# Tutorial 1 - Complex Numbers

1. Compute the following
   (a) Compute the following:
      i. $(\sqrt{2}-i)-i(1-\sqrt{2}i)$
      ii. $(2,-3)(-2,1)$
   (b) Solve the equation $z^2+z+1=0$.
   (c) $\frac{1+2i}{3-4i}+\frac{2-i}{5i}$
2. Let $z_1, z_2$ be any two nonzero complex numbers, show that
$$(z_1+z_2)^n=\sum_{k=0}^{n}\binom{n}{k}z_1^k z_2^{n-k}, \quad \text{for } n=1,2,3,\ldots.$$
3. Show that $\sqrt{2}|z|\geq|\operatorname{Re}(z)|+|\operatorname{Im}(z)|$.
4. Show that
   (a) $|z_1-z_2|\geq|z_1|-|z_2|$
   (b) $|z_1\pm z_2|\leq||z_1|+|z_2||$
   (c) $|z_1\pm z_2|\geq||z_1|-|z_2||$
   (d) If $|z|=2$ then $\left|\frac{1}{z^4-4z^2+3}\right|\leq\frac{1}{3}$.
5. Show that $z\overline{z}=|z|^2$
6. Show that $|z-z_0|=R$ can be written as $|z|^2-2\operatorname{Re}(z\overline{z_0})+|z_0|^2=R^2$.
7. Find the principal argument $\operatorname{Arg}(z)$ when
   (a) $z=-\frac{i}{2+2i}$
   (b) $z=(\sqrt{3}-i)^6$
8. Show that $\cos 3\theta=\cos^3\theta-3\cos\theta\sin^2\theta$.
9. Find the value $\theta$ such that $|e^{i\theta}-1|=2$.
10. Compute $(1+i)^{2021}$.
11. Compute $\displaystyle\sum_{k=0}^{7}\left(\frac{1-i}{\sqrt{2}}\right)^k$.
12. Solve $16^{1/4}$ in complex plane.
13. Evaluate $i^{0.5}$.

14. Find all the sixth roots of $z = -1 - i\sqrt{3}$ and sketch the roots on $z$-plane.

15. Evaluate $\lim_{n\to\infty} \left( \frac{1}{n^3} + i \right)$.

16. Evaluate $\lim_{n\to\infty} \left( -2 + i\frac{(-1)^n}{n^2} \right)$.

17. Show that $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} z^{2n+1}$ is absolutely convergent for all $z \in \mathbb{C}$.

18. For each of the following, sketch the region and determine whether the set is domain, open/closed/neither, bounded/unbounded:

    (a) $|z - 2 + i| \leq 1$

    (b) $|2z + 3| > 4$

    (c) $\mathrm{Im}(z) > 1$

    (d) $0 \leq \arg(z) \leq \frac{\pi}{4}, z \neq 0$