

STAT 332: Sampling and Experimental Design

Professor Riley Metzger
L^AT_EXer Iris Jiang

Spring 2020

Contents

| | | |
|----------|--|----------|
| 1 | PPDAC | 2 |
| 1.1 | Problem | 2 |
| 1.2 | Plan | 2 |
| 1.3 | Data | 2 |
| 1.4 | Analysis | 2 |
| 1.5 | Conclusion | 2 |
| 1.6 | Errors | 3 |
| 2 | Models | 3 |
| 2.1 | Model I | 3 |
| 2.2 | Independent vs. Dependent Groups | 3 |
| 2.2.1 | Ways of Creating Dependency | 4 |

1 PPDAC

Problem, Plan, Data, Analysis, Conclusion

1.1 Problem

Define the problem:

- Target Population (T.P.): The group of units referred to in the problem step
- Response: The answer provided by the T.P. to the problem
- Attribute: statistic of the response

Example 1.1. What is the average grade of students in STAT 101?

Solution.

- T.P.: All STAT 101 students
- Response: Grade of a STAT 101 student
- Attribute: Average grade

□

1.2 Plan

How?

- Study population (S.P.): The set of units you **can** study

Example 1.2. Problem: Does a drug reduce hair loss

Solution. You can not use untested drug directly on people out of ethical concerns

T.P.: People

S.P: Mice

□

- Sample: A subset of the study population

1.3 Data

Collect the data, according to the plan.

1.4 Analysis

Analyse the data.

1.5 Conclusion

Refers back to the problem.

1.6 Errors

- Study Error: The attribute of the T.P. differs from the parameter of the S.P.

Example 1.3. $a(T.P.) - \mu$

- Sample Error: The parameter differs from the sample statistic (estimate).

Example 1.4. $\mu - \bar{x}$

- Measurement Error: The difference between what we want to calculate and what we do calculate.

2 Models

Definition 2.1 (Model). A model relates a parameter to a response.

2.1 Model I

$$Y_j = \mu + R_j, R_j \sim N(0, \sigma^2)$$

- y_j : The response of unit j , it is random.
- μ : S.P. mean, it is not random and it is unknown
- R_j : The distribution of responses about μ

Note.

1. R_j 's are always independent.
2. Gauss's Theorem: Any Linear combination of normal R.V.s is normal
3. $Y_j \sim N(\mu, \sigma^2)$,

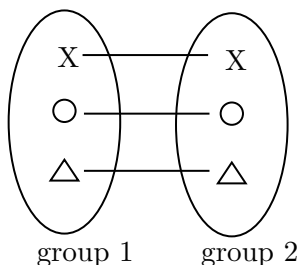
$$E(Y_j) = E(\mu + R_j) = E(\mu) + \mu + 0 = \mu$$

$$V(Y_j) = V(\mu + R_j) = V(R_j) = \sigma^2$$

Example 2.1. Average grade of STAT 101: $Y_j = \mu + R_j, R_j \sim N(0, \sigma^2)$

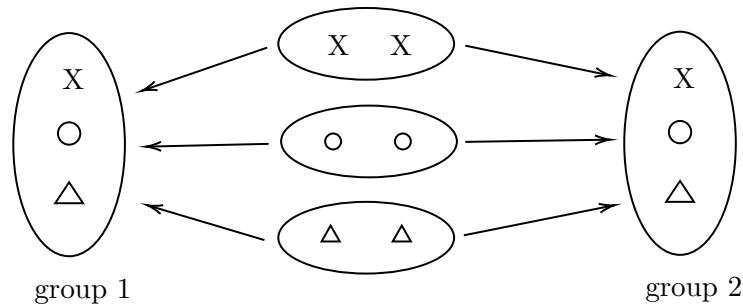
2.2 Independent vs. Dependent Groups

Definition 2.2 (Dependent). We randomly select one group and we find a match, having the same explanatory variates, for each unit of the first group.

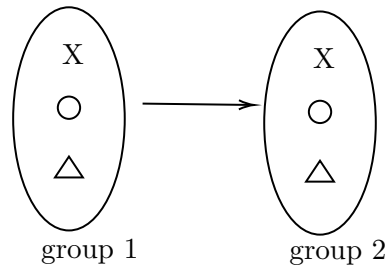


2.2.1 Ways of Creating Dependency

- Twins



- Reuse



Definition 2.3 (Independent). Are formed when we select units at random from mutually exclusive groups.

- No relationship between chosen groups

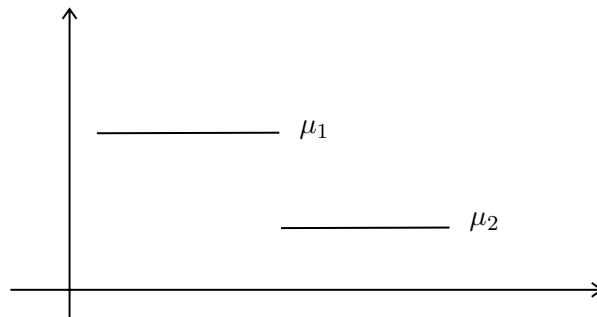
Example 2.2. Broken parts and non-broken parts

2.3 Model 2A

Independent groups where we assume the groups have the same standard deviation.

$$Y_{ij} = \mu_i + R_{ij}, R_{ij} \sim (0, \sigma^2)$$

- Y_{ij} : Response of unit j in group i
- μ_i : Mean for group i ; not random; unknown
- R_{ij} : The distribution of responses about μ_i



2.4 Model 2B

Independent groups but $\sigma_1 \neq \sigma_2$

$$Y_{ij} = \mu_i + R_{ij}, R_{ij} \sim N(0, \sigma_i^2)$$

