UNISA
college of science, engineering and technology

UNIVERSITY EXAMINATIONS

OCTOBER/NOVEMBER 2024

MAT1512

CALCULUS A

Welcome to the MAT1512 examination

Date: 17 October 2024
Time: 08: 00
Hours: 2

Examiner name: PROF Z I ALI
Internal moderator name: PROF T NAZIR

This paper consists of 5 pages (including the cover page).

Total marks: 100

Number of pages: 5
Instructions:

- Include reference to additional information sheets if applicable.
- ANSWER ALL QUESTIONS
- You are **NOT** allowed to use any calculator.
- No typed solutions will be accepted and the script will be cancelled
- You need to declare your honesty regarding writing this paper.
- This exam is IRIS invigilated. You must have your camera on for entire duration of the
- exam. Failure to have your camera on for the duration of the exam, will lead to disciplinary
- action taken against you.

Additional student instructions
1. Students must upload their answer scripts in a single PDF file (answer scripts must not be password protected or uploaded as "read only" files)
2. Incorrect file format and uncollated answer scripts will not be considered.
3. NO emailed scripts will be accepted.
4. Students are advised to preview submissions (answer scripts) to ensure legibility and that the correct answer script file has been uploaded.
5. Incorrect answer scripts and/or submissions made on unofficial examinations platforms (including the invigilator cell phone application) will not be marked and no opportunity will be granted for resubmission. Only the last answer file uploaded within the stipulated submission duration period will be marked.
6. Mark awarded for incomplete submission will be the student's final mark. No opportunity for resubmission will be granted.
7. Mark awarded for illegible scanned submission will be the student's final mark. No opportunity for resubmission will be granted.
8. Submissions will only be accepted from registered student accounts.
9. Students who have not utilised the proctoring tool will be deemed to have transgressed Unisa's examination rules and will have their marks withheld. If a student is found to have been outside the proctoring tool for a total of 10 minutes during their examination session,

they will be considered to have violated Unisa's examination rules and their marks will be withheld. For examinations which use the IRIS invigilator system, IRIS must be recording throughout the duration of the examination until the submission of the examinations scripts.

10. Students have 48 hours from the date of their examination to upload their invigilator results from IRIS. Failure to do so will result in students deemed not to have utilized the proctoring tools.

11. Students suspected of dishonest conduct during the examinations will be subjected to disciplinary processes. Students may not communicate with any other person or request assistance from any other person during their examinations. Plagiarism is a violation of academic integrity and students who plagiarise, copy from published work or Artificial Intelligence Software (eg ChatGPT) or online sources (eg course material), will be in violation of the Policy on Academic Integrity and the Student Disciplinary Code and may be referred to a disciplinary hearing. Unisa has a zero tolerance for plagiarism and/or any other forms of academic dishonesty.

12. Listening to audio (music) and making use of audio-to-text software is strictly prohibited during your examination session unless such usage of the software is related to a student's assistive device which has been so declared. Failure to do so will be a transgression of Unisa's examination rules and the student's marks will be withheld.

13. Students are provided 30 minutes to submit their answer scripts after the official examination time. Students who experience technical challenges should report the challenges to the SCSC on 080 000 1870 or their College exam support centres (refer to the Get help during the examinations by contacting the Student Communication Service Centre [unisa.ac.za]) within 30 minutes. Queries received after 30 minutes of the official assessment duration time will not be responded to. Submissions made after the official assessment time will be rejected according to the examination regulations and will not be marked. Only communication received from your myLife account will be considered.

14. Non-adherence to the processes for uploading assessment responses will not qualify the student for any special concessions or future assessments.

15. Queries that are beyond Unisa's control include the following: a. Personal network or service provider issues
    b. Load shedding/limited space on personal computer
    c. Crashed computer
    d. Non-functioning cameras or web cameras
    e. Using work computers that block access to the myExams site (employer firewall challenges)
    f. Unlicensed software (eg license expires during exams)

    Postgraduate students experiencing the above challenges are advised to apply for an aegrotat and submit supporting evidence within ten days of the examination session. Students will not be able to apply for an aegrotat for a third examination opportunity. Postgraduate/undergraduate students experiencing the above challenges in their second examination opportunity will have to reregister for the affected module.

16. Students suspected of dishonest conduct during the examinations will be subjected to disciplinary processes. UNISA has a zero tolerance for plagiarism and/or any other forms of academic dishonesty.

17. Students experiencing network or load shedding challenges are advised to apply together with supporting evidence for an Aegrotat within 3 days of the examination session.

QUESTION 1

(a) Calculate the limits of the following functions (if there exist):

(i) (3)

$$\lim_{x \to 0} \frac{x^2 - 5x + 6}{2x + 8}.$$

(ii) (3)

$$\lim_{x \to 0} \frac{\sqrt{2x + 3} - \sqrt{3x + 3}}{\sqrt{x + 4} - \sqrt{2x + 4}}.$$

(iii) (3)

$$\lim_{x \to -\infty} \sqrt{x^2 - 2x} + x.$$

(iv) (3)

$$\lim_{x \to -\infty} \frac{1 + \sin^3 x}{\cos^2 x}.$$

(v) (3)

$$\lim_{x \to 0} \frac{\sin(4x) \cos(2x)}{\tan(5x)}.$$

(vi) (5)

$$\lim_{x \to 0} \frac{\sin(4x) \cos(2x) + \cos(3x) \sin(2x)}{\tan(5x)}.$$

(vii) (3)

$$\lim_{x \to -\infty} \left(\frac{\sqrt{3x^2 + 6}}{5 - 2x} + 5x + \sqrt{25x^2 + x} \right).$$

(b) Use the Squeeze Theorem to determine (4)

$$\lim_{x \to \infty} \left(\frac{\sin(e^x)}{x} + \frac{\sin(e^x)}{\sqrt{x^2 + 3}} \right).$$

(c) We consider the function F defined by (4)

$$F(t) := \begin{cases} 4b, & \text{if } t \leq -2 \\ 3t^2, & \text{if } -2 < t \leq 1 \\ t + a & \text{if } t > 1 \end{cases}$$

(i) determine the limits

A. $\lim_{t \to -2^-} F(t)$ and $\lim_{t \to -2^+} F(t)$ (3)

B. $\lim_{t \to 1^-} F(t)$ and $\lim_{t \to 1^+} F(t)$ (3)

C. Find the values of a and b so that F is continuous at $t = 1$ and at $t = -2$. (3)

[**40**]

QUESTION 2

(a) Use the first principles of differentiation to determine $f'(x)$, the first derivative of $f(x)$ for $\qquad$ (5)

$$f(x) := \frac{x}{x+1}$$

at $x = 1$.

(b) Use the rules of differentiation to determine the derivatives of the function defined below

(i) $\qquad$ (4)

$$H(x) = \frac{\sin x}{e^x}$$

(ii) $\qquad$ (4)

$$F(x) = \int_{\sin(-x^2)}^{-\sqrt{x+1}} \cos(t^2 - 1)dt$$

(c) Consider the curve given by the equation

$$\sin(3x) - \sin(4y) = 0, \; -\frac{\pi}{4} \le x \le \frac{\pi}{4}, 0 \le y \le \frac{\pi}{8}.$$

(i) Determine $\frac{dy}{dx}$. $\qquad$ (3)

(ii) Determine the equation of the tangent at $x = \frac{\pi}{12}$ $\qquad$ (4)

$\qquad$ **[20]**

QUESTION 3

(a) Evaluate the integrals:

(i) $\qquad$ (3)

$$\int \left[e^{-3x+1} - \tan x \right] ds$$

(ii) $\qquad$ (4)

$$\int \frac{t}{t+1}dt + \int \frac{2t}{-t+1}dt$$

(iii) $\qquad$ (3)

$$\int \frac{\cos(-3x)}{1 - 8\sin(-3x)}dx$$

(iv) $\qquad$ (3)

$$\int \frac{\cos(-2x)}{\sin x - \cos(x)}dx$$

(v) $\qquad$ (3)

$$\int \cos^2(t)\sin^2(t)dt$$

[TURN OVER]

(vi) (2)

$$\int_0^{\frac{\pi}{4}} \tan x \sec^2 x \, dx$$

(b) We consider the functions given by

$$\begin{aligned} f_1(x) &= x - 2, \\ f_2(x) &= -x, \\ f_3(x) &= -x^2 + 2x + 4. \end{aligned}$$

   (i) Sketch the graphs of the above functions on the same plane (3)

   (ii) Find the area of the region above the functions f_1 and f_2 and below the function f_3. (4)

[**25**]

QUESTION 4

(a) Solve the initial value problem given by (5)

$$e^{-y}\frac{dy}{dx} + \left(x^2 - \cos x\right) = -1; y(0) = -1$$

(b) Consider the given function

$$f(x,y) = \tan\left(\frac{x}{y}\right), \ \textbf{with } x = u^2 + v^2, \ \textbf{and } y = u^2 - v^2.$$

   (i) Use the appropriate rules of differentiation to determine $\frac{\partial f}{\partial u}$ and $\frac{\partial f}{\partial v}$. (4)

(c) Consider the function F defined by (6)

$$F(x,y) := 7y^2 + \cos(-3x) - 12 + y^4.$$

Use implicit differentiation to find $\frac{dy}{dx}$ whenever $F(x,y) = 0$.

[**15**]

TOTAL MARKS: [100]

First examiner	Dr ZI Ali
External examiner	Prof Y Hardy