CS 476: Numeric Computation for Financial Modeling

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Contents

1. Week 1

Outline

(1) General definition of a financial derivative contract
(2) Standard options
(3) Payoff function

1.1 General Derivative Contracts

Definition 1.1 — Financial options.
A **financial option/derivative** is a financial contract stipulated today at $t = 0$. The value of the contract at the future *expiry* T is determined exactly by the market price of an *underlying asset* at T.

We don't know the value of the underlying asset at T, but we know the contract's value in relation to the underlying asset price ar T.

- The underlying asset can be: stock, commodity, market index, interest rate/bond, exchange rate

Notation 1.1 (S_t)**.** *Let S_t or $S(t)$ denote the underlying price at time t, a stochastic process.*

Holder — S_t, T, Payoff(S_T) — Writer

Knowing the future value of the contract in relation to the underlying allows it to be used as an insurance.

1.1.1 European calls and puts

Definition 1.2 — European call.
A **European call** option is the *right* to *buy* underlying asset at a preset strike price K.

The right can only be exercised at the expiry T.

Asymmetry: holder has the option to exercise. Writer has the obligation

Definition 1.3 — European put.
A **European put** option is the *right* to sell underlying asset at a preset strike price K. The right can only be exercised at the expiry T.

Definition 1.4 — American put.
An American call option is the *right* to *sell* underlying asset at a preset strike price K. The right can be exercised any time from now to the expiry T.

- Holder: Buyer of the option, enters a **long** position
- Writer: Seller of the option, enters a *short* position

Notation 1.2 ($V(S(t), t)$ or V_t). *Let* $V(S(t), t)$ *or* V_t *denote the option value at time* t. $V_T = payoff(S_T)$

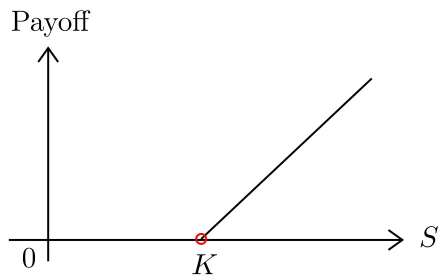
Central Question

- What is the fair value of V_0 of the option today?
- How should a writer hedge risk?

What are the payoff functions $V_T = \text{payoff}(S_T)$ for calls/puts?

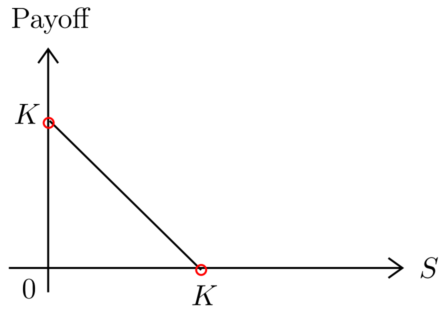
1.1.2 Call value at the expiry T

If $S_T \leq K$, holder should not exercise the call. $V_T = 0$
If $S_T \geq K$, holder exercises the right $V_T = S_T - K$
$\implies V_T = \text{payoff}(S_T) = \max(S_t - K, 0)$



1.1.3 Put value at the expiry T

If $S_T \leq K$, holder should exercise the put. $V_T = K - S_T$
If $S_T \geq K$, holder should not exercises
$\implies V_T = \text{payoff}(S_T) = \max(K - S_t, 0)$



■ Example 1.1
- Bulb wholesaler can purchase a call to have the option of buying tulip \$0.5 a dozen at a fixed price in 3 months.
- Bulb growers can purchase a put to allow selling tulip \$1 a dozen at a fixed price in 3 months.

A bet on the underlying price can be done by trading either S_t or V_t.

Note 1.1 Option is more risky compared to the underlying (leverage effect):

$$\left|\frac{S_T - S_0}{S_O}\right| << \left|\frac{V_T - V_0}{V_0}\right|, \; v_t = 0 \implies \frac{V_T - V_0}{V_0} = -100\%$$

When option expires out of money, 100% loss for the option holder.

We focus on stock option with expiry $T \le 1$ and interest rate randomness is reasonably ignored. (because we will only look into short term contracts in this course)
- Stock: a share in ownership of a company
- Dividend: payment to shareholder from the profits

Note 1.2 When stock pays dividend to the shareholder, holder of option on the stock receives nothing. Option is said to be *dividend protected*.