INTRODUCTION
TO
RATEMAKING AND RESERVING

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Contents

Chapter 1

Introduction

References that will be used for this subject are:

1. Main References:
   (a) Friedland, J., Fundamentals of general insurance actuarial analysis.
   (b) Grossi, P. and Kunreuther, H., Catastrophe Modeling: A New Approach to Managing Risk. Boston.
2. Additional References:
   (a) Basic Ratemaking
   (b) Estimating Unpaid Claims Using Basic Techniques

Both main references are available in *Mary KUOK Pick Hoo Library, UTAR*, while both additional references are available freely online.

1.1 Introduction to Insurance

Insurance organization is a mechanism for transferring risks and re-distributing losses and insurance is:

1. The promise to pay for a loss;
2. In exchange for a premium;
3. Based on the occurrence of an insurable event.

Insurable event needs to be *predictable to the insurer*, *random and undesirable for the insured* to avoid moral hazard. In short, insurance mechanism can be viewed as: Policyholder pays premium for the coverage/protection over an insurable event, if the insurable event happens during the coverage period then policyholder will be compensated for this loss.

Insurance product is different with some other physical products such as notebook, chair and etc. An insurance product is sold before the *exact* cost is known to the company, hence, an estimation is needed. This can be done based on various methods and for the same reason, because the product is priced by using an estimated cost, the company needs to set aside some amount of money as reserves. This whole process is called **pricing** (or **ratemaking**) and **reserving**.

> What is the differences between an insurance products and a tangible items, say laptop?

There are two types of insurance, life insurance and general insurance (also known as non-life insurance).

1.1.1 Life Insurance

The most commonly known insurance is life insurance. Life insurance covers the risk of death (or mortality), examples are:

1. Whole Life Insurance;
2. Term Insurance;
3. Investment Linked Insurance;
4. Group Insurance.

Typically, life insurance provides coverage over many years. For example, if a policyholder of age 25 purchases a whole life insurance today, the insurance company pays the sum insured only when he/she dies, which could be probably 30 years later.Life insurance products are not the main topic here. In the next subsection, we will look at few common general insurance products.

> For life insurance, the exposure period is usually longer but claim settlement could be relatively faster. While the exposure period for general insurance is shorter and claim may take more years to develop.

1.1.2 General Insurance

General insurance are categorized by personal line and commercial line and the contracts are usually one year exposure. However, the claim settlement can be a long process, and thus, reserves are required. Another type of classification of general insurance is by its nature, long-tailed or short tail.

A **long-tailed** line of business is where the claim payments will usually taking a long time to settle, while **short tail** line of business will take a shorter time to settle. For example, fire insurance is short tail as usually insurer will be able to fully settle the claim in a year or two; liability insurance is long-tailed as usually this type of claim takes longer time to develop. Figure 1.1 shows the relation of reported claims (as a percentage to ultimate loss) and development year[1].

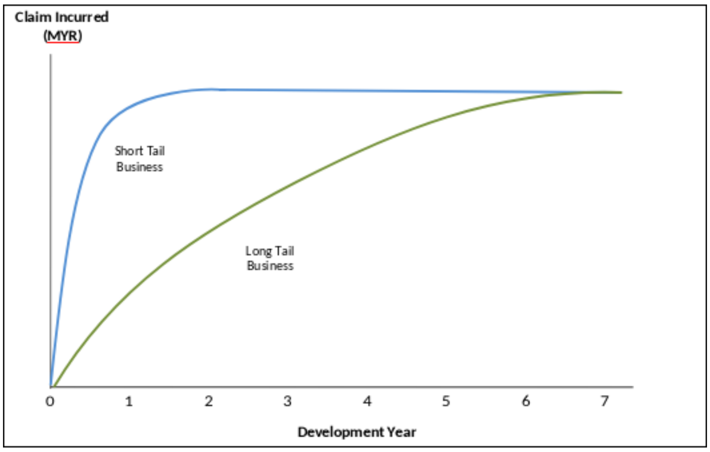


Figure 1.1: Percentage of reported claims vs development year

Motor Insurance

Motor insurance products has three different types of cover: Third Party, Third Party Fire and Theft and Comprehensive. Figure 1.2 below illustrates the differences between different types of coverage.

[1]**Development Year** can be viewed as "age" of a claim.

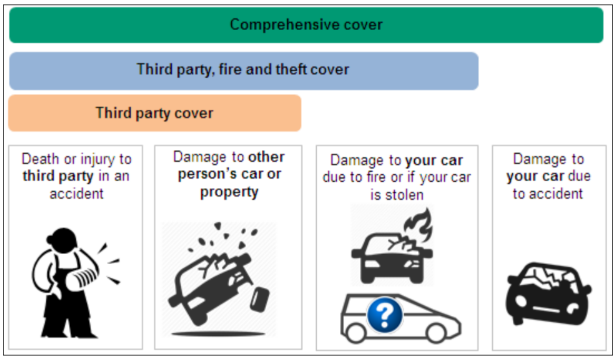


Figure 1.2: From Bank Negara Malaysia website

Third Party product has two components, Bodily Injury (TPBI) and Damage to Others (TPPD), while Third Party Fire and Theft (TPFT) has extra component - Fire and Theft. Comprehensive product covers Own Damage (OD) too.

Example 1.1.1. If Mr. A was driving and hit into Mr. B's car. Given that Mr. A purchased comprehensive cover and Mr. B purchased third party cover. There are two possible scenarios:

1. It was Mr. A's fault - Mr. A can file a claim to his insurer for his car damage (OD), Mr. B's car's damage (TPPD) and possibly Mr. B's injury;
2. It was Mr. B's fault - Mr. B can file a claim to his insurer for Mr. A's car damage (TPPD) only.

The above three types of covers can be viewed as different combinations of TPBI, TPPD, TPFT and OD. Note that TPBI is also called as Act cover, which is the minimum cover corresponding to the requirements of the Road Transport Act 1987. You could read more information from PIAM website.

Engineering

Engineering Insurance provides economic safeguard to the risks faced by the ongoing construction project, installation project, and machines and equipment in project operation. The coverage period is the same as construction period and hence the claims are long-tailed.

Fire

Fire Insurance protects policyholder's home, furniture and belongings against loss/damage by natural disasters. There could be other product named as **Homeowner Insurance** that has other coverage and at the same time also includes fire insurance. This type of claims are usually short tail.

Workmen Compensation

Workmen Compensation Insurance is paid by employer to provide medical care, salary replacement or death benefits to an employee, if he/she is injured or killed on the job. This type of claims are usually long-tailed as policies written today may have payouts for many decades.

Medical and Personal Accident

Medical Insurance covers medical costs, the details vary widely depending on the product and insurer. Medical insurance claim is a long-tailed business.
Personal Accident covers the events of death, injuries or disablement arising from accident. Personal accident claim is usually short tail.

1.1.3 Operation of General Insurers

Underwriting

Underwriter will look into the policy and evaluate the risk profile before accepting the risk.

Claim

There are two different status for any claim in insurance company:

1. **Open Claim** - Claim that is still active and not fully settled;
2. **Closed Claim** - Claim that has been fully settled and closed by claim department.

There are different types of claims that you will usually hear in a general insurance company.

1. **Paid Claims** - Amount paid to insured;
2. **Case Reserves**[2] - An estimate of the amount to fully settle a particular claim;
3. **Reported Claims**[3] - Cumulative Paid Claims + Case Reserve;
4. **Ultimate Claims** - Total amount needed to fully settle the claim.

Example 1.1.2. The following dates and events show the development of a claim:

- Jan 20, 2017 - Date of loss (DOL);
- Mar 1, 2017 - Claim amount of RM10,000 reported;
- Apr 1, 2017 - First payment of RM1,500 made by insurer;
- Sep 1, 2017 - Second payment of RM3,000 and case reserve is updated to RM6,500;
- Jan 30, 2018 - Final payment of RM6,000.

Calculate the cumulative paid claims and reported claims.

Solution:

Example 1.1.2 shows the processes of a claim, until the claim is closed. The **accident date** or **date of loss** in this example is Jan 20, 2017, while Mar 1, 2017 is known as **report date**.

Refer to 1.1.2, we illustrate some terminologies:

1. From DOL until the claim is reported, the claim is **Incurred But Not Yet Reported**;
2. Prior to final payment, we can see that the claims is **Incurred But Not Enough Reported**;
3. Once the final payment is made (i.e. claim will be closed after this payment), the Case Reserves must be equal to RM0 and we have **Claim Development** of -RM500.

[2] Also known as **case estimates**.
[3] Also known as **Incurred Claims**.

Thus, we know that the ultimate amount for any claim consists of the few components below:

- Incurred But Not Yet Reported Claim (IBNYR);
- Claim Development, or Incurred But Not Enough Reported (IBNER);
- Case Reserves;
- Cumulative Paid Claims.

Most of the time IBNR is being used to represent the total of INBYR and IBNER.

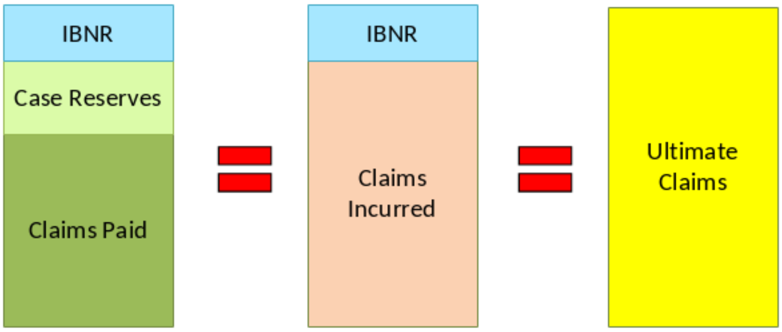


Figure 1.3: Component of Ultimate Claims

There are different types of ultimate values, such as

- Claims amount;
- Claim-related expenses;
- Claims count;
- Average severity.

Why are estimates of ultimate values important?

We summarize some terminologies below:

1. Exposure
    - Basic risk unit that underlies the premium;
    - Example, one car insured for half year can be considered as half an exposure.
2. Premium
    - Amount of the insured pays for the coverage.
3. Claim
    - A demand to the insurer for the indemnification under the policy.
4. Claimant
    - Individual who makes the claim.
5. Accident date
    - Also known as date of loss (DOL), date of the event that caused the loss.
6. Report date

- Date when the claimant reports the claim to insurer.

7. IBNR Claim
   - Incurred But Not Reported claim, loss that has occurred but not yet reported to insurer.
8. Reported Claim
   - After the report date, the claim is known to insurer and is classified as reported;
   $$\text{Reported Claims} = \text{Cumulative Paid Claims} + \text{Case Reserves}. \tag{1.1}$$
9. Ultimate Claim
   - Amount required to close and settle all claims for a defined group of policies;
   $$\text{Estimated Ultimate Claims} = \underbrace{\text{Paid} + \text{Case Reserves}}_{\text{Reported Claims}} + \underbrace{\text{IBNYR} + \text{IBNER Reserves}}_{\text{IBNR Reserves}}. \tag{1.2}$$
10. Loss Adjustment Expense (LAE)
    - Expenses incurred in settling claims;
    - Can be separated to ALAE (Allocated LAE) and ULAE (Unallocated LAE);
    - **ALAE** is claim-related expenses that can be allocated to a specific claim, e.g. fees paid to loss adjuster;
    - **ULAE** is claim-related expenses that can not be directly allocated to a specific claim, e.g. salaries of claims department personnel.
11. Underwriting Expenses
    - Expenses incurred in acquisition and servicing of policies. Four common categories of underwriting expenses: commissions and brokerage, other acquisition, general expenses, taxes, licenses, and fees.

Some of the terminologies will be discussed in more details in later sections/chapters.

Reinsurance

Insurers (cedent) can choose to cede out some of the risks that they have written to pass some of the risks to reinsurers by paying some of the premium received to reinsurers. Below are some basic types of reinsurances:

1. **Treaty** - covers the specified share of all the insurance policies issued by cedent;
   (a) **Proportional Treaty** - Prorated share (e.g. reinsurer will share 10% of loss while at the same time cedent will pass 10% of premium to reinsurer), two common types - Quota Share and Surplus;
   (b) **Non-Proportional Treaty** - Only covers losses above a specific amount;
2. **Facultative** - covers only a specific risk.

Database

Two types of data are available, *internal* and *external.* There are few common types of internal data available, for example:

1. Premium Data - Information of a *policy*, such as policyholder's name, location, type of cover and etc;
2. Claims Data - Information of a *claim*, such as date of loss, claim amount, type of incident and etc;
3. Accounting Data - Information that is *not policy/claim specific*, such as rental, CEO's salary and etc.

Examples of external data are *competitor rate filings*, *industry data*, *economy data* and *geo-demographic data.*

1.2 Some Basic Insurance Ratios

Some basic insurance ratios are introduced in this section. These will helpful for analyzing portfolio's performance.

1. Frequency
   - Frequency $= \frac{\text{No. of Claims}}{\text{No. of Exposures}}$;
   - Defined as how many claims per exposure;
   - To measure occurrence or identify trends in claim occurrence.
2. Severity
   - Severity $= \frac{\text{Total Loss}}{\text{No. of Claims}}$;
   - Defined as how much is the loss per claim;
   - We can further define **Reported Severity** and **Paid Severity**, by using either reported claims or paid claims;
   - To identify loss trends.
3. Pure Premium, $\bar{L}$
   - Pure Premium $= \frac{\text{Total Loss}}{\text{No. of Exposure}} =$ Severity $\times$ Frequency;
   - It is also "average loss";
   - ALAE and/or ULAE may be included
   - To identify overall loss trend.
4. Average Premium
   - Average Premium $= \frac{\text{Total Premium}}{\text{No. of Exposures}}$;
   - Both premium and exposure must be on the same basis, that is, written, earned or in-forced;
   - To identify changes in business mix.
5. Loss Ratio
   - Loss Ratio $= \frac{\text{Total Loss}}{\text{Total Premium}} =$ Pure Premium $\times$ Average Premium;
   - Can also define **Ultimate Loss Ratio**, which is $\frac{\text{Total Ultimate Loss}}{\text{Total Premium}}$;
   - To check the adequacy of rates.
6. Loss Adjustment Expense Ratio
   - LAE Ratio $= \frac{\text{Total LAE}}{\text{Total Losses}}$;
   - Monitor cost associated with claim settlement procedures.
7. Underwriting Expense Ratio
   - UW Expense Ratio $= \frac{\text{Total UW Expense}}{\text{Total Premium}}$;
   - Monitor underwriting costs
8. Operating Expense Ratio
   - Operating Expense Ratio $= \frac{\text{LAE}}{\text{Earned Premium}}$;
   - Monitor operating expenses and review overall profitability.
9. Combined Ratio
   - Combined Ratio = Loss Ratio + Operating Expense Ratio;
   - Similar to Operating Expense Ratio, review overall profitability.

10. Retention Ratio

    - Retention Ratio $= \frac{\text{No. of Policies Renewed}}{\text{No. of Potential Renewal Policies}}$;
    - Monitor the competitiveness of rates;
    - Closely monitored after rate changes or major changes in service;
    - Key parameter in projecting future premium volume.

11. Close Ratio

    - Close Ratio $= \frac{\text{No. of Accepted Quotes}}{\text{No. of Quotes}}$;
    - Monitored by product management and marketing departments;
    - Used to determine competitiveness of rates for new business.

Example 1.2.1. Suppose that company ABC has written 20,000 annual policies of motor insurance last year. The total collected premium is RM7 millions. If the company experienced 10,000 number of claims, with a total reported claims (loss) of RM3 millions. If one annual policy is considered as one exposure, compute the following

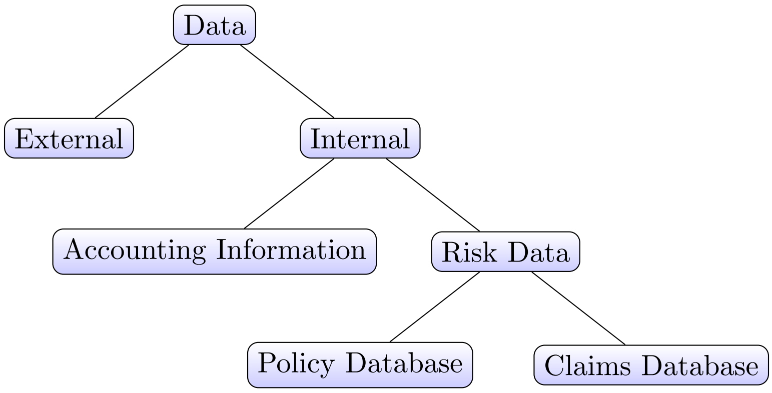
- Average premium;
- Frequency;
- Severity;
- Pure premium;
- Reported loss ratio

Solution:

Chapter 2

Data

Data is one of the most important element in any analysis. Figure below shows the types of data used during the process of ratemaking or reserving.



2.1 Types of Data

2.1.1 Policy Database

Policy database collects the information of the policy such as coverage, policy effective date, policyholder's information and etc.

Example 2.1.1. Table below is an example of what policy database (or premium data) may contain, sorted by policy, instead of transaction date.

Policy	Ori. Eff. Date	Ori. End. Date	Trans. Date	Ded	Terr	W. Premium
A	01/01/10	12/31/10	01/01/10	250	1	1,100
B	04/01/10	03/31/11	04/01/10	250	2	600
B	04/01/10	03/31/11	12/31/10	250	2	-150
C	07/01/10	06/30/11	07/01/10	500	3	1,000
C	07/01/10	06/30/11	01/01/11	500	3	-500
C	07/01/10	06/30/11	01/01/11	250	3	600

Table 2.1: Example of premium data of motor insurance

Observe that policy B has original effective date of Apr 1, 2010 and policy end date (termination date) is Mar 31, 2011. First record of policy B reflect the information at inception date, while second record is showing that policy B is cancelled before its expiration.

2.1.2 Claims Database

Claims database contains the details of a particular claim. Some of the typical fields in claims database are:

1. Policy number;
2. Risk identifier;
3. Claim identifier;
4. Claimant identifier;
5. Relevant loss dates - Accident, report, and transaction date;
6. Claim status - Open/Close;
7. Claim count - Number of claims associated with an occurrence;
8. Paid claims - Payment made;
9. Event identifier - Identify any event involving claim;
10. Case reserve ;
11. ALAE - Allocated Loss Adjustment Expense;
12. Salvage - Recoveries from the sale of damaged property or from third party;
13. Claim characteristics - type of injury, etc.

Example 2.1.2. Table below is an example of paid claims data of motor insurance.

Policy No.	Pol. Eff. Date	Insured	Vechicle No.	Claim No.	Claim Type	Gross Claim	Vehicle
A034833	10-29-2018	Mr. A	ABC111	A076	TPD	8,497.00	Car
A035193	11-01-2018	Ms. B	XYZ123	A072	OD	982.14	Car
A035193	11-01-2018	Ms. B	XYZ123	A072	OD	213.15	Car
A035597	11-26-2018	Mr. C	WWW789	A075	TPI	3,224.13	Car

Table 2.2: Example of paid claims data for motor insurance

We can see from these two examples that policy and claims data are sorted differently and their transaction dates could be in different calendar years. Thus, there might be mismatch between two data at a certain cutoff point.

2.1.3 Accounting Information

In an insurance company, there other data that are required for ratemaking or reserving but not specific to any policy or claim. Generally, expenses can be categorized into:

1. Underwriting expenses;
2. Loss adjustment expenses (LAE);
   (a) Allocated loss adjustment expenses (ALAE);
   (b) Unallocated loss adjustment expenses (ULAE).

2.1.4 External Data

Examples of external data are

1. Aggregated industry data;
2. Competitor rate filings;
3. Other third-party data such as consumer price index, government bond yield rate.

These types of data might be useful in pricing new line of business, or when a company is venturing into unexplored territory. It can also be used when pricing for existing line of business.

2.2 Data Aggregation

Information about the risk data (premium and claims data) are usually stored policy/claim specific but most of the time analysis on reserving or ratemaking are done in aggregate basis. Thus, grouping of data is needed. The main objectives of data aggregation are:

1. Accurately match claims and premium for the policy;
2. Use the most recent data available;
3. Minimize the cost of data collection and retrieval.

There are four common types of data aggregation, which are *calendar year*, *accident year*, *policy year* and *report year*.

1. Calendar Year
   (a) Transactions that occur in that twelve months period
   (b) Earned premium/exposure = All premium/exposure earned during that twelve months period
   (c) Paid Claims = Total amount paid for claims during the calendar year
   (d) Reported Claims = Paid Claims + Change in Case Reserve
2. Accident Year
   (a) Aggregate losses that occur in that twelve months period
   (b) Accident year is not closed (fixed) at the end of the year
   (c) Earned premium/exposure = Same as Calendar Year method
   (d) Paid Claims = All claim payments for losses that occur in that year
   (e) Reported Claims = Paid claims + case reserve (for losses occur in that year)
3. Policy Year
   (a) Premium, exposure and losses on policies in that were underwritten in that year
   (b) Earned premium/exposure = All premium/exposure earned for policies written in that year
   (c) Exposures may take up to 24 months to be fully earned (e.g. a policy written 12/31/2016 may have claim at 12/30/2017 and this is included in policy year 2016)
   (d) Paid Claims = All claim payments for policies written in that year
   (e) Reported Claims = Paid claims + case reserve (for policies written in that year)
4. Report Year
   (a) Losses that are reported in that twelve months period
   (b) Earned premium/exposure = Same as calendar year
   (c) Paid Claims = All claim payments for losses reported year
   (d) Reported Claims = Paid claims + case reserve (for losses reported in that year)

Example 2.2.1. An insurance company started writing annual policies in 2015. Given the following information for claims associated with policies written in 2015:

Accident Year	Calendar Year	Payments	End of Year Case Reserve
2015	2015	1,000,000	500,000
	2016	300,000	300,000
	2017	250,000	100,000
	2018	50,000	
2016	2015		
	2016	1,500,000	1,000,000
	2017	700,000	200,000
	2018	100,000	50,000

Table 2.3: Loss in 2015 and 2016

1. Calculate the *calendar year reported claims* for 2016.
2. Calculate the *accident year reported claims* for 2016 evaluated as of Dec 31, 2016.
3. Calculate the *accident year reported claims* for 2016 evaluated as of Dec 31, 2017.
4. Calculate the *policy year reported claims* for 2015 evaluated as of Dec 31, 2018.

Solution:

Note that in previous example, the amount of accident year and policy year reported claims evaluated as of end of 2016 and 2017 are different, while calendar year reported claims are fixed once calendar year ends. The comparison of different methods of aggregation are summarized below:

Aggregation Method	Advantages	Disadvantages
Calendar Year	Available quickly once calendar year ends; All premiums, exposures and claims value are fixed.	Greater mismatch between exposures and claims
Accident Year	Better match of exposures and claims.	Must estimate future development of claims.
Policy Year	Exact match between exposures and claims.	Data takes longer to develop.
Report Year	No. of claims is fixed at close of the year	Greater mismatch between exposures and claims.

Table 2.4: Comparison of different aggregation methods

2.3 Exposures

Exposure varies according to line of business, and some of the examples are:

- Homeowners insurance - house year;
- Workers compensation - annual payroll;
- Motor insurance - car year.

A good exposure base should be:

1. Proportional to expected loss;
2. Practical;
3. Historical precedence.

For exposures data, there are four ways of measuring them:

1. **Written Exposures** - total exposures arising from policies written during a specified period of time.
2. **Earned Exposures** - the portion of the written exposures for which coverage has already been provided as of a certain point in time.
3. **Unearned Exposures** - the portion of the written exposures for which coverage has not yet been provided as of that point in time.
4. **In-force Exposures** - the number of insured units that are exposed to loss at a given point in time.

Example 2.3.1. Suppose that an annual motor insurance is considered as one exposure and exposures are earned uniformly. By using Example 2.1.1, compute the

1. Written exposures, earned exposures, unearned exposures and in-force exposures for policy B, as at 6/30/2010 and 12/31/2010.
2. Repeat the above calculations for premium.

Solution:

2.3.1 Aggregation of Exposures

There are two methods available for exposures aggregation:

1. Calendar year aggregation (also known as calendar-accident year);
2. Policy year aggregation.

Example 2.3.2. Given the data below:

Policy	Effective Date	Expiration Date	Premium
A	10/01/10	09/30/11	100
B	01/01/11	12/31/11	200
C	04/01/11	03/31/12	350
D	07/01/11	06/30/12	400
E	10/01/11	09/30/11	100
F	01/01/12	12/31/12	150

Assume that the exposure base is earned years and the earning pattern is uniform. By using calendar year and policy year aggregation, compute the written exposures for the year 2010, 2011 and 2012.

Solution:

Will the written exposures for calendar year and policy year 2010, 2011 and 2012 remain the same if policy D was canceled on Mar 31, 2012?

Example 2.3.3. Refer to Example 2.3.2, compute the earned exposures for both calendar and policy year 2010, 2011 and 2012.

Solution:

Both written and earned exposures are measured for a specific period of time, such as calendar year or even calendar quarter, but unearned and in-force exposures are measured at a given point of time.

Example 2.3.4. By using the data from Example 2.3.2, compute the

1. Unearned exposures at the end of calendar and policy year 2010 and 2011;
2. In-force exposures as at 6/15/2011.

Solution:

For individual policy, we have the following relationship:

$$\text{Written Exposures} = \text{Earned Exposures} + \text{Unearned Exposures}. \tag{2.1}$$

For policy year aggregation, the formula above follows immediately. However, for calendar year aggregation of *annual policies*, we need to consider the unearned exposures at the beginning of the calendar year:

$$\begin{aligned}\text{CY Unearned Exposures} &= \text{CY Written Exposures} - \text{CY Earned Exposures} \\ &\quad + \text{Unearned Exposures as of beginning of CY}. \end{aligned}\tag{2.2}$$

For policy year aggregation, both written and earned exposures are the same if the policy year has complete. Thus, unearned exposures is zero.

Example 2.3.5. Repeat Example 2.3.4 by using Equation 2.2.

Solution:

2.4 Premium

If you purchased a pen with RM2 and you would like to sell it to your friend with a profit margin of 10%, how much should you sell it?

It can be easily seen that the equation for profit is

$$\text{Price} = \text{Cost} + \text{Profit}. \tag{2.3}$$

Putting Equation 2.3 into insurance context, we have the following

$$\text{Premium} = \text{Losses} + \text{LAE} + \text{UW Expenses} + \text{UW Profit}. \tag{2.4}$$

Equation above is also known as **Fundamental Insurance Equation**. From insurer's perspective, Equation 2.4 can also be viewed as:

$$\text{Premium} = \text{Losses} + \text{Expenses} + \text{Profit and Contingencies}, \tag{2.5}$$

where contingencies is the difference between expected cost and actual cost that can not be eliminated by any component of the rate.

Note that ratemaking process is prospective, that is, premium are calculated (or estimated) before the exact cost is known. This actuarial process of determining insurance premium is called **ratemaking**.

Example 2.4.1. What are the risks in a ratemaking process?

Solution:

Ratemaking is a complicated process that involves numerous considerations, for example: marketing, competition, legal and regulatory. So, there are four principles for ratemaking[1]:

1. A rate is an estimate of the expected value of future costs
2. A rate provides for all costs associated with the transfer of risk
3. A rate provides for the costs associated with an individual risk transfer
4. A rate is reasonable and not excessive, inadequate, or unfairly discriminatory if it is an actuarially sound estimate of the expected value of all future cots associated with an individual risk transfer

2.4.1 Rating Manuals

From individual insured's perspective, Equation 2.4 can be viewed as

$$\text{Premium} = \underbrace{\text{Exposures} \times (\text{Base rate adjusted by rating factor})}_{\text{Manual Premium}} \times \underbrace{(\text{Adjustment for individual risk rating})}_{\text{Individual risk rating}}. \tag{2.6}$$

In order to compute the manual premium for an individual, a rating manual is needed. The base rate adjusted by rating factor is described in a rating manual. A rating manual is used to classify and calculate the rate for a risk, it usually contains the following categories:

1. Rating rules;
2. Rate pages;
3. Rating algorithm;
4. Underwriting guidelines.

Rating rules contain

1. Directions for the assignment of a base rate and rating
2. Factors needed to calculated the premium, include definitions of terminology, classification of risk characteristics
3. Others like minimum or maximum values of the total amount of discount or surcharge, calculation of refund in the event of policy cancellation
4. Rating factors are developed to reflect the risk characteristics that are relevant to the specific line of business
5. Some rating factors are multiplicative, some are additive, some are added together before they are applied in multiplicative manner
6. Selection of rating factors depends on:
   (a) Data available;
   (b) Regulatory restrictions;
   (c) Operational constraints;
   (d) Marketing objectives.

Rate pages are are tables summarizing the base rates and rating factors required for the determination of premium for a particular insured. Some examples of "tables" in automobile insurance's rate pages:

1. Driver classification factor table reflecting marital status, gender and age;

[1] Read: Statement of Principles Regarding GI Ratemaking

2. Base rates for each different coverage;
3. Policy term factors;
4. Vehicles/driver/policy discounts and surcharges.

The rating algorithm describes how to combine the various components in the rules and rate pages to calculate the overall premium charged. It is very specific and includes explicit instructions, such as:

1. The order in which rating variables should be considered
2. How the effect of rating variables is applied in the calculation of premium (e.g., multiplicative, additive, or some unique mathematical expression)
3. The existence of maximum and minimum premiums (or in some cases the maximum discount or surcharge that can be applied)
4. Specifics associated with any rounding that takes place.

Underwriting guidelines are a set of company-specific criteria that can affect decisions made prior to calculating a rate, it includes:

1. Decisions to accept, decline or refer risk
2. Company placement
3. Tier placement
4. Schedule rating credits (downwards)/debits (upwards)

Example 2.4.2. Suppose that an insurance company is underwriting workmen compensation insurance for a small training center. The **class rates** are given below:

Class	Rate per RM100 of Payroll
Clerical	1.50
Teaching staff	2.00
All other employees	3.00

It is believed that underwriter can understand the business nature better, and hence underwriter can apply his/hers professional judgment to modify the manual premium based on the **schedule rating** below (subject to maximum credit or debit of 10%):

Premises	Medical Facilities	Safety Devices
±10%	±5%	-5% to 0%

Additional premium credits can also be applied to insureds:

Factor	Credit
Pre-employment Drug Screening	5%
Employee Assistance Program	10%
Return-to-Work Program	5%

This insurance company incurs a constant expenses of RM150 per policy. It is also known that the minimum premium of each policy is RM2,500.

Compute the total premium of this WC insurance if:

Class	Payroll
Clerical	RM35,000
Teaching staff	RM75,000
All Other Employees	RM25,000

It is known that the training center has "Pre-employment Drug Screening" program and underwriter has provided the following information:

Premises	Medical Facilities	Safety Devices
−5%	0%	-2.5%

Solution:

2.4.2 Aggregation of Premium

Similar to exposures, there are four ways of measuring premium as well:

1. **Written Premium** - total premium associated with policies that were written/issued during a specified period.
2. **Earned Premium** - the portion of the written premium for which coverage has already been provided as of a certain point in time.
3. **Unearned Premium** - the portion of the written premium for which coverage has not yet been provided as of that point in time.
4. **In-force Premium** - the total amount of full-term premium that are still in effect at a given point in time.

Premium aggregation's method is the same as exposures aggregation.

Example 2.4.3. Using the data given in Example 2.3.2, compute:

1. Written premium for calendar and policy year 2010, 2011 and 2012;
2. Earned premium for calendar and policy year 2010, 2011 and 2012;
3. Unearned premium at the end of calendar year 2010 and 2011;
4. In-force premium as at 6/15/2011.

Solution:

2.5 Calculation of Block Policies

Previous sections illustrated how to compute the exposures or premium with individual policies. However, data sometimes might be summarized on monthly or quarterly basis. In order to perform analysis with this type of data, calculation of the exposures for the **block policies** are needed.

For block policies, effective dates for each individual policies are not known, thus it is common to assume that the company writes all policies uniformly, i.e. "on average" the day of writing this block of policies is on the mid-point of the period.

Hence, if monthly data is provided, the assumed effective date would be on 15^{th} of the month and this is referred to as the "15^{th} of the month" rule or the "24^{th}s" method. This approximation is acceptable if the company is writing policies uniformly and the period is short.

Example 2.5.1. Assume that a company write annual policies in 2018 and write 240 exposures in each month.

1. Find the in-force exposures as at 7/1/2018, 1/1/2019 and 7/1/2019 using "24^{th}s" method.
2. Now compute the earned exposure for calendar year 2018 and 2019 using "24^{th}s" method.

Solution:

Similar technique can be applied on calculation of block premium.

2.6 Adjustment to Premium

Example 2.6.1. Consider a simplified scenario, if one bought a pen with RM2 last year and bought another pen this year with the price of RM4.

By using the latest price (RM4 per pen) and since amount spent on purchasing pens is RM6, thus, we can conclude that the quantity of pens we have is RM6/RM4 = 1.5.

Or, we can also use the earliest price and conclude that the quantity of pens we have is RM6/RM2 = 3.

Calculations in Example 2.6.1 are obviously wrong. This is because there is an adjustment in price between the two purchases.

Similarly, for ratemaking process, historical data should be adjusted to **current rate level** in order to capture the effect of historical rate changes.

If we fail to capture the rate changes already implemented, then we will end up with incorrect new rate.

For example, if the historical rate (for the year 2017) was RM100 and there is a rate change of 10% in 1/1/2018. After the ratemaking analysis was done, the indicated rate for year 2020 is RM120.

If the analyst did not consider the 10% rate change, the analyst will end up with the conclusion that, the rate needs to be increased by 20%. In fact, the increase should be 9.09%.

The process of premium adjusting is known as *adjusting premium to current rate level* or *bringing premium on-level*. There are two methods available, extension of exposures and parallelogram method.

2.6.1 Extension of Exposures

Extension of exposures involves recalculating the rates for *all* policies written during historical period. This method is suitable for lines of business that has premium rates calculated based on a specific rating formula.

Example 2.6.2. Given the following rate change history:

Date	Overall average rate change	Base rate per exposure	Class Teens	factor Adults	Policy fee
Initial	–	1,000	2.00	1.00	50
10/1/2015	5%	1,100	2.00	1.00	50
1/1/2016	10%	1,100	2.50	1.00	50
1/1/2017	-5%	1,000	2.50	1.00	50
4/1/2018	15%	1,000	2.50	1.50	100

If there is a policy of 1 exposure (class teens), and was effective on 2/1/2016, then the premium charged was RM2,800 ($1 \times \text{RM}1,100 \times 2.50 + \text{RM}50$). To bring this premium to *on-level*, recalculate the premium based on the current base rate, rating factors and policy fees.

$$\text{On-Level Premium} = 1 \times 1,000 \times 2.50 + 100 = 2,600.$$

To compute the on-level premium for whole line of business, one has to recalculate individual premium for all policies and aggregate them together.

2.6.2 Parallelogram Method

The parallelogram method does not compute individual premium, but on group basis. Thus, this method is less accurate compares to extension of exposures. The main assumption is that the exposures are written and earned uniformly.

Example 2.6.3. Refer to Example 2.6.2, compute the calendar year 2016 earned premium at present rate. Assume that all policies are annual and historical earned premium for calendar year 2016 is RM50,000.

Solution:

Redo Example 2.6.3 with the assumption that the policies are all half-yearly.

Chapter 3

Projecting Ultimate Claims

> "The Bondy method seems to underestimate, it should be increased, the easiest thing to do is to multiply the development portion by two."
>
> Joseph Boor, *Estimating Tail Development Factors: What to do When the Triangle Runs Out*

Recall that ultimate claims consist of:

1. Cumulative paid claims;
2. Case reserves;
3. Incurred but not enough reported;
4. Incurred but not yet reported.

The last two items can be grouped as **Incurred but not reported (IBNR) claims**. Thus, we have

$$\text{Ultimate Claims} = \underbrace{\text{Cumulative Paid Claims} + \text{Case Reserves}}_{\text{Reported Claims}} + \text{IBNR}. \tag{3.1}$$

In an insurance company, there are more than several types of ultimate values, such as:

1. Ultimate counts;
2. Frequency;
3. Severity;
4. Claims-related expenses.

In this chapter, we shall focus on ultimate claims but the techniques of estimating ultimate values are similar for each of the above.

Note that **valuation date** (date of which the estimation of ultimate claims are evaluated) is important when it comes to projection of ultimate values.

3.1 Claim Development Triangles

Example 3.1.1. Consider the following incremental paid claims data:

Transaction Year	AY	Amount (RM)
2016	2016	100
2017	2016	50
2017	2017	110
2018	2016	20
2018	2017	51
2018	2018	115

Table 3.1: Incremental paid claims data for the year 2016 to 2018

From Table 3.1, we can see that the *cumulative paid claims for accidents happened in 2016* is different at different valuation date:

- 12/31/2016 - 100;
- 12/31/2017 - 150;
- 12/31/2018 - 170.

3.1.1 Claims Development Triangle

A **development triangle** is a table that shows changes in the value of various cohorts over time. Table 3.1 can be represented as:

AY	12/31/2016	12/31/2017	12/31/2018
2016	100	50	20
2017		110	51
2018			115

Table 3.2: Incremental paid claims data by accident year as at different accounting dates

In fact, most of the time, we are more concerned with the cumulative paid claims. Thus, we can rewrite Table 3.2 as:

AY	12/31/2016	12/31/2017	12/31/2018
2016	100	150	170
2017		110	161
2018			115

Table 3.3: Cumulative paid claims data by accident year as at different accounting dates

In projection of ultimate claims, we will look at **development year or age** instead of accounting year:

AY	1	2	3
2016	100	150	170
2017	110	161	
2018	115		

Table 3.4: Claims development triangle

There are three important dimensions in a development triangle:

1. Rows - Represent each accident year
2. Columns - Age of the claims
3. Diagonals - Each diagonal represents a successive valuation date

In example earlier, first diagonal represents CY2016 and second, third diagonal represents CY2017 & CY2018, respectively. From the claim development triangle, we can see that the cumulative paid claims for

AY2016, as of Dec 31, 2017 is 150 and as of Dec 31, 2018 is 170.

Development triangles are important in analyzing pattern or development of historical claims. Besides paid claims, there are also some commonly used triangles for analysis of ultimate claims:

1. Reported claims, or Incurred claims;
2. Closed count;
3. Reported count.

Most companies use accident year in organizing claims data, however, it is common for reinsurers to use *underwriting year*.

Usually cumulative numbers are used instead of incremental numbers because cumulative numbers are generally more stable compared to incremental.

Example 3.1.2. You are given the following data:

Paid claims data:

AY	2012	2013	2014	2015
2012	75,000	137,500	75,500	49,000
2013		50,000	115,000	145,000
2014			115,000	123,000
2015				85,000

End of year case reserves:

AY	2012	2013	2014	2015
2012	188,000	115,000	74,000	35,000
2013		175,000	94,000	45,000
2014			115,000	68,000
2015				208,000

1. Construct the cumulative reported claims triangle;
2. Calculate paid claims for calendar year 2015;
3. Calculate the change in case reserves for calendar year 2015.

Solution:

3.2 Development Triangle as a Diagnostic Tool

Development triangle is also commonly used as a diagnostic tool to investigate the abnormality of data. Some of the triangles that are used widely are:

1. Reported loss ratio;
2. Paid loss ratio;
3. Paid to reported ratio;
4. Closed to open claim count ratio;
5. Average reported/paid/case reserves.

Example 3.2.1. Given the following information:

Claim ID	Accident Date	Paid in 2007	Case Reserve at 12/31/07	Paid in 2008	Case Reserve at 12/31/08	Paid in 2009	Case Reserve at 12/31/09
1	01/01/07	75	250	50	250	300	0
2	07/01/07	25	250	50	200	200	0
3	01/01/08			0	500	50	600
4	07/01/08			100	50	100	0
5	01/01/09					105	645

Table 3.5: Paid claims and case reserves in 2007 to 2009

1. Construct the cumulative paid and reported development triangles.
2. Is there any unusual pattern? Consider the paid-to-reported ratio triangle.

Solution:

We will not discuss much about the analysis of abnormality of claims data here. Generally, we look at the columns of development triangle vertically and in a stable environment, we should expect consistency within the same column. Normally analysis should be done before any projection of ultimate claims.

3.3 Projecting Ultimate Claims

There are various methods in projecting ultimate claims, in this chapter, we focus on four commonly used methods, *Development method, Expected Claims method, Bornhuetter-Ferguson method* and *Frequency-severity method*.

3.3.1 Development Method

Development method[1] is one of the most frequently used techniques used in projecting ultimate claims. It can be applied to various types of data such as claims, ALAE, recoveries, counts, premium, average values and ratios. It is also known as *Chain Ladder Method* or *Link Ratio Method*.

Regardless of the type of data used, Development method has the following two key assumptions:

[1] It is also known as **Chain Ladder Method**.

1. Historical experience is predictive of future experience; and
2. Activity observed to date is relevant for projecting future activity.

With the two assumptions above, we have some implicit assumptions when we are using this technique on projecting ultimate values, for example:

1. If we are using development method on projecting *claims*, then we are in fact assuming that the claims process remains the same throughout the period of study. That is, there is no speeding up in claim payments and etc.
2. If we are using development method on projecting *premium*, then we are assuming that there is no change in mixture of portfolio, exposures and etc.

The process of applying development triangle can be summarized in the following steps:

1. Display data in development triangle;
2. Calculate age-to-age factors;
3. Calculate average age-to-age factors;
4. Select age-to-age factors for each maturity age interval;
5. Select a tail factor;
6. Calculate cumulative development factors;
7. Project ultimate value.

We illustrate the procedure described above using the data from Example 3.1.2:

Example 3.3.1. Given the following paid and reported claims data:

AY	1	2	3	4
2012	75,000	212,500	288,000	337,000
2013	50,000	165,000	310,000	
2014	115,000	238,000		
2015	85,000			

AY	1	2	3	4
2012	263,000	327,500	362,000	372,000
2013	225,000	259,000	355,000	
2014	230,000	306,000		
2015	293,000			

Age-to-age factor

Age-to-age factor is also known as **loss development factor (LDF)** or **link ratio**. It is calculated by dividing reported claims amount at the next valuation date by reported claims amount at current valuation date. For example,

$$\text{Year 1 - 2 development factor for AY 2012} = 327,500/263,000 = 1.2452.$$

Repeating this calculation for all factors, we obtain the LDFs triangle:

Solution:

Average age-to-age factor

There are several values in each of the column in a LDFs triangle. We need to compute the averages for each of these columns. Some of the common averages that are being used are simple average, weighted average, geometric average and medial average. In this example, we shall compute simple, weighted and geometric average.

Solution:

In fact, if the data history is longer, we can also choose to use latest n-year average, that is, the average of latest n accident years.

Select age-to-age factors for each maturity age interval

Consider all the averages that we have computed so far, we can see that all averages for Year 1-2 are close to each other, while for Year 2-3 age-to-age factors, AY 2013 is much higher than AY 2012, by looking at the numbers in reported claims triangle, we can see that 259,000 is much lower than the other two numbers. Thus, we can choose weighted average as our age-to-age factor for all three years.

	1-2	2-3	3-4	4-Ult
Simple	1.2423	1.2380	1.0276	
Weighted	1.2430	1.2225	1.0276	
Geometric	1.2401	1.2309	1.0276	
Selected	1.2430	1.2225	1.0276	?

For all the *selected LDFs* that we have, they represent the development from beginning to age 4 only. It does not represent the development of claims until "ultimate".

Select a tail factor

A **tail factor** is development factor from last development stage to ultimate costs. There are four commonly used techniques in determining tail factor:

1. Bondy method;
2. Sherman-Boor method;
3. Curve fitting method;
4. Benchmark data.

Original Bondy method Let the last observed LDF to be $(1+d)$ and assume that the development portion of the LDFs are decreasing by half at each development period. Then

$$\begin{aligned}
&\left(1+\frac{d}{2}\right)\times\left(1+\frac{d}{4}\right)\times\dots\\
&1+\left(\frac{1}{2}+\frac{1}{4}+\dots\right)d+O(d^2)\\
&1+d+O(d^2)
\end{aligned}$$

It can be seen that this approach can be viewed as exponential decay with decay constant of $\frac{1}{2}$. In the case where d is small, it is safe to omit the terms $O(d^2)$. So, the tail factor is simply set to be the last observed age-to-age factor. It should be noted that this method might work well for short-tailed business but perhaps underestimate the remaining development for long-tailed lines.

Modified Bondy method As the original Bondy method underestimates the long-tailed lines, we could apply modified Bondy method, which takes $(1 + 2d)$ as the tail factor. In fact, this method is equivalent to *exponential decay with a decay coefficient of* $\frac{2}{3}$. Although this is better estimate for long-tailed line but it is still not considered as conservative approach.

Sherman-Boor method Sherman-Boor method is one of the algebraic methods and it involves determining the ratio of case reserves to paid claims for the oldest year in the triangle, then adjusting the case reserves by an estimate o f the ratio of the unpaid claims to carried case reserves. In essence, the case reserves of the oldest accident year are 'grossed up' to estimate the true unpaid claims using a factor. The estimate of the $\frac{\text{true unpaid claims}}{\text{case reserves}}$ factor is based on how many dollars of payments are required to 'eliminate' one dollar of case reserves.

1. Construct the case reserves triangle:

AY	2012	2013	2014	2015
2012	188,000	115,000	74,000	35,000
2013	175,000	94,000	45,000	
2014	115,000	68,000		
2015	208,000			

2. Construct the incremental paid claims:

AY	2012	2013	2014	2015
2012	75,000	137,500	75,500	49,000
2013	50,000	115,000	145,000	
2014	115,000	123,000		
2015	85,000			

3. Construct the incremental case reserves disposed of:

   Solution:

4. Divide incremental paid claims by incremental case reserves disposed of:

   Solution:

5. Choose adjustment factor for case reserves disposal:

   Solution:

6. Compute the tail factor:

Solution:

Curve fitting method The development portion of LDFs can also be fitted into an **inverse power curve**, LDF $= 1 + at^b$, to extrapolate the additional tail development. By simple algebra, we can see that

$$\ln(d) = \ln(a) + b\ln(t)$$

where d is the development portion of the LDFs.

Dev. Year, Y	LDF	Dev. Portion, d	$\log(d)$	$\log(Y)$
1	1.2430	0.2430	0.2175	0.0000
2	1.2225	0.2225	0.2009	0.6931
3	1.0276	0.0276	0.0272	1.0986

Fitting last two columns into linear regression, we have that

$$\ln(a) = -1.1057 \implies a = e^{-1.1057} = 0.3310, \qquad b = -1.7805.$$

Year	1	2	3	4	5	6	7	8	9	10
LDFs	1.2430	1.2225	1.0276							
Fitted	1.3310	1.0963	1.0468	1.0280	1.0188	1.0136	1.0104	1.0082	1.0066	1.0055
Error	0.0880	-0.1262	0.0192							

Taking the product of fitted LDFs from year 4 to 10. We have **1.0945**. Adjusting this number to have exact fit to last observed LDF, we have

$$\text{Tail factor} = 1 + 0.0945 \times \frac{0.0276}{0.0468} = 1.0557.$$

Method	Tail Factor
Bondy Method	1.0276
Modified Bondy Method	1.0552
Algebraic Method	1.0959
Curve Fitting Method	1.0557

Summary of Tail Factor In this example, the triangle has only four years of data and if the line of business is long-tailed business, then Bondy and Modified Bondy method will under-estimate the tail factor. While curve fitting method with only three data points might not be too accurate. So, to be conservative, we choose the tail factor to be 1.0959.

Calculate cumulative development factors

To calculate the cumulative development factors, we start from tail factor and successively multiply each selected age-to-age factor from right to left. Each cumulative factor also represents the expected development that remains at a particular maturity age and can be calculated by:

$$\text{Development Pattern} = \frac{1}{\text{Cumulative Factor}}.$$

Selected	1.2430	1.2225	1.0276	1.0945
Cumulative	1.7091	1.3750	1.1247	1.0945
% Reported	58.51%	72.73%	88.91%	91.37%

Project ultimate value

What we have done so far, can in fact be represented in one table:

AY	1	2	3	4	Ultimate Claims
2012	263,000	327,500	362,000	372,000	
2013	225,000	259,000	355,000		
2014	230,000	306,000			
2015	293,000				
Selected LDF	1.2430	1.2225	1.0276	1.0945	
CDF	1.7091	1.3750	1.1247	1.0945	
% Reported	58.51%	72.73%	88.91%	91.37%	

We would like to fill up the last column of this table. We can use the Cumulative Development Factor (CDF) to project each AY claims to ultimate value. For example, the ultimate claim for AY2012 equals:

$$\text{Ult Claims}_{AY2012} = 372,000 \times 1.0945 = 407,154.$$

Repeat for all accident years:

AY	Reported-to-date	CDF	Ultimate Claims
2012	372,000	1.0276	407,154
2013	355,000	1.0560	399,271
2014	306,000	1.2910	420,736
2015	293,000	1.6047	500,758

3.3.2 Expected Claims Method

Ultimate claims can also be estimated by using **expected loss ratio (ELR)**, that is:

$$\text{Ultimate Claims} = \text{Expected Loss Ratio} \times \text{Earned Premium}. \tag{3.2}$$

The key assumption of this calculation is that unpaid claims can better be estimated based on an priori estimate than using experience observed to date. That means Expected Claims method is suitable for:

1. Immature experience periods, particularly long-tailed line of business;
2. New GI products with limited/no historical experience;
3. Entry into a new geographical area;
4. Wide-ranging of changes, either internally or external environments.

Example 3.3.2. Given the following reported claims information:

AY	1	2	3	Earned Premium
2016	5,630	7,106	8,282	12,380
2017	6,380	8,051		13,430
2018	7,348			14,280
CDF	1.570	1.250	1.070	

- The annual loss ratio trend is 7.0%;
- Use Expected Claims method to compute the ultimate claims for accident year 2018.

This process is summarized into:

1. First estimate the ultimate claim using the cumulative LDFs;
2. Then apply the trend to each accident year's ultimate claims;
3. Compute the loss ratio for each accident year:
4. Choose the expected loss ratio.
5. Selected Expected Loss Ratio = 80.97%
6. Projected Ult Claims = 80.97% $\times 14,280 = 11,563$

Solution:

3.3.3 Bornhuetter-Ferguson Method

While Chain Ladder method assumes the historical pattern will be repeating in the future and Expected Claims method totally ignore the development pattern of claims, **Bornhuetter-Ferguson** method is blend of these two methods. It splits ultimate claims into two components - *actual reported* and *expected unreported claims*

$$\text{Ultimate Claims} = \text{Actual Reported Claims} + \text{Expected Unreported Claims},$$

with credibility on historical data (actual reported claims) to be:

$$Z = \frac{1}{\text{CDF}}. \tag{3.3}$$

Note that Equation 3.3 equals percentage of paid (or reported, if reported claims triangle is used). Hence, for later accident years, we are assigning lower credibility to the historical data, which makes complete sense. Similar to Chain Ladder method, Bornhuetter-Ferguson method can also be applied to various types of data, such as paid claims, claims reported, count data.

Formula below is being used to compute the ultimate claims:

$$\begin{aligned}\text{Ultimate Claims} &= \text{Actual Reported} + \text{Expected Unreported} \\ &= \text{Actual Reported} + \text{Expected Claims} \times \%\ \text{Unreported} \\ &= \text{Actual Reported} + \text{Expected Claims} \times \left(1 - \frac{1}{\text{CDF}}\right)\end{aligned} \tag{3.4}$$

where expected claims can be calculated by using previous method (Expected Claims Method).
As this method does not solely depend on historical development pattern, it is preferred over Chain Ladder and Expected method when:

1. There are random fluctuations early in life of an (accident) year;

2. The data is extremely thin and/or volatile.

Example 3.3.3. Given the following data:

AY	1 Months	2 Months	3 Months	Earned Premium
2006	5,630	7,106	8,282	12,380
2007	6,380	8,051		13,430
2008	7,348			14,280
CDF	1.570	1.250	1.070	

Assume that there is no loss trend observed and the expected loss ratio is 80.97%. We can compute the ultimate claims using Equation 3.4:

Solution:

3.3.4 Frequency-Severity Method

Frequency-Severity method uses Chain Ladder technique with *claims count*[2] and *severity* data. It involves projecting the ultimate claims count and ultimate severity.

Key assumptions of this method are:

1. Claims count and severity will continue to develop in similar pattern in the future;

2. Individual claims count that are being grouped has consistent definition and they are reasonably homogeneous.

Example 3.3.4. This example will demonstrated using Microsoft Excel. The template can be downloaded from here.

[2] It can be either *reported count* or *closed count*

3.4 Impact of Changing Conditions on Projection Methods

As we can see from previous section, each method is dependent on specific underlying conditions. In order to determine which methods are more suitable, we should consider the following:

1. Claims Settlement Pattern;
2. Mix of Business;
3. Data Constraints;
4. Exogenous Influences;
5. Outwards Reinsurance.

To illustrate the importance of understanding the data or changing of condition, consider the following five scenarios:

1. Scenario A - Steady state;
2. Scenario B - Steady-state volume and deteriorating claims ratios;
3. Scenario C - Increasing volume and steady-state claim ratios;
4. Scenario D - Steady-state volume and claim experience, strengthening in adequacy of case reserves;
5. Scenario E - Increasing volume, steady-state claim experience and strengthening in adequacy of case reserves.

For each of these scenarios, we examine the effect of changing conditions to the projection of ultimate claims using:

1. Expected Claims method;
2. Chain Ladder method using reported claims;
3. Chain Ladder method using paid claims;
4. Bornhuetter-Ferguson method using reported claims;
5. Bornhuetter-Ferguson method using paid claims.

3.4.1 Scenario A - Steady-state

This is an "ideal and perfect" scenario that insurer is able to capture the development of claims perfectly from data, and expected loss ratio is also being estimated accurately. This serves as a base scenario for comparison of changing of conditions. In this scenario, all methods manage to estimate the ultimate claims accurately.

3.4.2 Scenario B - Steady-state volume and deteriorating loss ratios

In this scenario, the business volume is assumed to be constant but *ultimate loss ratio* is deteriorating, that is, ultimate loss ratio is increasing for later accident years. In this case, methods that are using expected loss ratio as an input, will underestimate the ultimate claims.

3.4.3 Scenario C - Increasing volume and steady-state loss ratios

The situation when business volume is increasing but having constant ultimate loss ratio will not distort the projection of ultimate claims.

3.4.4 Scenario D - Steady-state volume and claim experience, strengthening in adequacy of case reserves

Strengthening case reserves implies that insurer is writing higher case reserves for new claims. This will result in higher reported claims and hence the reported development pattern will be distorted. Note that this does not affect the actual ultimate claims. Thus, Chain Ladder and Bornhuetter-Ferguson method using reported claims will be overestimating the ultimate claims.

3.4.5 Scenario E - Increasing volume, steady-state claim experience and strengthening in adequacy of case reserves.

From Scenario C and D, we know that steady-state business volume does not affect the projection and the use of Chain Ladder and Bornhuetter-Ferguson method using reported claims under the situation of strengthening case reserves will lead to overestimation of ultimate claims.

3.4.6 Summary

We can summarize these discussion in following table:

Scenario	Expected	Dev. (Rpt)	Dev. (Paid)	BF (Rpt)	BF (Paid)
A	-	-	-	-	-
B	Underestimate	-	-	Underestimate	Underestimate
C	-	-	-	-	-
D	-	Overestimate	-	Overestimate	-
E	-	Overestimate	-	Overestimate	-

Is Chain Ladder using paid claims data the best method?

3.5 Evaluating and Selecting Estimates of Ultimate Claims

As shown in previous sections, we know that there are different assumptions under different methods of projection. Thus, one method might be working well for one particular accident year but might not be working well for other years. Similarly, for the same accident year, we might need different methods to estimate the ultimate values for different line of business.

One should understand the data such as the trends, change in insurance operations, or change in external environment, before selecting the best estimate of ultimate values.

A **best estimate**[3] is defined as "an estimate that represents an expected value over the range of reasonably possible outcomes". One common way of selecting the best estimate of ultimate values is to use the average of ultimate values produced by several methods.

However, it should be noted that some methods might be biased or distorted and these method should be excluded from the average.

[3] Also known as central estimate.

Chapter 4

Financial Reporting

In previous chapter, we learned various techniques on projecting ultimate claims and it is also known that

$$\text{Case Reserves} + \textit{Estimated}\ \text{IBNR} = \textit{Estimated}\ \text{Ultimate Claims} - \text{Cumulative Paid Claims}.$$

In this chapter, we will look at how to estimate claims related expenses and what is claims and premium liabilities.

4.1 Claims Related Expenses

Recall that Loss Adjustment Expenses (LAE) can be categorized into Allocated Loss Adjustment Expenses (ALAE) and Unallocated Loss Adjustment Expenses (ULAE). The process of estimation of the **unpaid loss adjustment expenses** is different for ALAE and ULAE.

Examples of ALAE are legal and expert witness expenses. ULAE include salaries, rent and computer expenses for claims department. Due to the different characteristics of ALAE and ULAE, different techniques are required.

4.1.1 Estimating ALAE

As ALAE are claims specific, we can arrange our data into development triangle and apply the techniques used for projecting ultimate claims. Two commonly used techniques are:

1. Development technique for reported and paid ALAE;
2. Development technique applied to ratio of paid ALAE to paid claims.

Different companies might have different ways of handling ALAE. For example, some companies do not keep records of case reserves of ALAE (only paid ALAE), some companies combine the claims data with ALAE and estimate the ultimate values together. This approach seems easier but it is not suitable for all line of businesses, as the claims payment and LAE may be paid at a different point of time. For example, third party liability, the LAE may be on an ongoing-basis during the investigation period, and before any claim payments was being made.

Example 4.1.1. Given the following cumulative paid claims and paid ALAE data:

Cumulative paid claims:

AY	1	2	3	4
2014		9,200	9,300	9,300
2015	8,000	8,900	9,060	9,060
2016	9,200	9,900	9,980	
2017	8,300	9,400		
2018	9,500			

Cumulative paid ALAE:

AY	1	2	3	4
2014		690	753	764
2015	500	760	853	861
2016	550	650	710	
2017	555	770		
2018	630			

The selected ultimate claims are:

AY	1998	1999	2000	2001	2002
Selected Ultimate	9,300	9,060	9,980	9,520	10,680

Development technique for paid ALAE

Solution:

Development technique applied to ratio of paid ALAE to paid claims

Solution:

We could also combine both triangles in Example 4.1.1 and estimate the unpaid claims *plus* ALAE together.

It can be seen the example above that the estimation of unpaid ALAE using paid-to-paid ratios depends on the estimated ultimate claims. Thus, any error in the estimation of ultimate claims will affect the estimation of unpaid ALAE. Besides that, paid-to-paid ratios might not be suitable for line of business where ALAE may be spent on claims that ultimately settle with no payment.

The main advantage of using paid-to-paid ratios is that it analyses the relationship between paid claims and paid ALAE. With paid-to-paid ratios, an actuary can easily apply his/her actuarial judgment in the projection. See the Excel Example here.

When the paid-to-paid ratios are small, additive approach seems to be more stable. It is also important to check that all assumptions for Chain Ladder method, are also applicable to the projection of ALAE.

4.1.2 Estimating ULAE

On the other hand, ULAE are not projected to ultimate for financial reporting; instead, estimate of unpaid ULAE are calculated directly. Most techniques for estimating unpaid ULAE can be categorized into

1. Dollar-based methods:
   (a) Classical Paid-to-Paid method;
   (b) Kittel's Refinement to Classical method;
   (c) Conger and Nolibos method;
   (d) Mango-Allen Refinement.
2. Count-based methods:
   (a) Classical method;
   (b) Wendy-Johnson method;
   (c) Mango-Allen Claim Staffing method;
   (d) Rahardjo;
   (e) Spalla.

We will only discuss Classical Paid-to-Paid and Wendy-Johnson method in this section.

Classical Paid-to-Paid Method

Generally, there are four steps in Classical Paid-to-Paid method:

1. Calculate historical ratios;
2. Review historical ratios for trends or patterns;
3. Select a ratio of ULAE-to-claims;
4. Apply 50% of the selected ULAE ratio to case reserves and 100% to IBNR.

Example 4.1.2. Given the following data of company XYZ as of Dec 31, 2018, estimate the Unpaid ULAE:

- Case Reserves = 603,000;
- IBNR = 316,000;
- Paid ULAE and paid claims for the past six calendar years:

Calendar Year	Paid ULAE	Paid Claims
2013	14,352	333,000
2014	15,321	358,000
2015	16,870	334,000
2016	17,112	347,000
2017	17,331	391,000
2018	14,352	333,000

Calculate the total unpaid ULAE.

Solution:

The ratio applied to case reserves is not necessarily to be 50%. If company thinks that more (less) resources are needed for handling existing claims, the percentage can be higher (lower).

Wendy-Johnson Method

There are two major drawbacks for all types of Dollar-based methods:

1. Does not regconize the fact that ULAE doe not solely depend on claim amount;
2. Unpaid ULAE determined will fluctuate in response to changes in estimate of ultimate claims.

More specifically, the Classical Paid-to-Paid method is not suitable when external environment is changing. For example, during the time of business growth or inflation.

Wendy-Johnson method would be able to resolve these issues but more data are required:

1. Triangles of reported and closed counts;
2. Selected ultimate counts;
3. Paid ULAE.

The major steps in applying Wendy-Johnson method are:

1. Estimate counts for newly reported and closed in each CY, counts remaining open at the end of each CY;
2. Select average ULAE per weighted count;
3. Project unpaid ULAE.

Example 4.1.3. Given the following information:

- Reported and ultimate counts:

AY	1	2	3	4	5	Ult
2014	2,133	2,252	2,266	2,271	2,271	2,271
2015	2,037	2,149	2,174	2,182		2,184
2016	1,620	1,675	1,681			1,694
2017	1,239	1,307				1,323
2018	1,248					1,310

- Incremental closed count that were determined using frequency-severity closure method:

AY	1	2	3	4	5
2014	1,338	484	238	141	70
2015	1,347	430	227	98	82
2016	1,090	305	135	86	78
2017	773	287	135	72	56
2018	890	274	70	26	50

- Paid ULAE for 2016, 2017 and 2018 are RM1,619, RM1,846 and RM1,867, respectively;
- Trend factor for ULAE is 3% per year.

Assume that the effort needed for handling newly reported, open and closed counts are 20%, 70% and 10%, respectively. There is no development after year 5, calculate the total unpaid ULAE.

1. Estimate counts for newly reported and closed in each CY, counts remaining open at the end of each CY:

   (a) Compute the ratio of *Reported Count to Ultimate Count* and select a ratio for each development years:

   Solution:

   (b) Using the selected ratio and complete the square:

   Solution:

   (c) Compute the incremental reported counts:

   Solution:

   (d) Compute the newly reported, closed and open claims count for each CY:

   Note that newly reported count is the sum of diagonal of the incremental triangles and open count for any calendar year is the difference of total reported-to-date and total closed-to-date.

For example, for CY2016:

$$\text{Newly Reported}_{2016} = 14 + 112 + 1,620 = 1,746$$
$$\text{Closed-to-date}_{2016} = 1,338 + 484 + 238 + 1,347 + 430 + 1,090 = 4,927$$
$$\text{Open}_{2016} = (2,266 + 2,149 + 1,620) - \text{Close-to-date}_{2016} = 1,108$$

For calendar years after 2016, we can compute the open count using:

$$\text{Open}_{CY_t} = \text{Open}_{CY_{t-1}} + \text{Newly Reported}_{CY_t} - \text{Closed}_{CY_t}$$

Solution:

2. Select average ULAE per weighted count:

   (a) Compute the trended average ULAE for histroical years:

   Solution:

   (b) Using simple average for latest 2 years, compute the selected avg. ULAE:

   $$\text{Selected} = 1.7989$$

3. Project unpaid ULAE:

Solution:

4.2 Claims Liabilities

Generally, **claims liabilities** is the reserves for unpaid claims and it should include:

1. Case reserves;
2. Provision for future development on case reserves;
3. Provision for IBNYR (also known as pure IBNR);
4. Provision for reopened claims;

5. Claims handling expenses (ALAE and ULAE).

It should includes all accident years and in most countries, actuaries are required to estimate the CL on both gross and net of reinsurance basis. In some countries, claims liabilities should be booked at 75th percentile of claims liability. The difference between the 75th percentile and best estimate of claims liabilities is known as **risk margin** or **provision for adverse deviation (PRAD)**.

Example 4.2.1. Table below shows the estimated unpaid claims for each accident year.

AY	Ult Claims	Paid Claims	Case Reserves	IBNR	Est. Unpaid Claims
Prior Year			1,500	0	1,500
AY1	33,595	32,936	0	659	659
AY2	29.790	28.782	0	1,008	1,008
AY3	26,196	24,652	848	696	1,544
Total	89,581	86,370	848	2,363	3,211

Assume that the company has projected the ALAE together with claims amount and the estimated unpaid loss adjustment expenses is RM600 then the best estimate of total unpaid claims is RM3,811. If PRAD for this line of business is RM700. Then 75th percentile claims liabilities of this line of business is RM4,511.

4.2.1 Provision for Adverse Deviation

There are various methods on estimating the 75th percentile of claims liabilities, such as: Mack method, Bootstrap method (modeled by Over-Dispersed Poisson distribution) and Stochastic Chain Ladder. Mack method is analytic while Bootstrap and Stochastic Chain Ladder method requires simulation of "future payment". These methods have the following limitations:

1. Assumptions of independence and identical distribution of claims pattern;
2. Over-parameterization;
3. Does not work with very sparse data;
4. Estimation of variance *beyond* triangle.

In this section, we will only focus on the calculation of 75th percentile of claims liabilities using Mack method. This method was introduced by Thomas Mack in 1993[1] and improved his method in 1999[2] to include the impact of tail factor. In fact, Mack method:

1. Works well with pure Chain Ladder method only;
2. is a distribution-free approach to arrive at the prediction error;

We need to compute the prediction error by using Mack method, assume that the loss distribution follows *Lognormal distribution* with parameters μ and σ. The 75th percentile can then be calculated using:

$$75^{th} \text{ percentile of claims liabilities} = e^{Z_{0.75}\times\sigma+\mu}.$$

To compute the prediction error of Chain Ladder estimation, we defined the following variables:

1. $C_{i,k}$ to be the accumulated claims amount of accident year i and paid/reported up to development year k;
2. $F_{i,k} = \frac{C_{i,k+1}}{C_{i,k}}$, which is the LDF;

[1] Distribution-Free Calculation of the Standard Error of Chain Ladder Reserve Estimates, ASTIN Bulletin, Vol. 23, No. 2, 1993

[2] The Standard Error of Chain Ladder Reserve Estimates: Recursive Calculation and Inclusion of a Tail Factor, ASTIN Bulletin, Vol. 29, No. 2, 1999

3. $\hat{f}_k = \dfrac{\sum_{i=1}^{n-k} w_{i,k} C_{i,k}^{\alpha} F_{i,k}}{\sum_{i=1}^{n-k} w_{i,k} C_{i,k}^{\alpha}}$, where $\alpha = 0, 1, 2$ and $0 \leq w_{i,k} \leq 1$, to be the selected LDF for development year k;

4. $\hat{C}_{i,k+1} = \hat{C}_{i,k} \hat{f}_k$, which is the estimated claims amount of accident year i and paid/reported up to development year $k+1$;

5. $\hat{R}_i$ to be the unpaid claims for accident year i.

Let n be the last development year, then we can see that:

$$\text{Var}\left(\hat{R}_i\right) = \text{Var}\left(\hat{C}_{i,n}\right)$$

and $\text{Var}\left(\hat{C}_{i,n}\right)$ can be calculated by:

$$\begin{aligned}\text{Var}\left(\hat{C}_{i,n}\right) = \text{Var}\left(C_{i,n}|D\right) + \left(E\left(C_{i,n}|D\right) - \hat{C}_{i,n}\right)^2 &= \hat{C}_{i,n}^2 \sum_{k=n+1-i}^{n-1} \frac{\hat{\sigma}_k^2}{\hat{f}_k^2} \left(\frac{1}{\hat{C}_{i,k}} + \frac{1}{\sum_{j=1}^{n-k} w_{j,k} C_{j,k}^{\alpha}}\right) \\ &= \hat{C}_{i,n}^2 \sum_{k=n+1-i}^{n-1} \left(\text{Var}\left(F_{i,k} \widehat{|C_{i,1}}, \ldots, C_{i,n}\right) + \text{Var}(\hat{f}_k)\right)\end{aligned}$$

where $\hat{\sigma}_k^2$ is the error term:

$$\hat{\sigma}_k^2 = \frac{1}{n-k-1} \sum_{i=1}^{n-k} w_{i,k} C_{i,k}^{\alpha} \left(F_{i,k} - \hat{f}_k\right)^2, \quad k = 1, 2, \ldots, n-2.$$

and

$$\text{Var}\left(F_{i,k} \widehat{|C_{i,1}}, \ldots, C_{i,n}\right) = \frac{\hat{\sigma}_k^2}{w_{i,k} C_{i,k}^{\alpha}} \qquad \text{which is process variance, and}$$

$$\text{Var}(\hat{f}_k) = \frac{\hat{\sigma}_k^2}{\sum_{j=1}^{n-k} w_{j,k} C_{i,j}^{\alpha}} \qquad \text{which is estimation error.}$$

The error term for $k = n-1$, can be calculated by assuming *loglinear regression*:

$$\hat{\sigma}_{n-1}^2 = \min\left(\frac{\hat{\sigma}_{n-2}^4}{\hat{\sigma}_{n-3}^2}, \min(\hat{\sigma}_{n-2}^2, \hat{\sigma}_{n-3}^2)\right)$$

and for $k = n$, actuary who develops the tail factor, f_{ult} should be able to develop an estimate for σ_n^2. The variance for each accident year can be calculated recursively:

$$\text{Var}\left(\hat{C}_{i,k+1}\right) = \hat{C}_{i,k}^2 \left(\text{Var}\left(F_{i,k} \widehat{|C_{i,1}}, \ldots, C_{i,n}\right) + \text{Var}(\hat{f}_k)\right) + \text{Var}\left(\hat{C}_{i,k}\right) \hat{f}_k^2$$

with $\text{Var}\left(\hat{C}_{i,n+1-i}\right) = 0$. Similarly, for total unpaid claims, we have the following:

$$\begin{aligned}\text{Var}\left(\sum_{i=n+1-k}^{n} \hat{C}_{i,k+1}\right) = \sum_{i=n+1-k}^{n} \left(\hat{C}_{i,k}^2 \text{Var}\left(F_{i,k} \widehat{|C_{i,1}}, \ldots, C_{i,n}\right)\right) + \left(\sum_{i=n+1-k}^{n} \hat{C}_{i,k}\right)^2 \times \text{Var}(\hat{f}_k) \\ + \text{Var}\left(\sum_{i=n+2-k}^{n} \hat{C}_{i,k}\right) \hat{f}_k^2\end{aligned}$$

For those who are interested, an Excel template which is using parameters $w_{i,j} = \alpha = 1$ can be downloaded from here. For model that does not included tail factor, can be easily downloaded from CAS's website.

4.3 Premium Liability

Premium Liabilities is the estimated value of claim and expense payments to be made after the accounting date. It should include an amount for future claims and all future LAE, expenses incurred in maintaining unexpired policies. There are two common approaches in determining premium Liabilities.

1. Premium approach;
2. Claims approach.

4.3.1 Premium Approach

The best estimates of the premium liabilities equals to unearned premium reserves (net unearned premium) less deferred acquisition cost and profit margin. **Deferred acquisition cost** is the acquisition cost incurred when premium is written but earned over the term of the policy. Under US statutory accounting, all DAC should be fully earned at the inception of the policy. Thus,

$$\text{PL} = \text{UPR} - \text{DAC} - \text{Profit Margin}. \tag{4.1}$$

4.3.2 Claims Approach

This approach calculates the best estimate of premium liabilities by estimating the future claims and LAE as well as expenses incurred in maintaining unexpired policies. Future claims may be estimated as unearned premium multiplied by an expected loss ratio and usually expected claims includes ALAE. The expected future claims is also known as **unexpired risk reserves (URR)**.

In some countries, premium liabilities is defined as the higher of unearned premium reserves and unexpired risk reserves at 75th percentile.

Example 4.3.1. Given the following information:

LOB	Unearned Premium	AY1	AY2	AY3	BF Expected AY3	Business Plan
Property	882	73.00%	70.00%	74.00%	73.00%	70.00%
Automobile	876	68.00%	70.00%	71.00%	72.00%	70.00%

Assume that the ULAE ratio is 10%, maintenance expenses and commissions are 5.3% and 3.6%, respectively. Calculate the total premium liabilities if risk margin is 10% and the company has diversification discount of 20%.

Solution:

Chapter 5

Trending

In previous chapters, we have seen the use of trend factors in projecting ultimate claims. Trend acts as a tool to close the gap between time periods due to measurable differences in the cost level of claims. The purpose of estimating claim trend is to adjust claim experience to a *cost level* associated with a *common point in time*. Two common points in time that are frequently used by actuaries are:

- Latest year in the experience period (used this in projecting ultimate claims);
- Average accident date underlying the period for which the proposed rates will be in effect (for ratemaking purposes).

To identify trend in data, it is very common for actuaries to use *ratio*. Some of the commonly used ratios are:

1. Paid claims to reported claims ratio;
2. Paid claims to on-level earned premium;
3. Open counts to reported counts ratio;
4. **Frequency**;
5. **Severity**;
6. **Pure premium**.

5.1 Frequency and Severity Trend

Recall that

$$\text{Pure Premium} = \text{Frequency} \times \text{Severity}$$

and pure premium is average loss per exposure. It is always preferred to analyze the pure premium trend using its components, that is *frequency* and *severity*. Since pure premium equals to product of frequency and severity, we have

$$\text{Pure Premium Trend} = (1 + \text{Frequency Trend})\,(1 + \text{Severity Trend}) - 1. \tag{5.1}$$

In this section, we will focus on analyzing trend in frequency and severity. Examples of changing characteristics that could affect the frequency and/or severity include:

1. Deductibles;
2. Policy Limits;
3. Coverage;
4. Others such as territories or target markets.

We need to capture the impact of these changes in the analysis and projection as any data used for development analysis and ultimate claims projection should be grouped by homogeneous categories.

The procedures in estimating trend factor can be summarized into:

1. Aggregate claims, premiums and exposures data quarterly, half-yearly or annually for analysis;
2. Compute the frequency, severity and pure premium;
3. Compute the historical rate of changes;
4. Estimate the future trend factor;
5. Trend is usually estimated by fitting historical data using linear or exponential regression.

Example 5.1.1. An actuary is performing analysis on frequency and severity trend. He has fitted annual frequency and severity using *exponential* trend based on data for the 12 months ending each quarter evaluated through December 31, 2018 are as follows:

Number of Points	Frequency Exponential Fit	Severity Exponential Fit
20	-2.9%	3.4%
16	-3.2%	3.0%
12	-2.5%	2.8%
8	-0.5%	2.9%
6	3.0%	3.1%
4	2.8%	3.3%

As we can observe from the table above, when we include more data points into the analysis, the frequency trends are changing from positive to negative. This example shows that it may not be always suitable to include as much data as possible.

How many data points do we need to include in our analysis? It is a consideration of stability versus responsiveness. The longer period it is, the more stable result we have; however, shorter period will be more responsive. Thus, for short tail business (e.g. TPD) - shorter period can be used; while longer period can be used when we are analyzing long tail business. However, we need to be aware with any potential changes in coverage, change in policy limits and etc.

5.1.1 Linear and Exponential Regression

It is common to use linear or exponential regression to model frequency or severity. The general formula for linear regression is given as:

$$y = ax + b + \epsilon, \tag{5.2}$$

and for exponential regression:

$$y = be^{ax+\epsilon} \tag{5.3}$$

where y represent the frequency or severity, x is the accident year and $\epsilon \sim N(0, \sigma^2)$. Note that when data used is not in yearly basis, we can "annualize" it. For example,

$$\text{Annual Trend} = (1 + \text{Semi-Annual Trend})^2 - 1.$$

Example 5.1.2. Given the following data:

AY	Ultimate Severity	Ultimate Frequency
1	24,473	0.57%
2	28,158	0.63%
3	24,343	0.57%
4	27,427	0.67%
5	22,812	0.63%
6	27,129	0.65%

Fitting the above data into linear and exponential regression, we have:

$$\begin{aligned}
\text{Linear Regression:}& \\
y_s &= 9.3143x + 25,691.0667 \\
y_f &= 0.0001x + 0.0057 \\
\text{Exponential Regression:}& \\
y_s &= 25,641.9423e^{0.0001x} \\
y_f &= 0.0057e^{0.0234x}
\end{aligned}$$

Thus, the annual trend implied by both regressions are:

$$\begin{aligned}
\text{Linear Regression:}& \\
\text{Annual Severity Trend} &= \left(\frac{y_s \text{ at AY6}}{y_s \text{ at AY1}}\right)^{1/6} - 1 = 0.03\% \\
\text{Annual Frequency Trend} &= \left(\frac{y_f \text{ at AY6}}{y_f \text{ at AY1}}\right)^{1/6} - 1 = 1.94\% \\
\text{Exponential Regression:}& \\
\text{Annual Severity Trend} &= \left(\frac{y_s \text{ at AY6}}{y_s \text{ at AY1}}\right)^{1/6} - 1 = 0.01\% \\
\text{Annual Frequency Trend} &= \left(\frac{y_f \text{ at AY6}}{y_f \text{ at AY1}}\right)^{1/6} - 1 = 1.97\%
\end{aligned}$$

If the frequency or severity of accident year i , y_i, is modeled by exponential regression, then its changes across accident years are constant. This can be shown easily by applying the relation $\hat{y}_i = be^{ax_i}$ and $x_{i+1} = x_i + 1$.

Proof:

Since y_i follows exponential regression, we have that

$$\begin{aligned}
\text{Trend} = \frac{\hat{y}_{j+1}}{\hat{y}_j} - 1 &= \frac{be^{ax_{j+1}}}{be^{ax_j}} - 1 \\
&= e^{a(x_{j+1}-x_j)} - 1 \\
&= e^a - 1.
\end{aligned}$$

Since a is a constant found by using least square method, $e^a - 1$ must be a constant also. There is no nice form for linear regression. □

Since the frequency/severity trend for each period can be calculated by

$$\text{Trend} = \frac{y_{j+1}}{y_j} - 1, \tag{5.4}$$

we can also model the *change of ultimate frequency/severity* rather than *ultimate frequency/severity* itself.

Example 5.1.3. Given the following data, estimate the pure premium trend.

Accident Year	Earned Exposures	On-Level Earned Premium	Ultimate Loss and ALAE	Ultimate Count
2016 Q1	101	22,250	19,740	14
2016 Q2	125	28,000	25,794	18
2016 Q3	115	27,500	24,701	17
2016 Q4	99	23,750	22,245	15
2017 Q1	147	26,923	32,956	22
2017 Q2	148	27,692	33,286	22
2017 Q3	155	28,846	35,121	23
2017 Q4	135	25,000	30,880	20
2018 Q1	163	29,235	37,440	24
2018 Q2	170	29,118	39,375	25
2018 Q3	178	30,353	41,366	26
2018 Q4	172	29,118	40,150	25

Solution:

Accident Year	Ultimate Frequency	Ultimate Severity	Frequency Trend	Severity Trend
2016 Q1	13.86%	1,410		
2016 Q2	14.40%	1,433	3.89%	1.63%
2016 Q3	14.78%	1,453	2.66%	1.40%
2016 Q4	15.15%	1,483	2.50%	2.06%
2017 Q1	14.97%	1,498	-1.22%	1.01%
2017 Q2	14.86%	1,513	-0.68%	1.00%
2017 Q3	14.84%	1,527	-0.18%	0.93%
2017 Q4	14.81%	1,544	-0.16%	1.11%
2018 Q1	14.72%	1,560	-0.61%	1.04%
2018 Q2	14.71%	1,575	-0.12%	0.96%
2018 Q3	14.61%	1,591	-0.67%	1.02%
2018 Q4	14.53%	1,606	-0.49%	0.94%

In this case, we can observe that the frequency trend in year 2016 is not consistent with year 2017 and 2018. Thus, we can choose the simple average of last 8 quarters as the frequency and severity trend.

Annualize the trends:

For larger data set, Excel file can be downloaded from here.

5.2 Premium Trend

Similar to claims data, we have to be aware with the changing characteristics of the portfolio:

1. Deductibles;
2. Policy Limits;
3. Mix of business and etc.

Even though the same techniques that we learned in previous section can also be applied to premium data, but this may not be necessary all the time. This is because premium trend is usually quite consistent as compared to loss trend. Changes/trend measured usually are historical average premium rather than changes/trend in total premium and for premium data, we can use either earned premium or written premium.

An Excel example can be downloaded from here.

5.3 Trend and Time Periods

Once trend factor is selected, we adjust our data to be "trended" so that they are applicable in our analysis.In order to adjust our data, we need to determine the time interval for which trend will be applied. Since data are usually aggregated in blocks rather than individual data, we introduce the following concepts:

1. Experiece Period - Period of time to which historical data used for analysis;
2. Forecast Period - Future time period to which the historical data are projected;
3. Trending Period - The time over which the trend is applied in projecting from experience to forecast period.

Generally, there are two methods of trending - one-step trending and two-step trending, which are applicable to both claims and premium data.

5.3.1 Loss Trending

Loss trend period is defined as the period from the average loss occurrence date of each *experience period* to the average loss occurrence date of *forecase period* in which the rate will be in effect. Usually experience period is aggregated by accident year while forecast period is aggregated by policy year.

If the trend factor in experience period is believed to be different with forecast period, **two-step loss trending** can be used. We start by introducing the **one-step loss trending**, of which the trend factor is believe to be the same for both experience and forecast period.

One-Step Loss Trending

In performing loss trending, we assume the policies are written uniformly and losses also occur uniformly.

Example 5.3.1. We look at an example on how to determine the loss trend period. Assume that

- Losses to be trended are from accident years 2011;
- Company writes annual policies;
- Proposed effective date is Jan 1, 2015;
- The length of time the rates are expected to be in effect is one year.

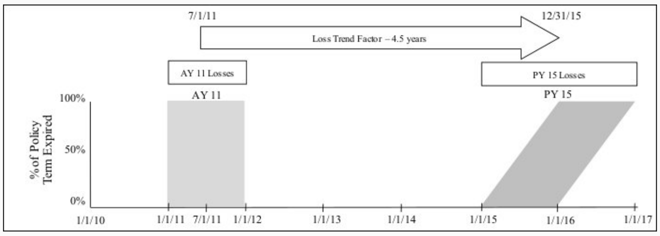
To determine the average loss occurrence date of experience and forecast period, and hence the loss trend period, we first note that we are trending from AY2011 to PY2015, and by assuming that losses occur uniformly. For average loss occurrence date of AY2011:

$$\text{“Trend From” Date} = 7/1/2011 \text{ (losses occur uniformly from } 1/1/2011 \text{ to } 12/31/2011\text{).}$$

For average loss occurrence date, policies are written uniformly from 1/1/2015 to 12/31/2015. So, losses occur uniformly from 1/1/2015 to 12/30/2016.

$$\text{“Trend To” Date} = 12/31/2015.$$

We can use time diagram with AY2011 and PY2015 to illustrate this:



Thus, Loss Trend Period is 4.5 years.

Example 5.3.2. Redo Example 5.3.1, assume that the policy term is 6 months.

Solution:

Example 5.3.3. Redo Example 5.3.1, if the historical data were aggregated by policy year.

Solution:

Two-Step Loss Trending

It is common that the loss trend in historical experience period and the expected trend for forecast period are not identical. Under this scenario, two-step trending will be useful. The trending period is being split into two separate periods:

1. Historical period;

2. Forecast period.

Example 5.3.4. Refer to Example 5.3.1 with the following additional information:

- Last data point in loss trend data is the twelve months ending fourth quarter 2013;
- The current trend is estimated to be -1%;
- The loss projection trend selected is 2%.

Determine the current trend period, projected trend period and total loss trend factor.

Solution:

5.3.2 Premium Trending

Similar to loss trending, premium trend can be calculated by using one-step and two-step trending. As mentioned earlier, we can use either earned or written premium for trending, but which is more suitable?

Earned premium is used in most parts of actuarial analysis, while written premium is more responsive. It is a trade-off between "more relevant" versus "more responsive". Trends observed in written premium will eventually emerge in earned premium and hence trends observed in written premium can be applied to earned premium.

Usually the trend period is from *average written/earned date of* ***premium earned*** in historical period to *average written/earned date of* ***policies written*** during the time the rates will be in effect. Note that for the analysis, all written premium has to be adjusted to current rate level.

One-Step Premium Trending

For premium trend, whether we are using written or earned premium, we have two different *average date* to be used:

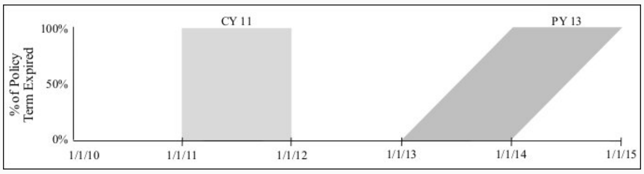
1. Average written date;

2. Average earned date.

Example 5.3.5. Assume that:

- Calendar year 2011 earned premium is used for analysis;
- Policies are to be written during the period of 1/1/2013 through 12/31/2013;
- Policies are annual.

Similar to loss trending, we first prepare the diagram for the historical and projected (or forecast) period:

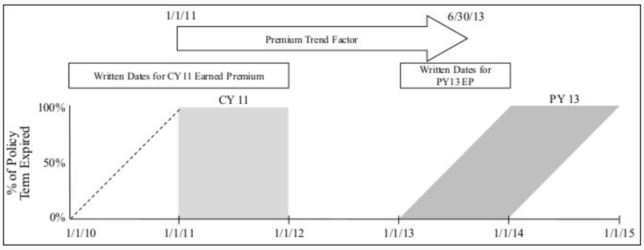


Note that we are looking at average **written** date of the earned premium in calendar year 2011, and this number are calculated from premium written in calendar year 2010 and 2011. Thus,

$$\text{Average Written Date for CY2011} = 1/1/2011.$$

The projected period is in calendar year 2013, i.e. policies are written uniformly from Jan 1 to Dec 31, 2013. Thus,

$$\text{Average Written Date for Projected Period} = 6/30/2013.$$



Thus, trending period is 2.5 years.

Example 5.3.6. Refer to Example 5.3.5, determine the trending period by using average earned date.

Solution:

Two-Step Premium Trending

It is not uncommon that company expects premium trend to be changing over time, actuaries might believe that trend factor during the forecast period is different with historical period. **Current premium trend factor** can be calculated by:

$$\text{Curr. Prem. Trend Factor} = \frac{\text{Latest Ave. WP at Curr. Rate Level}}{\text{Hist. Ave. EP at Curr. Rate Level}},$$

which is used to adjust the historical premium to the current trend level.

Example 5.3.7. Using Example 5.3.5 with the following extra information:

- Average earned premium for calendar year 2011 is RM740;
- Average written premium for calendar quarter (CQ) 4Q2011 is RM753;
- Assume that the projected annual premium trend is of -1%;
- On-level earned premium for calendar year 2011 is RM1,440.

We can determine the current premium trend factor by taking the ratio of average written premium of 4Q2011 to average earned premium of calendar year 2011:

$$\text{Current Premium Trend Factor} = \frac{753}{740} = 1.0176.$$

Then the length of historical period is the average written date for the EP of RM740 to average written date for the WP of RM753:

$$\begin{aligned} \text{“Trend from” date} &= 1/1/2011 \\ \text{“Trend to” date} &= 11/15/2011 \\ \text{Length of period} &= 0.875 \text{ years.} \end{aligned}$$

Next, for forecast period, we have:

$$\begin{aligned} \text{“Project from” date} &= 11/15/2011 \\ \text{“Project to” date} &= 6/30/2013 \\ \text{Length of period} &= 1.625 \text{ years.} \end{aligned}$$

Note that the *current premium trend factor* is the total trend factor for a period of 0.875 years and *forecast premium trend factor* is annual, thus

$$\begin{aligned} \textbf{Total trend} &= 1.0176 \times (1 - 0.01)^{1.625} = 1.0011 \\ \text{Projected premium at current rate level} &= 1,440 \times 1.0011 = 1,441.58. \end{aligned}$$

Remark: We can also compute the annualized current trend factor before *total trend* is calculated, which is equal to $1.0176^{\frac{1}{0.875}} = 1.0201$. By doing so, we can actually compare this number with other analysis and hoose the current trend factor to be 2%.

Example 5.3.8. Given the following information:

- The experience period is 2014 to 2016;
- Effective date for rate changes is 1/1/2020;
- Rates are reviewed annually;
- All policies are of annual term.

What is the total premium trend factor

1. for each calendar year by using one-step trending method with 3% annual trend?

   Solution:

2. for CY2014 by using two-step trending method with projected annual trend to be 3% and 4Q2016 average written premium is RM860, calendar year 2014 average earned premium is RM820?

   Solution:

Chapter 6

Ratemaking

Ratemaking is the actuarial process of determining insurance premium. As ratemaking is a complicated process that "cost of good sold" is not known at the time products are being priced, there are certain risks in pricing, for example:

1. Underwriting Risk;
2. Pricing Risk;
3. Product Design Risk.

Ratemaking is a complicated process that involves numerous considerations, for example: marketing, competition, legal and regulatory consideration. There are four principles[1] for ratemaking:

1. A rate is an estimate of the expected value of future costs;
2. A rate provides for all costs associated with the transfer of risk;
3. A rate provides for the costs associated with an individual risk transfer;
4. A rate is reasonable and not excessive, inadequate, or unfairly discriminatory if it is an actuarially sound estimate of the expected value of all future cots associated with an individual risk transfer.

Recall that the fundamental insurance equation is:

$$\text{Premium} = \text{Loss} + \text{LAE} + \text{UW Expenses} + \text{UW Profit}.$$

Notation for premium components above:

1. The average expected loss and LAE = $\overline{L}$ and $\overline{E}_L$
2. Fixed expense = $\overline{E}_F$
3. Variable expense that is expressed as a percentage of premium = V
4. Profit margin = Q_T

Rewriting the fundamental insurance equation, we have:

$$P = \overline{L} + \overline{E}_L + \left(\overline{E}_F + V \times P\right) + Q_T \times P. \tag{6.1}$$

6.1 Expenses and Profit

Before we introduce how expenses can be estimated, we recall the following ratios that are frequently used in ratemaking process:

- Ultimate Loss Ratio;

[1] See: "Statement of Principles Regarding Property and Casualty Insurance Ratemaking"

- Frequency and Severity;
- Pure Premium;
- ULAE is usually shown as a percentage of total claims.
- Variable Permissible Loss Ratio (VPLR):
  - VPLR $= 1 -$ Variable Expense % $-$ Target Profit % $= 1 - V - Q_T$;
  - Portion of each dollar of premium to be spent on Projected Loss, LAE & Fixed Expense.
- Total Permissible Loss Ratio (TPLR):
  - TPLR $= 1 -$ Total Expense % $-$ Target Profit % $= 1 - F - V - Q_T$;
  - Portion of each dollar of premium to be spent on Projected Loss & LAE.