Problem Chosen	2023 MCM/ICM Summary Sheet	Team Control Number
A		2300000

Output Power Maximization Model of Wave Energy Device

Based on the analysis of the heave and pitch motion of the wave energy device in seawater, the motion states of the float and vibrator from static to heave and pitch motion are obtained through the force analysis of the float and vibrator and the differential equation is established. Then, by adjusting the relevant parameters of the damping coefficient of the linear damper and the rotary damper, the relevant parameters of the damping coefficient which make the maximum output power of the PTO system are calculated, and the energy conversion efficiency of the wave energy device is improved.

In view of problem 1, we only consider the dangling motion of the device, excluding the influence of the pitching force, etc., which can be considered as a linear system with multiple degrees of freedom. According to the meaning, the device in the initial state is balanced in still water, and the linear expression of the restoring force in still water can be obtained by assuming that the float will not be completely immersed in water or exposed to the conical part during the hanging motion thereafter. The force analysis of the float and oscillator is carried out, the multivariate differential equation is established, and the ode45 function in MATLAB is used to solve it. Finally, the vertical displacement and velocity of the float and oscillator within 40 wave cycles are obtained. The data at 10s, 20s, 40s, 60s and 100s under their respective conditions are shown in Table 1 and Table 2 in Article 5.1 below.

For problem 2, since it is to find the damping coefficient that maximizes the average output power of the PTO system, it may be useful to simplify the calculation by using the motion of the float and oscillator under steady state. When the damping coefficient is a certain constant, the relative velocity of the float and the oscillator under steady state can be obtained by using the motion frequency of the float and the oscillator to be equal to the sea wave. Thus, the expression of the average output power of the PTO system can be written as follows:

$$\mathrm{PTO} = \frac{1}{2}\frac{\omega^6 m_z^2 f^2 \eta_i}{(C \cdot D\eta)^2 + \omega^2 \left[E\eta_l + B\lambda_d\right]^2} \tag{0.1}$$

It follows that when

$$\eta_{\text{i.best}} = \sqrt{\frac{c^2 + \omega^2 B^2 \lambda^2}{D^2 + \omega^2 E^2}} \tag{0.2}$$

it has the maximum power. In addition, we can also use the obtained numerical results to calculate the work done by the PTO system every 0.2 seconds, and divide the trapz trapezoidal numerical integration by the total time to obtain the approximate average output power under a certain damping coefficient.

Then, we can get the maximum output power of 229.334W by iterating the value of the damping coefficient 37193.812. When the damping coefficient is proportional to the power of the absolute value of the relative velocity, it is difficult to get the analytical solution. Referring to the results of the first small question, it can be seen that the precision of trapz trapezoidal numerical integration meets the requirements. Therefore, the former is directly used to calculate, and the maximum output power is 230.127 W, the damping proportional coefficient is 9.75×10^4, and the power value is 0.370.

For problem 3. The floats and oscillators not only swing but also pitch. After careful review of the problem, it was found that the problem did not specify the relative rotation axis of the pitching excitation moment, the wave damping moment and other moments. After analyzing the forces and application points on the rotation axis and the central axis, it was found that it was too complicated and a lot of information was unknown, so a reasonable approximation of the model was needed. Through the preliminary estimation of the data given in the attached table, it is obtained that the angular displacement of the float and oscillator is small, so the influence brought by the pitch motion can be ignored when considering the vertical motion. When considering pitch motion, since the calculated relative vertical displacement of the float and oscillator is very small, the float and oscillator can be regarded as a whole to calculate their rotation in seawater. Then, a translational non-inertial reference frame with the rotating axis as the origin is constructed to analyze the rotation of the oscillator. Finally, the vertical displacement and velocity as well as pitch angular displacement and velocity of the float and vibrator within 40 wave cycles are obtained.

For problem 4, only considering the constant damping coefficient and referring to the solution of problem 2, it can be considered that trapz trapezoidal numerical integration method is accurate enough. MATLAB is used to make the global traversal diagram and the maximum output power under this condition is 322.128 W, and the damping coefficient of linear damper is 5.93×10^4. The damping coefficient of the rotary damper is 100000.

Through the stability analysis of the model, the model is valid for most wave excitation force and wave excitation moment. The model has good robustness. In general, the model has a high originality and reasonable simplification, and the results are consistent with the initial hypothesis: in the third and fourth models, due to the small amount of approximation, certain errors may occur when the waves are large, and the accuracy may decrease. The model can also be extended to study the influence of spring stiffness and torsional spring stiffness, or the mass ratio of float and oscillator on the maximization of output power, so as to improve the conversion efficiency of wave energy devices.

Key Words: linear systems with multiple degrees of freedom; multivariate differential equations; translational non-inertial systems

Contents

1 Restatement of the Problem

1.1 Problem Background

Today, the world is increasingly depleted of fossil fuels and environmental problems are becoming more and more serious. Wave energy is renewable, abundant and easy to obtain, which is expected to be a major help for mankind to solve the current energy crisis and environmental pollution. In order to use wave energy as an important Marine renewable energy on a large scale, it is necessary to solve the problem of energy conversion efficiency of wave energy devices.

The principle of a device that uses wave energy is that the movement of the float under the action of the wave drives the movement of its internal oscillator, and through the PTO (energy output system) composed of a spring and a damper, the relative motion of the two drives the damper to do work, so as to realize the conversion to other energy sources.

1.2 Question Raising

A wave energy device consists of four parts, which are a hollow shell float with uniform mass, a central shaft sealed inside the float, a vibrator pierced on the central shaft and a PTO containing a spring and a damper. The float is composed of a covered bottomless cylindrical shell and a conical shell. The base of the central axis is fixed at the center of the separation layer of the connecting part of the two shells, and is connected with the oscillator through the PTO. Under the action of waves, the float moves on the sea surface. The relative motion of the float and the oscillator makes the damper do work, converting the wave energy into usable energy. Meanwhile, the energy conversion efficiency of the device is expected to be improved by adjusting the damping coefficient of the damper.

Based on the above background information and attachments, a mathematical model should be established to solve the following problems:

Problem 1: Considering the case that the float only moves in vertical swing and the central axis is fixed, and the vibrator is connected to the base by PTO and moves only along the central axis, the damping force it receives is positively correlated with the relative velocity of the float. The device is initially balanced in still water, and then begins to move under the influence of the vertical excitation force. A mathematical model is established to calculate the vertical displacement and velocity of the float and oscillator within 40 wave cycles after the device is affected by the vertical excitation force when the damping coefficient of the damper is fixed at 10000N· s/m and the damping coefficient is proportional to the power of the absolute relative velocity of the float and oscillator (the proportional coefficient is 10000 and the power index is 0.5).

Problem 2: Other conditions are the same as in (1). Solving the optimal damping makes the conversion efficiency of the device optimal. The maximum average output power and corresponding damping coefficient are calculated for the constant damping coefficient within [0.100000], the proportional coefficient within [0 100000] and the power coefficient within the interval of [0, 1] respectively.

Problem 3: Consider the case of simultaneous heave and pitch motion of the device. At this time, the central shaft is hinged with the base, and a rotary damper and a torsion spring are installed at the rotating shaft. See the attachment for the parameters. It is required to establish a mathematical model and calculate the heave displacement and velocity of the float and the pitch angular displacement and velocity of the oscillator within 40 cycles under the combined action of heave exciting force and pitch exciting moment for the device balanced in water.

Problem 4: Calculate the maximum output power and corresponding damping coefficient under the condition of heave and pitch of the float at the same time by taking the values of linear damper and rotary damper within [0.100000].

2 Problem analysis

This paper mainly deals with the optimization of energy conversion efficiency of a wave energy device. Problems 1 and 2 are to solve the motion of float and oscillator when the device only does heave motion and how to determine the optimal damping coefficient to make the highest average output power, while problems 3 and 4 are to solve the motion of float and oscillator when the device only does heave and pitch motion and to achieve the optimization of average output power by adjusting the damping coefficient.

2.1 Analysis of Problem 1

The first problem is to consider that the float in the wave only do swing motion, the central axis and the base fixed connection. When t = 0, the device is balanced in still water: when t0, the float is stimulated by the vertical force to drive the oscillator to move and reach a steady state after a period of time. Since it has been assumed that the float only does heave motion, the effect of pitching excitation force is not considered. According to the conditions given by the topic, the force of each part of the device is analyzed first, and the dynamic equations of the float and oscillator are established.

The first question: When the damping is fixed, the damping coefficient of 10000N – s/m is taken into the listed two simultaneous differential equations, and the numerical solution can be calculated using MATLAB.

The second question: given that the damping coefficient is proportional to the power of the absolute relative velocity between the float and the oscillator, then the dynamics equation is modified, and the proportional coefficient 10000 and the power index 0.5 are put in, which are also solved by using the meter MATLAB.

2.2 Analysis of Problem 2

Problem 2 requires to determine the optimal damping coefficient under the condition of problem 1 to achieve the maximum average power of the PTO system. For the optimal damping coefficient, on the one hand, the complex solution can be used to solve the response function, obtain the relation between PTO power and damping coefficient, and determine its analytical solution. On the other hand, the numerical solution of the optimal damping coefficient can be obtained by computer enumeration.

The first question is to obtain the analytical solution and the numerical solution. If the difference between the two solutions is small, it can be concluded that the two methods can accurately obtain the optimal damping coefficient.

Second question: Because complex solution method is difficult to solve nonlinear equations, in the case of unknown proportional coefficients and power exponents, the analytical solution may not be written down. In this case, it is necessary to consider replacing the analytical solution with numerical solution. If the difference between the two solutions in the first small question is small, the numerical solution can be determined to be accurate enough, and the numerical solution obtained by computer enumeration is the optimal solution in the case of the basic small question.

2.3 Analysis of Problem 3

Problem 3 considers that the float only makes heave and pitch motion in water, the central shaft is hinged with the base, and a rotary damper and a rotating spring are added to the rotating shaft. Other conditions are the same as in Problem 1.

It is assumed that the angular displacement of the float and the oscillator is very small, so when considering the heave motion, the influence caused by the pitch motion can be ignored: When considering the pitch motion, the relative vertical displacement of the float and oscillator calculated is very small, so the float and oscillator can be regarded as a whole to calculate the rotation, and then a translational non-inertial reference frame with the rotating axis as the origin is constructed to analyze the rotation of the oscillator.

Two dynamic equations and two rotational equations are listed respectively, and the differential equations are solved by MATLAB.

2.4 Analysis of Problem 4

In problem 4, the value of linear damper and rotary damper within [0.100000] is needed to obtain the maximum output power and corresponding damping coefficient under the condition of both heave and pitch of the float. Similar to problem 2, the system PTO is solved by numerical integration method, and then the optimal damping coefficients of linear damper and rotary damper are traced from 0-100000 to calculate the output PTO under different conditions. The optimal parameters are found by subdividing the target interval several times.

3 Model Assumptions

1. Suppose that sea water is inviscid and rotateless:

2. The mass of base, central shaft, compartment, PTO and all friction are not considered:

3. Ignore the size, thickness and height of the base, central shaft, central shaft frame, rotating shaft and other structures not given in the attachment; 4. The sea level is large enough and the float movement has no effect on the sea level height;
4. The tip of the float will not be submerged and the cone will not be above the water;
5. In solving problem 2, it is assumed that the model will stabilize in a finite time, and eventually the circular frequencies of all generalized coordinates will be equal to the frequencies of the excitation force circles:
6. In solving problem 3, the relative displacement of the oscillator and the float's heave motion can be regarded as a small quantity compared to the heave motion of the float:
7. When solving problem 3, the angular displacement of the pitch motion between the float and the oscillator is small.

4 Symbol Description

The initial distance between the float's center of mass and its axis of rotation: Initial state the distance between the center of mass of the float and the oscillator and the axis of rotation

5 Model Establishment and Solution

5.1 Establishment and solution of Problem 1 Model

5.1.1 Establishment of model

Considering the heave motion of the wave energy device on the sea surface, it is necessary to first exclude the condition that the sea level is higher than the float roof or the cone part of the float is above the water surface because the linear condition of the restoring force of still water is no longer applicable in this case. Therefore, the distance between the top of the float and the sea level in the equilibrium state is calculated for comparison. Suppose that the distance between the top of the float and the water surface is ho, and assume that the float is in a normal dangling motion, then the amplitude must be less than h at the same time and the difference between the length of the cylindrical shell and ho.

In the initial state, that is, $t = 0$, it can be considered that the movement of the float will not cause changes in the sea level because the sea level is large enough. Due to the force balance in this state, the following equation can be obtained:

$$\rho g \left[\frac{1}{3} \pi R_f^2 h_z + \pi R_f^2 \left(h_y - h_0 \right) \right] = \left(m_f + m_j \right) g \tag{5.1}$$

We can obtain:

$$Ho = 0.99999 \, \mathrm{m} \tag{5.2}$$

The amplitudes are then compared to verify that the floats conform to the hypothesis.

Next, we need to analyze the force and motion of the float and oscillator. Let's call the vertical displacement of the float x1 and the vertical displacement of the oscillator x2

Within 40 wave cycles, the float is subjected to periodic heave exciting force, and its motion will also be affected by static water restoring force (which is positively correlated with its displacement x), heave wave damping force (which is positively correlated with its velocity) and heave additional inertia force. In the device, because the float and the oscillator are not rigidly connected, the relative motion of the two generates the corresponding spring force (which is positively related to the relative displacement of the two) and the damping force (which is positively related to the relative velocity of the two). Consider that in the vertical direction, the float receives the wave excitation force, the hydrostatic recovery force, the wave damping force and the additional inertia force and the elastic force and damping force of the TPO system. If the displacement of the float is x1 and the displacement of the oscillator is x2, the equation of motion of the system can be obtained after the force analysis.

When the damping coefficient is fixed, the equation of motion of the float and oscillator is as follows:

$$\begin{cases} \left(m_f+\mu\right)\ddot{x}_1+\eta_i\left(\dot{x}_1-\dot{x}_2\right)+\lambda_1\dot{x}_1+k\left(x_1-x_2\right)+\rho g\pi R_f^2 x_1=f \\ m_z\ddot{x}_2+\eta_i\left(\dot{x}_2-\dot{x}_1\right)+k\left(x_2-x_1\right)=0 \end{cases} \tag{5.3}$$

When the damping coefficient is proportional to the power of the absolute relative velocity of the float and the abstracts, the equation of motion of the float and the oscillator is as follows:

$$\begin{cases} \left(m_f+\mu\right)\ddot{x}_1+\eta\left(\dot{x}_1-\dot{x}_2\right)\left|\dot{x}_1-\dot{x}_2\right|^{\frac{1}{2}}+\lambda_1\dot{x}_1+k\left(x_1-x_2\right)+\rho g\pi R_f^2 x_1=f \\ m_z\ddot{x}_2+\eta\left(\dot{x}_2-\dot{x}_1\right)\left|\dot{x}_1-\dot{x}_2\right|^{\frac{1}{2}}+k\left(x_2-x_1\right)=0 \end{cases} \tag{5.4}$$

The change of displacement and velocity of the float and oscillator with time can be obtained by solving the two sets of equations respectively the root of the problem.

5.2 Solution of the first small question model (when the damping coefficient is fixed)

ode45 is a kind of function in MATLAB specially used for solving differential equations. It is a medium order, adaptive step size, used to solve non-rigid ordinary differential equations. For the formula (1) that has been obtained, code was written and numerical solution was obtained. ode45 was used to solve the non-rigid differential equation. See the attachment result1-1.xlsx. The obtained results were visualized to visually reflect the movement of the float and oscillator over time, and the following figure was obtained The vertical displacement and velocity of the float and oscillator at 10s, 20s, 40s, 60s and 100s are shown in the table below:

Table 1: Vertical displacement and velocity table of float and oscillator (where t is in unit s, displacement is in unit m, and velocity is in unit m/s)

t	x_1	v_1	x_2	v_2	x_1-x_2
10	-0.190591947	-0.640564278	-0.211549275	-0.69360835	0.020957328
20	-0.590531163	-0.240467502	-0.634068234	-0.27228741	0.043537071
40	0.285456229	0.313435653	0.296609879	0.333247927	-0.01115365
60	-0.31443603	-0.479107064	-0.33135498	-0.51555596	0.01691895
100	-0.083590128	-0.604065973	-0.084036856	-0.64304386	0.000446728

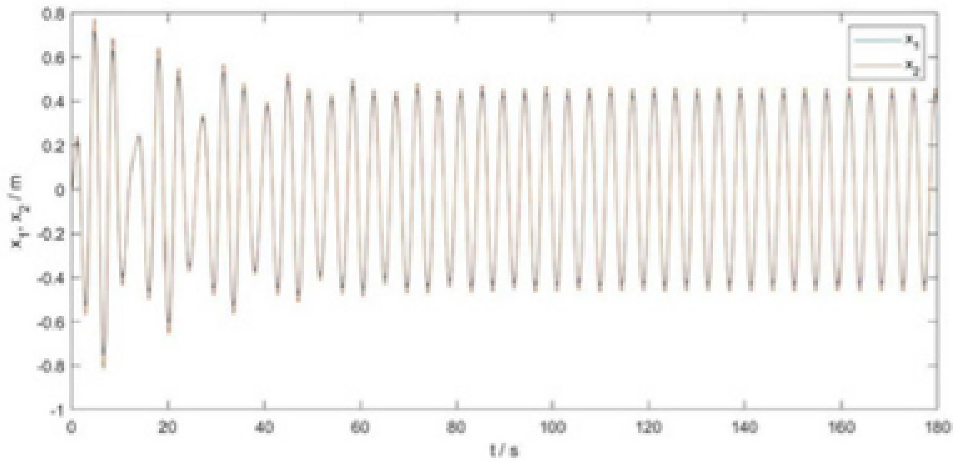


Figure 1: Variation curve of float and oscillator displacement x1 and x2 with time t.

5.2.1 Solution of the second minor question model (the damping coefficient is proportional to the power of the absolute relative velocity of the float and the oscillator)

Similarly, ode45 is used to solve formula (2), and the data obtained is shown in the attachment result1-2.xlsx.

The obtained results were visualized to visually reflect the movement of the float and oscillator over time, and the following figure was obtained:

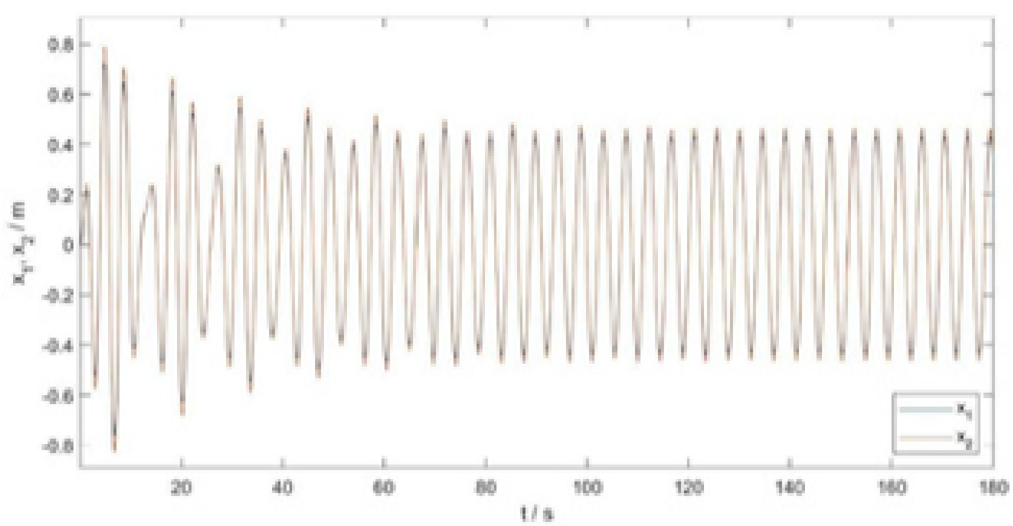
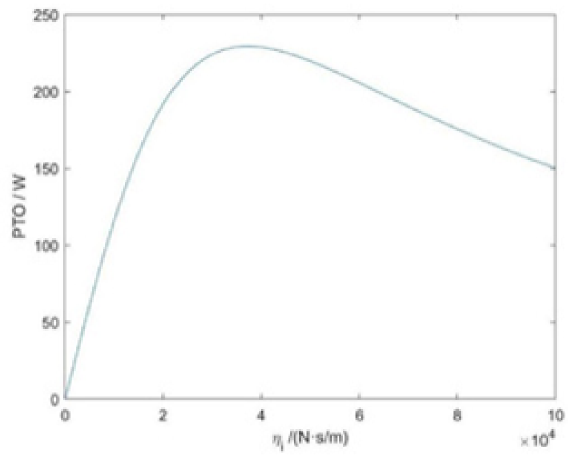


Figure 2: Variation curves of float and oscillator displacement x1 and x2 with time t.

It is not difficult to find that the maximum amplitude of x1 is smaller than h. To satisfy the assumption that the water surface will not exceed the top cover of the float and that part of the conical shell of the float will not come out of the water.

Table 2: Displacement and velocity table of the float and oscillator's gravity swing (where t is in unit s, displacement is in unit m, and velocity is in unit m/s)



5.3 Establishment and solution of Problem 2 model

Since it is to find the damping coefficient that maximizes the average output power of PTO system, it may be useful to simplify the calculation by using the motion of float and oscillator in steady state. When the damping coefficient is a constant, the relative velocity of the float and the vibrator in steady state can be obtained by using the motion frequency of the float and the oscillator to be equal to the sea wave. In this way, the expression of the average output power of the PTO system can be written, and the analytical solution of the damping coefficient which maximizes the output power can be obtained. In addition, we can also use the obtained numerical results to calculate the work done by the PTO system every 0.2 seconds, and divide the trapz trapezoidal numerical integration by the total time to obtain the approximate average output power under a certain damping coefficient. Then, we can get the maximum output power and the value of the damping coefficient at this time by traversing the value of the damping coefficient. When the damping coefficient is proportional to the power of the absolute value of the relative velocity, it is difficult to obtain the analytical solution. Referring to the results of the first question, if the precision of trapz trapezoidal numerical integration meets the requirements, the former can be directly used to calculate the maximum output power, damping proportional coefficient and power value.

5.3.1 Solution of the First small question model (when the damping coefficient is a certain value)

Analytical solution of the first small question Since the damping work is only related to the damping coefficient and the relative motion between the float and the oscillator, take PTO as the energy conversion power of the device, and when the damping coefficient is a fixed value, the relation can be expressed as:

$$PTO = \frac{1}{2}\eta_i \omega^2 \left| x_1 - x_2 \right|^2 \tag{5.5}$$

Where, x1 and x2 at this time represent complex amplitude.

Therefore, the equation of relative motion displacement of float and oscillator must be determined first.

First try the formula solution. The response function is solved by complex solution. As shown in 5.1, the equation of motion when the damping coefficient is fixed is shown in (1). For (1), the motion frequency of the float and oscillator is equal to that of the sea wave in the steady state, which can be rewritten as:

$$\begin{bmatrix} -\omega^2\left(m_f+\mu_1\right)+i\omega\left(\lambda_1+\eta_i\right)+k+\rho g\pi R_f^2 & -i\omega\eta_i-k \\ -i\omega\eta_i-k & -\omega^2 m_z+i\omega\eta_i+k \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} f \\ 0 \end{bmatrix} \tag{5.6}$$

Get:

$$\mathrm{Y}=\begin{bmatrix} -\omega^2\left(m_f+\mu_1\right)+i\omega\left(\lambda_1+\eta_i\right)+k+\rho g\pi R_f^2 & -i\omega\eta_i-k \\ -i\omega\eta_i-k & -\omega^2 m_z+i\omega\eta_i+k \end{bmatrix} \tag{5.7}$$

Then:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=Y^{-1}\begin{bmatrix} f \\ 0 \end{bmatrix}=\frac{f}{|Y|}\begin{bmatrix} -\omega^2 m_z+i\omega\eta_i+k \\ i\omega\eta_i+k \end{bmatrix} \tag{5.8}$$

So that:

$$x_1-x_2=\frac{f}{|Y|}\left[-\omega^2 m_z\right] \tag{5.9}$$

get:

$$\mathrm{A}=-\omega^2\left(m_f+\mu_1\right)+k+\rho g\pi R_f^2, \quad \mathrm{B}=-\omega^2 m_z+k \tag{5.10}$$

Then:

$$x_1-x_2=\frac{-\omega^2 m_z f}{AB-k^2-\omega^2\lambda_1\eta_t+i\omega\left[A\eta_t+B\left(\lambda_1+\eta_i\right)-2k\eta_t\right]} \tag{5.11}$$

get:

$$\mathrm{C}=\mathrm{AB}-k^2, \mathrm{D}=-\omega^2\lambda_1, \mathrm{E}=\mathrm{A}+\mathrm{B}-2\mathrm{k} \tag{5.12}$$

To simplify to:

$$\left|x_1-x_2\right|^2=\frac{\omega^4 m_z^2 f^2}{\left(C-D\eta_i\right)^2+\omega^2\left[E\eta_i+B\lambda_1\right]^2} \tag{5.13}$$

Plug in to get:

$$PTO=\frac{1}{2}\frac{\omega^6 m_z^2 f^2\eta_t}{\left(C-D\eta_i\right)^2+\omega^2\left[E\eta_t+B\lambda_l\right]^2} \tag{5.14}$$

Derivative to get:

$$\eta_{i_{\text{bett}}}=\sqrt{\frac{C^2+\omega^2B^2\lambda_1^2}{D^2+\omega^2E^2}} \tag{5.15}$$

To get the maximum power Plug in the data, get the best

$$\eta_{i\_\text{best}}=37193.8119\ \mathrm{N}\cdot\mathrm{s/m} \tag{5.16}$$

The corresponding maximum power is 229.3339W.

Numerical solution of the first question The numerical solution adopts the trapz trapezoidal method to perform the numerical integration operation. By dividing a region into trapezoids containing multiple regions that are easier to calculate, the approximation of the integral within the interval is calculated.

Considering that the wave energy device does not reach a steady state of sag for some time after the initial state, the sampling interval should start at an appropriate moment.

The integral interval is 0-400s, the sampling interval is 200-400s, and the step size is 0.2s. When n is within the range of [0,100000], the function relation of PTO with respect to n is shown in the figure:

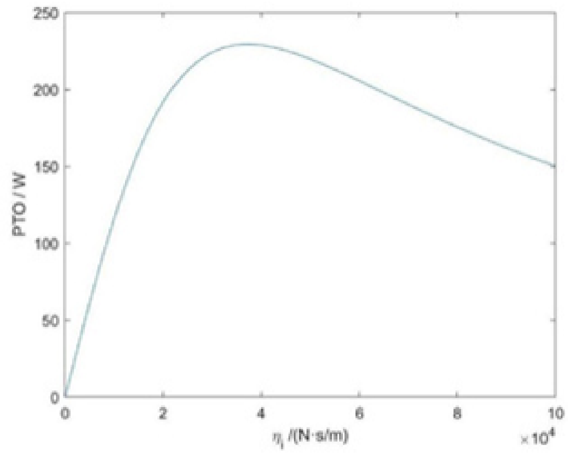


Figure 3: 3PTO function with respect to m Figure 1.

It can be seen from the observation of the image that the peak value of PTO appears in the interval where n is located at [30000,40000], and the graph can be drawn in this interval as follows:

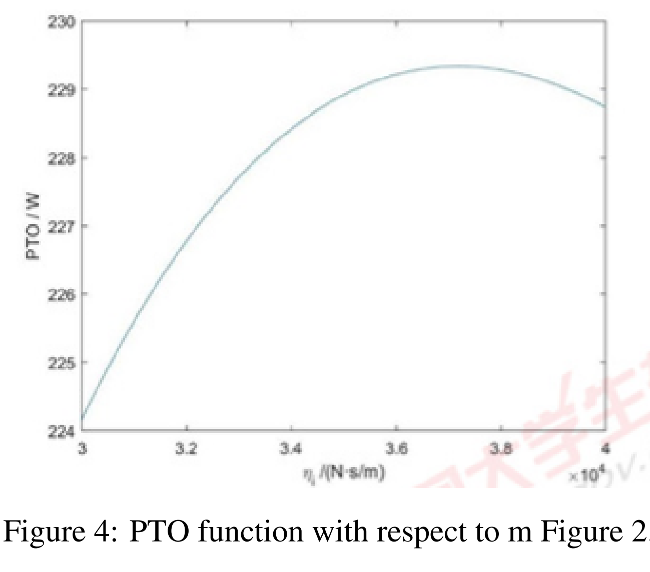


Figure 4: PTO function with respect to m Figure 2.

After sampling the peak value, the maximum power is 229.497276437679W, and the damping coefficient is 3.72x10N-s/m.

5.3.2 Solution of the second minor question model (damping coefficient is proportional to the power of the absolute relative velocity of the float and the oscillator)

The model is established in the same way as in 5.2.1. When the damping coefficient is proportional to the power of the absolute relative velocity of the float and the oscillator, set the power index as α and the proportionality coefficient as n, then the relationship can be expressed as:

$$PTO = \frac{1}{2}\eta\omega^2 \left|x_1 - x_2\right|^{2+\alpha} \tag{5.17}$$

Regarding = 1/2 | x - x1 as power for alpha is unknown, it is difficult to calculate the analytical solution of the equation.

In 5.2.1, the comparison shows that, by the result 229.49728W obtained by trapz Trapz method after integration and enumeration is very close to the 229.33394W obtained by the analytical solution obtained by the previous mathematical method. Therefore, it can be considered that the results obtained by using the numerical method are quite accurate. When the analytical solution is difficult to be obtained, the numerical method can be used to obtain the results.

Consider using numerical solution to solve this problem.

Using the same idea as 5.2.1, the integral interval is 0-400s, the sampling interval is 200-400s, and the step size is 0.2s. MATLAB is used for two-layer traversal, and the PTO function about the scaling coefficient η and the power exponent α is made.

After visualization, when η is in the range of [0,100000] and α is in the range of [0,1], the functional relationship between PTO and α is shown as follows:

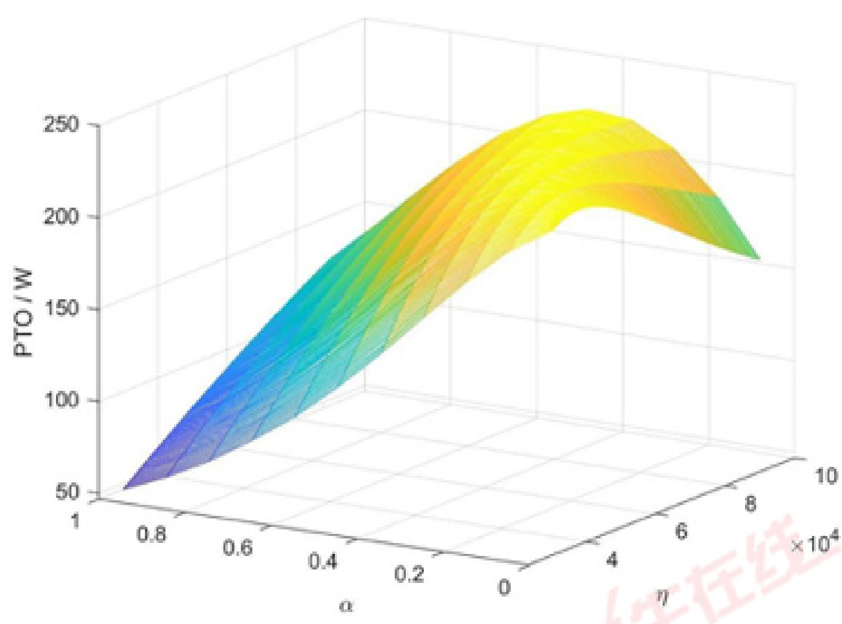


Figure 5: PTO function with respect to proportionality coefficient and power exponent α FIG. 1

It can be seen from the image that PTO peaks when η is in the range of [20000,100000] and α is in the range of [0.3, 0.5].

5.4 Establishment and solution of Problem 3 model

5.4.1 Determination of rotation amount of each part

The area of each part of the float shell is needed to calculate the moment of inertia and the distance between the center of mass under the balance drag, so the area of the following three parts should be calculated first. Side area of float cylinder part:

$$S_1 = \pi R_f h_y = 6\pi \tag{5.18}$$

Area of the cone section of the float:

$$S_2 = \pi R_f \sqrt{R_f^2 + h_z^2} = \frac{\sqrt{41}}{5}\pi \tag{5.19}$$

The area of the float bucket cover:

$$S_3 = \pi R_f^2 = \pi \tag{5.20}$$

In order to simplify the expression, in the following formulas are directly represented by its value instead of symbol, no longer appear area. The distance between the vibrator's center of mass and the rotation axis O in equilibrium state:

$$h_1 = L_0 + \frac{1}{2}H_z - \frac{m_z g}{k} \tag{5.21}$$

The distance between float center of mass and rotation axis O in equilibrium state:

$$h_2 = \frac{1}{35+\sqrt{41}}\left[\sqrt{41}\left(\frac{-h_z}{3}\right) + 30\left(\frac{h_y}{2}\right) + 5h_y\right] \tag{5.22}$$

The distance between the center of mass of the oscillator and the float and the rotation axis O in equilibrium state:

$$h_3 = \frac{1}{m_z + m_f}\left(m_z h_1 + m_f h_2\right) \tag{5.23}$$

The moment of inertia of the vibrator with respect to the horizontal axis of the vibrator's center of mass:

$$I_z = \frac{1}{4}m_z R_z^2 + \frac{1}{12}m_z H_z^2 \tag{5.24}$$

The moment of inertia of the float with respect to the horizontal axis of the float's center of mass:

$$I_f = \frac{m_f}{35+\sqrt{41}}\left[\sqrt{41}\left(\frac{h_z^2}{6} + \frac{R_f^2}{4} + \frac{2}{3}h_z h_2\right) + 30\left(\frac{R_f^2}{2} + \frac{h_f^2}{12}\right) + 5\left(\frac{R_f^2}{4} + \left(h_y - h_z\right)^2\right)\right] \tag{5.25}$$

5.4.2 Solution of heave motion and pitch motion

Based on the preliminary estimation of the values given in the attached table, it is assumed that the angular displacement of the float and oscillator is small, so the influence of pitch motion is not needed to be considered when calculating the heave motion of the float. It follows that:

$$\begin{cases} \left(m_f + \mu\right)\ddot{x}_1 + \lambda_1 \dot{x}_1 + \rho g \pi R_f^2 x_1 = f\cos\omega t + kx_2 + \eta_t \dot{x}_2 \\ m_z \ddot{x}_z + y_t \dot{x}_2 + kx_2 = m_z\left[g\left(1-\cos\theta_2\right) - \ddot{x}_1\cos\theta_2 + \ddot{\theta}_1 h_3 \sin\left(\theta_2 - \theta_1\right) - h_3\dot{\theta}_1^2\cos\left(\theta_2 - \theta_1\right)\right] \end{cases} \tag{5.26}$$

Where, the definition of x1 is the same as in 5.1, indicating the displacement of the float in the vertical direction, and the positive direction is set as vertical upward; The definition of x2 is slightly changed (as shown in FIG. 9). It is defined as the displacement of the oscillator along the central axis in a translational non-inertial system with the rotating axis O as the origin, and the positive direction is up along the central axis, which can be approximately understood as the relative displacement of the float and the oscillator. It is not difficult to find that the relative displacement x2 between the float

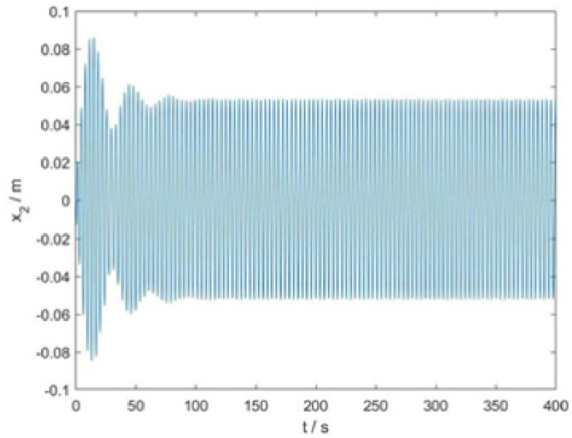


Figure 6

and the oscillator is a small quantity. Therefore, when calculating the pitch motion of the float, it is advisable to consider the float and the oscillator as a whole to calculate the displacement, velocity, angular displacement and angular velocity of the float. After obtaining the displacement, velocity, angular displacement and angular velocity of the float, the motion of the oscillator in this system can be calculated by taking the rotating axis 0 as the origin. Take the deflection Angle of the float as 01, the deflection Angle of the oscillator (equal to the swing Angle of the central axis) as 02, the displacement of the float x1 and the displacement of the oscillator x2, and the positive direction is shown in the figure.

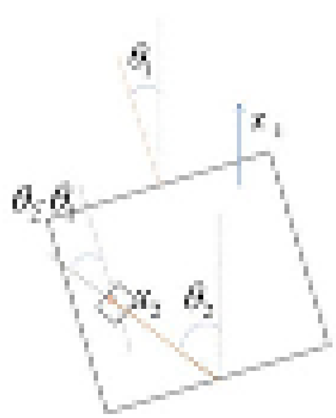


Figure 7

Considering the force and torque balance of the analysis float and oscillator in this system, the following simultaneous equations can be obtained:

$$\begin{cases} \left[I_s + I_f + M_s(h_3 - h_1)^2 + M_f(h_3 - h_2)^2\right]\ddot{\theta}_1 + \lambda\dot{\theta}_1 + C_w\theta_1 = L\cos\omega t \\ \left[I_s + m_I(h_1 + x_2)^2\right]\ddot{\theta}_2 + C_k(\theta_2 - \theta_1) + \eta_j\left(\dot{\theta}_2 - \dot{\theta}_1\right) \\ = m_1\left[(g + \ddot{x}_1)\sin\theta_2 + h_3\dot{\theta}_1^2\sin(\theta_2 - \theta_1) + \ddot{\theta}_1 h_3\cos(\theta_2 - \theta_1)\right](h_1 + x_2) \end{cases} \tag{5.27}$$

The functional function in MATLAB is used to solve the non-rigid differential equation. See the attached result3.xlsx for the results. It is not difficult to find that all are small quantities, satisfying the

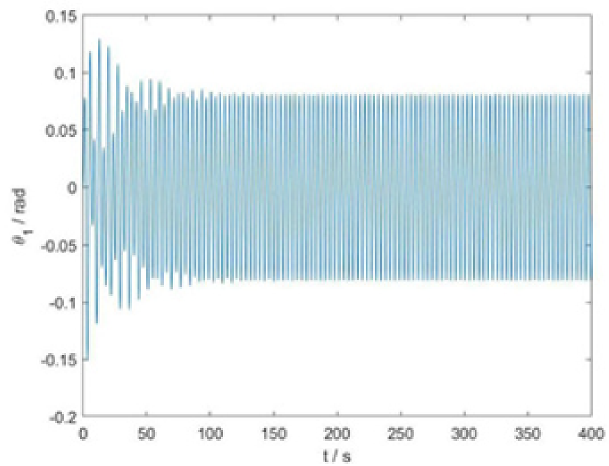


Figure 8

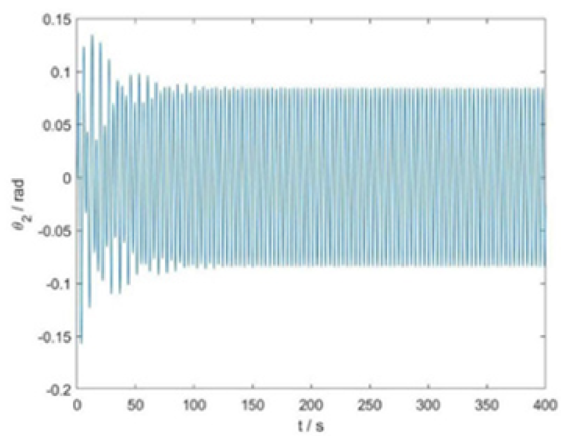


Figure 9

initial assumption.

The vertical displacement and velocity and pitch angular displacement and velocity of the float and vibrator at 10s, 20s, 40s, 60s and 100s are shown in Table 3 and Table 4, respectively. The unit of each value is the standard unit.

Table 3

表 3 浮子垂荡位移和速度以及纵摇角位移和角速度

t	x_1	v_1	θ_1	ω_1
10	-0.52313	0.98326	-0.05299	-0.13555
20	-0.69838	-0.24489	0.12253	0.02588
40	0.37500	0.76586	-0.05116	-0.06105
60	-0.31531	-0.72160	0.07359	0.09148
100	-0.04792	-0.94736	0.03163	0.13157

Table 4

表 4 振子垂荡位移和速度以及纵摇角位移和角速度

t	x_2	v_2	θ_2	ω_2
10	-0.59350	1.05454	-0.05529	-0.14150
20	-0.76358	-0.28834	0.12784	0.02742
40	0.39883	0.85610	-0.05293	-0.06373
60	-0.33537	-0.79616	0.07664	0.09485
100	-0.04062	-1.03652	0.03294	0.13739

Similarly, it is not difficult to find that the maximum amplitude of x1 is smaller than that of h0, which satisfies the assumption that the water surface will not exceed the float top cover and that part of the float conical shell will not surface.

Similarly, it is not difficult to find that the maximum amplitude of x1 is smaller than that of h0, which satisfies the assumption that the water surface will not exceed the float top cover and that part of the float conical shell will not surface.

5.5 Establishment and solution of Problem 4 model

The idea behind problem four is the same as problem two. Since it was also difficult to obtain the analytical solution, the trapz trapezoidal method was adopted to perform the numerical integration operation, calculate the approximate value of the integral within the interval, and obtain the numerical solution.

The global traversal diagram of PTO about n and n is as follows: After studying the data, it is found that PTO changes monotonically with respect to n, so the value of rotation damping coefficient n when the maximum PTO is obtained is 100000. Then, for n, take 100000 local traversal, solve. The local picture is as follows:

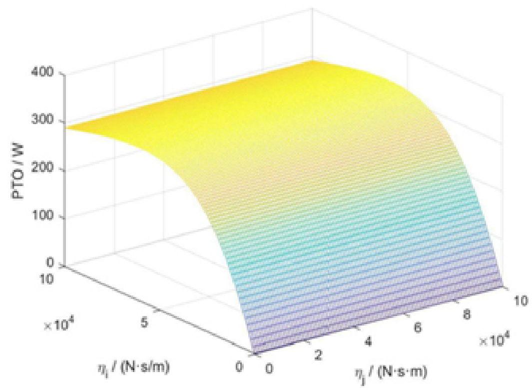


Figure 10

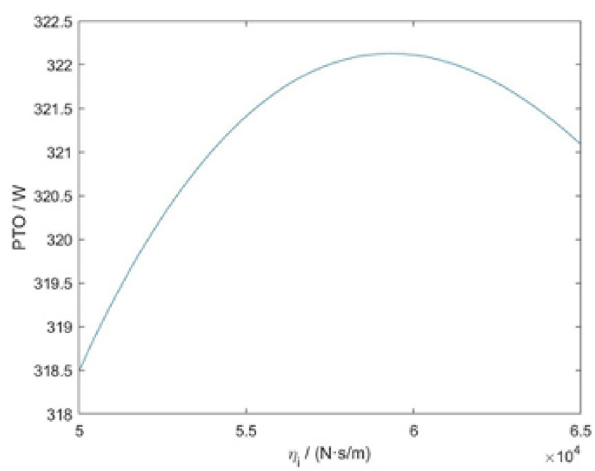


图 13 PTO 关于η_i的局部遍历图

Figure 11

6 Model analysis and test

6.1 Model Test

6.1.1 Test that the top of the float will not be submerged and the conical part will not be above the water surface

The float displacement data obtained from the four problems are all smaller than the height (1.00m) between the top and the water surface at the starting equilibrium. The hypothesis is true.

6.1.2 Test of "The system tends to steady state within a limited time"

Using the signal analyzer of MATLAB, the frequency spectrum of the vertical displacement data in question 1 is analyzed and the power spectrum is obtained. The second half of the movement is taken (tends to be stable), and the signal circle frequency is

$$0.0891 \times 5 \times \mathrm{pi} = 1.399\mathrm{rad/s} \tag{6.1}$$

Approximately equal to the frequency of the excitation circle. The hypothesis is true.

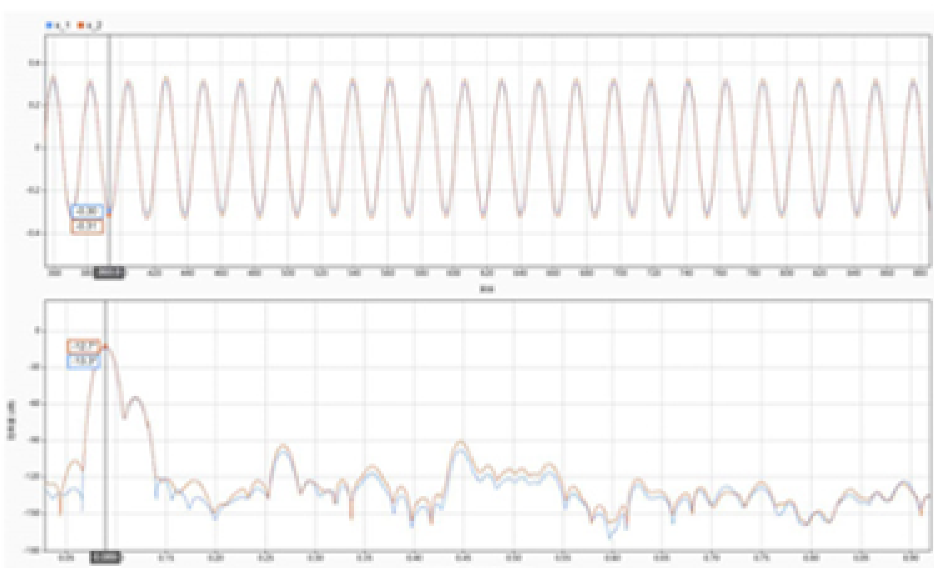


Figure 12: Frequency analysis.

6.1.3 The test of "The relative displacement of the oscillator and float in vertical motion can be regarded as a small quantity"

According to the data of the four questions, the ratio of relative displacement to float heave displacement is within the range of [0.09, 0.10], which can be regarded as a small quantity.

6.1.4 Test of “The angular displacement of the pitch motion of the float and oscillator is small”

According to the data in problem 3, the maximum angular displacement of the float is 6.88°, and the maximum angular displacement of the oscillator is 7.73°, both of which can be regarded as small angles.

6.2 Model stability analysis

6.2.1 Stability analysis of wave excitation force by the model

The displacement of the oscillator relative to the float is calculated for different wave excitation forces. It can be concluded that the displacement of the oscillator relative to the float satisfies the

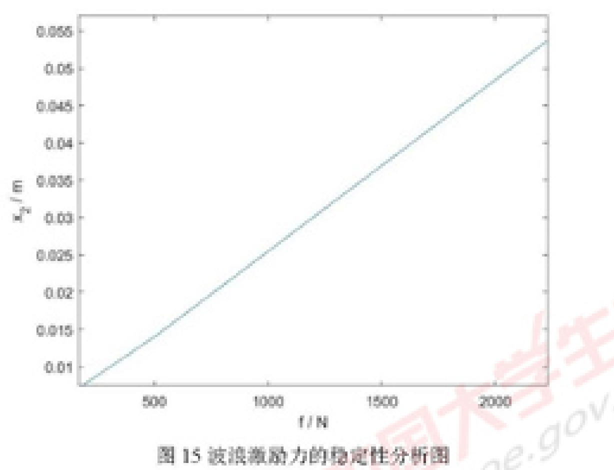


Figure 13: Stability analysis.

assumption of small quantity when the wave excitation force changes within a certain range.

6.2.2 Stability analysis of wave excitation moment by the model

The angular displacement of the oscillator relative to the float is calculated for the wave excitation moments of different sizes. It can be concluded that the wave excitation moment varies within a certain range, and the displacement of the oscillator relative to the float satisfies the assumption of small quantity.

6.2.3 Stability analysis of the model for torsion spring stiffness

For different sizes of torsion spring stiffness, calculate the output power.

It can be seen that the torsion spring stiffness changes in the field of the data given in the topic, the model is valid, and the system output is normal. When the torsion spring stiffness is less than a certain value, the model collapses.

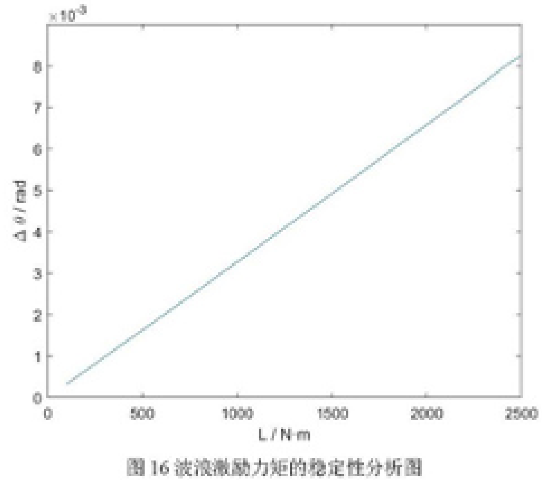


Figure 14: Stability analysis.

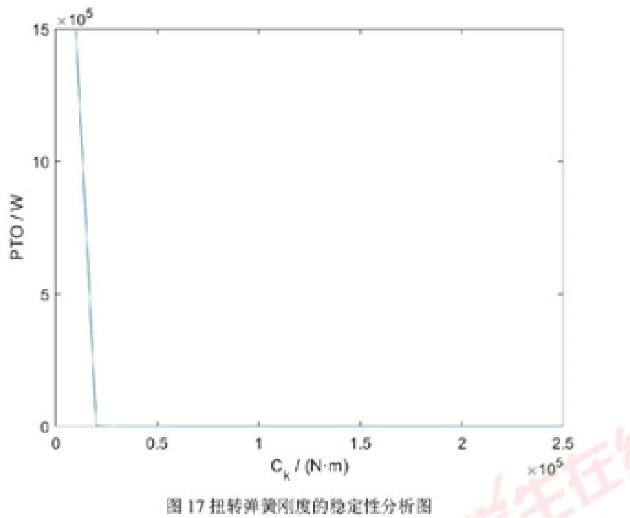


Figure 15: Stability analysis.

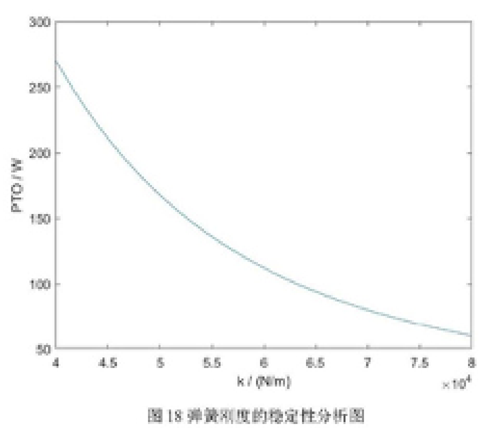


Figure 16: Stability analysis.

6.2.4 Stability analysis of the model to the spring stiffness

For different sizes of spring stiffness, calculate the output power. It can be seen that the spring stiffness changes in the field of the data given in the topic, the model is valid, and the system output is normal.

7 Evaluation of the Model

7.1 Advantages of the Model

1. High originality, all models in this paper are independently established;
2. In problem 2, two solutions, namely analytic solution and product decomposition, are given with higher accuracy;
3. In Problem 3, the model is reasonably simplified to avoid the discussion of the forces (moments) on the rotating and central axes;
4. In problem 3, the translational non-inertial frame is cleverly selected to make the model easy to understand and calculate;

7.2 Shortcomings and improvements of the model

1. The model ignores the influence brought by the volume, mass and friction of the secondary components in the wave energy device, so it is different from the actual situation in production and life;
2. The models of Problem 3 and Problem 4 are built on the basis that the pitch Angle of the float and oscillator is small and the relative displacement is small. When the wave is large, there may be large errors. See ”VI. Analysis and Test of the Model” for details;
3. In Problem 2 and Problem 4, integral method is adopted to solve numerical solutions, and the formula of analytical solutions is not given, which may slightly reduce the accuracy. Meanwhile, recalculation is required when the model is extended and used;

Possible improvements are:

1. Influence of volume, mass and friction of minor components;
2. Fully analyze the force (moment) at the rotating axis and central axis, analyze the oscillator and float separately, and relist the differential equation;
3. Give more accurate integration algorithm as far as possible, or find higher theoretical solutions to analytical solutions

7.3 Model Extension

In this paper, the energy conversion process of a wave energy device is analyzed theoretically and a mathematical model is established. By analyzing the different values of linear damping coefficient and rotary damping coefficient, the damping coefficient and its form which maximizes the output power are obtained. Similarly, the model can be used to analyze when the spring stiffness and torsional spring stiffness, or the mass ratio of float and oscillator change, to seek the spring stiffness or mass ratio that maximizes the output power, so as to improve the conversion efficiency of the wave energy device.

8 Reference

[1] LI Huibin, Vibration Theory and Engineering Application. First edition, Beijing Institute of Technology Press, September 2006.

[2] ZHENG Xiongbo, ZHANG Liang, MA Yong, Hydrodynamic Calculation and Energy Conversion Characteristic Analysis of Double Floating Body Wave Energy Device [Knife Science & Technology Review, 2014, 32 (19) : 26 – 30.

9 Source Code

Code for Question 1:

```
clc
%Code for Question 1
m_f=4866;
mu_l=1335.535;
eta_i=10000;
lambda_i=656.3616;
k=80000;
rho=1025;
g=9.8;
R_f=1;
f=6250;
omega=1.4005;
m_z=2433;
x_0=[0 0 0 0];
tspan=[0:0:2:180];
%%num
[t,x]=ode45(@(t,x)funcl(t,x),tspan,x_0);
%%plot
plot(t,x(:,1),'-',t,x(:,3),'-');
```

Code for Question 2:

```
function dxdt = funcl(t,x)
    m_f = 4866;
    mu_l = 1335.535;
    eta_i = 10000;
    lambda_i = 656.3616;
    k = 80000;
    rho = 1025;
    g = 9.8;
    R_f = 1;
    f = 6250;
    omega = 1.4005;
    m_z = 2433;
    dxdt = zeros(4,1);
    dxdt(1) = x(2);
end
f = figure(1);
stem(range_n, xb_1, 'o', 'LineWidth', 1, 'MarkerSize', 8);
hold on;
stem(range_n, xb_2, '*', 'LineWidth', 1, 'MarkerSize', 8);
stem(range_n, xb_3, '^', 'LineWidth', 1, 'MarkerSize', 8);
plot(range_n, xb_1, ':b'); % for better illustration of xb_1
hold off;
xlabel('$n$', 'Interpreter', 'LaTeX', 'FontSize', 12);
ylabel('$x_b[n]$', 'Interpreter', 'LaTeX');
legend('$f=0.0625$', '$f=0.4375$', '$f=0.5625$', 'Interpreter', 'LaTeX');
saveas(f, '2_t.eps', 'epsc');
```

Code for Question 3:

```
% Non-Central Chi-Square Distribution
y = @(x, K) (K + 1) * exp(-K - (K + 1) * x) .* besseli(0, 2 * sqrt(K * (K + 1) * x));
x = 0:0.01:5;
y1 = y(x, 0);
y2 = y(x, 3);
y3 = y(x, 80);
figure(1);
plot(x, y1, 'LineWidth', 2);
hold on;
plot(x, y2, 'LineWidth', 2);
hold on;
plot(x, y3, 'LineWidth', 2);
title('Non-Central $\chi^2$ Distribution with $\Omega_p=1$', 'Interpreter', 'LaTeX');
legend('$K=0$', '$K=3$', '$K=80$', 'Interpreter', 'LaTeX');
fig = gcf;
fig.Position(3:4) = [500, 300];
```