Tutorial 7 - Further topic in geometry

1. Let $z = x + iy$. Find the image of the half-plane $x > 0$ under the transformation $w = iz + i$.
2. What is the image of the line $(1 + 2i)z + (1 - 2i)\overline{z} + 3 = 0$ under reciprocation? Leave your answer in terms of w.
3. Find the image of the half-plane $x \geq c$ for $c > 0$ under the transformation $w = 1/z$.
4. Find the image of the infinite strip $0 < y < \frac{1}{2c}$ under the transformation $w = \frac{1}{z}$.
5. Show that linear fractional transformation maps any circles and lines onto circles and lines.
6. Find the linear fractional transformation that maps the points $z_1 = -i, z_2 = 0, z_3 = i$ onto the points $w_1 = -1, w_2 = i, w_3 = 1$. Into what curve is the imaginary axis $x = 0$ transformed?
7. Show that for $a, b, c, d \in \mathbb{R}$, the linear fractional transformation $T = \frac{az+b}{cz+d}$ maps upper half-plane $\operatorname{Im} z > 0$ onto $\operatorname{Im} z > 0$ if $ad - bc = 1$.