Macro Factor Mimicking Portfolios [a]

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Abstract

The estimation of risk factors and their replication through mimicking portfolios are of critical importance for academics and practitioners in finance. We propose a general optimization framework to construct macro factor mimicking portfolios that encompasses existing portfolio mimicking approaches, such as two-pass cross-sectional regression models (Fama and MacBeth, 1973) and maximal correlation approaches (Huberman et al., 1987, and Lamont, 2001). We incorporate empirical estimation improvements through machine learning methodologies. We provide an application to the construction of tradable portfolios mimicking three global macro factors, namely growth, inflation surprises, and financial stress indicators. We show how these macro mimicking factors can be used to improve the risk-return profile of a typical endowment multi-asset portfolio.

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1 - Introduction

Factor investing is driving a significant change in the way investors approach asset management. In a broad sense, factors are the fundamental drivers of asset returns (Ang, 2014). Investment returns are rewards that investors harvest for holding assets that expose them to some systematic source of risks. The main advantage of the factor representation lies in the dimensionality reduction of the portfolio construction problem investors face. Providing that the estimated factor model fits reasonably well the variability (variance) of asset returns, investors can focus on a narrower set of instruments (the factors) rather than having to forecast all portfolio components individually. The systematic factor investing approach is not new and has been present in the academic literature for a long-time, as exemplified by the CAPM or the APT models.

From theory to practice, investors need to identify empirically the underlying factors. While a lot of work has been devoted to statistical factors such as Principal Component Analysis (Roll and Ross, 1980; Connor and Korajczyk, 1988) or fundamental factors such as value, size and momentum factors (Fama and French, 1992 and 1993; and Carhart, 1997); macroeconomic factors have been somehow less popular. While it is clear that macroeconomic conditions and regimes are pervasive for the dynamics of financial asset returns (Chen et al., 1986; Ang and Bekaert, 2004), one of the obvious practical limitations of macro factors is that they are not tradable. A standard way to tackle this issue is to try to associate some investible asset classes to macroeconomic indicators, such as growth with equities or inflation with commodities (Greenberg et al., 2016).

While simple and sensible, this approach remains arbitrary as the choice of asset classes aiming to replicate macroeconomic factors is not motivated by a statistical efficiency criterion. A more general solution consists to follow the approach proposed in the asset pricing literature of Factor Mimicking Portfolios (FMPs henceforth) which is a portfolio of assets representation of the underlying non-traded factors.

Our contribution is fivefold. First, we propose a general minimum variance portfolio construction framework that encompasses all the popular factor mimicking portfolio approaches, such as two-pass cross-sectional regressions (Fama and MacBeth, 1973; and Lehman and Modest, 1988) and maximum correlation portfolio approach (Huberman et al., 1987; and Lamont, 2001). One of the benefits of our general construction framework is that it allows investors to easily integrate practical portfolio constraints such as short-sales or liquidity restrictions.

Second, we develop a new machine learning estimation procedure for factor mimicking portfolios. Indeed, one of the major problems with macro factor models are that macroeconomic variables are noisy and imprecise, and that they do not completely span the asset return space. In that perspective, macro factors can be deemed as being "weak" factors, which leads to the well-known errors-in-variables and omitted variables econometric problems that biases FMP weight estimates. To avoid these problems, we adopt the instrumental variables (IV) method proposed by Connor and Korajczyk (1991) and Giglio and Xiu (2019) that is using the first principal components (PCs) of the asset returns as instruments to estimate the betas of the factor models. We improve the Two-Stage Least-Squares IV FMP estimator by introducing several machine-learning technique enhancements. In the first-stage regressions, we use a supervised principal components method rather than a standard PCA to extract the statistical factors. This allows us to take into account the predictive power of asset returns on each targeted macro factor, similarly to Bai and Ng (2008). In the second-stage regression, we also replace the standard OLS regression by the OLS post-LASSO approach. This allows for a better identification of the relevant factors, while circumventing the over-shrinkage limitations of LASSO regressions (Belloni and Chernozhukov, 2013).

Third, we use a set of base assets that are representative of cross-asset portfolios to construct our macro factor mimicking portfolios. To the best of our knowledge, most of previous papers use instead one single asset class (equities or, less frequently, corporate bonds) to construct factor mimicking portfolios. The chosen base assets can be invested through liquid and cost-efficient vehicles such as futures/swaps derivatives or ETFs, leading to straightforward implementations.

Fourth, we compare the ability of various factor-mimicking methodologies to replicate the characteristics of three major global macro factors (growth, inflation surprises and financial stress) over nearly fifty years spanning different economic regimes, both in-sample and out-of-sample. Overall, the results prove the superiority of our new machine learning FMP approach over the usual factor mimicking methodologies.

Fifth, we present a case study where we illustrate how the estimated macro FMPs can be used to hedge the macro risk exposures of an institutional investor portfolio. We find that macro-hedging can improve the investor risk-return profiles significantly.

The remainder of this paper is organized as follows. In Section 2, we introduce the general macro FMP analytical framework. In Section 3, we discuss the estimation of the macro FMPs in the presence of measurement errors and omitted factors. In Section 4, we detail the empirical estimation of macro FMPs before illustrating their potential use for hedging a representative institutional portfolio in Section 5. We conclude in Section 6.

2 – Factor Mimicking Portfolio Analytical Framework

Factor Mimicking Portfolios (FMPs) have been the subject of a vast academic literature (see Balduzzi and Robotti, 2008). In general terms, they consist in forming portfolios of investable assets (so-called "base assets") that replicate ("mimick") the behavior of one or several (potentially non-investable) factors. In the asset pricing literature, two main approaches have been proposed. Both approaches have in common to rely, for a set of N assets, on factor models of the type:

$$\boldsymbol{R}_t = \boldsymbol{\mu} + \boldsymbol{B}\boldsymbol{F}_{\boldsymbol{t+h}} + \boldsymbol{\varepsilon}_t \tag{1}$$

where $\boldsymbol{R}_t$ and $\boldsymbol{\mu}$ are the $(N \times 1)$ vectors of base asset returns and associated expected returns, respectively; $\boldsymbol{F}_{t+h}$ is the set of K zero-mean factors that we want to replicate through FMPs[1]; $\boldsymbol{B}$ is the $(N \times K)$ matrix of asset loadings to factors; and $\boldsymbol{\varepsilon}_t$ is the $(N \times 1)$ vector of mean-zero disturbances.

On the basis of the factor model (1), the first popular FMP approach consists in two-pass cross-sectional regressions (CSR), following the seminal work of Fama and MacBeth (1973). In the first-step, the asset loadings of the factor model (1) are estimated via time-series multivariate regressions

[1] The horizon h will depend on applications. For macro factors, h will tend to be superior to 0 as financial assets tend to anticipate the behavior of macro variables.

of base asset's returns onto the factors. In the second step, the risk premium of each factor is obtained by regressing cross-sectionally the returns of the base assets onto the estimated betas. While the primary objective of CSR is to assess whether factors are "priced", it is well-known (Fama, 1976) that each coefficient obtained in the second regression is equivalent to the return of a portfolio replicating the factors considered in the first regression. The specificity of these K factor mimicking portfolios is that they have unit-beta (i.e. unit exposure) to the factor they aim to replicate and zero-beta to the other factors. In Fama-MacBeth approach, the second-pass cross-sectional regression is performed through Ordinary Least Squares (OLS), meaning that the base asset returns are equally weighted. To deal with heteroskedasticity and cross-correlations of residuals in (1), several authors have recommended the use of different weighting schemes such as Weighted Least Squares (WLS; Litzenberger and Ramaswany, 1979) or Generalized Least Squares (GLS; Lehman and Modest, 1988).

The second FMP approach, pioneered by Huberman et al. (1987) and Breeden et al. (1989), aims to maximize the (squared) correlation (MCP) between each FMP and the associated factor of interest. Each FMP is then determined individually, meaning that the asset loadings of the factor model (1) are estimated separately for each factor through univariate regressions. Once the univariate asset loadings to the factors have been estimated, the individual FMPs are then determined as the solutions of a minimum variance program respecting specific univariate beta-target constraints. A related approach, denoted the Economic Tracking Portfolio (ETP), has been proposed by Lamont (2001). While the starting point is different, as the individual FMPs are obtained in this case by projecting each factor onto the full set of base assets, it can be shown that the ETP's weights are proportional to the MCP's ones, with the coefficient of proportionality given by the coefficient of determination of the multivariate regression of each factor on the base asset returns.

In this paper, we elaborate on the work of Huberman et al. (1987) who have shown that any factor mimicking portfolio can be retrieved as solutions of a specific portfolio optimization problem. For a given factor, we suggest obtaining FMP as the solution to the following general constrained variance-minimization problem:

$$\text{Min}\,\frac{1}{2}\boldsymbol{w}_k^{\boldsymbol{T}}\,\boldsymbol{\Omega}\,\boldsymbol{w}_k \tag{2}$$

$$s.t.\,\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{w}_k = \boldsymbol{\beta}_k$$

where $\boldsymbol{\Omega}$ is the $(N \times N)$ variance-covariance matrix of base assets and $\boldsymbol{\beta}_k$ is the $(K \times 1)$ vector of risk exposures associated with the mimicking portfolio on the k-th factor. By convention, we assume that each FMP has unit-beta exposure to the factor it aims to replicate and pre-specified exposures to other factors; meaning that the vector $\boldsymbol{\beta}_k$ has 1 in k-th entry and β_{kl} for $l \neq k$.

As there are generally more investable assets than factors $(N \gg K)$, there are an infinite number of portfolios that can replicate the desired target factor exposures. Among all those portfolios, program (2) states that the FMP portfolio must be the one that maximizes the explanatory power of the factor model representation (1). Indeed, constraining factor exposures and minimizing variance simultaneously is equivalent to minimize the portfolio's idiosyncratic risk[2]. In general

[2] As shown in Appendix 2, substituting the residual risk matrix $\boldsymbol{\Omega}_\varepsilon$ for the total risk matrix $\boldsymbol{\Omega}$ in the objective function (2) leads to the same FMP solution.

terms, this means that FMPs are well-diversified portfolios that target some specific risk exposures with respect to a set of factors of interest.

Solving program (2), the portfolio weights for the full set of the FMPs are given by the column vectors of the $(N \times K)$ weighting matrix (see Appendix 1):

$$\boldsymbol{W}_K = \Omega^{-\mathbf{1}}\boldsymbol{B}\left(\boldsymbol{B}^{\boldsymbol{T}}\Omega^{-\mathbf{1}}\boldsymbol{B}\right)^{-\mathbf{1}}\boldsymbol{B}_{\boldsymbol{K}} \tag{3}$$

where $\boldsymbol{B}_{\boldsymbol{K}}$ is the $(K \times K)$ matrix collecting the target exposures for the various FMPs, with $\boldsymbol{B}_{\boldsymbol{K}} = (\boldsymbol{\beta}_1\boldsymbol{\beta}_2 \ldots \boldsymbol{\beta}_K)$. Given FMP weights, the returns of the FMPs are obtained as:

$$\boldsymbol{FMP}_t = \boldsymbol{W}_K^{\boldsymbol{T}}\boldsymbol{R}_t \tag{4}$$

where $\boldsymbol{FMP}_t$ is the $(K \times 1)$ vector of FMP returns at time t. Note that the FMP factor exposures are by construction equal to the targeted ones, i.e. $\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{W}_K = \boldsymbol{B}_{\boldsymbol{K}}$. Note however that covariances between the FMPs returns are generally different from the covariance between the factors being mimicked, since $\boldsymbol{Cov}\left(\boldsymbol{FMP}_t, \boldsymbol{FMP}_t\right) = \boldsymbol{B}_{\boldsymbol{K}}{}^{\boldsymbol{T}}\left(\boldsymbol{B}^{\boldsymbol{T}}\Omega^{-\mathbf{1}}\boldsymbol{B}\right)^{-\mathbf{1}}\boldsymbol{B}_{\boldsymbol{K}}$.

In Table 1, we show how to retrieve the main FMP approaches as special cases of the general formula (3). In particular, we can see that all these methods can be distinguished by three types of specifications:

(i) the variance-covariance matrix of base assets $\boldsymbol{\Omega}$ that can be identity (OLS-CSR; Fama-McBeth), diagonal (WLS-CSR) or full (GLS-CSR or all MCP);

(ii) the factor-model that can be univariate (all MCP) or multivariate (all CSR);

(iii) the target exposure constraint matrix that be identity (all CSR) or general (all MCP).

Three remarks are worth mentioning regarding our general framework. Firstly, we note that the use of a variance minimization program to obtain FMP portfolios is not new. To the best of our knowledge, Huberman et al. (1987) are the first to have advocated such approach. More recently, this has also been considered by Melas et al. (2010), Roll and Srivastava (2018) or Pukthuangthong et al. (2019a). However, these authors always relate it to some specific FMP cases, while we show here that we can cast all the proposed FMP methodologies into one single generic formula (3).

Secondly, in the program (2) or in the applications below, we do not impose any additional portfolio constraints other than target exposures $\boldsymbol{\beta}_k$. In particular, by default, FMPs are long-short portfolios. It is straightforward to add in the portfolio optimization program (2) various forms of constraints. For instance, the incorporation of long-only constraints, minimum or maximum per base asset, or even the addition of scores (based on liquidity, alpha etc…) to weight base assets are feasible as long as these constraints respect the convex nature of the optimization program. Obviously, in most of these cases, analytical solutions such as (3) are not available anymore.

Thirdly, while in the analytical expressions above or in our empirical applications, we are considering only static betas (e.g. FMP weights), our general framework can easily accommodate dynamic asset loadings through the use of lagged macroeconomic or asset specific instrumental variables as in Ferson and Harvey (1991).

Table 1. Factor Mimicking Portfolio Construction Methodologies

Methodology	Properties	Specifications	Portfolio Weights W_K
Two-Pass Cross-Sectional Regression Approaches (CSR)			
OLS CSR (Fama and Macbeth, 1973)	Minimum variance portfolio with unit beta to factor of interest and zero beta to other factors	$\boldsymbol{\Omega} = \sigma \boldsymbol{I}_N$: Uncorrelated assets with constant variance $\boldsymbol{B}$: Multivariate $\boldsymbol{B_K} = \boldsymbol{I_K}$ (identity matrix)	$\boldsymbol{B}\left(\boldsymbol{B^T B}\right)^{-1}$
WLS CSR (Litzenberger and Ramaswany, 1979)	Idem	$\boldsymbol{\Omega} = Diag(\boldsymbol{\sigma}^2)$: Uncorrelated assets $\boldsymbol{B}$: Multivariate $\boldsymbol{B}_K = \boldsymbol{I}_K$(identity matrix)	$Diag(\boldsymbol{\sigma}^{-2})\boldsymbol{B}\left(\boldsymbol{B^T} Diag(\boldsymbol{\sigma}^{-2})\boldsymbol{B}\right)^{-1}$
GLS CSR (Lehman and Modest, 1988)	Idem	$\boldsymbol{\Omega}$: Unconstrained $\boldsymbol{B}$: Multivariate $\boldsymbol{B}_K = \boldsymbol{I}_K$ (identity matrix)	$\boldsymbol{\Omega^{-1} B}\left(\boldsymbol{B^T \Omega^{-1} B}\right)^{-1}$
Maximum Correlation Approaches (MCP)			
MCP (Huberman et al., 1987, and Breeden et al., 1989)	Minimum variance portfolio with pre-specified beta to factor of interest and associated non-null beta to other factors	$\boldsymbol{\Omega}$: Unconstrained $\boldsymbol{B}$: Univariate $\boldsymbol{B_K} = \boldsymbol{B^T \Omega^{-1}} \mathbf{B}$ (Target matrix)	$\boldsymbol{\Omega^{-1} B}$
MCP Unit Beta (Grinold and Kahn, 2000)	Minimum variance portfolio with unit beta to factor of interest and associated non-null beta to other factors	$\boldsymbol{\Omega}$: Unconstrained $\boldsymbol{B}$: Univariate $\boldsymbol{B}_K = \left(\boldsymbol{B^T \Omega^{-1} B}\right) Diag\left(\boldsymbol{B^T \Omega^{-1} B}\right)^{-1}$ (Target matrix)	$\boldsymbol{\Omega^{-1} B}\, Diag\left(\boldsymbol{B^T \Omega^{-1} B}\right)^{-1}$
ETP (Lamont, 2001)	Minimum variance portfolio with univariate beta to factor of interest equal to the R^2 of the factor projection on base assets and associated non-null univariate beta to other factors	$\boldsymbol{\Omega}$: Unconstrained $\boldsymbol{B}$: Univariate $\boldsymbol{B}_K = \left(\boldsymbol{B^T \Omega^{-1} B}\right) Diag(\boldsymbol{\Omega}_K)^{-1}$ (Target matrix)	$\boldsymbol{\Omega}^{-1}\mathbf{Cov}(\boldsymbol{R}, \boldsymbol{F})$

Notes. The Table summarizes the various specifications of the FMPs. $\boldsymbol{\Omega}$ is the variance-covariance matrix of base assets, $\boldsymbol{\Omega}_K$ is the variance-covariance matrix of factors, $\boldsymbol{B}$ is the matrix of betas of base assets to macro factors, and $\boldsymbol{B}_K$ is the matrix of target betas.

3 – Machine-Learning Factor Mimicking Portfolios Estimates

In theory, expression (3) provides the Best Linear Unbiased estimator of the FMPs. However, in practice, factor risk exposures are estimated imprecisely due to the presence of measurement errors for the selected factors or the omission of additional relevant factors. These problems are especially important for macro factors as economic variables are measured imprecisely and even difficult to define[3]. This induces the well-known errors-in-variables problem that biases the traditional OLS-FMP weight estimates.

In order to adjust for this bias, we follow the instrumental variables (IV) estimation approach proposed by Connor and Korajczyk (1991) and Giglio and Xiu (2019) that is using the first principal components (PCs) of the asset returns as instruments to estimate the factor loadings in (1). This IV procedure works as follows. In the first step, the observed real factors $(\boldsymbol{F_{t+h}})$ are projected onto the L first principal components PCs of asset returns $(\boldsymbol{F}_{t+h}^{PC})$, with $L \leq N,$ to obtain the fitted values of the factors $(\widehat{\boldsymbol{F}}_{t+h})$. In the second step, the base asset returns are regressed on the fitted values of the factors $\widehat{\boldsymbol{F}}_{t+h}$ to obtain the asset factor loading IV estimates $\boldsymbol{B}^{PCIV}$.

Contrary to standard OLS estimates, this Two-Stage Least-Squares IV regression procedure provides an unbiased estimator for the factor loadings, since the first PCs can recover the true factor space of asset returns subject to a rotational indeterminacy, as shown by Bai and Ng (2002).

[3] See Ghysels et al. (2018) for an analysis of the impact of revisions on macroeconomic statistics. More generally, following Stock and Watson (2002), a vast literature analyzes the issue of the measurement of the business cycle. The nowcasting approach presented in Beber et al. (2015) defines an alternative way to measure business cycle. The diversity of measures or the recurrent debates between economists of how to measure GDP or define inflation illustrate how difficult it can be to measure economic variables.

Intuitively, because base asset returns vary altogether with the latent risk factors, one can use the information contained in individual asset returns to recover the true factor structure and improve the FMP estimates by removing the measurement errors from the observed factors[4].

We then extend the Two-Stage Least-Squares FMP estimator (5) by introducing several machine-learning technique enhancements. In the first-stage regressions, we replace the traditional unsupervised PCA methodology by a supervised statistical approach that incorporates explicitly the predictive capabilities of base assets to each macro factor when extracting the statistical factors In the machine learning literature, there are at least three statistical extraction methods that utilize the predictive power of the regressors. The first one is to use Partial Least squares (PLS) approach to extract the statistical factors, as shown recently by Kelly and Pruitt (2015). The second one, named scale PCA (sPCA; Huang et al., 2018), extracts the PCs from a modified set of base asset returns that are scaled according to their individual predictive power for the considered factor. The third one is the target PCA (tPCA; Bair et al., 2006, and Bai and Ng, 2008) which extracts the PCs from a subset selection of base assets that have strong forecasting power for the target factor by using hard or soft thresholding rules. We follow this last approach by using a soft-thresholding LASSO penalty rule in order to estimate the fitted values of the observed factors ($\widehat{\boldsymbol{F}}_{t+h}$). The soft-thresholding rule consists in selecting the LASSO shrinkage parameter which minimizes the Bayes Information Criterion. In the second-stage regression, we also replace standard OLS regressions by OLS post-LASSO regressions. This allows for a better identification of the relevant factors

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[4] A related IV estimator for FMP weights consists in projecting the observable factors directly onto all the base assets and then use the fitted values from the regressions as instruments. However, this IV estimator becomes increasingly inefficient when the number of assets increases, or even infeasible when the number of assets becomes larger than the number of observations.

through the LASSO regularization while circumventing usual over-shrinkage limitations of LASSO regressions (Belloni and Chernozhukov, 2013).

The machine-learning factor mimicking portfolios are then obtained by replacing the OLS betas by the machine-learning beta estimates into the OLS-CSR formula (ML-CSR).

4 – Empirical FMP of macro factors

As for any FMP, building macro factor mimicking portfolios requires a set of base assets. In our case, these base assets need not only to be investable but also to be available over a long history as our empirical setting starts in the early 70's. Table 2 lists the nine selected base assets. They cover major asset classes such as equities, government bonds, credit, inflation-linked bonds[5], commodities and foreign exchange. For base assets that are not spread of indices, returns are computed as excess returns over cash rates (USD Libor 1 month). For most investors, they can be invested through liquid and cost-efficient vehicles such as derivatives (futures or swaps) or ETFs.

[5] U.S. Treasury inflation-linked bonds (TIPS) have been launched only in 1997. To replicate the return of TIPS before that date, we follow the methodology developed by Swinkels (2018) based on real rates. Consistently with the author, we obtain a correlation of 0.6 between the simulated returns and observed returns over the common sample post-1997.

Table 2 – Definition of base assets

Base asset	Acronym	Indices used
Equities	WEQ	MSCI World in USD
Treasuries (nominal)	GLT	Bloomberg Barclays U.S. Treasury
Credit	CRE	Bloomberg Barclays U.S. Credit Baa index vs Aaa index
Inflation-Linked Bonds	ILB	From April 1997 onwards, Bloomberg Barclays U.S. TIPS vs US Treasury All maturities. Before that date, spread return based on estimated real yields changes.
Gold	GOLD	Gold
Industrial Metals	INM	From February 1977 onwards, S&P GSCI Industrial Metals. Before that date, equally-weighted basket made of Aluminum, Copper, Lead, and Zinc
Energy commodity	ENG	From February 1983 onwards, S&P GSCI Energy. Before that date, equally-weighted basket made of Crude Oil and Natural Gas
U.S. Dollar	DXY	US dollar trade-weighted index
Commodity vs safe heaven currencies	FXCS	Spread of returns (against USD) between commodity currencies (equally-weighted basket made of Canadian dollar, Norwegian krona, Australian dollar) vs safe-heaven currencies (equally-weighted basket made of Japanese yen, Swiss franc)

Notes. Sources for data are Bloomberg and World Bank.

Regarding macro factors, we follow Ang (2014) and consider three global macro factors: growth, inflation surprises and financial stress[6]. Following the literature, we rely on innovations to macro variables rather than their levels. Growth is measured through the OECD Composite Leading Indicator (CLI), which is a popular predictive measure for global economic activity, as it focuses on indicators able to capture turning points in global business cycle. Furthermore, the indicator removes the effect of long-term trends, providing a measure of innovations in future growth for a large part of the world economy. Inflation surprises are proxied by the six-month ahead difference

[6] See also the discussion in Section IV by Pukthuanthong et al. (2019b).

between the realized and lagged past year OECD year-on-year inflation rate, hence assuming sticky expectations. Finally, for financial stress, we use an equal-weight combination of two popular indicators, the Chicago Fed's National Financial Conditions Index (NFCI) and the turbulence index developed by Chow et al. (1999)[7].

To facilitate their interpretation, the growth, inflation surprises and financial stress indicators are first normalized to z-scores by substracting their full-sample average from each observation and dividing by their full-sample standard deviations. We then scale these indicators to a 1% monthly volatility. The sample period spans from January 1974 to June 2018 (534 monthly observations). These near-50-years data window covers different business cycles with various macroeconomic events such as the oil shock of the 70's or several economic recessions and well-known episodes of market crashes.

To construct macro FMPs, we first need to estimate factor models linking base asset returns to the macro factors we want to mimick. For this, we use returns over 12-month rolling samples in order to account for the fact that financial assets can react to economic events ahead of their advent. In total, we regress the base assets excess returns from t-12 to t on the macro variables observed in month t.

[7] The NFCI index is a weighted average of a large spectrum of individual measures of financial conditions, mixing measures of liquidity and volatility in financial markets (such as VIX, swaptions or TED spread) with commercial banking conditions (such as FRB Senior Loan Officer Survey or Consumer Credit Outstanding) or health of financial institutions. The turbulence indicator is computed as the Mahalanobis distance metric applied to the monthly returns of the base asset listed in Table 1, and smoothed using a six-months rolling windows.

Table 3 reports the results of the OLS estimates of these multivariate regressions over the full sample (534 observations). T-statistics are corrected for the effect of overlapping due to the rolling windows through Newey-West estimates with a lag of twelve. All regressions are statistically significant, with R^2 ranging from 4% to 33%. The set of base assets are reacting very differently to the various macro factors. As emphasized by Lehman and Modest (1988), it is critical that the base assets used to form factor mimicking portfolios display sufficient dispersion in their factor loadings to replicate them efficiently. Higher growth benefits to equities, credit, and industrial metals and impacts negatively nominal bonds performance. Inflation surprises are beneficial to inflation-linked bonds versus nominal bonds, gold and energy and negatively impact the U.S. dollar. Financial stress negatively impacts growth-assets (equities, credit) and benefits to gold, energy and U.S. dollar.

Table 3 – OLS regressions of base asset returns on macro factors

	Growth	Inflation Surprises	Financial Stress	Adjusted R²
	t-stat	t-stat	t-stat	p-value
WEQ	**0.68**	-0.02	**-0.29**	32.3%
	5.72	-0.10	**-2.49**	
GLT	**-0.14**	-0.05	0.02	10.7%
	-2.50	-1.12	0.37	
CRE	**0.09**	-0.01	**-0.12**	16.5%
	1.89	-0.15	**-2.71**	
ILB	0.15	**0.26**	-0.04	17.1%
	1.03	**2.33**	-0.29	
GOLD	0.07	**0.92**	**0.76**	29.5%
	0.33	**5.12**	**2.13**	
INM	**1.40**	0.30	0.46	25.3%
	2.81	0.55	1.35	
ENG	0.34	**1.40**	**1.79**	22.5%
	0.46	**1.92**	**2.24**	
DXY	0.01	**-0.26**	0.11	11.8%
	0.15	**-3.61**	1.27	
FXCS	0.19	0.00	0.02	4.5%
	1.35	0.03	0.14	

Notes: The table represents individual monthly multivariate OLS regressions of base assets past yearly returns over realized macro factors. For each base asset, large-font numbers represent beta estimates and adjusted R^2, while small-font numbers represent t-stat and Fisher statistic p-value. T-stats are based on heteroscedasticity and autocorrelation consistent Newey-West standard errors with twelve lags.

As explained in Section 3, poorly estimated factor models can undermine the quality of the macro factor portfolios. This is particularly relevant for macro factor models since economic variables are inherently noisy.

To correct for the errors-in-variables econometric problem, we use the machine-learning factor model estimation framework described in the previous section. For each macro factor, we first extract the latent factors by performing a target PCA where the relevant individual asset components are selected by a LASSO regression (so-called supervised PCA)[8] and the tPCA-fitted macro variables estimated accordingly. Finally, for each base asset, we regress the excess returns on these tPCA-fitted macro variables. The time series regression is an OLS post-LASSO that corrects for the underestimation of true estimates in the LASSO regression (Belloni and Chernozhukov, 2013).

The regression results of the machine learning factor model are displayed in Table 4. We observe that the explanatory power of the machine learning factor model is much larger than the equivalent OLS model in Table 3, with R^2 doubled or more for each base asset. We also note that most of the PC-fitted macro factors are retained, despite the application of the selection process inherent in the LASSO model. This confirms the importance of the considered macro factors for the base asset returns.

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[8] The LASSO smoothing parameter is here automatically selected according to BIC information criteria.

Table 4 – Machine learning macro factor models

	Growth	**Inflation Surprises**	**Financial Stress**	**Adjusted R^2**
	t-stat	t-stat	t-stat	p-value
WEQ	**1.31**		**-0.46**	60.3%
	7.06		**-2.22**	
GLT	**-0.27**	**-0.15**		22.6%
	-3.89	**-2.02**		
CRE	0.07	0.09	**-0.31**	33.2%
	0.84	0.72	**-2.71**	
ILB	-0.25	**1.31**	**-0.50**	55.6%
	-1.57	**6.18**	**-3.19**	
GOLD	**-0.54**	**3.40**	**0.99**	80.6%
	-2.27	**10.54**	**2.43**	
INM	**3.75**		**2.11**	60.0%
	8.27		**6.27**	
ENG	0.84	**4.01**	**3.86**	56.3%
	1.07	**3.94**	**2.95**	
DXY	**0.79**	**-1.52**	**0.87**	60.5%
	6.14	**-11.45**	**6.55**	
FXCS	**0.36**			9.9%
	2.69			

Notes: The table represents individual multivariate OLS post-LASSO regressions of base assets over target PCA-fitted macro factors. Sample period is January 1974 to June 2018. For each base asset, large-font numbers represent beta estimates and adjusted R^2, while small-font numbers represent t-stat and Fisher statistic p-value. T-stats are based on heteroscedasticity and autocorrelation consistent Newey-West standard errors with twelve lags.

Equipped with factor loading estimates, one can then build macro FMPs based on the various methodologies presented above[9]. We consider five specific FMP candidates: (i) the OLS-CSR Fama-McBeth model; (ii) the FMP with GLS correction to estimate the variance-covariance matrix of factor model residuals; (iii) the equivalent model with WLS correction; (iv) the unit-beta MCP; (v) the ML-CSR where we use machine learning beta estimates in the OLS-CSR Fama-McBeth model.

In Figure 1, we present the composition of these mimicking portfolios for the three global macro factors. Some base assets display consistent exposures across the different FMP approaches. Growth FMPs are long equities, industrial metals and inflation-linked bonds and are short nominal treasuries. Inflation surprises FMPs are short nominal treasuries and U.S. dollar, and long equities, inflation-linked bonds, credit and gold. Financial stress FMPs are long nominal treasuries, energy and U.S. dollar, and short equities and credit. Other capital exposures vary according to the FMP approach considered.

In Table 5, we present different characteristics of these portfolios. Through leverage (measured as sum of absolute positions), we observe that ML FMPs are much less extreme, for all macro factors. As leverage is often associated to risk or limited for some investors, this is a key advantage. At the opposite, FMP methods that correct for the variance-covariance matrix of assets, such as WLS-CSR or GLS-CSR, lead to highly leveraged portfolios. The (full sample) volatility of the portfolios varies significantly across specifications. Here again, the ML-CSR approach leads to more

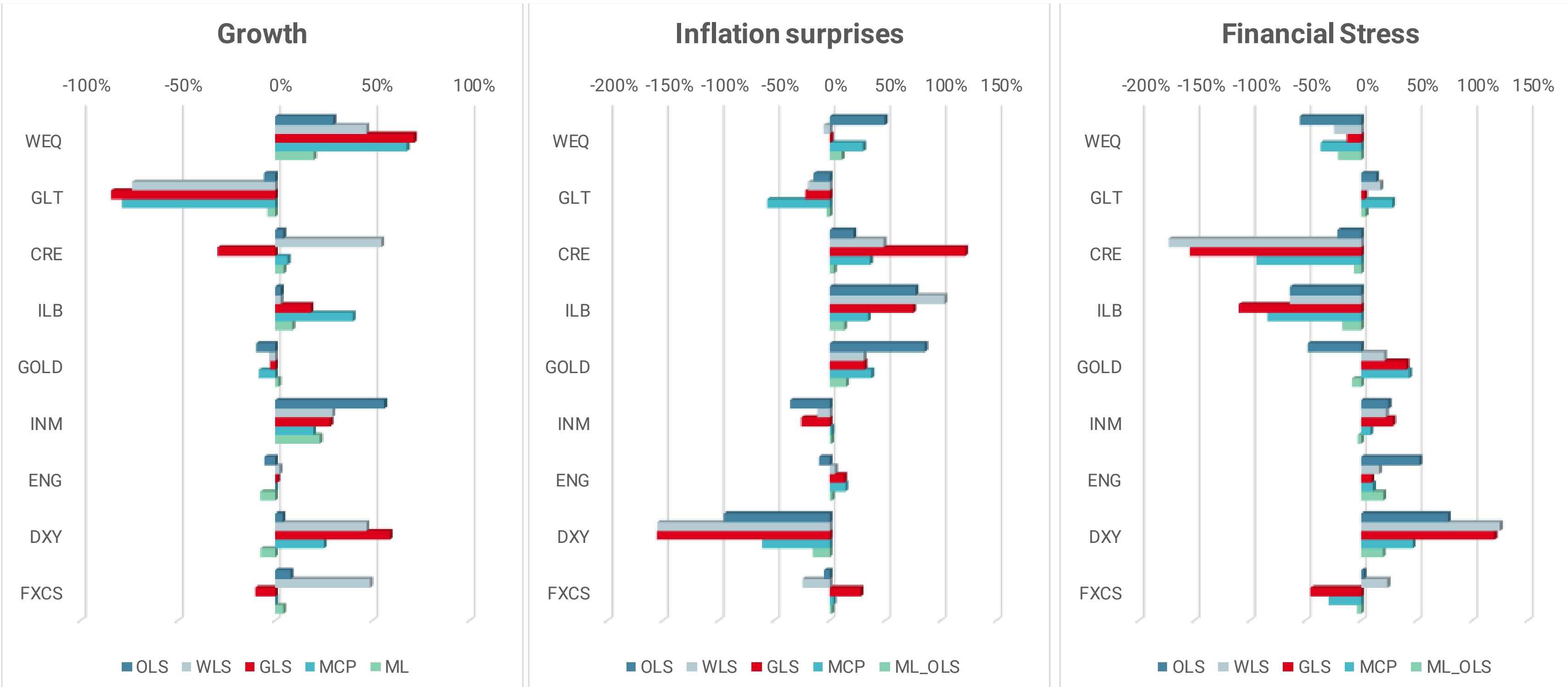
[9] To build these macro FMPs, we use monthly excess returns of base assets. These returns are consistent with returns an investor would get in unfunded instruments (e.g. futures, swaps,…) replicating associated indices.

reasonable numbers but also closer to the original volatility of the macro factors (set at 1% per month by construction). We then display a set of statistics aiming at evaluating the goodness of fit of the FMPs relatively to macro factors over the full sample, and more precisely the correlation of FMPs with their underlying factors, RMSE (Root Mean Square Errors) and MAE (Mean Absolute Error). While correlation metrics do not lead to a clear hierarchy, measures of average errors consistently point to the superiority in-sample of the ML-CSR approach, followed by MCP while OLS-CSR (Fama McBeth) is frequently offering the worst fit.

We also display risk premia (defined as the annualized average excess return associated to each macro FMP) and associated Sharpe ratios. Across different macro factors, OLS-CSR and ML-CSR are the only method to build portfolios with positive returns. Inflation surprises premia are the highest and the most consistent across methodologies. Risk premia associated to growth also tend to be positive, while being frequently negative for financial stress. This might indicate that while financial stress is affecting assets, it is not necessarily rewarded by a positive premia and that investors looking for positive returns might be better-off shorting financial stress FMP than buying it. As we will see in the next section, a typical endowment portfolio seems to be short financial stress while being long growth and inflation surprises.

In Figure 2, we represent the time-series evolution of the realized returns of the growth, inflation surprises and financial stress ML-CSR FMPs, jointly with the respective underlying macro factors. We use one-year moving average of realized machine learning FMPs to reduce the inherent noise in monthly returns. At such horizon, ML-CSR FMPs seems to track even better the evolution of the macro factors while short-term (monthly) macro factors will be more noisy.

Figure 1 – FMP composition as estimated by various methods



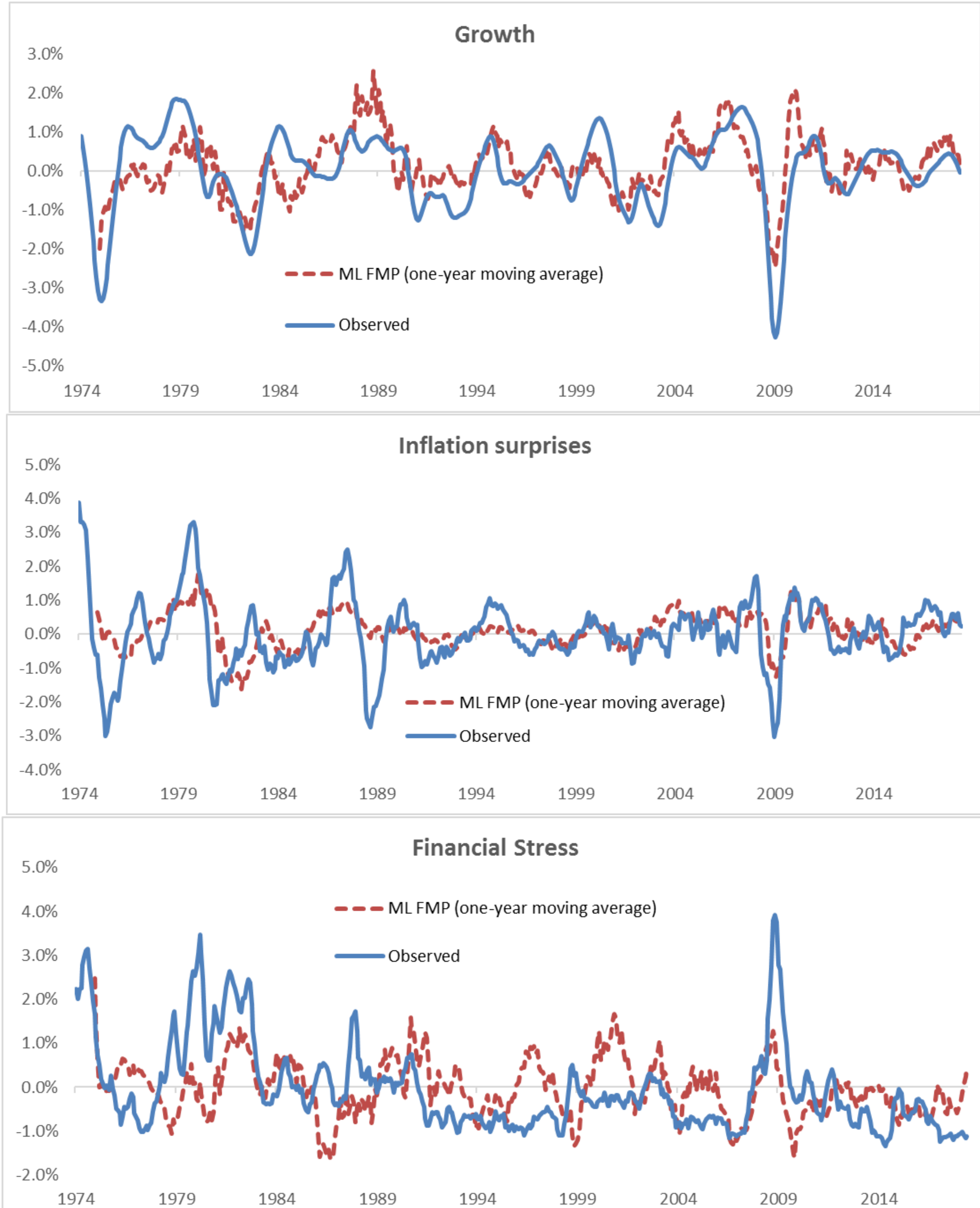
Notes: The figure represents the weights of the macro FMPs estimated through five methodologies: OLS-CSR, WLS-CSR, GLS-CSR, Unit Beta MCP, and ML-CSR. Sample period is January 1974 to June 2018.

Table 5 – In-sample characteristics of FMPs

	Growth					Inflation Surprises					Financial Stress				
Method:	**OLS**	**WLS**	**GLS**	**MCP**	**ML**	**OLS**	**WLS**	**GLS**	**MCP**	**ML**	**OLS**	**WLS**	**GLS**	**MCP**	**ML**
Leverage	126%	309%	305%	246%	81%	394%	403%	473%	275%	66%	360%	487%	524%	381%	102%
Volatility	4.2%	4.3%	3.9%	3.7%	2.1%	6.5%	5.3%	5.4%	4.5%	1.4%	5.9%	4.2%	4.2%	3.6%	2.0%
Correlation	0.14	0.11	0.13	0.12	0.10	0.20	0.23	0.21	0.26	0.26	0.04	0.10	0.14	0.23	0.10
RMSE	4.20%	4.31%	3.89%	3.73%	2.26%	6.38%	5.18%	5.33%	4.33%	1.51%	5.93%	4.25%	4.19%	3.51%	2.15%
MAE	3.09%	3.21%	2.95%	2.74%	1.67%	4.82%	4.00%	4.01%	3.19%	1.12%	4.55%	3.21%	3.14%	2.59%	1.63%
Risk premia	2.46%	2.48%	-1.84%	-0.54%	1.20%	3.63%	3.66%	8.80%	5.04%	1.39%	0.74%	-3.37%	-4.04%	-0.80%	0.13%
Sharpe ratio	0.17	0.17	-0.14	-0.04	0.16	0.16	0.20	0.47	0.33	0.28	0.04	-0.23	-0.28	-0.06	0.02

Notes: The figure represents the characteristics of the macro FMPs estimated through five methodologies: OLS-CSR, WLS-CSR, GLS-CSR, Unit Beta MCP, and ML-CSR. Sample period is January 1974 to June 2018. Leverage is estimated as the sum of absolute weights. Volatility is the full sample (monthly) volatility. Correlation is the correlation between each FMP and the underlying observed macro factor. RMSE is the Root Mean Squared Errors statistics where errors are defined as the difference between each macro FMP and underlying observed macro factor. MAE is the Mean Absolute Error statistics where errors are defined as the difference between each macro FMP and underlying observed macro factor. Risk premia is the annualized average excess return associated to each macro FMP. Sharpe ratio is the ratio between the Risk premia and the (annualized) volatility.

Figure 2 – ML-CSR macro FMPs vs underlying macro factors



Source: OECD, Chicago Fed, Bloomberg. ML-CSR are machine learning macro factor-mimicking portfolios where macro factor models are estimated by the machine-learning factor model presented in Section 3. Macro factors are studentized to a 1% (monthly) volatility.

So far, the results are based on estimation over the full sample. To gauge how methodologies fare on an out-of-sample basis, we run two different types of tests. In the first out-of-sample test, we estimate the FMPs over the first half of the sample (from January 1974 to March 1996) and then analyze the goodness-of-fit metrics over the second half of the sample. In the second test, starting from April 1996, we re-estimate the model every month by expanding the sample and apply the new estimated FMPs to the next month. The results for these two out-of-sample tests are summarized in Table 6.

For almost all methodologies and all macro factors, we observe as expected a deterioration out-of-sample in all the goodness-of-fit metrics. Indeed, some correlations even turn negative, particularly for financial stress, while all RMSEs and MAEs are larger. Looking at the different methodologies, the ML FMP approach seems to be the most robust across the different out-of-sample tests. This confirms the dominance of the methodology already observed over the full sample.

Table 6 – Out-of-sample characteristics of FMPs

	Growth					Inflation Surprises					Financial Stress				
Method:	**OLS**	**WLS**	**GLS**	**MCP**	**ML**	**OLS**	**WLS**	**GLS**	**MCP**	**ML**	**OLS**	**WLS**	**GLS**	**MCP**	**ML**
OOS #1 - Two subperiods															
Correlation	0.03	-0.04	-0.07	-0.09	0.01	0.08	0.18	0.14	0.18	0.21	-0.23	-0.10	-0.13	-0.02	-0.14
Volatility	3.8%	4.7%	5.5%	5.2%	2.4%	5.3%	4.4%	7.2%	5.5%	1.2%	4.4%	3.4%	4.3%	3.5%	1.5%
RMSE	3.89%	4.80%	5.68%	5.39%	2.56%	5.27%	4.37%	7.15%	5.44%	1.27%	4.66%	3.56%	4.48%	3.63%	1.82%
MAE	2.94%	3.19%	3.60%	3.34%	1.93%	4.08%	3.48%	5.23%	4.04%	0.97%	3.50%	2.72%	3.37%	2.60%	1.41%
OOS #2 – Expanding sample															
Correlation	0.04	0.07	-0.21	-0.01	0.15	-0.03	0.16	0.04	-0.06	0.20	-0.01	0.19	-0.10	-0.08	0.18
Volatility	4.0%	5.7%	5.2%	4.5%	4.9%	4.1%	6.2%	5.1%	4.3%	4.5%	2.3%	1.3%	1.7%	2.5%	1.4%
RMSE	4.06%	5.72%	5.42%	4.66%	4.84%	4.26%	6.21%	5.08%	4.43%	4.46%	2.51%	1.37%	1.95%	2.75%	1.45%
MAE	3.08%	4.37%	4.13%	3.14%	3.76%	2.86%	4.62%	3.64%	2.95%	3.40%	1.86%	1.01%	1.50%	1.85%	1.06%

Notes: The table displays the out-of-sample metrics associated to the macro FMPs estimated through five methodologies: OLS-CSR, WLS-CSR, GLS-CSR, Unit Beta MCP, and ML-CSR. We run two types of out-of-sample tests. In the first test (OOS #1), the macro FMP models are estimated over the first half of the sample and maintained over the second subperiod where we estimate the metrics. In the second test (OOS #2), we re-estimate the model every month over an expanding window – starting from half of the sample – and applying the estimated weights to the next month. Correlation is the correlation between each macro FMP and the underlying observed macro factor. Volatility is the volatility of FMP. RMSE is the Root Mean Squared Errors statistics where errors are defined as the difference between each macro FMP and underlying observed macro factor. MAE is the Mean Absolute Error statistics where errors are defined as the difference between each macro FMP and underlying observed macro factor. Risk premia is the annualized average excess return associated to each macro FMP. Sharpe ratio is the ratio between the Risk premia and the (annualized) volatility.

5 – Practical use: Hedging macro risks for an endowment portfolio

One of the main potential advantage of investable macro factor mimicking portfolios is to allow investors to hedge macro risks they represent, i.e. recession, inflation surprises and financial stress in our case. We provide such an illustration with a representative endowment portfolio, with the following portfolio allocation [10]: 35% Global Equities (MSCI World), 10% Global Treasuries (Bloomberg Barclays Global Treasuries), 5% US High Yield (Bloomberg Barclays US High Yield), 5% Commodities (Bloomberg Commodities Total Return), 20% Hedge Funds (Hedge Fund Research Fund of Funds), 10% Real Estate (NCREIF Property index) and 15% Private Equity (Cambridge Associates Private Equity).

To analyze this portfolio through a macro angle, we report in Table 7 regressions of the endowment portfolio excess returns on the three macro FMPs. To limit in-sample bias, the FMPs are estimated recursively on an expanding window (similarly to the OOS test in the previous section).

The macro model explains close to 90% of the total variance of the endowment portfolio quarterly excess returns. We also see that this portfolio has an alpha equivalent to close to 2.5% per year. In Figure 3, we represent the breakdown of the portfolio in terms of risk contributions, risk being defined in terms of volatility. The portfolio is dominated by financial stress first, then growth factor and inflation surprises, while the idiosyncratic risk is near 10% of the total portfolio risk.

[10] This allocation is inspired by the 2018 version of Nacubo-TIAA study of endowments.

Table 7 – Exposure of the endowment portfolio to Machine-Learning macro FMP

	Coefficients	t-stat
Intercept	0.60%	4.52
Growth	0.14	3.71
Inflation	0.55	10.24
Stress	-0.80	-11.60
Adjusted R^2 0.89		F-stat 298.43 (p-val: 0.00)

Notes: The table summarizes the regression results of the quarterly returns of the endowment portfolio on the quarterly returns of the ML-CSR macro FMPs over the period January 1990 – June 2018.

Figure 3 – Macro Risk Contributions

Notes: The figure displays the contributions to the volatility of the endowment portfolio, as obtained through the quarterly regression of the endowment returns on ML-CSR macro FMPs. Sample period is January 1990 – June 2018.

The previous results suggest that an investor might be seeking to hedge the macro risks to improve her risk-return profile. A practical way to do such optimal hedging is to determine a combination of the endowment portfolio with macro FMPs that minimizes the variance of the combined portfolio. More precisely, we assume that the investor determines the weights $\boldsymbol{\omega}$ so that:

$$\min_{\boldsymbol{\omega}} T^{-1} \sum_{t=1}^{T} \boldsymbol{H}_t^2, \tag{5}$$

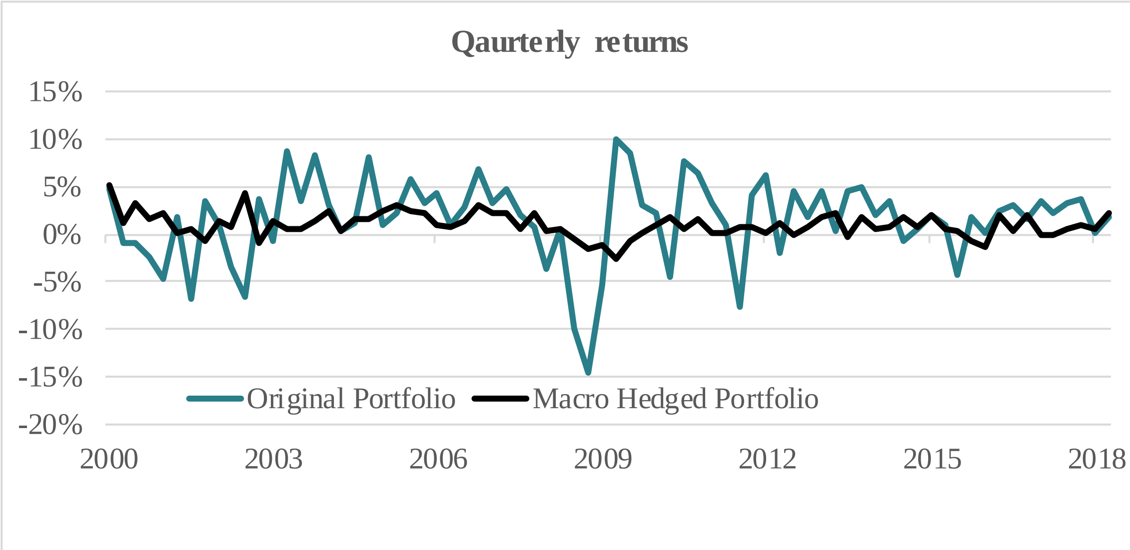
$$\boldsymbol{H}_t \equiv \boldsymbol{ENDO}_t - \boldsymbol{\omega}^{\boldsymbol{T}} \boldsymbol{FMP}_t, \tag{6}$$

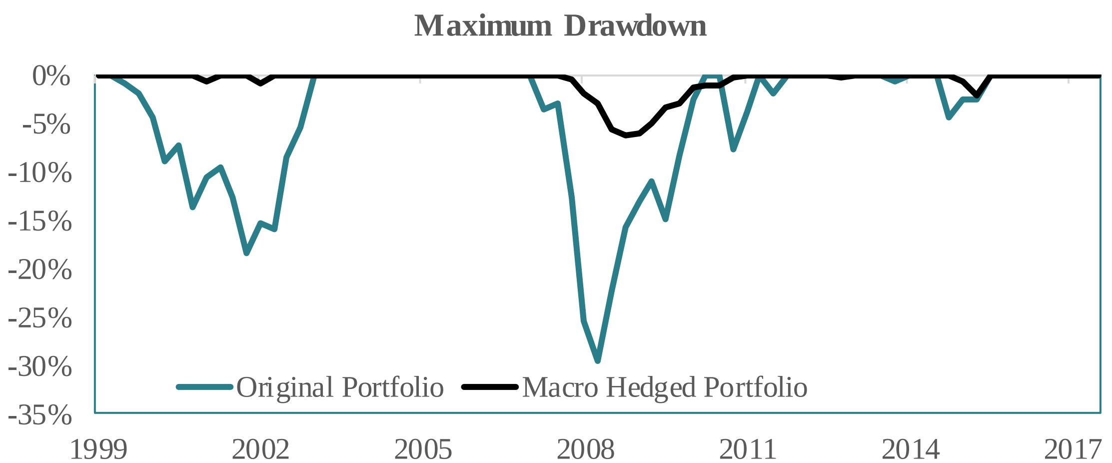
where $\boldsymbol{ENDO}_t$ are the endowment portfolio returns and $\boldsymbol{FMP}_t$ are the macro FMPs returns[11]. To make the exercise more practically orientated, we run it as an out-of-sample exercise as follows. For each quarter, the optimization problem (5) is solved over the previous 10 years (40 quarterly observations) for FMP estimated on an expanding basis up (but excluding) the current quarter. The first out-of-sample hedged portfolio $\boldsymbol{H}_1$ is obtained in 2000Q1 (with data from 1990Q1 to 1999Q4) and the last one in 2018Q2, leading to 74 observations ($T = 74$).

In Figure 4, we jointly represent the quarterly returns of the endowment portfolio and its (minimum variance) macro-hedged counterpart on the top of the chart, while we represent their respective maximum drawdowns on the bottom chart.

[11] Note that the macro FMPs are self-financed as the base assets are adjusted for cash returns when they are not defined as return spreads.

Figure 4 – Endowment portfolio and its macro-hedged version: Quarterly returns and Maximum Drawdowns





Notes: The figure jointly represents the endowment portfolio quarterly returns and its (minimum variance) macro hedged version. Sample period is January 2000Q1 – 2018Q2.

Table 8 displays descriptive statistics on both portfolios. While the average returns are slightly reduced, the macro-hedging allows the investor to achieve more consistent returns over time, with a much lower volatility or maximum drawdown. The distribution looks much more favorable with a positive rather than negative skewness and a reduced kurtosis. Overall, the risk-adjusted metrics (Sharpe and Calmar ratios) are significantly improved.

Table 8 – Endowment portfolio and its macro-hedged version: Portfolio performance metrics

	Original	**Macro-hedged**
Annualized return	5.9%	4.1%
Volatility	8.8%	2.6%
Minimum	-14.6%	-2.7%
Maximum	10.1%	5.1%
Skewness	-1.03	0.22
Kurtosis	5.10	4.22
Maximum Drawdown	29.5%	6.3%
Sharpe Ratio	0.41	0.70
Calmar ratio	0.09	0.14

Notes: The table represents the performance metrics of the endowment portfolio and of its macro-hedged version. Sample period is 2000Q1 –2018Q2.

6 – Conclusion

Investors frequently form views on macro factors. Still, acting on the basis of these views remains a challenge as macro factors are not directly investable. In the asset pricing literature, the issue of non-tradability of economic factors has been handled through a variety of different methodologies grouped under the common label of "Factor Mimicking Portfolios" (FMPs). In this article, we introduce a general FMP framework that encompasses existing factor mimicking approaches, such as the two-pass cross-sectional regressions (CSR) and maximum correlation portfolio (MCP) approach.

We also show how investors can improve the estimation of macro FMPs by combining machine-learning methods such as supervised principal component analysis and LASSO regressions. We apply our macro factor mimicking framework to three major global macro factors (growth, inflation surprises and financial stress) over the period 1974-2018, and show how the estimated machine learning macro FMPs can be used to improve a typical endowment portfolio risk-return profile through macro-hedging.

Empirical applications of our methodological framework can naturally be extended to deal with macro factor correlation dynamics or practically relevant portfolio constraints such as transaction costs, liquidity or regulatory guidelines. We leave these extensions to future research.

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Appendix 1. General Factor Mimicking Portfolio Optimization Program

Formally, the *k*-th FMP can be obtained as the solution of the following generic beta-constrained variance minimization problem:

$$Min \frac{1}{2} \boldsymbol{w}_k^{\boldsymbol{T}} \boldsymbol{\Omega}\, \boldsymbol{w}_k \tag{A.1.1}$$

$$s.t. \boldsymbol{B}^{\boldsymbol{T}} \boldsymbol{w}_k = \boldsymbol{\beta}_k$$

where $\boldsymbol{\Omega}$ is the positive definite $(N \times N)$ covariance matrix, $\boldsymbol{B}$ is the $(N \times K)$ matrix of factor loadings (univariate or multivariate) and $\boldsymbol{\beta}_k$ is the $(K \times 1)$ vector of systematic risk exposures associated with the mimicking portfolio on the *k*-th factor, with by construction 1 in *k*-th position. The Lagrangian function of the system (A.1.1) is given by

$$L(\boldsymbol{w}_k; \lambda_k) = \frac{1}{2} \boldsymbol{w}_k^{\boldsymbol{T}} \boldsymbol{\Omega}\, \boldsymbol{w}_{\boldsymbol{k}} - \lambda_k^{\boldsymbol{T}} (\boldsymbol{B}^{\boldsymbol{T}} \boldsymbol{w}_k - \boldsymbol{\beta}_k) \tag{A.1.2}$$

where $\boldsymbol{\lambda}_k$ is the $(K \times 1)$ vector of the Lagrangian multipliers. The first-order condition of (A.1.2) is given by:

$$\frac{\partial L(\boldsymbol{w}_k; \lambda_k)}{\partial \boldsymbol{w}_{\boldsymbol{k}}} = \boldsymbol{\Omega}\, \boldsymbol{w}_k - \boldsymbol{B} \lambda_k = 0 \tag{A.1.3}$$

Premultiplying by the inverse matrix $\boldsymbol{\Omega}^{-\boldsymbol{1}}$ and solving for the optimal weights, we have:

$$\boldsymbol{w}_k = \boldsymbol{\Omega}^{-\boldsymbol{1}} \boldsymbol{B} \lambda_k \tag{A.1.4}$$

Premultiplying (A.1.4) by $\boldsymbol{B}^{\boldsymbol{T}}$ leads to:

$$\boldsymbol{B}^{\boldsymbol{T}} \boldsymbol{w}_k = \boldsymbol{B}^{\boldsymbol{T}} \boldsymbol{\Omega}^{-\boldsymbol{1}} \boldsymbol{B} \lambda_k = \boldsymbol{\beta}_k \tag{A.1.5}$$

We infer:

$$\lambda_k = \left(\boldsymbol{B}^{\boldsymbol{T}} \boldsymbol{\Omega}^{-\boldsymbol{1}} \boldsymbol{B}\right)^{-\boldsymbol{1}} \boldsymbol{\beta}_k \tag{A.1.6}$$

Equation (A.1.4) can then be rewritten as:

$$\boldsymbol{w}_k = \boldsymbol{\Omega}^{-\boldsymbol{1}} \boldsymbol{B} \left(\boldsymbol{B}^{\boldsymbol{T}} \boldsymbol{\Omega}^{-\boldsymbol{1}} \boldsymbol{B}\right)^{-\boldsymbol{1}} \boldsymbol{\beta}_k \tag{A.1.7}$$

Taken together, the Factor-Mimicking Portfolio weight matrix is:

$$\boldsymbol{W}_K = \boldsymbol{\Omega}^{-1}\boldsymbol{B}\left(\boldsymbol{B}^T\boldsymbol{\Omega}^{-1}\boldsymbol{B}\right)^{-1}\boldsymbol{B}_K \tag{A.1.8}$$

where $\boldsymbol{B}_K$ is the $(K \times K)$ target factor beta matrix with $\boldsymbol{B}_K = (\boldsymbol{\beta}_1\ \boldsymbol{\beta}_2\ \dots\ \boldsymbol{\beta}_K)$.

Appendix 2. Equivalence between Specific Risk and Total Risk FMP Solutions

Following the approach of Grinblatt and Titman (1987), define:

$$\boldsymbol{A} = \boldsymbol{\Omega}^{-1}\boldsymbol{B}\left(\boldsymbol{B}^T\boldsymbol{\Omega}^{-1}\boldsymbol{B}\right)^{-1} \tag{A.2.1}$$

which implies:

$$\begin{cases} \boldsymbol{B}^T\boldsymbol{A} = \boldsymbol{I}_N \\ \boldsymbol{\Omega}\,\boldsymbol{A}\left(\boldsymbol{B}^T\boldsymbol{\Omega}^{-1}\boldsymbol{B}\right) = \boldsymbol{B} \end{cases} \tag{A.2.2}$$

Substituting $\boldsymbol{\Omega} = \boldsymbol{B}\,\boldsymbol{\Omega}_{\boldsymbol{K}}\,\boldsymbol{B}^{\boldsymbol{T}} + \boldsymbol{\Omega}_{\boldsymbol{\varepsilon}}$, where $\boldsymbol{\Omega}_K$ is the variance-covariance matrix of factors and $\boldsymbol{\Omega}_\varepsilon$ is the variance-covariance of idiosyncrasic risks, into the latter equation of (A.2.2) and using (A.2.1) yields:

$$\boldsymbol{B}\,\boldsymbol{\Omega}_{\boldsymbol{K}}\left(\boldsymbol{B}^T\boldsymbol{\Omega}^{-1}\boldsymbol{B}\right) + \boldsymbol{\Omega}_{\boldsymbol{\varepsilon}}\boldsymbol{A}\left(\boldsymbol{B}^T\boldsymbol{\Omega}^{-1}\boldsymbol{B}\right) = \boldsymbol{B} \tag{A.2.3}$$

That is:

$$\boldsymbol{A} = \boldsymbol{\Omega}_{\boldsymbol{\varepsilon}}^{-1}\boldsymbol{B}\left[\left(\boldsymbol{B}^T\boldsymbol{\Omega}^{-1}\boldsymbol{B}\right)^{-1} - \boldsymbol{\Omega}_{\boldsymbol{K}}\right] \tag{A.2.4}$$

Premultiplying by $\boldsymbol{B}^{\boldsymbol{T}}$ and using (A.2.1) yields:

$$\left(\boldsymbol{B}^T\boldsymbol{\Omega}_{\boldsymbol{\varepsilon}}^{-1}\boldsymbol{B}\right)^{-1} = \left[\left(\boldsymbol{B}^T\boldsymbol{\Omega}^{-1}\boldsymbol{B}\right)^{-1} - \boldsymbol{\Omega}_{\boldsymbol{K}}\right] \tag{A.2.5}$$

Substituting this into (A.2.4) leads to the desired result, that is:

$$\boldsymbol{\Omega}^{-1}\left(\boldsymbol{B}^T\boldsymbol{\Omega}^{-1}\boldsymbol{B}\right)^{-1} = \boldsymbol{\Omega}_{\boldsymbol{\varepsilon}}^{-1}\boldsymbol{B}\left(\boldsymbol{B}^T\boldsymbol{\Omega}_{\boldsymbol{\varepsilon}}^{-1}\boldsymbol{B}\right)^{-1} \tag{A.2.6}$$

So that:

$$\boldsymbol{W}_K = \boldsymbol{\Omega}^{-1}\boldsymbol{B}\left(\boldsymbol{B}^T\boldsymbol{\Omega}^{-1}\boldsymbol{B}\right)^{-1}\boldsymbol{B}_K = \boldsymbol{\Omega}_{\boldsymbol{\varepsilon}}^{-1}\boldsymbol{B}\left(\boldsymbol{B}^T\boldsymbol{\Omega}_{\boldsymbol{\varepsilon}}^{-1}\boldsymbol{B}\right)^{-1}\boldsymbol{B}_K \tag{A.2.7}$$

Appendix 3. Principal Components Based Instrumental Variables Estimator

We assume that the individual asset returns have a factor structure that can be represented as:

$$\boldsymbol{R}_t = \boldsymbol{\mu} + \boldsymbol{B}\boldsymbol{F}_{t+h} + \boldsymbol{\varepsilon}_t \tag{A.3.1}$$

where $\boldsymbol{R}_t$ and $\boldsymbol{\mu}$ are the $(N \times 1)$ vector of the base asset returns and associated expected returns, respectively. $\boldsymbol{B}$ is the $(N \times K)$ matrix of asset loadings to factors; $\boldsymbol{F}_{t+h}$ is the $(K \times 1)$ vector of zero-mean observable factors; and $\boldsymbol{\varepsilon}_t$ is the $(N \times 1)$ vector of zero-mean disturbances.

Let $\overline{\boldsymbol{R}}$ correspond to the $(N \times T)$ matrix of asset centered returns, $\boldsymbol{F}$ denote the $(K \times T)$ matrix of observed factors and $\boldsymbol{\varepsilon}$ be the $(N \times T)$ matrix of disturbances. The factor model can then be represented in matrix form as:

$$\overline{\boldsymbol{R}} = \boldsymbol{B}\boldsymbol{F} + \boldsymbol{\varepsilon} \tag{A.3.2}$$

Given the base asset returns and the factors of interest, the Principal Components Instrumental Variables FMP weight estimator proceeds as follows:

1. **PC extraction step:** Extract the first L principal components of asset returns by conducting the PCA of the VCV matrix $\boldsymbol{\Omega} = (T)^{-1}\overline{\boldsymbol{R}}\overline{\boldsymbol{R}}^{\boldsymbol{T}}$ so that

$$\boldsymbol{F}^{PC} = \boldsymbol{E}_L^{\boldsymbol{T}}\,\overline{\boldsymbol{R}} \tag{A.3.3}$$

where $\boldsymbol{F}^{PC}$ is the $(L \times T)$ matrix of PCs and $\boldsymbol{E}_L$ corresponds to the $(N \times L)$ eigenvector matrix associated with the largest L eigenvalues of $\boldsymbol{\Omega}$, with $L \leq N$.

2. **Fitted Factor Estimation step:** Regress the K observed factors $\boldsymbol{F}_{t+h}$ onto the L statistical factors extracted from the PCA to obtain the fitted values of the observed factors and their multivariate factor exposures, that is

$$\widehat{\boldsymbol{F}} = \widehat{\boldsymbol{B}}_{\boldsymbol{K}}\boldsymbol{F}^{PC} \text{ and } \widehat{\boldsymbol{B}}_{\boldsymbol{K}} = \boldsymbol{F}\boldsymbol{F}^{PC^{\boldsymbol{T}}}(\boldsymbol{F}^{PC}\boldsymbol{F}^{PC^{\boldsymbol{T}}})^{-1} \tag{A.3.4}$$

The observable fitted factors and their multivariate exposures can be written equivalently more compactly as

$$\widehat{\boldsymbol{F}} = \boldsymbol{F}\boldsymbol{P} \tag{A.3.5}$$

where $\boldsymbol{P}$ is a $(T \times T)$ projection matrix, with $\boldsymbol{P} = \boldsymbol{F}^{PC^{\boldsymbol{T}}}(\boldsymbol{F}^{PC}\boldsymbol{F}^{PC^{\boldsymbol{T}}})^{-\mathbf{1}}\boldsymbol{F}^{PC}$.

3. **Times Series Regression Step:** Regress the excess asset returns $(\boldsymbol{R}_{t+h} - \boldsymbol{\mu})$ onto the K fitted values of the observable factors to obtain the Principal Components Instrumental Variables Estimator (PCIV) of the individual factor loadings, that is:

$$\boldsymbol{B}^{PCIV} = \overline{\boldsymbol{R}}\,\widehat{\boldsymbol{F}}^{\boldsymbol{T}}(\widehat{\boldsymbol{F}}\widehat{\boldsymbol{F}}^{\boldsymbol{T}})^{-\mathbf{1}} \tag{A.3.6}$$

Substituting in (A.3.6) $\widehat{\boldsymbol{F}}$ by its expression (A.3.5) and rearranging leads to

$$\boldsymbol{B}^{PCIV} = \overline{\boldsymbol{R}}\,\boldsymbol{P}^{\boldsymbol{T}}\boldsymbol{F}^{\boldsymbol{T}}(\boldsymbol{F}\boldsymbol{P}\boldsymbol{P}^{\boldsymbol{T}}\boldsymbol{F}^{\boldsymbol{T}})^{-\mathbf{1}} \tag{A.3.7}$$

where $\boldsymbol{P}$ is the $(T \times T)$ projection matrix defined in step 2.

The instrumental variables estimator of the FMP weights is then obtained by substituting in (A.1.8) $\boldsymbol{B}$ by $\boldsymbol{B}^{PCIVE}$. That is:

$$\boldsymbol{W}_K = \boldsymbol{\Omega}^{-\mathbf{1}}\boldsymbol{B}^{PCIV}\left(\boldsymbol{B}^{PCIV\,\boldsymbol{T}}\boldsymbol{\Omega}^{-\mathbf{1}}\boldsymbol{B}^{PCIV}\right)^{-\mathbf{1}}\boldsymbol{B}_{\boldsymbol{K}} \tag{A.3.8}$$

where $\boldsymbol{B}_{\boldsymbol{K}}$ is the $(K \times K)$ target beta matrix.

One can remark that the Instrumental Variable FMP weight estimator (A.3.8) provides an unbiased estimator, since the first PCs can recover the true factor space subject to a rotational indeterminacy (see Bai and Ng, 2002).