UNIVERSITY EXAMINATIONS

OCTOBER/NOVEMBER 2023

MAT1512

CALCULUS A

Examiners:
First: MR S BLOSE
Second: DR ZA IDRISS

100 Marks
2 Hours

Welcome to the MODULE MAT1512 exam.

This paper consists of 5 pages.
Closed book examination.

Instructions:

- **This web based examination question paper remains the property of the University of South Africa and may not be distributed from the Unisa platform.**
- **Declaration: I have neither given nor received aid on this examination**.
- This is a closed book and online examination which you have to write within **2 hours** and submit online through the eAssessment using the link: https://cset.myexams.unisa.ac.za/
- This exam is IRIS invigilated (you cannot use a cell phone during this exam)
- This examination allows single PDF attachment only as part of your submission.
- Use of a any calculator is **NOT** allowed
- Answer All Questions and Submit within the stipulated timeframe.
- Late submission will not be accepted.
- ALL CALCULATIONS MUST BE SHOWN.

Additional student instructions: OCT/NOV 2023 online examination rules.

Students are expected to familiarise themselves with online examination rules before their examination sittings.

Examination sessions commence at the time indicated on the final examination timetable. You are required to adhere strictly to the specified times.

i. For file upload/take-home examinations:

1. Students must upload their answer scripts in a single PDF file on the official

myExams platform (answer scripts must not be password protected or uploaded as “read-only” files).

2. **NO** e-mailed scripts will be accepted.

3. Students are advised to preview submissions (answer scripts) to ensure legibility and that the correct answer script file has been uploaded.

4. Students are permitted to resubmit their answer scripts should their initial submission be unsatisfactory.

5. Incorrect file format and uncollated answer scripts will not be considered.

6. Incorrect answer scripts and/or submissions made on unofficial examination platforms (including the invigilator cellphone application) will not be marked and no opportunity will be granted for resubmission.

7. A mark awarded for an incomplete submission will be the student’s final mark. No opportunity for resubmission will be granted.

8. A mark awarded for illegible scanned submission will be the student’s final mark. No opportunity for resubmission will be granted.

9. Only the last file uploaded and submitted will be marked.

10. Submissions will only be accepted from registered student accounts.

11. Students who have not utilised invigilation or proctoring tools will be deemed to have transgressed Unisa’s examination rules and will have their marks withheld.

12. Students have 48 hours from the day of their examination to upload their recordings in the IRIS Invigilator App. Failure to do so will result in students deemed not to have utilised the IRIS invigilation or proctoring tools.

13. Students must complete the online declaration of their work when submitting. Students suspected of dishonest conduct during the examinations will be subjected to disciplinary processes. Students may not communicate with other students or request assistance from other students during examinations. Plagiarism is a violation of academic integrity, and students who do plagiarise or copy verbatim from published work will be in violation of the Policy on Academic Integrity and the Student Disciplinary Code and may be referred to a disciplinary hearing. Unisa has zero tolerance for plagiarism and/or any other forms of academic dishonesty.

14. Students are provided 30 minutes to submit their answer scripts after the official examination time. Students who experience technical challenges should report to the SCSC on 080 000 1870 or their College exam support centers (refer to the Get help during the examinations by contacting the Student Communication Service Centre (unisa.ac.za)) within 30 minutes. Queries received after one hour of the official examination duration time will not be responded to. Submissions made after the official examination time will be rejected by the examination regulations and will not be marked.

15. Non-adherence to the processes for uploading examination responses will not qualify the student for any special concessions or future assessments.

16. Queries that are beyond Unisa's control include the following:
    a. Personal network or service provider issues
    b. Load shedding/limited space on personal computer
    c. Crashed computer
    d. Using work computers that block access to myExams site (work firewall challenges)
    e. Unlicensed software (eg license expires during exams)

    Postgraduate students experiencing the above challenges are advised to apply for an aegrotat and submit supporting evidence within ten days of the examination session. Students will not be able to apply for an aegrotat for a third examination opportunity. Postgraduate/Undergraduate students experiencing the above challenges in their second examination opportunity will have to reregister for the affected module.

17. Students experiencing technical challenges should contact the SCSC on 080 000 1870 or via e-mail at Examenquiries@unisa.ac.za or refer to the Get help during the examinations by contacting the Student Communication Service Centre (unisa.ac.za) for the list of additional contact numbers. Communication received from your myLife account will be considered.

TURN OVER

QUESTION 1

(a) Determine the following limits (if they exist):

(i) $\lim_{\theta\to 0}\left(\dfrac{\sin\theta+\tan 3\theta}{\theta}\right)$ (3)

(ii) $\lim_{x\to 1}\dfrac{1-\sqrt{x}}{1-x}$ (3)

(iii) $\lim_{t\to 3}\left(\dfrac{5+3t}{t^3+2t^2-3}\right)$ (3)

(iv) $\lim_{x\to -2^+}\left(\dfrac{x^2-4}{|x|-2}\right)$ (3)

(v) $\lim_{t\to -\infty}\left(\dfrac{t^2-at-bt+ab}{b^2+2abt-a^2t^2}\right)$, a, b are real numbers. (4)

(b) Given the function

$$h(x)=\begin{cases} x+2 & \text{if} \quad x>-1 \\ x^2 & \text{if} \quad -1\geq x<1 \\ 3-x & \text{if} \quad x\geq 1 \end{cases}$$

Determine at which value(s) of x is $h(x)$ continuous (4)

(c) Use the Squeeze Theorem to determine the following limit (5)

$$\lim_{k\to -\infty}\frac{5k^2+\sin(3k)}{10-k^2}$$

[25]

QUESTION 2

(a) Using the first principle of differentiation, determine the derivative of
$f(x)=2x^2-3x+5$ at $x=2$. (5)

(b) Find the derivatives of the following functions by using the appropriate rules for differentiation:

(i) $h(x)=(x^2-ax)(\sqrt{ax^2+1})$, $a\geq 0$ (3)

(ii) $y(t)=\tan\sqrt{\cos(8t)}$ (3)

(iii) $g(x)=\dfrac{\sqrt{x}-x^2}{\cos x\tan x}$, $x>0$. (3)

(c) Use the Fundamental Theorem of Calculus to find the derivative of

$$K(x)=\int_{\cos x}^{sinx} t(5+t^2)\,dt$$

(4)

[TURN OVER]

(d) Given $y\cos(2x) = x\sin(2y)$ find

(i) $\dfrac{dy}{dx}$ using implicit differentiation. (4)

(ii) the equation of the tangent line to the curve $y\cos(2x) = x\sin(2y)$ at the point $(\frac{\pi}{4}, \frac{\pi}{2})$. (3)

[25]

QUESTION 3

(a) Determine the following integrals by making a direct substitution and change of limit where necessary:

(i) $\displaystyle\int \sec^2\left(\frac{\theta}{5}\right)\, d\theta$ (3)

(ii) $\displaystyle\int \left(\frac{3}{x^2} - x\right)\left(x + \frac{2}{x^2}\right) dx$ (3)

(iii) $\displaystyle\int 3t\sqrt{5+t^2}\, dt$ (3)

(iv) $\displaystyle\int \frac{1}{\left(4+\sqrt{3}v\right)^4}\, dv$ (5)

(v) $\displaystyle\int_0^{\frac{\pi}{4}} \sec^3 r \tan^3 r\, dr$ (5)

(b) Determine the area of the region enclosed by $f(x) = -|x|$ and the line $h(x) = x^2 - 2$.
Hint: Sketch the region enclosed by the given functions f and g. (6)

[25]

QUESTION 4

(a) Solve the following initial value problem

$$\frac{dy}{dt} - \frac{\cos^2 y}{4t-3} = 0, \qquad y(1) = \frac{3\pi}{4}.$$

(5)

(b) Given that

$$G(x,y) = 7y + \cos\left(x^2y^2\right) - x^3y.$$

(i) Find the first partial derivatives G_x and G_y. (3)

(ii) Write down $\dfrac{dy}{dx}$ using your answer to b(i) above. (2)

(iii) If $G(x,y) = 0$, confirm your answer in b(ii) above by using implicit differentiation. (5)

(c) Use the chain rule for partial derivatives to find $\dfrac{\partial z}{\partial u}$; when $u = 0$ and $v = 1$, if $z = \sin(xy) + x \sin y$, $x = u^2 + v^2$ and $y = uv$. (5)

(d) A bacterial culture starts with 1000 bacteria and after 2 hours there are 2500 bacteria. Assuming that the culture grows at a rate proportional to its size, find the population after 4 hours. (5)

[25]

Total: [100]

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