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Optimization and Sizing for Propulsion System of Liquid Rocket Using Genetic Algorithm

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Abstract

Flight vehicle conceptual design appears to be a promising area for application of the Genetic Algorithm (GA) as an approach to help to automate part of the design process. This computational research effort strives to develop a propulsion system design strategy for liquid rocket to optimize take-off mass, satisfying the mission range under the constraint of axial overload. The method by which this process is accomplished by using GA as optimizer is outlined in this paper. Convergence of GA is improved by introducing initial population based on Design of Experiments Technique.

Key words：liquid rocket；propulsion system；genetic algorithm；design of experiments

1 Introduction

Design optimization of liquid rocket propulsion system is still a challenging and labor-intensive process. There is little published information available on propulsion system design codes used in industry (for obvious competitive reasons), and many of these codes are known to employ the gradient-based schemes to optimize the continuous variables. Using a Genetic Algorithm (GA) as a non-gradient based global search method allows optimization-like techniques to be applied in the conceptual phase of design, which traditionally has been dominated by qualitative or subjective decision making. Features of the GA provide several advantages over the traditional gradient-based schemes for conceptual design including: the ability to combine discrete, integer and continuous variables, the population-based search, no requirement for an initial design, and the ability to address non-convex, multi-modal and discontinuous functions. The conceptual design stage of propulsion system is crucial to the success of the total design process and the resulting vehicle system. It has been estimated that at least 80 percent of a vehicle's life-cycle cost is locked in by the concept that is chosen.

Liquid Rocket Propulsion Systems (LRPSs) are the most popular form of rocket propulsion when relatively high specific impulse and high thrust level are required. Performance of a liquid rocket depends greatly on the continuous variables like chamber pressure and oxidizer mass fuel rate; however, the integer and discrete variables like the number and shape of fuel ports and choice of oxidizer and fuel system, also impact the rocket performance . A multidisciplinary optimization for expandable solid launcher are successfully solved by GA in Ref.[1]. A comprehensive overview of GA applied in propulsion and other aerospace disciplines are accounted for in Ref.[2]. Another application of GA for hybrid rockets is given in Ref.[3]. Most of the published information available on propulsion system design consider solid motor sizing considering internal and external ballistics simultaneously.

To date it is not aware of any published application of the GA to the liquid rocket propulsion system design considering simultaneously internal

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and external ballistics. This factor has provided the motivation for the efforts described in this paper in which GA is used to optimize the liquid rocket propulsion system considering internal and external ballistics simultaneously.

2 Statement of the Problem

Liquid propellant rocket engine and propellant feed system form the propulsion system. Tandem propellant tanks with pressure feed system are selected for this study. A single stage ballistic missile capable to deliver a payload of 1000 kg at a mission range of 1500 km is considered. The structural mass other than propulsion system including rocket casing, electronic components /actuators, cable network and common mounting parts etc are taken from statistical data on similar rocket. The aerodynamic coefficients are obtained from Ref.[4]. Commercially available LOX/RP-1 is selected as propellant for propulsion system conceptual design problem.

2.1 Objective selection

The minimum take-off mass is taken as objective function. Since vehicle development costs tend to vary as a function of gross take-off mass, this minimum gross take-off mass vehicle may be considered as a minimum development cost concept.

Axial overload constraint is implemented to be restrict below 12 g. During launch maneuver, the maximum angle of attack is constrained to be below 8°. Weight to thrust ratio is constrained within allowable limits.

2.2 Design variables

Design variables define the design space which is explored through optimizer to get the objective function satisfying all implemented constraints. The design variables considered for this study are: combustion chamber pressure p_c, nozzle exit plane pressure p_e, case diameter d, thrust F and burning time t_k.

Table 1 shows the lower bound and upper bound for each design variable.

2.3 Liquid fueled missile's mass equation

The initial take-off mass of a rocket may be represented as the sum

$$m_0 = m_{pay} + m_d + m_p \tag{1}$$

Table 1 Ranges of design variables

Parameter	Lower bound	Upper bound
p_c / Pa	50	150
p_e / Pa	0.45	0.75
t_k / s	50	150
d / m	1.0	2.5
F / kN	200	600

Mass of "dry" shell m_d can be expressed as:

$$m_d = m_{ca} + m_{en} + m_t + m_{pt} \tag{2}$$

where m_{ca} is the mass of control apparatus, m_{en} is the mass of engine, m_t is the mass of tail section and m_{pt} is the mass of propellant tank.

In this study masses of the tail and the instrument section are taken from statistical data while component weight relationships have been developed for the propulsion system.

2.4 Propulsion system component sizing and weight relationships

The propulsion system component weight relationships are of prime importance for optimization procedure. Some empirical relations[5] are also used for this purpose.

The whole mass model depends upon the selected design variables. The expression for throat area may be expressed as follows

$$A_t = \frac{F}{p_c \cdot C_f} \tag{3}$$

The mass of combustion chamber

$$m_{ch} = \frac{\pi \rho_c d_c \delta_c}{\varepsilon_c} \left\{ \begin{array}{c} L^* - \frac{1}{3}\left(\frac{F}{p_c C_f}\right)^{1/2} \\ \left(\varepsilon_c^{1/2} - 1\right)\frac{\cos\theta}{\sin\theta} \end{array} \right\} \tag{4}$$

where

$$\delta_c = \frac{f p_c d_c}{2[\sigma]} \tag{5}$$

L*=1.143 m for (LOX/RP-1)

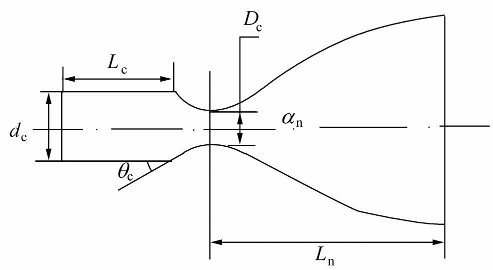

Fig.1 Elements of basic cylindrical combustion chamber

$$C_{\mathrm{f}}=\sqrt{\frac{2\gamma^2}{\gamma-1}\left[\frac{2}{\gamma+1}\right]^{\frac{\gamma+1}{\gamma-1}}\left[1-\left(\frac{p_{\mathrm{e}}}{p_{\mathrm{c}}}\right)^{\frac{\gamma-1}{\gamma}}\right]}+ \varepsilon\left[\frac{p_{\mathrm{e}}-p_{\mathrm{a}}}{p_{\mathrm{c}}}\right] \tag{6}$$

Nozzle Length

$$L_{\mathrm{n}}=0.8\sqrt{\frac{F}{\pi p_{\mathrm{c}}C_{\mathrm{f}}}}\left[\frac{\sqrt{\varepsilon_{\mathrm{c}}}-1+1.5(\sec\alpha_{\mathrm{n}}-1)}{\tan\alpha_{\mathrm{n}}}\right] \tag{7}$$

The mass of nozzle

$$m_{\mathrm{n}}=\frac{\pi\rho_{\mathrm{n}}f_{\mathrm{s}}(\varepsilon-1)}{2\sin\theta_{\mathrm{n}}}\sqrt{\frac{F}{p_{\mathrm{c}}C_{\mathrm{f}}}}0.67\sqrt{(\varepsilon-1)\left(\frac{p_{\mathrm{c}}+\sigma_{\mathrm{n}}}{E}\right)} \tag{8}$$

Total mass of propellant

$$m_{\mathrm{p}}=\frac{t_{\mathrm{k}}F}{C_{\mathrm{f}}}\sqrt{\frac{\gamma[2/(\gamma+1)]^{\frac{\gamma+1}{\gamma-1}}}{RT_{\mathrm{c}}}} \tag{9}$$

where p_{c}, T_{c} and L_{c} are the combustion chamber pressure, temperature and length respectively. Ellipsoidal tank end volume is calculated from propellant volume with 5% additional ullage volume,

$$V_{\mathrm{t}}=\pi r^2 l_{\mathrm{c}}+\frac{4\pi r^2 r_{\mathrm{e}}}{3} \tag{10}$$

where r is the radius of tank, r_{e} is the maximum radius of ellipsoidal end.

Mass of tank with two elliptical ends and a cylindrical part

$$m_{\mathrm{pt}}=\frac{\pi r^2 t_{\mathrm{c}}E'\rho_{\mathrm{t}}}{k}+2\pi r l_{\mathrm{c}}t_{\mathrm{c}}\rho_{\mathrm{t}} \tag{11}$$

where t_{c} is the thickness of the tank, E' is the design factor, k is ratio of tank radius to ellipsoidal end radius.

Pressurization tank volume

$$V_{\mathrm{L}}=\frac{W_{\mathrm{g}}R_{\mathrm{g}}T_{\mathrm{g}}}{P_{\mathrm{g}}} \tag{12}$$

where W_{g} is the the required pressurant weight in the propellant tank, T_{g} and P_{g} are the required temperature and pressure in pressurant tank.

Mass of pressurizing spherical storage tank

$$m_{\mathrm{pr}}=2(2\pi r_{\mathrm{p}}t_{\mathrm{p}}\rho_{\mathrm{p}}) \tag{13}$$

where r_{p}, t_{p} and ρ_{p} are radius, thickness and density of pressurant tank.

3 Software Algorithm

A set of design parameters is passed to weight and sizing for propulsion system. Fig.2 depicts a block diagram of the overall structure of the program and many of the components that are sized during this design process. As shown the entry of the program defines the mission and records the input data. Mission definition and input data are used to estimate sizing, mass and characteristic performance of the vehicle.

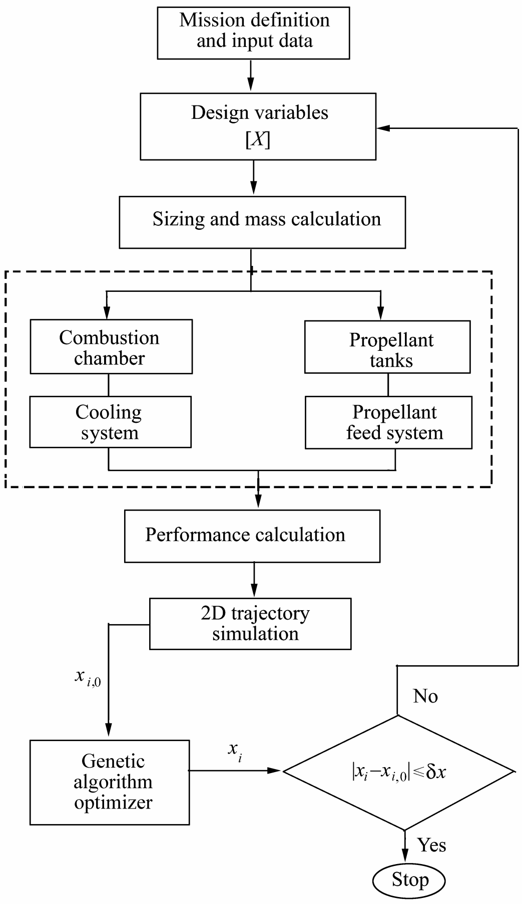

Fig.2 Flow chart of solution strategy

With the initial estimates of the vehicle characteristics and system design variables, a 2D trajectory is simulated by solving the equations of motions. Data obtained from the trajectory and initial estimates of the system design variables are now the primary inputs for the optimizer. GA optimizer is used to search for a combination of system design variables to minimize the initial take-off mass subjected to mission range and overload constraint. The principle outputs are vehicle component weights and the optimized system design variables. Linking the propulsion system code and trajectory simulation code to the genetic algorithm is done in modular fashion so that other modules could be later substituted for the ones used in the study.

4 Trajectory Simulation

Two degree of freedom (2DOF) missile trajectory is simulated using simplified equations of motion[6], assuming that:

(1) the rocket motion is symmetrical and takes place in a launch plane as shown in Fig.3

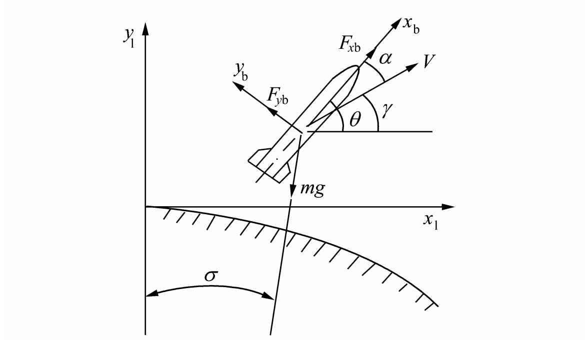

Fig.3 Planar motion of rocket

(2) the Earth is spherical and non-rotating; and no disturbances are included.

The motion differential equations are

$$\left.\begin{aligned}
\frac{\mathrm{d}V_{x1}}{\mathrm{d}t} &= \frac{1}{m}(F_{xb}\cos\theta - F_{yb}\sin\theta) + g_{x1} \\
\frac{\mathrm{d}V_{y1}}{\mathrm{d}t} &= \frac{1}{m}(F_{xb}\sin\theta + F_{yb}\cos\theta) + g_{y1} \\
\frac{\mathrm{d}x_1}{\mathrm{d}t} &= V_{x1} \\
\frac{\mathrm{d}y_1}{\mathrm{d}t} &= V_{y1} \\
\frac{\mathrm{d}m}{\mathrm{d}t} &= -\mu_{\mathrm{p}}
\end{aligned}\right\} \tag{14}$$

Where relational equations are

$$\left.\begin{aligned}
g_{x1} &= -\frac{\mu x}{r^3} \\
g_{y1} &= -\frac{\mu(R_{\mathrm{E}} + y_1)}{r^3} \\
r^2 &= x_1^2 + (R_{\mathrm{E}} + y_1)^2 \\
h &= r - R_{\mathrm{E}} \quad \text{or} \quad h \approx y_1 + \frac{x_1^2 + y_1^2}{2R_{\mathrm{E}}} \\
V_{xb} &= V_{x1}\cos\theta + V_{y1}\sin\theta \\
V_{yb} &= -V_{x1}\sin\theta + V_{y1}\cos\theta \\
V^2 &= V_{xb}^2 + V_{yb}^2 \\
\tan\alpha &= -\frac{V_{yb}}{V_{xb}} \\
\tan\gamma &= \frac{V_{y1}}{V_{x1}} \quad \text{or} \quad \gamma = \theta - \alpha
\end{aligned}\right\} \tag{15}$$

where R_{E} is radius of Earth, α is the angle of attack, θ is angle of pitch and γ is the flight path angle.

A standard flight program US Standard Atmosphere SA-76 is used during trajectory simulation. The motion equations are modeled and simulated in MATLAB.

5 Selection of Encoding for GA

Selection of the best encoding scheme of the GA depends on the nature of the problem undergoing optimization. When first presented to GA, binary encoding often is used to illustrate the ideas of GA and its operators. This might lead to the assumption of binary encoding being the best and most convenient encoding method for the optimization. According to several authors (Goldberg, Wright, Gen and Cheng) the presence of Hamming cliffs should be kept in mind when making the choice of encoding. Literature seems to agree to binary encoding handling Hamming cliffs poorly. In Ref.[7] the authors claims that the real-number encoding has been widely confirmed to perform better than both binary and Gray encoding. On the other hand research performed by Goldberg states that the decision is far from clear cut[8]. In his work Goldberg also give the recommendation of not agonizing over the coding, but simply deciding.

In Ref.[9] the author suggests real-coded GA, because it is better suited for the non-linear programming problems in view of the efficiency of search performance. Making a literature study on the choice of encoding seems, to some extend, to be a complex task but most of the practitioners of GA continually report successes using real genocodes (encodings). If the optimization results in being unstable, after multiple runs, or if the premature convergence is suspected, the encoding might be the cause of this, but there are other factors playing the roles. In this study real coded GA is used for optimization.

6 Genetic Algorithm

Genetic algorithm (GA) is stochastic global search and optimization method that mimic the metaphor of natural biological evolution[10]. GA operates on a population of potential solutions applying the principle of survival of the fittest to produce successively better approximations to a solution. At each generation of a GA, a new set of approximations is created by the process of selecting individuals according to their level of fitness in the problem domain and reproducing them using operators borrowed from natural genetics. This process leads to the evolution of populations of individuals which are better suited to their environment than the individuals from which they were created, just as in natural adaptation[11].

In this study, the GA is the controlling routine which calls the propulsion and trajectory performance code as needed. The GA passes down the design parameters to performance and sizing code which gives mass and mass flow rate to trajectory simulation that passes back a measure of how well the design performed in terms of minimum take-off mass of missile achieved. Linking the sizing code and trajectory simulation code to the genetic algorithm is done in modular fashion so that other modules could be later substituted for the ones used in the study.

To get the best results from the genetic algorithm, it is needed to experiment with different combinations of GA parameters. Selecting the best options for a problem involves trial and error, i.e., one of the most important factors that determine the performance of the genetic algorithm is the diversity of the population. If the average distance between individuals is large, the diversity is high; and if the average distance is small, the diversity is low. Getting the right amount of diversity is a matter of trial and error. If the diversity is too high or too low, the GA might not perform well. The GA parameters for this study are reported in Table 2.

The controls for the GA used in this study are fairly common. Historically GA has been run with higher population members to help maintain good diversity. Candidate designs violating axial overload constraints of 12 *g* are assigned a zero fitness level, so that it will learn not to try these designs in the future.

Table 2 Parameters for genetic algorithm

Mode/variable	Value
Maximum number of generations G	200
Population type	Double vector
Selection	Stochastic uniform
Crossover	Single point, $p_c = 0.8$
Mutation	Uniform, $p_m = 0.05$
Fitness scaling	Rank
Reproduction	Elite count=2
Population size	40
Termination criteria	Fitness limit of 13 500 kg

7 Design of Experiment

Design of Experiment techniques gives a set of experimental points, which allows estimation of the model with the maximum confidence by using just a fraction of the number of experimental runs[12]. In this study, this DoE method is used to introduce statistically selected initial population for GA and its performance is compared with that of simple GA.

Optimal design of experiments is formed using the following process. Lower and upper bound ranges of all design variables (factors) are divided into 16 equal levels forming a large pool of candidate set. An initial starting design of 40 (equal to population size) test points is chosen at random from the set of defined candidate points, *m* additional points chosen from the candidate set are added to the design randomly and *m* points are deleted from the design optimally, i.e., to minimize the prediction error variance (PEV). If the resulting design is better than the original, it is kept. This process is repeated until either (a) the maximum number of iterations is exceeded or (b) a certain number of iterations have occurred without appreciable change in the optimality value for the design.

A useful measure of the quality of an experiment design is its prediction error variance (PEV). V-optimal designs of experiments minimizes the average PEV, to obtain accurate prediction. The V-optimality value is calculated using the formula

$$V_{\mathrm{eff}} = \frac{1}{nc}\sum_{j} \boldsymbol{x}_j' (\boldsymbol{X}_{\mathrm{c}}'\boldsymbol{X}_{\mathrm{c}})^{-1}\boldsymbol{x}_j \tag{16}$$

Where $\boldsymbol{x}_j$ are rows in the regression matrix, $\boldsymbol{X}_{\mathrm{c}}$ is the regression matrix of all candidate set points, and n_{c} is the number of candidate set points.

For this study, model based calibration toolbox of MATLAB is used to generate V-optimal design of experiments. These selected designs as shown in Fig.4, act as initial population of GA instead of randomly generated first population. This methodology resulted in decreasing the average number of generations required for convergence from 76 to 46 only.

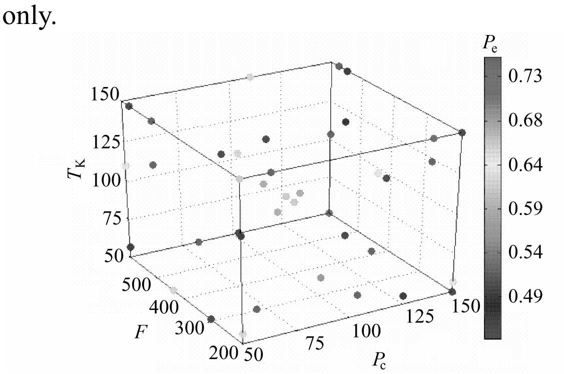

Fig.4 Optimal design of experiments (40 designs) as initial population for GA

8 Performance Results

GA performance for two different cases mentioned above is compared. The first case is based on simple GA with randomly created initial population. The second case is based on GA with its initial population created through V-optimal experimental design technique.

With the design problem and parameters completely defined, the two cases are executed (ten times each) until the minimum take-off mass is achieved. The comparison of overall performance of these two cases is illustrated in Table 3.

The most significant contribution is the drastic reduction in number generations required for convergence when GA is initialized with V-optimal design. The proposed scheme is especially beneficial

Table 3 Performance comparison

Parameter	Case-1 (GA)	Case-2 (GA+DoE)
Mean generations	76	46
Min generations	9	13
Max generations	140	124
Std deviation of gen	43	35
Number of exact Analysis (mean)	3 040	1 840

for complex non-linear optimization problems without the expense or effort of an enumerative search strategy. The number of generations needed to converge reflects the performances of these two cases in Fig.5 and Fig.6.

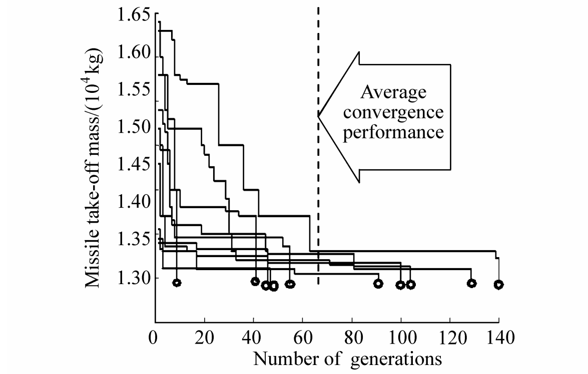

Fig.5 Case-1: performance of GA with random initial population

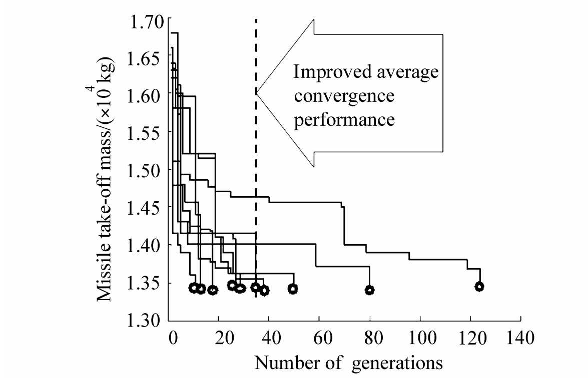

Fig.6 Case-2: performance of GA with initial population created from optimal design of experiments

Propulsion system design problem is posed to GA optimizer and it is successfully solved under the given conditions and constraints.

The optimal design is achieved after 46 generations. The optimal design variables and trajectory results are shown in Table 4. Trajectory simulation plots are presented in Fig.7.

Table 4 Optimum results

Design variable	p_c/Pa	p_e/Pa	t_k/s	d/m	F/kN
Value	112	0.71	103	1.12	301
Performance variable	m_0/kg	m_p/kg	m_d/kg	$\dot{m}$ /(kg • s^{-1})	υ_o
Value	13 500	10 125	2 325	98	0.44

υ_o = weight to thrust ratio

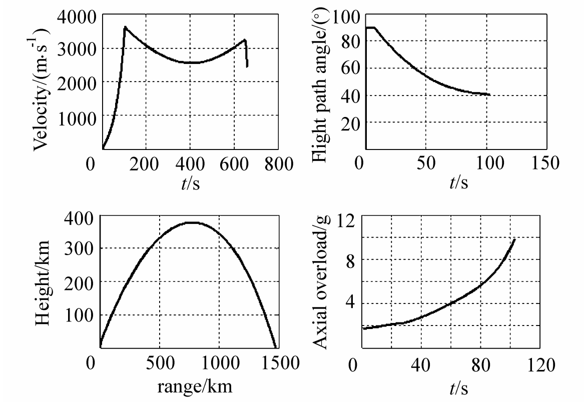

Fig.7 Trajectory performance of optimal design

9 Conclusions

A method of optimization using G A as optimizer for conceptual design of liquid rocket propulsion system are demonstrated in this paper. The optimization procedure is accompanied with a calculation of point mass trajectory. The method described in this paper provides the designer with a simple and powerful approach to the preliminary design. Simple analytical expressions are used for propulsion system sizing, which can be easily replaced by highly accurate code with more capabilities.

It is found that the genetic algorithms can be used to solve complex system design objective functions if genetic algorithm parameters are chosen to induce sufficient randomness. Performance enhancement by introducing Design of Experiments based initial population resulted in exciting gains in terms of improved convergence and much lower computational time.

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