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Teo goes to the ~~Freak Show~~ Zoo

SOME WIDESPREAD DISCRETIZATIONS

A. BÁEZ, F.X. TRIAS AND N. VALLE

«All things excellent are as difficult as they are rare» - Baruch Spinoza

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1. NSE discretizations according to the physical interpretation criterion
    1. The characteristics approach. Wave equation School.
    2. The Poisson equation approach. Heat equation School.
    3. Is there any common ground?
2. NSE discretizations according to the basis of functions spaces
    1. Finite Volume Methods
    2. Finite Differences Methods
    3. Finite Element Methods
    4. Spectral Methods
3. Freak Show.

Charactersitics lines approach. Waves equation School.

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = \left[\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x}\right]\left[\frac{\partial u}{\partial t} - c\frac{\partial u}{\partial x}\right] = 0$$

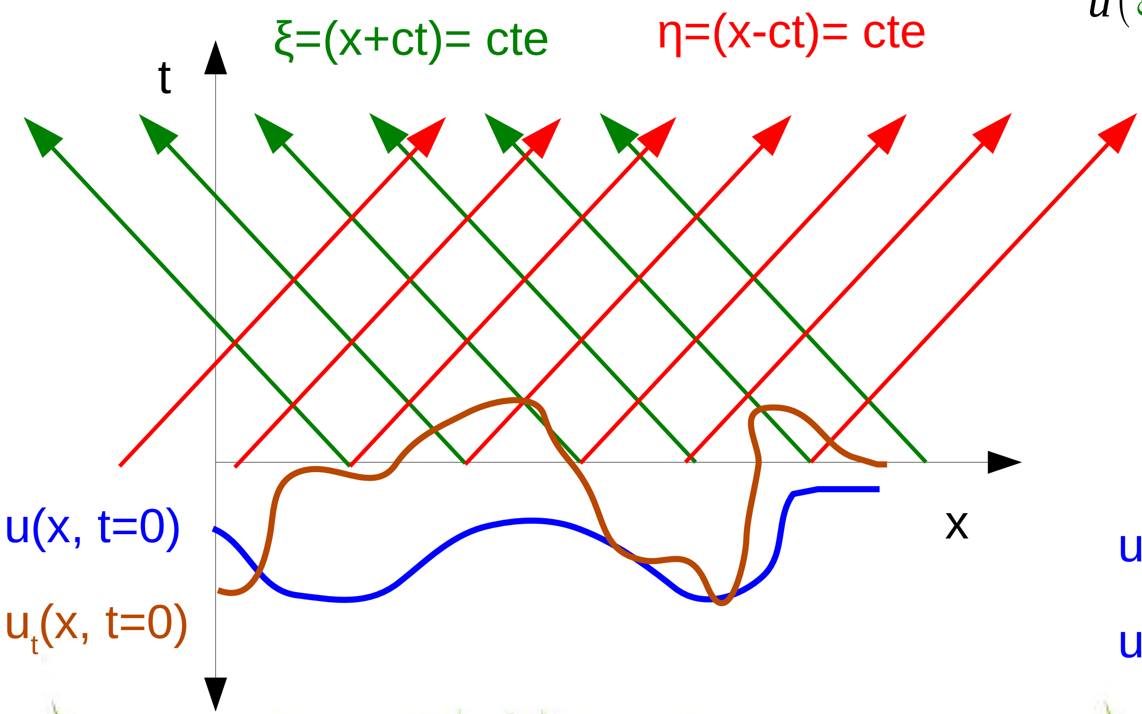
$$\frac{\partial^2 u}{\partial \xi \partial \eta} = 0 \rightarrow u = F(x+ct) + G(x-ct)$$

$$F(\xi) = \frac{u(\xi)}{2} - \frac{1}{2}c\int_{-\infty}^{x} u_t(x)\,dx + c$$

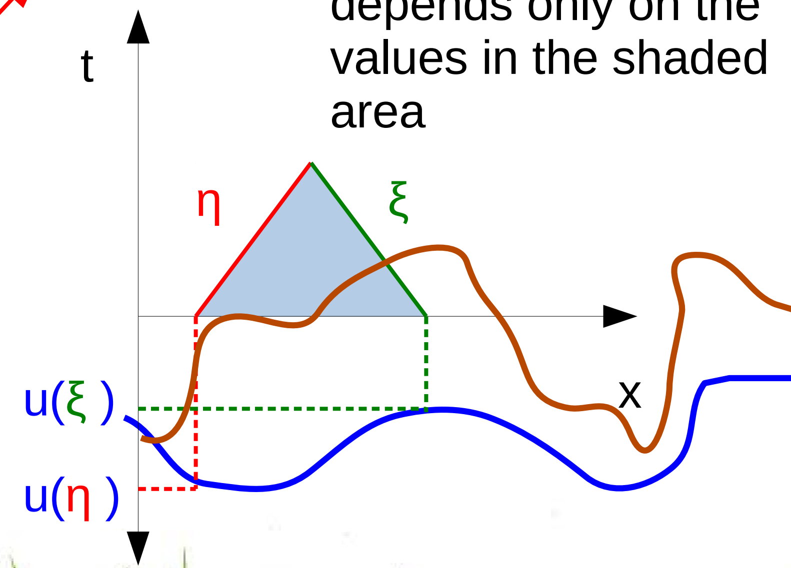
$$G(\eta) = \frac{u(\eta)}{2} + \frac{1}{2}c\int_{-\infty}^{x} u_t(x)\,dx$$

$$\downarrow$$

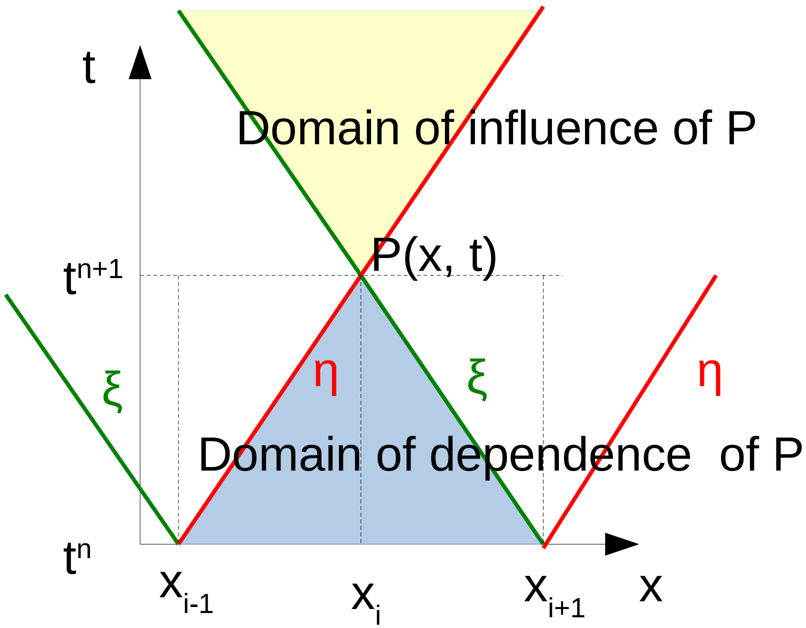
$$u(\xi,\eta) = \frac{u(\xi)+(\eta)}{2} + \frac{1}{2}c\int_{\eta}^{\xi} u_t(x,0)\,dx$$



The solution at (x, t) depends only on the values in the shaded area



Discretization according to characteristics

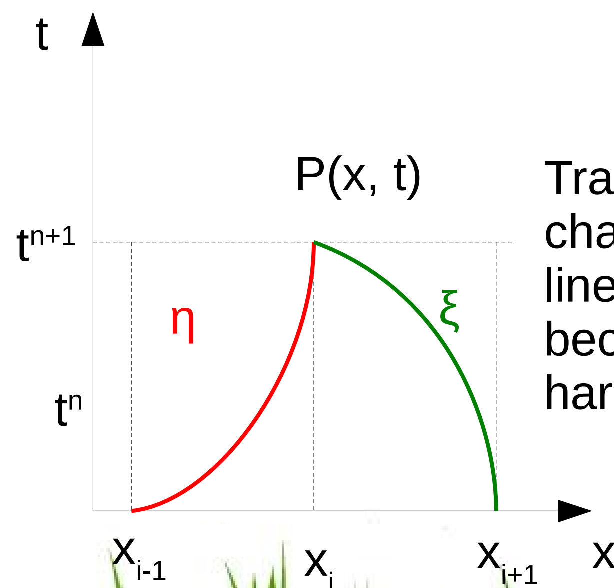


$$u(x_i, t^{n+1}) = F(x_{i+1}, t^n) + G(x_{i-1}, t^n)$$

And this is an exact solution with an ad-hoc discretization

But, if the problem is not so simple (non-linearities, multidimmensions, system of equations...) e.g. Euler eqs in comp. Flow

$$\frac{\partial^2 u}{\partial t^2} - u^2 \frac{\partial^2 u}{\partial x^2} = \left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right]\left[\frac{\partial u}{\partial t} - u\frac{\partial u}{\partial x}\right] = 0$$



Tracking the characteristic lines becomes harder.

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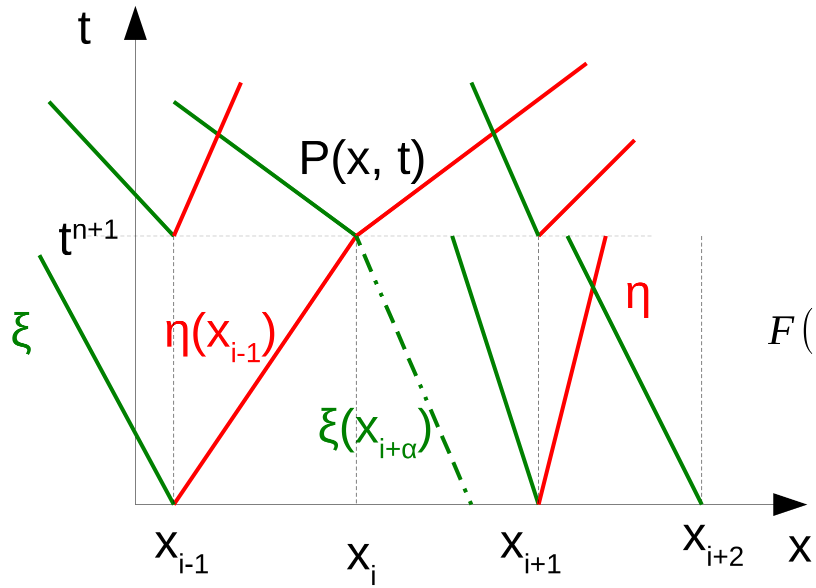
The discretization becomes characteristic-slope restricted.



$$u(x_i, t^{n+1}) = F(x_{i+\alpha}, t^n) + G(x_{i-1}, t^n); \quad \alpha \in [0,1]$$

So an exact evaluation of

$$F(x_{i+\alpha}, t^n)$$

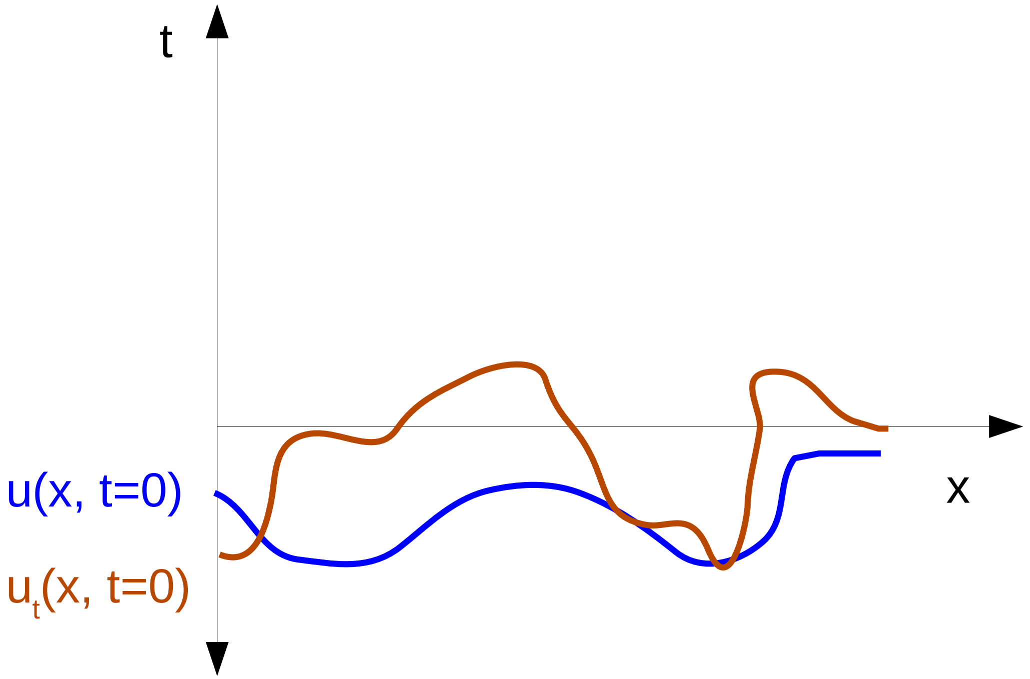
would lead, again, to an exact solution.

It can be calculated with expressions like

$$F(x_{i+\alpha}, t^n) \simeq F(x_{i+1}, t^n) + \frac{F(x_{i+1}, t^n) - F(x_i, t^n)}{x_{i+1} - x_i}(x_{i+\alpha} - x_{i+1}) + \ldots$$

And the family of characteristics line approach methods arises.

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$$



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A study case

MATHEMATICS AS THE LANGUAGE OF PHYSICS

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1. Statement of the problem
    1. Engineering problem
    2. Physical problem
    3. Scales and sizes
2. Understanding the equations and systems of equations
    1. What determines the solution properties?
    2. Heat equation, Waves equation and Poissson equation revisited
3. Towards the discretization