*1 mark*) After simplification, how many strategies are left for the row player? [46]

a) none
b) one
c) two
d) three
e) four

43. (*1 mark*) After simplification, how many strategies are left for the column player? [47]

a) none
b) one
c) two
d) three
e) four

Questions 44 to 47 refer to the game matrix below.

	b_1	b_2	b_3
a_1	$2,6$	$0,4$	$4,4$
a_2	$3,3$	$0,0$	$1,5$
a_3	$1,1$	$3,5$	$2,3$

44. (*1 mark*) Which plays by the row player are best responses to the column player's b_1? [48]

a) a_1 only
b) a_2 only
c) a_3 only
d) there are two best responses
e) there are more than two best responses

45. (*1 mark*) Which plays by the column player are best responses to the row player's a_3? [49]

a) b_1 only
b) b_2 only
c) b_3 only
d) there are two best responses
e) there are more than two best responses

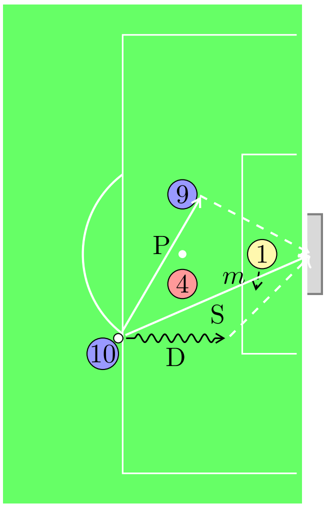
46. (*2 marks*) Which plays by the row player are best responses to the column player's mixed action $\frac{1}{3}b_1\frac{1}{3}b_2\frac{1}{3}b_3$? [51]

a) a_1 only
b) a_2 only
c) a_3 only
d) there are two best responses
e) there are more than two best responses

47. (*1 mark*) Which plays by the column player are best responses to the row player's mixed action $\frac{1}{2}a_1\frac{1}{4}a_2\frac{1}{4}a_3$? [52]

a) b_1 only
b) b_2 only
c) b_3 only
d) there are two best responses
e) there are more than two best responses

Questions 48 to 53 refer to the problem below.



Consider the football situation shown above, where Alice (yellow #10) has three options:

P pass to her team-mate (yellow #9);
D dribble closer to goal before shooting; or
S shoot from where she is.

The chances of scoring if Alice passes (P) to her team-mate are 3 in 10. Her chances of scoring by first dribbling closer (D) to goal and then shooting are 5 in 10. Her chances of scoring by shooting from where she is (S) are 2 in 10.

Bob, the goal-keeper (blue #1), can choose to move (m) toward the ball as shown to reduce Alice's scoring chances if she dribbles to 1 in 10 at the expense of increasing her scoring chances by passing and shooting respectively to 5 and 3 in 10.

48. (*1 mark*) Which is Alice's *Maximin* pure action? [53]

a) P
b) D
c) S
d) both P and D
e) none of the above

49. (*1 mark*) Which is Bob's *Maximin* pure action? [54]

a) $\overline{m}$
b) m
c) both $\overline{m}$ and m
d) neither $\overline{m}$ nor m
e) none of the above

50. (*2 marks*) How many pure strategy equilibria does this game have? [56]

a) 0
b) 1
c) 2
d) 3
e) none of the above

51. (*2 marks*) Assuming that this situation were repeated many times (*i.e.*, mixed strategies are allowed), the lowest value to which Bob could restrict Alice's best response is: [58]

a) 7 in 10
b) 6 in 10
c) 5 in 10
d) 4 in 10
e) none of the above

52. (*1 mark*) Let $p = P(m)$ be the probability that the goal-keeper will move. Which value of p would restrict Alice's best response to the least chance of scoring? [59]

a) $p = \frac{1}{3}$
b) $p = \frac{3}{5}$
c) $p = \frac{2}{3}$
d) $p = \frac{2}{5}$
e) none of the above

53. (*1 mark*) If mixtures are allowed for both players, which of the following is an equilibrium? [60]

a) $(\frac{1}{3}\mathrm{P}\frac{2}{3}\mathrm{D}, \frac{1}{3}\overline{m}\frac{2}{3}m)$
b) $(\mathrm{P}, \overline{m})$
c) $(\mathrm{D}, \frac{1}{3}\overline{m}\frac{2}{3}m)$
d) $(\frac{1}{2}\mathrm{P}\frac{1}{2}\mathrm{D}, \frac{2}{3}\overline{m}\frac{1}{3}m)$
e) none of the above

End of exam

Total questions:	53
Total marks:	60