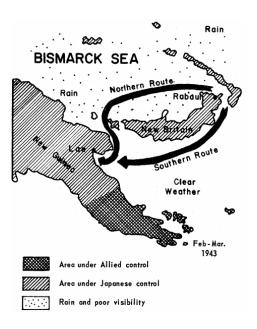
## GSOE9210 Engineering Decisions

## Problem Set 09

1. Consider the 'Battle of the Bismarck Sea' discussed in lectures.



- (a) Can this be represented as a zero-sum game?
- (b) Represent the game in extensive form (i.e., as a game tree).
- (c) Represent the game in normal (strategic) form (i.e., as a game matrix).
- (d) Simplify the problem by eliminating dominated strategies.
- (e) Which, if any, are the rational 'solutions' to the game?
- 2. Consider the 'Jailbreak game' from lectures. Suppose that neither the prisoner (P) nor the guard (G) know the other's move.
  - (a) Is the game zero-sum?
  - (b) Draw the game tree for this game.
  - (c) Convert this to extensive form, with the prisoner as the row player.
  - (d) Simplify the game using dominance.
  - (e) Repeat the above for the game in which the prisoner knows which wing the guard will patrol (i.e., N or S).
- 3. Use dominance to solve the following zero-sum game (payoffs are for the row player):

	$b_1$	$b_2$	$b_3$	$b_4$
$a_1$	0	1	7	7
$a_2$	4	1	2	10
$a_3$	3	1	0	25
$a_4$	0	0	7	10

4. Use dominance to reduce the following zero-sum games:

	$b_1$	$b_2$	$b_3$		$b_1$	$b_2$	$b_3$	$b_4$
$a_1$	3	8	3	$a_1$	1	2	3	3
$a_2$	0	1	10	$a_2$	1	5	0	0
$a_3$	3 0 3	6	5	$a_3$	1 1 1	6	4	1

5. Consider the following zero-sum game, in which mixed strategies are allowed.

- (a) Which, if any, strategies can be eliminated by using dominance?
- (b) Show that if player A had a possible strategy  $a^*$ , with payoffs 2 and 3 in response to player B's strategies  $b_1$  and  $b_2$  respectively, then  $a^*$  would not be dominated.
- 6. Use dominance to solve the following matrix representation of a two-player non strictly competitive game.

7. Two companies, X and Y, produce a similar product which earns a profit of \$1 per unit sold. The two companies compete for a total annual market of 4000 units. However, if either company (or both) advertises, the total annual market will increase by 50%.

If neither or both companies advertise then they split the market evenly. If only one advertises, then the one that advertises gains two-thirds of the market.

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Company X is deciding whether to close production (exit this market), or continue, and if so, whether to advertise or not.

Company Y is committed to this market (i.e., it won't leave), but is monitoring whether ot not company X stays in the market before deciding whether ot not to advertise.

If X stays, both companies must decide whether or not to advertise this year before they know whether the other will.

- (a) Draw the extensive form of this game.
- (b) Draw the corresponding game matrix from the perspective of player X.
- (c) Reduce this game to identify possible solutions.
- (d) Repeat the above for the case where the annual cost of advertising for each company is \$1000.