Problem Chosen	2023	Team Control Number
A	MCM/ICM Summary Sheet	2301663

Drought-Stricken Plant Communities

Drought is one of the most widespread and destructive natural disasters to human production activities in the world, and it is of great significance to the development and succession of plant communities.This paper aims at modeling drought effects on plant community succession to investigate which plant communities are more resistant to drought and the interaction between plants and environment under drought conditions.

Three models are mainly established: Model 1: Multi-species Growth Model; Model 2: Drought Frequency Model; Model 3: Drought Impact Model.

Before all the models are established, we analyzed the mechanisms of drought effects on plants in order to better study the impact of drought on plant communities. Then we consulted papers on plant interactions to make the analysis more comprehensive.

For Model 1: we used a auto-adaptive **Population Growth Model** based on typical Logistic models to describe the growth of individual species. We introduced the correlation factor to normalize the dimension and combine simulations for multiple species to obtain the growth models of different species.We found that when a variety of organisms exist in a community at the same time, the environmental capacity of species with weaker viability will be improved.

For Model 2: we established **Drought Frequency Model** by constructing the joint distribution function of drought variables to analyze the frequency and extent of drought.

Through the data of soil moisture percent in south of America, we use the kernel density estimation method to fit the standardized soil moisture index (SSMI).Through the Run Theory, the frequency of drought events of different degrees can be obtained.

For Model 3: we established **Drought Impact Model**. In this model, we built a function affected by drought conditions, and the parameter r represents the degree of system disturbance by drought. For a community composed of multiple species, the function of the combination coefficient of positive and negative correlation is used for fitting.

When the drought condition is imposed on the growth curve of the population, it can be clearly seen that the growth curve of the population fluctuates, and the magnitude of the fluctuation reflects the extent to which the community is affected by drought.See **Section 5.2** for the results.

Additionally, we have taken into account the effect of different degrees of drought and the influence of habitat reduction and pollution.We added noise interference to the original drought impact model to represent the adverse environmental factors of pollution and habitat loss, compared with the original image under the same drought conditions, and finally drew a conclusion. See **Section 6.3** for the results.

Finally, sensitivity analysis takes the lag effect of drought,drought-stricken of different frequency and intensity,and pollution of different extent into account.we found that species are very sensitive to pollution.If pollution's intensity more than a constant,all species will gradually die.

Key Words: Logistic Model,Species Growth,Drought Frequency Analysis ,Kernel Density Estimation,Run Theory,Nonlinear Systems,Noise Interference.

Contents

1 Introduction

1.1 Background

Drought is widely distributed and one of the most destructive natural disasters to human production activities in the world. It has the characteristics of slow occurrence, long duration and far-reaching impact on time and space.Different plants have a variety of adaptations to drought due to difference in water requirements of plants.At the same time, due to the difference in plant types and plant numbers in the plant community, the ability of the plant community to resist drought will also vary.

For this reason,considering the impact of the difference in the soil's ability to hold water changes in water use efficiency and interaction of different plants,the establishment of models to analyze the relationship of drought adaptability with respect to the number of species in a plant community is useful for exploring the long-term viability of plant community.

1.2 Restatement of the Problem

This paper aims to build a proper model to explore and better understand the relationship of drought adaptability with respect to the number of species in a plant community.

- **Establish a model** to predict how a plant community changes over time. As the plant community is exposed to weather cycles which are various and irregular, the model ought to consider times of drought when precipitation should be abundant. What's more, in communities with only one species of plant, the generations that follow are not well adapted to drought conditions, so the model ought to account for interactions between different species during cycles of drought.

- **Draw conclusions** after applying our model to the long-term interactions of a community of plants and the larger environment.

  ⇒ Figure out the number of different plant species are required for the community to benefit.

  ⇒ The influences of the types of species in the community to the results.

  ⇒ Solve for what should be done to ensure the long-term viability of a plant community and the impacts on the larger environment.

- **Analyze the changes** in model results according to the following situations:

  ⇒ As the number of species grows whether the number of different plant species will change.

  ⇒ A greater frequency and wider variation of the occurrence of droughts in future weather cycles or droughts are less frequent.

  ⇒ Other factors such as pollution and habitat reduction.

- **Sensitivity Analysis** for droughts from Gross Primary Production and evapotranspiration.

1.3 Our Work

The problem requires us to work out a proper model to figure out the relationship between drought adaptation and the number of species in a plant community.Considering the occurrence of drought frequency, the interaction of multi-species growing together and the impact of drought on multi-species plant communities our main work is as follows:

- In order to describe the growth of biological populations, based on the typical Logistic model, we use a self-defined adaptive population growth model to describe the growth of a single population under the assumptions in this paper.
- In order to increase the complexity of the plant community, we selected four plants as candidate species for this simulation, representing trees, shrubs, herbs, and mosses that are common in general ecosystems. By simulating this plant community, the effect of drought on plant community succession is studied.
- In order to develop a model for the analysis of drought frequency, we use the kernel density estimation method to fit the standardized soil moisture index to divide the drought level, and identify drought events and extract corresponding drought variables based on run theory. Therefore, we can calculate the probability of the occurrence of the event under different drought degrees.
- According to the probability obtained by the above model, create a drought event in the form of random number generation, which impacts the plant community. Adjust the type of plant community, the amount of vegetation, and observe the results. Then add pollution items to make vegetation more likely to die, observe and analyze the results.
- The sensitivity of the model was analyzed from the Hysteresis effects of drought on plant communities.

2 Preparation of the Models

2.1 Assumptions and explanations

In order to build the mathematical model to solve the problems, we make the following supplementary assumptions on the basis of the problem, following a reasonable explanation for each assumption.

- **Assumption 1:Only the effects of interspecific competition and drought are considered**

⇒**Explanation:** Since there are many factors affecting plant growth in the natural environment, we adopt the method of controlling variables to keep the light, oxygen and other conditions consistent, and only consider the effects of interspecific competition and drought on the growth of different types of plants, which can be seen more clearly Adaptation of different plant communities to drought and disturbance of interspecific competition.

- **Assumption 2:Allocation of Environmental Resources.**

⇒**Explanation:**In order to simplify the model, we assume that environmental resources are limited, and each individual in the same population allocates resources equally.

- **Assumption 3: Individual consistency.**

⇒**Explanation:**For the same population, the size and age of individuals in the population are not distinguished.This can reduce the influence of irrelevant factors and simplify the establishment of the model.

- **Assumption 4: Select four representative plants.**

⇒**Explanation:** The evolution process of the plant community is very complicated, so we chose four plants to represent various plant types. The effects of drought on different species were characterized by studying the response of these four plants to drought.

- **Assumption 5: No consideration of species invasion.**

⇒**Explanation:** Considering the complexity of biological communities in nature, we assume that different populations are evenly distributed in space and there is no species invasion.

- **Assumption 6: The species' ability to tolerate drought does not change.**

⇒**Explanation:** The drought tolerance of a single species is a limited fixed value, which does not change with the population size.

- **Assumption 7: Initial Soil Condition Assumptions.**

⇒**Explanation:** The initial soil conditions are good, which can meet the conditions for the growth of the lowest-level plant populations.

- **Assumption 8: Simplification of the relationship between growth rate and density.**

⇒**Explanation:** We believe that the change of the growth rate is a linear function relationship with the density, and there is an equilibrium density in the population.

2.2 Notations

The primary notations used in this paper are listed in Table 1.

Table 1: Notations

Symbol	Description
KDE	Kernel Density Estimation
PDF	Probability Density Function
$SSMI$	Standardized Soil Moisture Index
CDF	Cumulative Distribution Function
$USDM$	United States Drought Monitor

2.3 The Data

2.3.1 Data Collection

The problem doesn't give us data, so we need to collect data to build a model. To analyze the problem, we need to collect data on Standardized Soil Moisture Index, as well as the Gross Primary Production of different plant species. After the data is collected, certain processing and analysis are carried out to prepare for the subsequent model establishment.

The official websites of the National Agricultural Statistics Service and ESACCI website provide us with a lot of data about the Standardized Soil Moisture Index. And we collected data of the production of different plant species from the paper researching them. Detailed data sources are shown in the Table 2 below. (Additional data sources are included in the Reference.)

Table 2: Partial Data Websites

Database Names	Database Websites
the National Agricultural Statistics Service	https://www.usda.gov/wps/portal/usda/usdahome
US National Bureau of Statistics	https://www.fedstats.gov/
ESACCI Soil Moisture website	https://esa-soilmoisture-cci.org/

2.3.2 Data Processing

For the data of Standardized Soil Moisture Index for different regions, we need to fit the models for drought prediction. Then we established the growth model of the plant community by processing the increase data of the plant population. More details of data processing are shown in Figure 1 below.

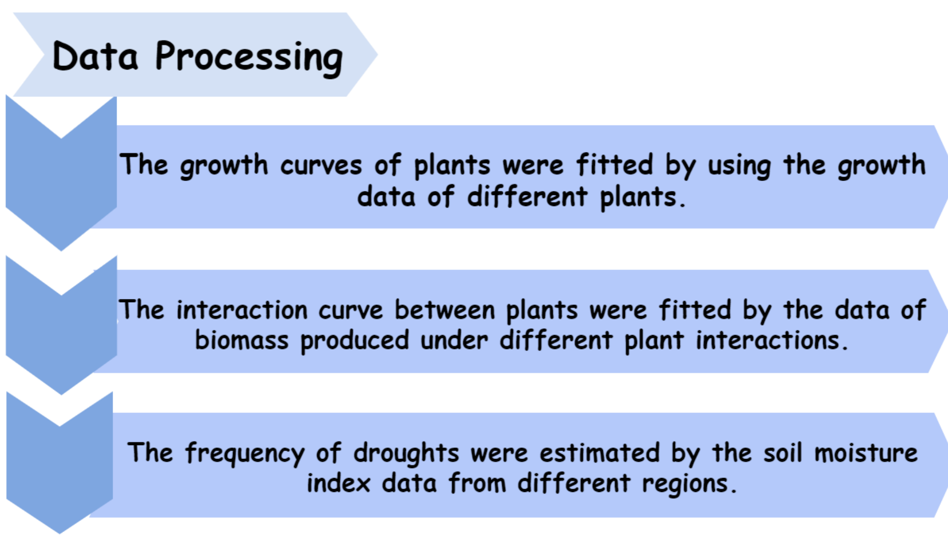


Figure 1: Data processing.

3 Model 1: Multi-species Growth Model

3.1 The Establishment of Multi-species Growth Model

3.1.1 Growth model of single species under natural conditions

In recent years, many different biological growth models have been proposed in the field of biology, which try to be suitable for the simulation of biological growth in different environments.

In an environment with limited living space and natural resources, the growth of any biological population is restricted by density. The classical model to quantitatively describe this dynamic process is Logistic Equation which is a famous population growth model put forward by mathematical biologist Pierre-Francois Verhulst. Its application extends from population growth model to many fields, widely used in biology and so on. However, the linear assumption of density-dependent effect in the model tends to bias the simulation results.

Based on the typical Logistic model, we proposed a self-defined adaptive population growth model to describe the growth of a single population under specific assumptions.

We defined $N(t), N_m$ and r to represent population size, environmental tolerance and potential specific growth rate parameters respectively, then the Logistic model could be expressed as

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = rN(t)\left(1 - N(t)/N_m\right) \tag{3.1}$$

The solution to this equation is as follows

$$N(t) = N_m / \left[1 + \left(\frac{N_m}{N_0} - 1\right) e^{-rt}\right] \tag{3.2}$$

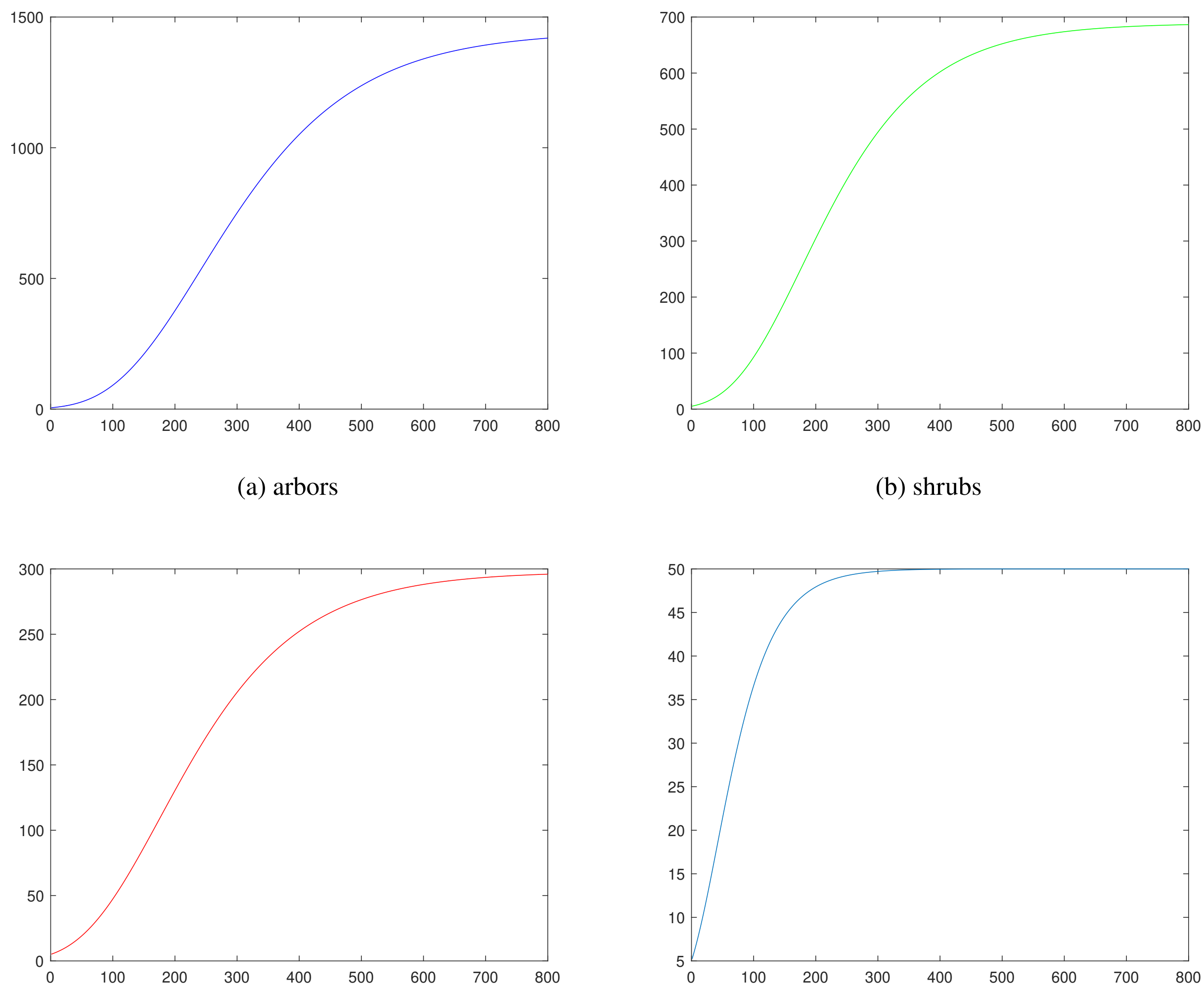
Where the image of the density-dependent effect function $f(x) = 1 - \frac{N_t}{N_m}$ is a lower-right ray.In fact, due to the biological characteristics of the population and the complexity of the environment, the true linear restriction effect is very rare, but the different degrees of convex or concave curve restriction effect is more common.We nonlinearize the linear constraint function, which is as follows

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = rN(t)\left(1 - (\frac{N(t)}{N_m})^s\right) \tag{3.3}$$

Where s is the density-dependent parameter, and its value range is $0 < s < +\propto$. When $s \to +\propto$, models tend to grow exponentially. When $s \to 0$, the population tends to stay the same. Thus, the solution of the nonlinear constrained population growth mathematical model can be written as

$$N(t) = N_m / \left[1 + \left(\frac{{N_m}^s}{{N_0}^s} - 1\right) e^{-srt}\right]^{\frac{1}{s}} \tag{3.4}$$

We selected four plant species as candidates for this simulation, representing four common species of arbors, shrubs, herbs and mosses in general ecosystems. The results of the first model, which contain the situations of their growth are shown in figures below.



(a) arbors (b) shrubs

(c) herbs (d) mosses

Figure 2: Single-species growth curves.

3.1.2 Growth model of multiple species under natural conditions

For multiple species, due to the interaction between multiple species during drought, a model different from 3.4 should be adopted, which is as follows

$$Y = \sum_{i=1}^{n} \alpha_i N_i(t) - \sum_{j=1}^{n} \beta_j N_j(t) \tag{3.5}$$

where $\alpha_i, \beta_j > 0$ represents the change in the growth rate of species B when species A is present. α_i are the combined growth coefficient of species with positive correlation, and β_j are the combined growth coefficient of species with negative correlation. We represent α_0, α_1, α_2, α_3 as the combined growth coefficients of moss, herb, shrub and arbor respectively.After querying the information, we found a positive growth relationship for the four species. So we normalized the dimensions and made $\alpha_0 = 3$, $\alpha_1 = 2$, $\alpha_2 = 1$, $\alpha_3 = 1$.

We performed combined simulations on the four candidate species selected above, and the results are shown in the figures below.

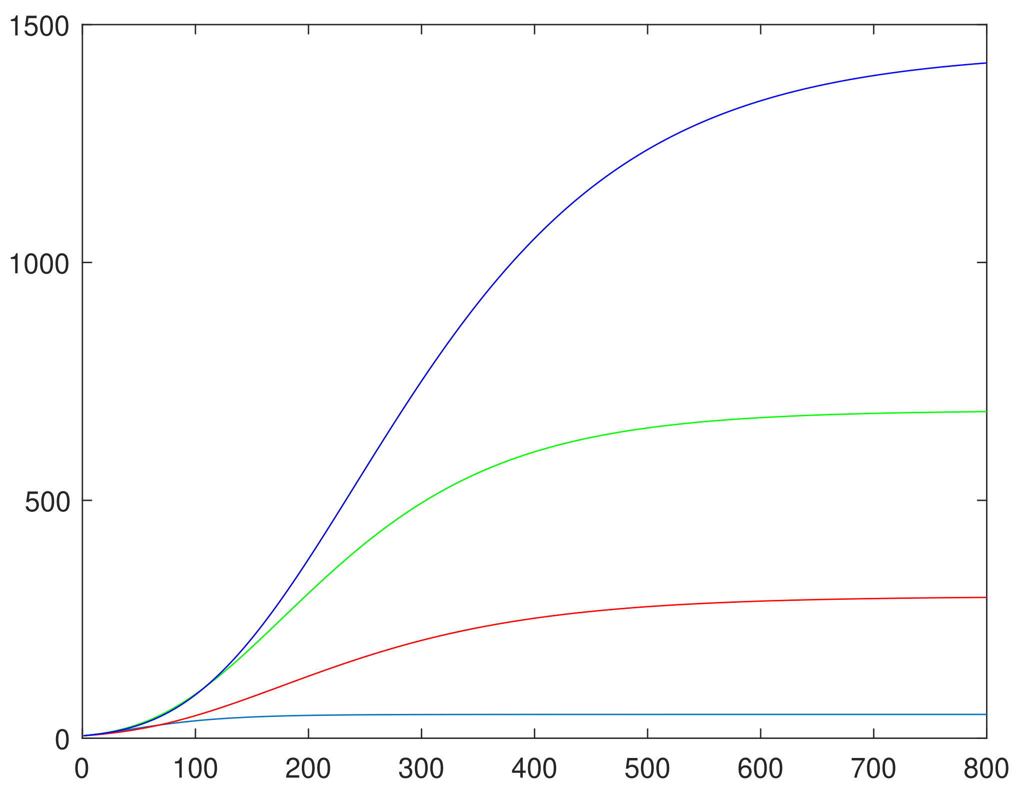


Figure 3: Growth curves of four species combinations.

The top curve represents the growth curve of the arbor in the combined simulation, the second curve from top to bottom represents the growth curve of the shrub, the third curve from top to bottom represents the growth curve of the herb, and the bottom curve represents the growth curve of the moss in the combined simulation.

3.2 Summary

When doing simulations of single species growth, different species grow at different rates and their growth curves is pretty different.

The growth model of multiple species under natural conditions has accounted for interactions between different species. Since the four species show a positively correlated growth relationship, so they all reach equilibrium earlier.

4 Model 2: Drought Frequency Model

4.1 Basic Ideas

Drought frequency analysis can calculate the occurrence probability and recurrence period of a specific drought event, which is one of the effective methods for drought risk assessment.

We used the standardized soil moisture index and the run course theory to identify the regional scale drought events, and extracted the corresponding drought duration, intensity and peak drought as variables of drought events. Then in order to quantify drought risk, we used the non-parametric kernel density estimation method to construct the joint distribution function of drought variables for drought frequency analysis.

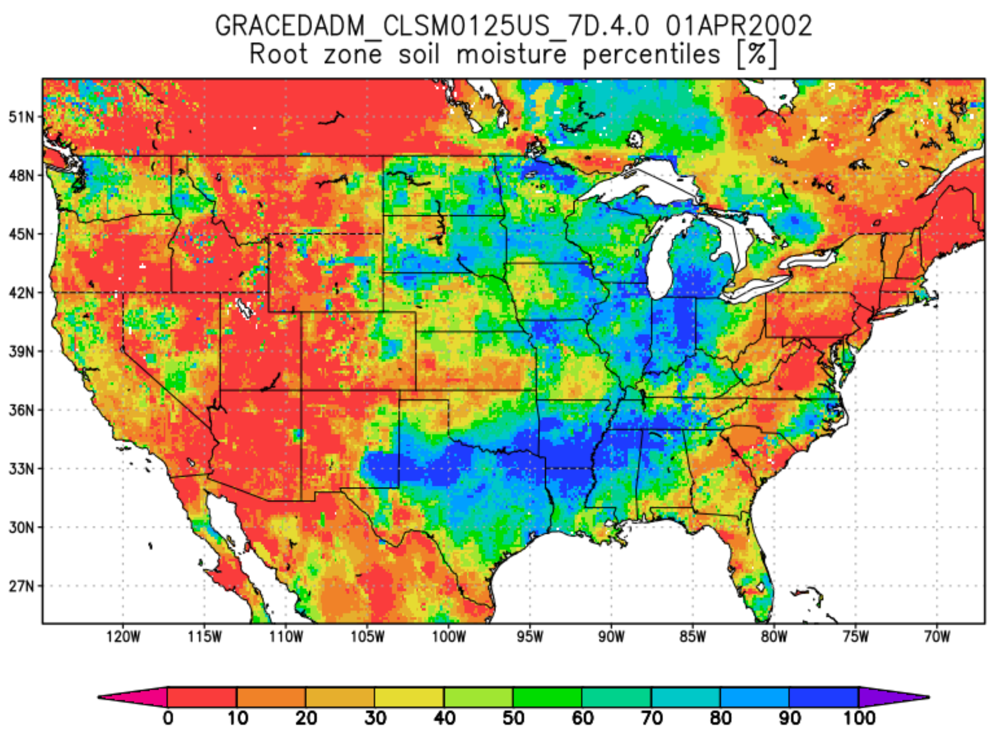


Figure 4: A map of soil moisture percentiles of America.

4.2 The Establishment of Drought Frequency Model

4.2.1 Standardized Soil Moisture Index

The standardized soil moisture index (SSMI) was proposed by Carrao et al. (2013) and modified by Samaniego et al. (2013), which is often used for drought monitoring.

Since soil moisture data presents the characteristics of multi-peak distribution (Vidal et al. 2010), using Kernel Density Estimation(KDE) can better fit the distribution of soil moisture.

For each month, the Probability Density Function (PDF) of soil moisture can be expressed as follows

$$f(x) = \frac{1}{Nh}\sum_{i=1}^{N} K\left(\frac{x-x_i}{h}\right) \tag{4.1}$$

where N represents the total number of years in the study period; the soil moisture x in N years in total was divided from x_{min} to x_{max} equidistantly, and x_i is the average value of soil moisture in each equidistant interval; $K(x)$ is the kernel function; h is bandwidth. The form of kernel function has less influence on the fitting effect of KDE (DiNardo et al. 2001; Rajagopalan et al. 1997; Weng et al. 2015), so select the most commonly used Gaussian kernel function as follows

$$K(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}} \tag{4.2}$$

The value of bandwidth has great influence on the fitting effect of KDE.After the frequency of different intervals f_{nj} was obtained ($j = 1, 2, .., n$), we obtained the bandwidth optimal solution as follows

$$g(h) = \sum_{j=1}^{N}\left[\int_{-\propto}^{\bar{x_j}} f(x|h)dx - f_{nj}\right]^2 \tag{4.3}$$

where $\bar{x_j}$ is the average of x in the j interval.

$$\frac{\partial g(h)}{\partial h} = 0 \tag{4.4}$$

It is verified that the minimum h is obtained.

Cumulative Distribution Function (CDF) was obtained by integrating $f(x)$

$$F(x) = \int_{-\propto}^{x} f(u)du \tag{4.5}$$

Finally, we use the quantile mapping method to calculate the SSMI whose mean value is 0 and variance is 1

$$\text{SSMI} = \varphi^{-1}(F(x)) \tag{4.6}$$

where $\varphi^{-1}(x))$ is the inverse Gaussian distribution function.Refer to the United States Drought Monitor (USDM) (Svoboda et al. 2002) for the corresponding drought levels in different value ranges, as shown in Table 3:

Table 3: Classification of drought levels

Value range of SSMI (quantile)	Drought grade
30 ~ 100%	-
20 ~ 30%	Abnormally dry
10 ~ 20%	Mild drought
5 ~ 10%	Moderate drought
2 ~ 5%	Severe drought
0 ~ 2%	Extreme drought

4.2.2 Extract drought variables and establish the joint distribution function

The run theory was proposed by Yevjevich (1967). The run is defined as a part of a time series that is higher or lower than the truncation level X_0.According to the run theory, because mild or worse droughts can have an effect on plants, so we took $x_0 = 20\%$ as threshold to obtain the drought events in different periods.

For a drought event, it has the characteristics of duration, intensity and peak. These features were extracted as drought variables. Drought duration D is the number of consecutive months when SSMI is lower than the truncation level X_0. Drought intensity S is the cumulative deficit of SSMI and truncation level X_0 in a drought event. The drought peak P is the maximum deficit between SSMI and truncation level X_0 in a drought event.

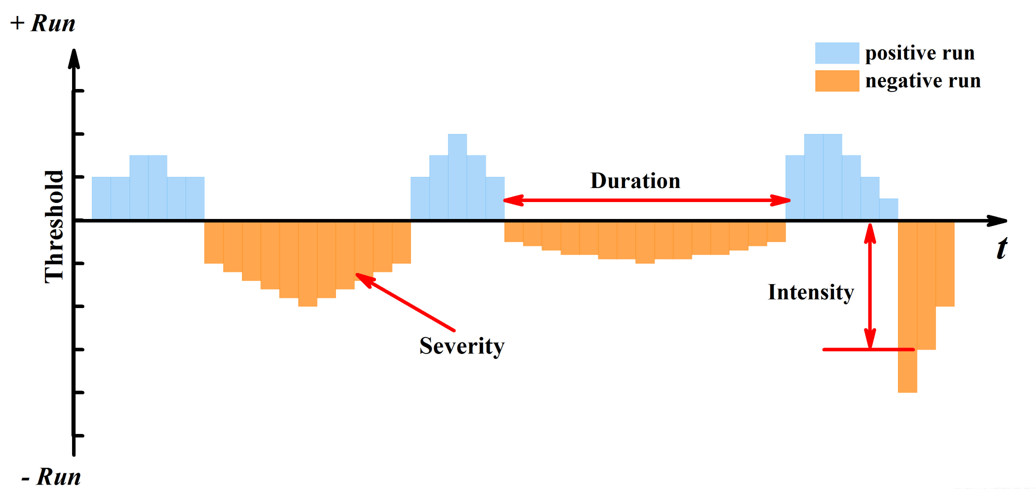


Figure 5: Drought events identification and extraction of drought variables using the run theory.

There is a very close relationship between drought variables. If the duration of drought is longer, the intensity or peak of drought will generally be higher, and vice versa.

The probability of D, S and P is calculated by the joint distribution function of two dimensions and three dimensions. Multi-dimensional KDE is used to obtain the joint probability density function of multi-drought variables as follows

$$f(x,y)=\frac{1}{nh_xh_y}\sum_{i=1}^{n}\left\{K\left(\frac{x-x_i}{h_x}\right)K\left(\frac{y-y_i}{h_y}\right)\right\} \tag{4.7}$$

$$f(x,y,z)=\frac{1}{nh_xh_yh_z}\sum_{i=1}^{n}\left\{K\left(\frac{x-x_i}{h_x}\right)K\left(\frac{y-y_i}{h_y}\right)K\left(\frac{z-z_i}{h_z}\right)\right\} \tag{4.8}$$

where x_i, y_i and z_i are drought duration, intensity and peak value of the i th drought event respectively. h_x, h_y and h_z are the bandwidth corresponding to drought duration, intensity and peak respectively. We also adopted Gaussian kernel function, and the Gaussian kernel optimal bandwidth calculation formula of multidimensional KDE is referred to Bowman et al. (1997) as follows

$$h_{d,opt}=\left(\frac{4}{n(p+2)}\right)^{\frac{1}{(p+4)}}\sigma_d \tag{4.9}$$

p is the number of dimensions of KDE, for example $p=2$ represents two-dimensional KDE;σ_d is the standard deviation of the sample in the d dimension.We figured out the optimal bandwidth.

According to the run theory, analyze the value of D, S, P.

$$\begin{cases} x=\sum_{SSMI\leq 20\%}1 \\ y=\sum_{SSMI\leq 20\%}SSMI \\ z=min\left(SSMI\leq 20\%\right) \end{cases} \tag{4.10}$$

where x, y and z are drought duration, intensity and peak value of the drought event respectively. When $SSMI \geq 20\%, x=0, y=0, z=0$. When $SSMI \leq 20\%$, calculate $F(x)$ which is as follows

$$F(t)=\int_V f(x,y,z)dxdydz \tag{4.11}$$

4.2.3 Frequency analysis of drought variables

The purpose of drought frequency analysis is to quantify drought risk, expressed in terms of the probability of drought occurrence and return period. Based on the calculated probability of drought occurrence, the return period of drought event can be obtained.

The probability of drought is further divided into "or" and ". The probability of drought occurrence in the case of "or" represents the probability that at least one drought variable exceeds the given threshold value, which is also called the joint surpass probability. The probability of drought occurrence in the case of "and" represents the probability that all the drought variables exceed the given threshold value, which is also called the co-occurrence surpass probability.

Taking duration and intensity of drought as an example, the formula for calculating bivariate drought probability in the case of "or" and "is as follows

$$\mathrm{P}_{\mathrm{DS}}^{\mathrm{or}}=\mathrm{P}(\mathrm{D}>\mathrm{d}\cup\mathrm{S}>\mathrm{s})=1-F_{DS}(d,s) \tag{4.12}$$

$$\mathrm{P}_{\mathrm{DS}}^{\mathrm{and}}=\mathrm{P}(\mathrm{D}>\mathrm{d}\cap\mathrm{S}>\mathrm{s})=1-F_D(d)-F_S(s)+F_{DS}(d,s) \tag{4.13}$$

Where $\mathrm{P}_{\mathrm{DS}}^{\mathrm{or}}$ and $\mathrm{P}_{\mathrm{DS}}^{\mathrm{and}}$ are bivariate drought probabilities under the condition of "or" and "respectively; d and s are the given thresholds of drought duration and intensity respectively. Similarly, the formula for calculating the probability of three-variable drought occurrence in the case of "or" and "and" is as follows

$$\mathrm{P}_{\mathrm{DSP}}^{\mathrm{or}} = \mathrm{P}(\mathrm{D} > \mathrm{d} \cup \mathrm{S} > \mathrm{s} \cup \mathrm{P} > \mathrm{p}) = 1 - F_{DSP}(d, s, p) \tag{4.14}$$

$$\mathrm{P}_{\mathrm{DSP}}^{\mathrm{and}} = \mathrm{P}(\mathrm{D} > \mathrm{d} \cap \mathrm{S} > \mathrm{s} \cap \mathrm{P} > \mathrm{p}) = 1 - F_D(d) - F_S(s) - F_P(p) + F_{DS}(d, s) + F_{DP}(d, p) + F_{SP}(s, p) - F_{DSP}(d, s, p) \tag{4.15}$$

Based on the probability of drought occurrence, the return period of drought event can be obtained, including the return period in the case of "or" (joint return period) and the return period in the case of "and" (co-occurrence return period)

$$\mathrm{T}_{\mathrm{DS}}^{\mathrm{or}} = \frac{N}{n\left(1 - F_{DS}(d, s)\right)} \tag{4.16}$$

$$\mathrm{T}_{\mathrm{DS}}^{\mathrm{and}} = \frac{N}{n\left(1 - F_D(d) - F_S(s) + \hat{F}_{DS}(d, s)\right)} \tag{4.17}$$

$$\mathrm{T}_{\mathrm{DSP}}^{\mathrm{or}} = \frac{N}{n\left(1 - F_{DSP}(d, s, p)\right)} \tag{4.18}$$

$$\mathrm{T}_{\mathrm{DSP}}^{\mathrm{and}} = \frac{N}{n\left(1 - F_D(d) - F_S(s) - F_P(p) + F_{DS}(d, s) + F_{DP}(d, p) + F_{SP}(s, p) - F_{DSP}(d, s, p)\right)} \tag{4.19}$$

The recurrence period and occurrence probability have opposite relationship numerically, that is, the shorter the recurrence period, the greater the occurrence probability.

4.3 Results

We used the data of Texas 2017-2019 to calculate the values of SSMI. The result of the second model,based on the data of each day in Texas from 2017 to 2019,can be made into diagrams better showing the flow of the process of the changes of SSMI in figure6.

According to the run theory, we took $x_0 = 20\%$ as threshold to obtain the drought events in different periods. Then we can calculate $F(x)$ which can be used in model 3.

5 Model 3:Drought Impact Model

5.1 The Establishment of Drought Impact Model

The model is based on the effects of drought on a single species and interactions between multiple species.

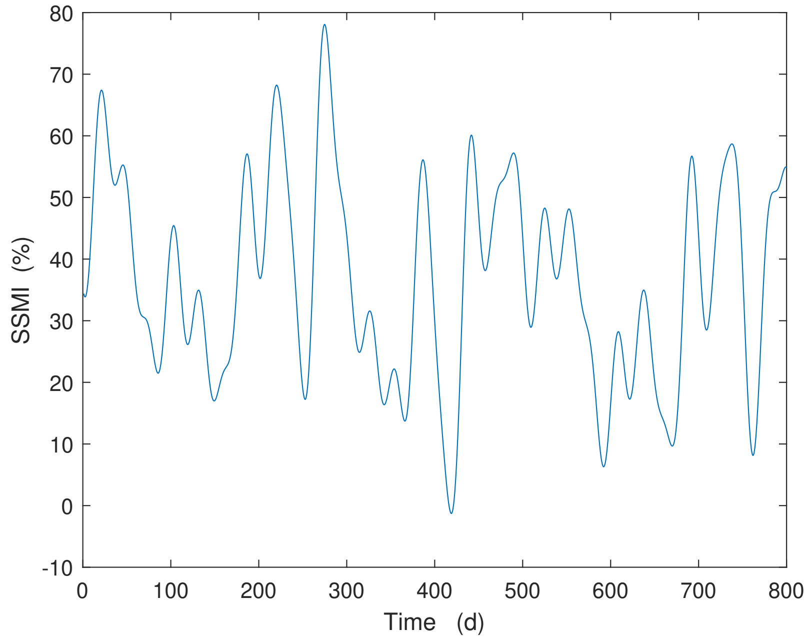


Figure 6: Drought curve.

5.1.1 Influences on Single Species

We have established a mathematical model of adaptive nonlinear constraints on population growth in (3.1). This equation is an optimal mathematical model to describe the law of population growth under the condition of limited resources, but it does not consider times of drought when precipitation should be abundant. Therefore,we assumed that the parameter r of the system is disturbed. We set the parameter r as a function of time, which is also affected by drought. The function is as follows

$$R(t) = (F(t) - x_0)t + r_0 \tag{5.1}$$

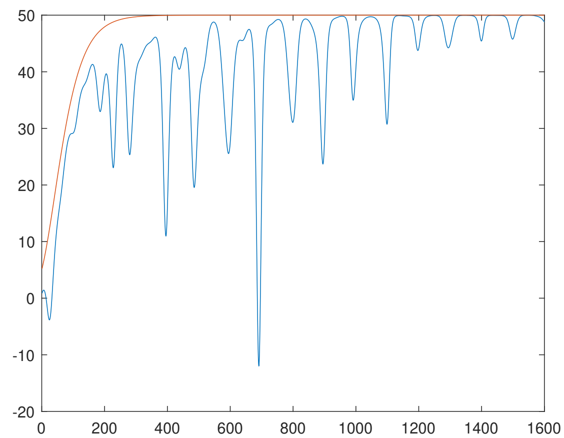
where $F(t)$ is the function related to drought in (4.11); x_0 is an appropriate value obtained by fitting the $F(x)$ curve with the relevant data of each day in Texas from 2017 to 2019; r_0 is the initial potential specific growth rate parameter without drought disturbance.

Thus, we obtained the nonlinear constrained population growth mathematical model which considers times of drought $N_2(t)$.

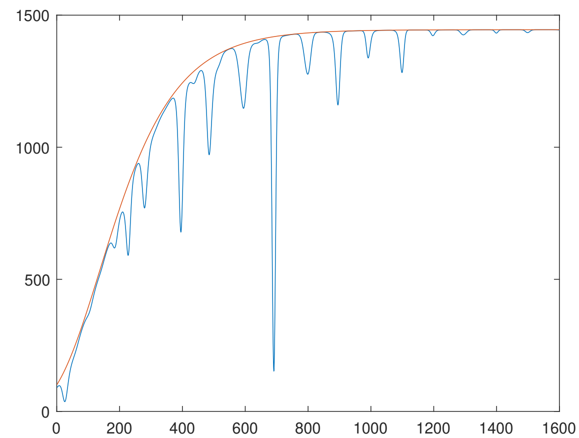
The result of the model,based on the data of mosses and arbors respectively,can be made into figure7.The red curve in the figure represents the growth curve of the plant community without external factors, and the blue curve shows how a plant community changes over time as it is exposed to various irregular weather cycles.

5.1.2 Influences on Multiple Species

For multiple species, since there are interactions between different species, it is necessary to extend the model above. We used the method similar to the formula (3.5), which is as follows



(a) moss



(b) arbor

Figure 7: Growth curves of single species.

$$Y_2 = \sum_{i=1}^{n} \alpha_i N_{2i}(t) - \sum_{j=1}^{n} \beta_j N_{2j}(t) \tag{5.2}$$

where $\alpha_i, \beta_j > 0$ represents the change in the growth rate of species B when species A is present. α_i are the combined growth coefficient of species with positive correlation, and β_j are the combined growth coefficient of species with negative correlation.

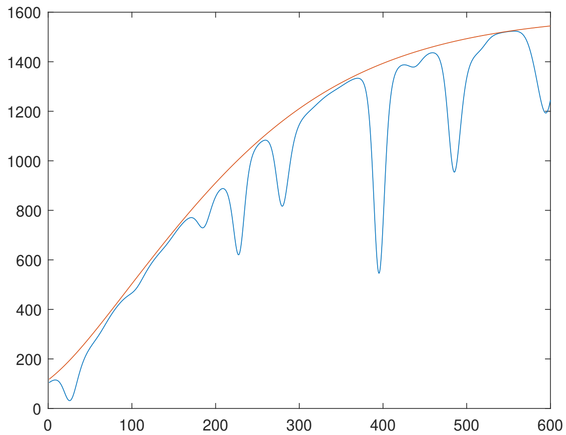
The result of the model, based on the data of mosses, herbs, shrubs, arbors and the combined growth coefficient of species among them,can be made into figure8. The red curve in the figure represents the growth curve of the plant community without external factors, and the blue curve shows how a plant community changes over time as it is exposed to various irregular weather cycles, accounting for interactions between different species during cycles of drought.

We simulated a plant community with two different species the community with mosses and arbors, and the community with mosses and shrubs respectively. In addition, we simulated a plant community with three and more different species for comparison.

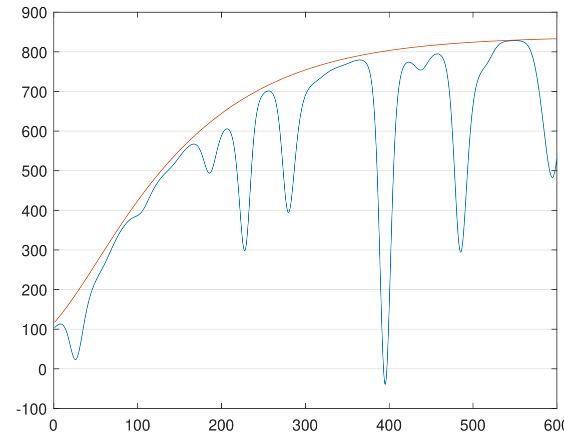
5.2 Results

By observing the above graphs showing the response of different communities to drought conditions we can conclude the following:

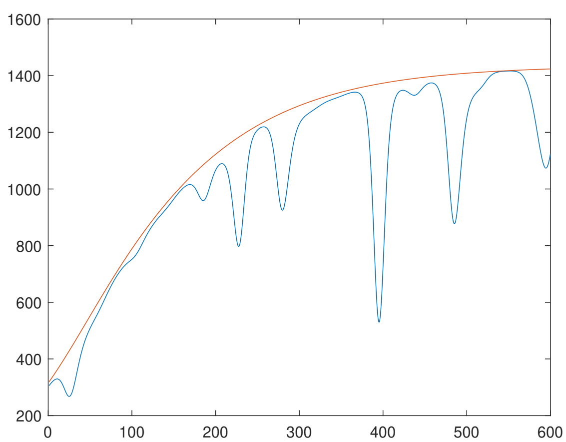
- When mosses and trees are affected by different degrees of drought individually, the biomass of moss populations will drop below zero when affected by mild drought, while the biomass of trees will only increase when the population is affected by very severe drought. It can be seen that the ability of trees to resist drought is far stronger than that of moss.
- Through the images of the response of plant communities of multiple species to drought, we can observe that the more species in the plant community, the smaller the impact on biomass when it is hit by drought.



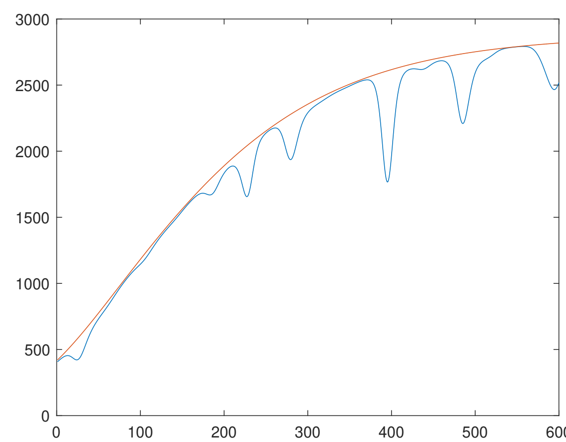
(a) moss-arbor



(b) moss-shrub



(c) moss-herb-shrub



(d) moss-herb-shrub-arbor

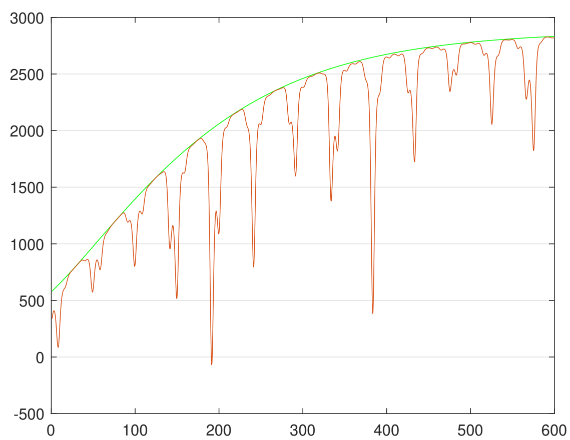
Figure 8: Growth curves of multiple-species.

- Comparing a community with only mosses and arbors to a community with only mosses and shrubs we can observe that when the species with strong drought resistance ability are included in the community, the drought resistance ability of the whole community is stronger.
- Comparing the biomass in several pictures, we can find that when there are only one or two plants in the community, the biomass of the community may be negative, and the community has the risk of extinction. But when there are three or more plants in the community, the biomass of the community will not be negative, and the more plant species, the more stable the biomass when it suffers from severe drought.

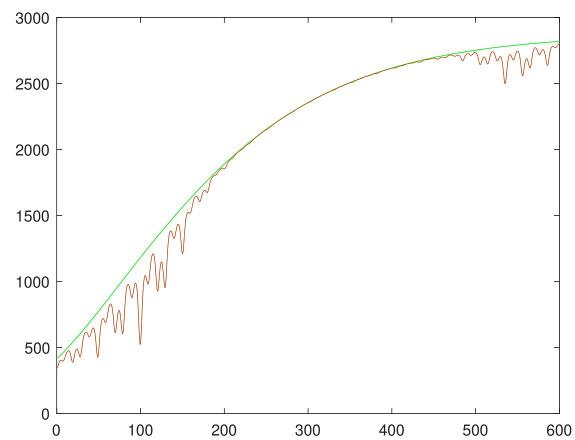
6 Extend Our Model

6.1 Different degrees of drought

The drought model was re-fitted with the data of regions with different degrees of drought to explore the effects of different degrees of drought on species growth.The green curves in the figure9 and figure10 represent the growth curves of the plant community without external factors.



(a) a greater frequency



(b) less frequent

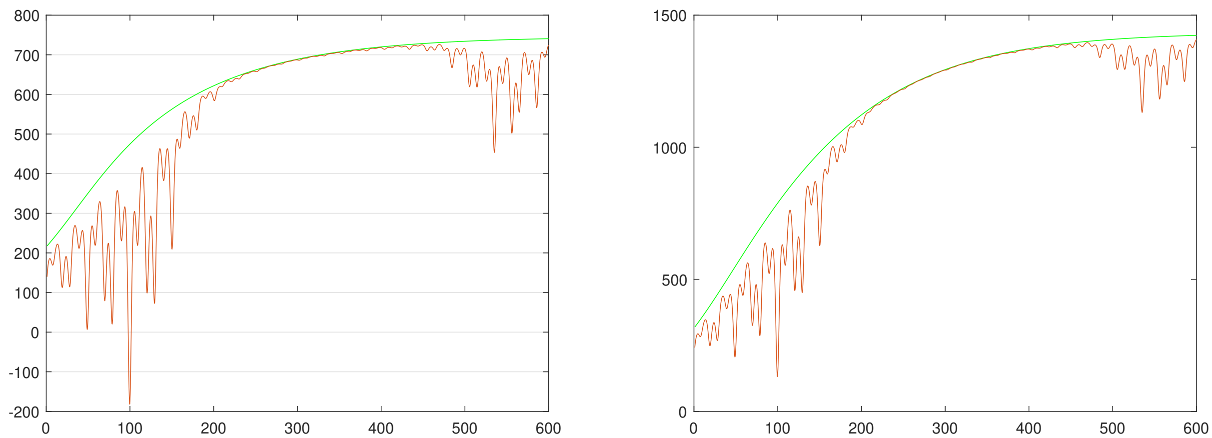
Figure 9: Influences of drought under Different degrees of drought.

Figure9 shows comparison between different situation of the plant community with 4 species under a greater frequency and wider variation of the occurrence of droughts and less frequent droughts.

Figure10 shows that plant community with two different species without arbors and shrubs, the two species still cannot benefit the community even under less drought frequency.

6.2 Other Factors

We introduce a pollution coefficient as a measure of the impact of pollution on species growth. Pollution indirectly affects the growth of species by affecting the environmental holding capacity of



(a) 2 species-without arbors and shrubs

(b) 3 species-without arbors

Figure 10: Influences of less frequent droughts.

species. After adding the pollution coefficient, the model is corrected as follows:

$$Y_3(t) = L(t)Y_2(t) \tag{6.1}$$

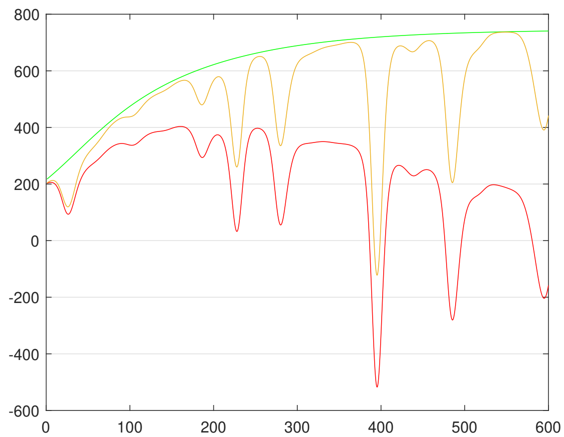
$$L(t) = P_m - \sum_{i=1}^{[t]} \{mod[i,T] == 0\} \tag{6.2}$$

where P_m is environmental pollution coefficient, T is a period of pollution.

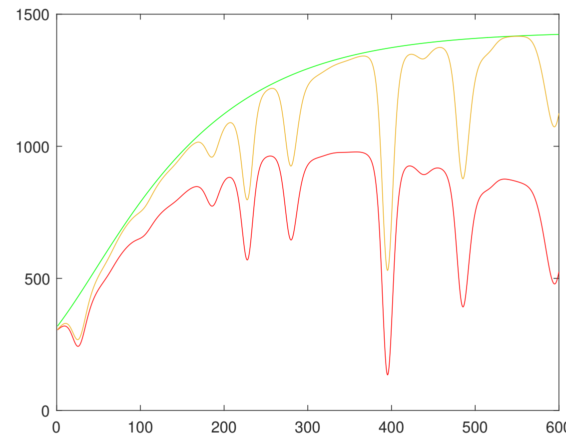
The green curves in the figure11 represent the growth curves of the plant community without external factors.The yellow line in the figure represents drought stress and the red line represents pollution stress.

6.3 Results

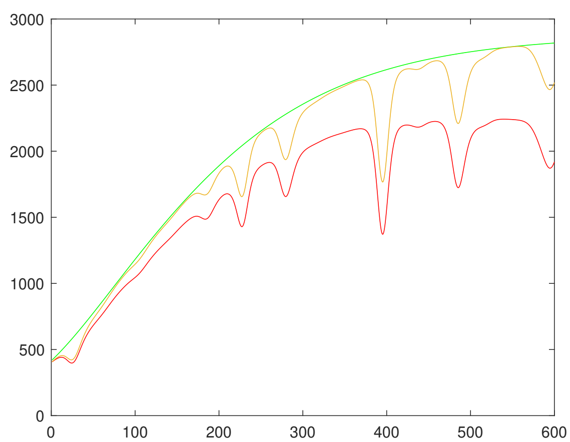
- Although from the above discussion, the plant population with four different species has a relatively strong ability to resist the negative impact of the outside world, but if there is a greater frequency and wider variation of the occurrence of droughts, the plant population is still facing the threat of extinction.
- Although it is under less drought frequency, plant community with only two different species still cannot benefit the community.
- In the case of the same frequency, the negative impact of pollution on the growth of plant communities is greater than that of drought stress. Although three different plant species could benefit the community under normal drought stress, the community was threatened with extinction when this it was faced with the same frequency of pollution.



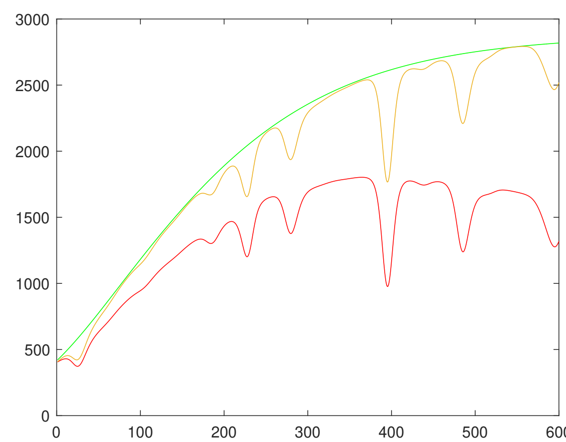
(a) light-pollution-two-plants



(b) light-pollution-three-plants



(c) light-pollution-four-plants



(d) severe-pollution-four-plants

Figure 11: Influences of pollution.

- However, the growth curves of the plant community with four plant species were relatively stable no matter the light pollution stress or severe pollution stress. Therefore, plant populations with richer biodiversity have stronger ability of anti-interference.

7 Sensitivity Analysis——The Lag Effect of Drought

Drought had a lag effect on Water Use Efficiency (WUE) of 70.87% land vegetation, and the average lag months was 4.12 months. In our model, the effect of drought on the water use efficiency of vegetation is immediate. If the lag effect of drought is considered, we only need to quantify the lag effect of drought on WUE by determining how many months lag SSMI has the highest correlation with WUE.

$$\mathrm{R}_i = \mathrm{cor}\left(\mathrm{SSMI}_i, \mathrm{WUE}\right), 0 \leq i \leq 12 \tag{7.1}$$

$$\left|\mathrm{R}_{\text{max-lag}}\right| = \max\left(\left|\mathrm{R}_i\right|\right) \tag{7.2}$$

where $SSMI_i$ is the SSMI time series with a lag of i months. For example, $i = 0$ is the original SSMI time series, and $i = 12$ is the original SSMI time series moved back by 12 months. R_i is the Pearson correlation coefficient between $SSMI_i$ and WUE, and only R_i that passes the 5% confidence level is retained. $R_{max-lag}$ is the R_i with the largest absolute value, ranging from -1 to 1, which represents the lag effect of drought on WUE. Further, the lag months corresponding to $R_{max-lag}$ are divided into four categories: no lag effect (lag months =0), short-term lag effect (1≤ lag months ≤4), medium-term lag effect (5≤ lag months ≤8), and long-term lag effect (9≤ lag months ≤12).

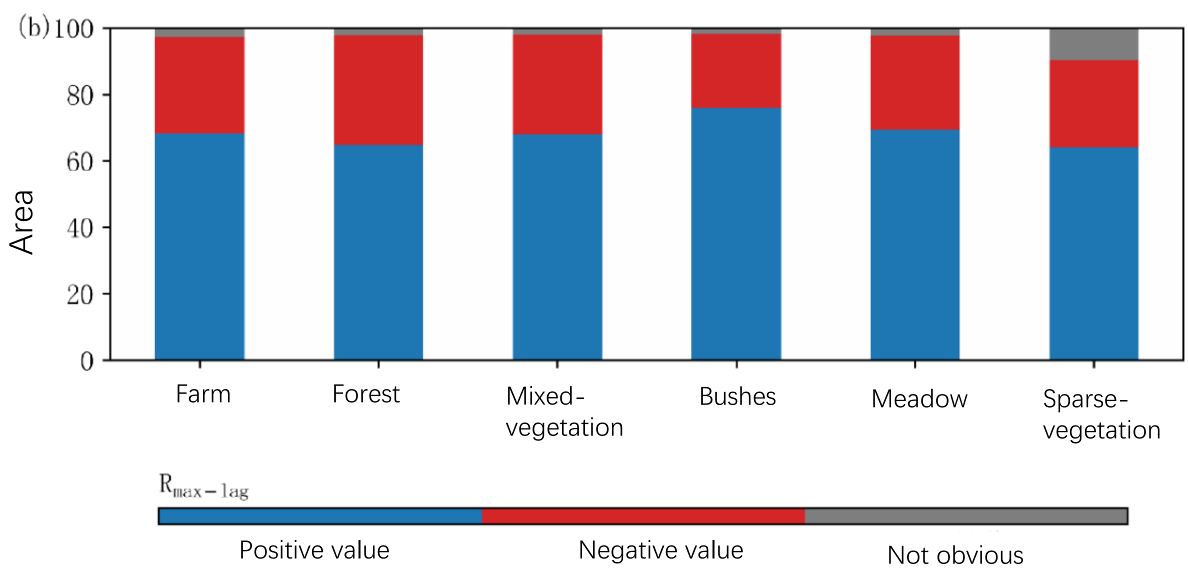


Figure 12: Area proportion of R_{max_lag}.

Among all land cover types, drought has a short-term lag effect on the WUE of shrub and sparse vegetation, while it mainly has a medium-term and long-term lag effect on the WUE of forest. This is consistent with our conclusion that four different plant species are required for the community to benefit.

8 Conclusion

Based on our work, it is not difficult to draw the following conclusions.

(1) Through our model simulations, it can be seen that a plant community needs at least three different plant species to benefit under the currently assumed drought conditions, and the community will be more stable and less likely to die out as the number of species increases. Comparing the biomass in several pictures, we can find that when there are only one or two plants in the community, the biomass of the community may be negative, and the community has the risk of extinction. But when there are three or more plants in the community, the biomass of the community will not be negative, and the more plant species, the more stable the biomass when it suffers from severe drought.

(2) Different plant types have different tolerances to drought. Our simulation results show that for drought tolerance, arbor > shrub > herb > moss. Comparing a community with only mosses and arbors to a community with only mosses and shrubs we can observe that when the species with strong drought resistance ability are included in the community, the drought resistance ability of the whole community is stronger.

(3)Although it is under less drought frequency, plant community with only two different species still have difficulty benefiting the community.

(4) Taking into account factors such as pollution and habitat, we found that pollution aggravates drought damage, and similarly, habitat loss also aggravates drought damage.And the heavier the degree of pollution, the greater the possibility of community extinction and the higher the degree of harm.

(5)However, the growth curves of the plant community with more plant species were relatively stable no matter the light pollution stress or severe pollution stress. Therefore, plant populations with richer biodiversity have stronger ability of anti-interference to some extent.

(6) To ensure the long-term viability of a plant community, necessary protective measures needs to be taken.

- Plants need areas to grow, and try to occupy the ground. Therefore, we should provide plants with sufficient area, taking measures such as returning farmland to forests.

- The times of drought when precipitation should be abundant have a pretty negative impact on the plant community, and even result in the extinction of some species. Therefore, we should do a good job of predicting drought and watering in time to reduce the negative impact of drought on plant communities.

- The negative impact of pollution on plant communities is much greater than that of drought. Therefore, in some suburbs or rural market towns, for some high-energy-consuming and high-pollution industries such as paper mills, effective management is required, and they are required to install sewage discharge measures and be able to effectively deal with some waste and waste residues in a timely manner.

Figure 13: Comparison.

(7)We believe that the plant community has positive impacts on the surrounding environment.Plant communities can improve the water-holding capacity of the soil, which can prevent soil erosion and desertification effectively. What's more, it can promote the growth of some surrounding plant communities, which can help improve the biodiversity of the ecosystem.

9 Evaluation of Strengths and Weaknesses

9.1 Strengths

- Both the population growth model and the drought model used in this paper are universal and can adapt well to different populations and different drought conditions.
- The method adopted in this paper can well take into account the influence of other conditions such as pollution on the growth of plant communities. And different conditions can be supplemented to obtain different results.
- We use the data obtained from the official website to fit the model to make the establishment of the model more reliable.

9.2 Weaknesses

- The hysteresis effect of drought on plant communities is not considered, which may have an impact on the results of the deduction.
- No consideration of other factors affecting drought. Since the nature is complicated, there are many factors may contribute to drought. We only choose soil moisture as indicators for measuring drought and that may be inaccurate.

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