ETW3450 Assignment 3

Part 1

1.1 Introduction & 1.2 Data preprocessing

Cryptocurrencies differ widely in how much electricity they consume. Ren and Lucey (2022) point out that Bitcoin and Ethereum use Proof-of-Work, an algorithm that requires large amounts of energy, so they are often described as “dirty.” By contrast, Cardano and Stellar rely on energy-saving consensus methods such as Proof-of-Stake and are viewed as “clean.” Exploring how these two groups move with monthly U.S. electricity prices can shed light on the links between energy markets and digital-asset valuations.

In this study, we collect monthly average U.S. electricity prices (all sectors) and daily prices for four major cryptocurrencies from authoritative online sources. Specifically, two "dirty" cryptocurrencies (Bitcoin and Ethereum) and two "clean" cryptocurrencies (Cardano and Stellar) are selected for analysis. The original periods available for each variable are as follows:

Table 1: Raw Data Availability for all variables

Variable	***Frequency***	***Available Period***
Electricity Price (All Sectors)	Monthly	2001-01 – 2025-03
Bitcoin Price	Daily	2010-07-18 – 2025-06-08
Ethereum Price	Daily	2015-08-07 – 2025-06-08
Cardano Price	Daily	2017-10-18 – 2025-06-08
Stellar Price	Daily	2014-08-06 – 2025-06-08

Sources: Cryptocurrency prices from TokenInsight

Electricity prices from U.S. Energy Information Administration

Our first job is to build a single, tidy dataset that we can feed straight into VAR or VECM without worrying about missing dates or mismatched frequencies. In other words, we want the largest sample we can get while keeping every series time and frequency aligned.

We decided to convert the daily crypto prices into monthly averages to match the format of the electricity data, which is already reported as a monthly average. This way, all the variables are aligned in frequency and reflect the same kind of information. Instead of focusing on one specific day, each value now represents the average price level during that month.

We use a shared time window from November 2017 to March 2025, which is the longest period where all five variables have complete data. This helps keep time consistent across every series. We chose November 2017 as the starting point because it is the first month with full data for Cardano, the most recently introduced cryptocurrency in our selection. The endpoint is March 2025, the latest month for which electricity prices are available. This gives us 89 months of complete and aligned observations, allowing us to include as much data as possible.

After converting all five variables into monthly time series objects with shared time window, we then combined them using the cbind() function to form a single multivariate time series named "multi_ts". We also check if there is any NA value in each series. The results showed no NA in any of the series, so we stored the complete dataset as "multi_clean". This final dataset is fully aligned and ready for further analysis. The full data processing procedure is shown in R Code Sections 1.1 and 1.2.

1.3 Preliminary analysis

1.3.1 Seasonality analysis

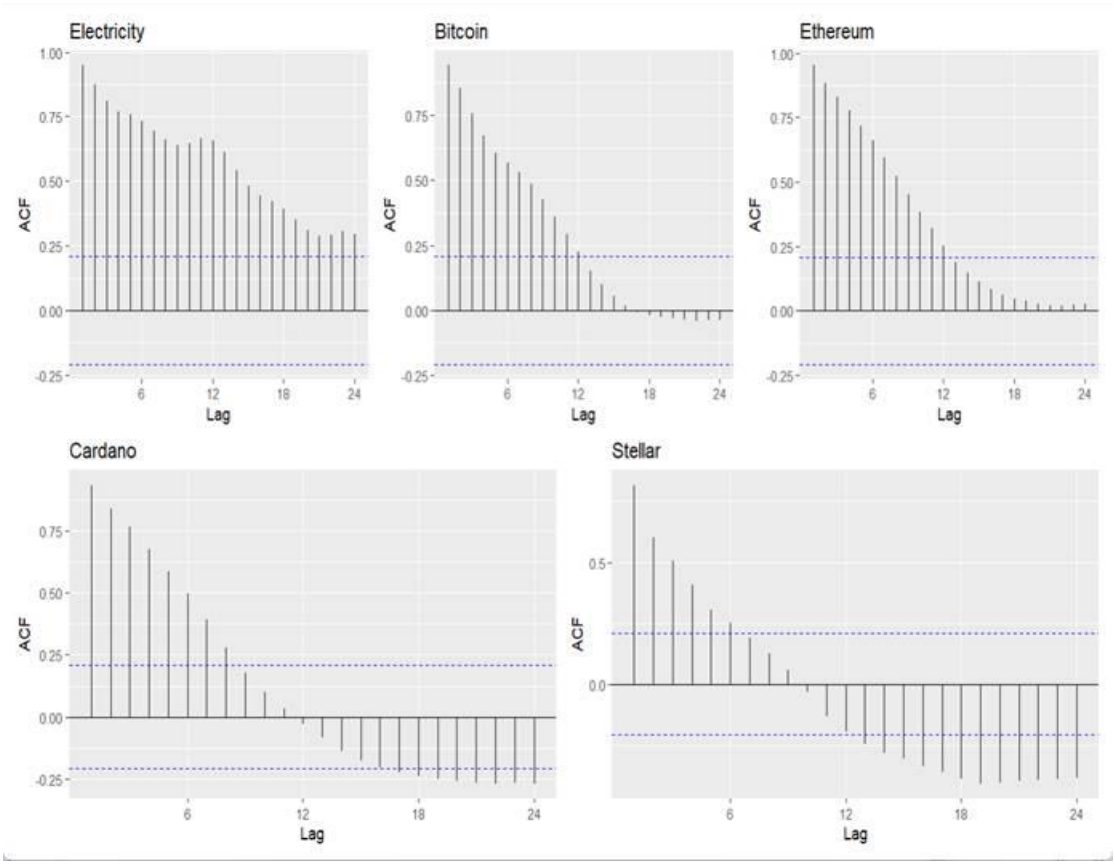

Figure 1: ACF of Monthly Prices for Electricity and Selected Cryptocurrencies

When a time series contains a trend, the ACF typically shows positive values that decay slowly with increasing lags, and when seasonality is present, the ACF will exhibit pronounced peaks at lags corresponding to multiples of the seasonal period (Hyndman & Athanasopoulos, 2018). The ACF plots in Figure 1 show a slow decline as the lag increases for all five series. This gradual decay is characteristic of trending behavior, suggesting the presence of trend in each series and therefore they are non-stationary. For the electricity price series, we also observe prominent spikes at seasonal lags such as 12 and 24, indicating the presence of seasonality in this series. In contrast, the ACFs for the four-cryptocurrency series do not show such seasonal spikes, implying that they do not exhibit clear monthly seasonality.

1.3.2 Unit root & Cointegration Analysis

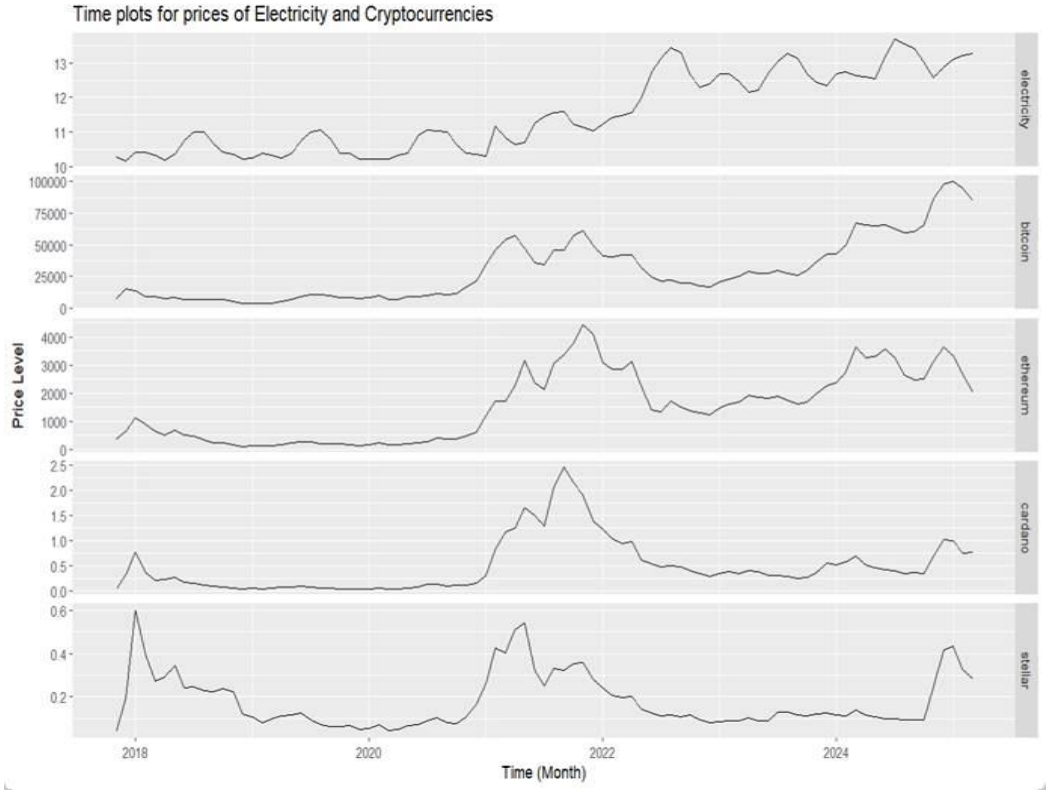

Figure 2: Time plots for monthly prices of Electricity and Cryptocurrencies

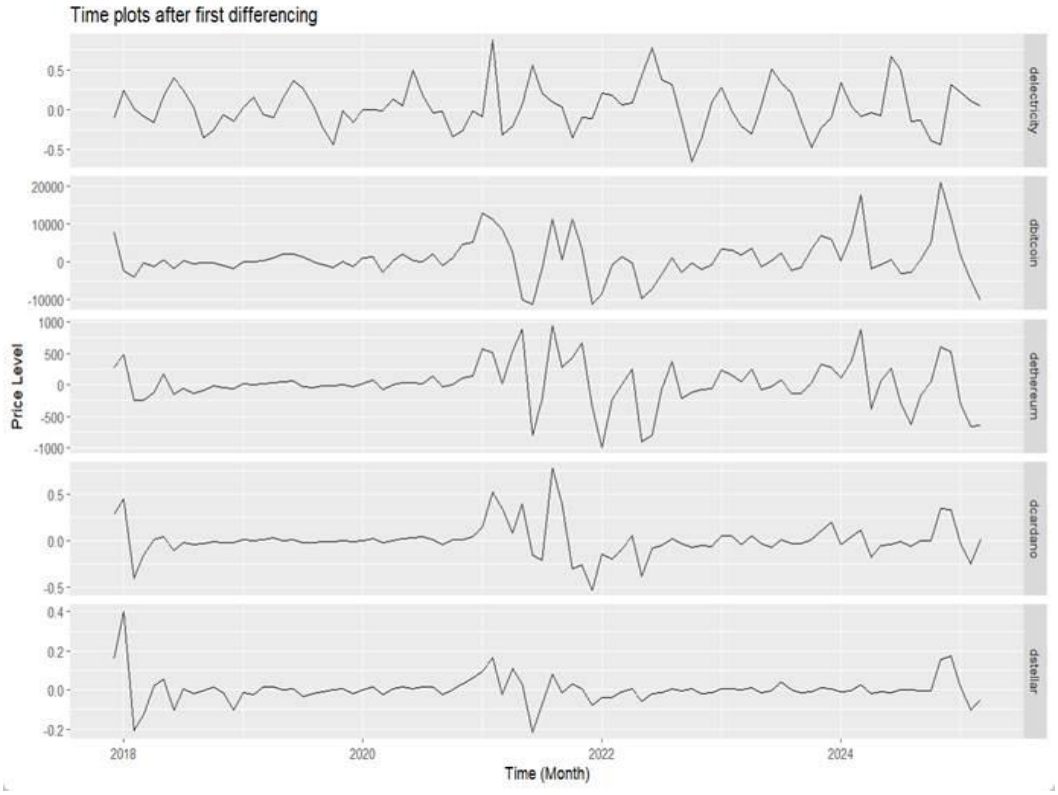

Figure 3: Time plots after first differencing

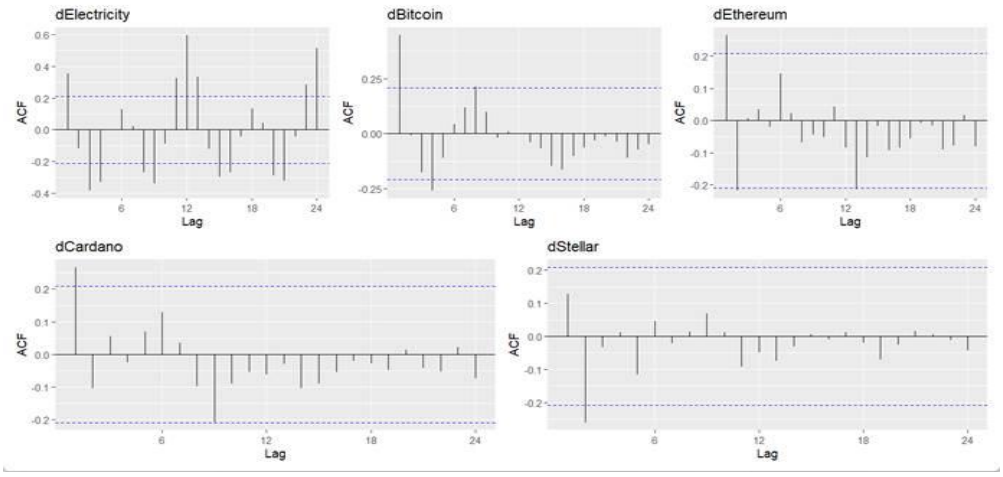

Figure 4: ACF plots of variables after first differencing

Unit Root Analysis (Informal & Fromal)

From Figure 1 and 2 above, we can see that none of the series have a deterministic trend, since they are not monotonically increasing with time. We can also observe that there is no constant mean across time for all 5 series and therefore they are non-stationary. As discussed earlier in 1.3.1, the ACF shows a slow decay from an initial value close to one for each series, which is a typical characteristic of a non-stationary process and further supports this conclusion. Since it is clear from visual inspection that none of time series above have a deterministic trend, any nonstationarity must be due to the presence of a stochastic trend or unit root. In addition, we can also observe that all 5 variables appear to have non-zero means.

Figure 3 displays the series after first differencing. We can observe that each series displays a mean-reverting behaviour and behaves like a white noise process (zero mean, relatively constant variance and no predictable pattern in the long run). As shown in Figure 4, the ACFs decline very rapidly for the series after first differencing, suggesting that they are already stationary. Since differencing the 5 variables once appears to make them stationary, we tentatively conclude that all 5 variables have a single unit root. Overall, all 5 variables seem like I(1) processes containing only stochastic trends. Additionally, we can also observe that all 5 first-differenced variables have approximately zero means.

Although the empirical evidence shows that the unit roots tests have a low power, we still apply the Augmented Dickey-Fuller (ADF) test to each series in both levels and first differencing forms to formally verify the presence of unit root and establish the order of integration for each variable.Basically, we first apply the ADF test to the level series of each variable. The null and alternative hypotheses for this test is that:

$H_0: \gamma = 0$ i.e y_t has at least one unit root

$H_1: \gamma < 0$ i.e y_t does not have a unit root and it is I(0)

We fail to reject the null hypothesis at 5% significance level for all 5 variables and conclude that all 5 variables have at least one unit root; they are at least I(1) process.

To formally test whether all variables are indeed I(1) process as suggested by the visual inspection, we then conduct the ADF test on the differenced series of each variable. The null and alternative hypotheses is that:

$$H_0: \gamma = 0 \text{ i.e } y_t' \text{ has at least one unit root}$$

$$H_1: \gamma < 0 \text{ i.e } y_t' \text{ does not have a unit root and it is I(0)}$$

We reject the null at 5% significance level for all 5 variables and conclude that all 5 variables after first differencing are I(0) process. Since all variables after first differencing is a I(0) process, we confirm that each variable is indeed a I(1) process.

In summary, through the ADF tests, we formally confirmed our conclusion from the visual inspection that all five variables are I(1) processes containing only stochastic trend. Refer to Appendix 01 for complete analysis of ADF tests for each variable. The related R code is provided in Section 1.3.4.

Cointegration Analysis (informal)

Since each series has been confirmed as an I(1) process with only a stochastic trend, we now explore whether there might be long-run co-movements between them. Looking at Figure 2, we can see that the prices of some cryptocurrencies, especially Bitcoin and Ethereum, often move together, rising and falling around the same time. Cardano and Stellar also tend to follow similar movements during periods of major spikes. This kind of visual pattern suggests the possibility of a long-run relationship among the cryptocurrency series. In contrast, electricity prices follow a more gradual and independent path, showing no strong co-movement with the crypto prices. While visual inspection gives us an initial sense of potential cointegration between variables, we will need formal statistical tests to confirm it.

1.3.3 Extreme Observations Analysis (spikes & outliers)

From Figure 2, we observe that all four cryptocurrencies reached their peak prices around April to May 2021. This surge occurs around late 2020 to early 2021 and appears to coincide with the global COVID-19 pandemic. During this time, massive monetary and fiscal stimulus, growing interest in alternative investments, and broader digital adoption may have contributed to increased speculative demand in the crypto market. In contrast, the electricity price series shows a steady upward trend without any sharp spikes.

We then applied the tsoutliers() function to identify any potential outliers within each time series. The outliers detected in each time series along with the suggested replacement values by this function are presented in the Appendix 02. Outliers can arise for different reasons. Some are due to data entry errors and should be corrected or removed. Others occur naturally when observations differ substantially from the rest of the data. In these cases, it may not be appropriate to discard them without further investigation. According to Hyndman and Athanasopoulos (2018), outliers that reflect real structural changes should be retained, as removing them may distort the true economic signal. Because we collect our data from credible sources, we believe that the outliers

detected by this function should be attributed to structural changes, so we do not do anything with the outliers detected.

Part 2

i)

```
> ### Section 2.1
> # Convert the variable in log form
> df <- log(multi_clean)
> colnames(df) <- c("ln_electricity","ln_bitcoin","ln_ethereum","ln_cardano","ln_stellar")
> head(df)
         ln_electricity ln_bitcoin ln_ethereum ln_cardano ln_stellar
Nov 2017       2.330200   8.965621    5.883118 -3.3257859 -3.2306381
Dec 2017       2.319442   9.653498    6.466967 -1.1318120 -1.6177475
Jan 2018       2.342767   9.506448    7.018961 -0.2597854 -0.5072540
Feb 2018       2.343727   9.148388    6.772884 -1.0026606 -0.9248566
Mar 2018       2.336020   9.120609    6.454574 -1.5407911 -1.3038559
Apr 2018       2.320425   8.976781    6.229930 -1.4880824 -1.2368580
```

Figure 5: Log transformation

Note:

(1) We have confirmed that each variable is I(1) process in report section 1.3.2

(2) The rationale for log transformation is explained in detail in Part 3 (i)

(3) All variables in the df remain I(1) processes, as the log transformation does not change the order of integration.

ii)

Since the analysis in Section 1.3.2 suggested a possible cointegration relationship between the five I(1) variables, we need to first rely on Granger Representation Theorem to help us determine the correct model we should use. Readers can refer to the report part 3 iv for the entire analysis process using Granger Representation Theorem. From the analysis in part 3 iv, we know that the rank of the Π matrix is 0 (i.e. it is a null matrix) or in other words, we cannot find a CIV for these five I(1) variables. Therefore, a VAR in first differences is the appropriate model for a set of I(1) variables that are not cointegrated. After determining the correct model, we took the following steps to find a suitable five-dimensional VAR model.

```
> ddf <- diff(df)
> colnames(ddf) <- c("dln_electricity","dln_bitcoin","dln_ethereum","dln_cardano","dln_stellar")
> head(ddf)
         dln_electricity dln_bitcoin dln_ethereum dln_cardano dln_stellar
Dec 2017   -0.0107580500  0.68787665    0.5838488  2.19397390  1.61289066
Jan 2018    0.0233246726 -0.14704950    0.5519944  0.87202660  1.11049351
Feb 2018    0.0009601537 -0.35806046   -0.2460774 -0.74287522 -0.41760260
Mar 2018   -0.0077071672 -0.02777885   -0.3183099 -0.53813048 -0.37899935
Apr 2018   -0.0155948580 -0.14382751   -0.2246432  0.05270867  0.06699788
May 2018    0.0165615086  0.06563182    0.2853910  0.18574281  0.17395764
```

Figure 6: First-differenced log transformation

We first take a first difference to all logged I(1) variables to make them I(0).

```
> summary(ddf)
 dln_electricity       dln_bitcoin         dln_ethereum       dln_cardano        dln_stellar
 Min.   :-0.0500682   Min.   :-0.400968   Min.   :-0.47705   Min.   :-0.74288   Min.   :-0.60092
 1st Qu.:-0.0101321   1st Qu.:-0.061898   1st Qu.:-0.09276   1st Qu.:-0.14130   1st Qu.:-0.14218
 Median : 0.0004801   Median : 0.007944   Median : 0.01317   Median :-0.02880   Median :-0.01160
 Mean   : 0.0029012   Mean   : 0.027120   Mean   : 0.01962   Mean   : 0.03465   Mean   : 0.02227
 3rd Qu.: 0.0166520   3rd Qu.: 0.136873   3rd Qu.: 0.16220   3rd Qu.: 0.16528   3rd Qu.: 0.09560
 Max.   : 0.0811634   Max.   : 0.687877   Max.   : 0.64785   Max.   : 2.19397   Max.   : 1.61289
> VARselect(ddf, lag.max = 10, type = "none")
$selection
AIC(n)  HQ(n)  SC(n) FPE(n)
     1      1      1      1
```

Figure 7: Summary and VARselect

Next, we use VARselect() to help us to determine the initial order of the VAR model. Since all variables have an approximately zero mean (can see from the Figure 3 or summary of variables above), we should not include an intercept for the VAR specification (type = “none”). All information criteria suggested a VAR (1) model.

```
> # Estimate VAR(1) model
> var1 <- VAR(ddf, p = 1, type = "none")
> # Test for serial correlation in residuals
> serial.test(var1, lags.pt = 16, type = "PT.asymptotic")

        Portmanteau Test (asymptotic)

data:  Residuals of VAR object var1
Chi-squared = 401.32, df = 375, p-value = 0.1676
```

Figure 8: Portmanteau test results

We initially estimate a VAR (1) model, as suggested by all information criteria and then use serial.test() function to test whether the errors in this model are vector white noise. The null and alternative hypothesis for this test is that:

$$H_0: \rho_1 = \rho_2 = \cdots = \rho_{16} = 0; \epsilon_t \text{ is a vector white noise}$$

$$H_1: \text{at least one } \rho_j \neq 0; \epsilon_t \text{ is not a vector white noise}$$

Since the p-value = 0.1676 > 0.05, we fail to reject the null at 5% significance level and conclude that the errors in this VAR(1) model are vector white noise. Therefore, we select 1 as the VAR order for this model. Our final 5-dimension VAR model is of the form:

$$\underset{(5\times1)}{x_t} = \underset{(5\times5)}{A_1}\,\underset{(5\times1)}{x_{t-1}} + \underset{(5\times1)}{\varepsilon_t}, \qquad \varepsilon_t \sim \text{VWN}(0, \Omega);$$

$$x_t' = (dx_{1t}, dx_{2t}, dx_{3t}, dx_{4t}, dx_{5t})'$$

Note that all variables are in log first-difference form

$$dx_{1t} = dlnelectricity_t \text{ ; } dx_{2t} = dlnbitcoin_t \text{ ; } dx_{3t} = dlnethereum_t$$

$$dx_{4t} = dlncardano_t; \; dx_{5t} = dlnstellar_t$$

Arranging this 5-dim VAR (1) model by variables, we obtain:

$$dx_{1t} = \alpha_{11,1}\, dx_{1t-1} + \alpha_{12,1}\, dx_{2t-1} + \alpha_{13,1}\, dx_{3t-1} + \alpha_{14,1}\, dx_{4t-1} + \alpha_{15,1}\, dx_{5t-1} + \varepsilon_{1t}$$

$$dx_{2t} = \alpha_{21,1}\, dx_{1t-1} + \alpha_{22,1}\, dx_{2t-1} + \alpha_{23,1}\, dx_{3t-1} + \alpha_{24,1}\, dx_{4t-1} + \alpha_{25,1}\, dx_{5t-1} + \varepsilon_{2t}$$

$$dx_{3t} = \alpha_{31,1}\, dx_{1t-1} + \alpha_{32,1}\, dx_{2t-1} + \alpha_{33,1}\, dx_{3t-1} + \alpha_{34,1}\, dx_{4t-1} + \alpha_{35,1}\, dx_{5t-1} + \varepsilon_{3t}$$

$$dx_{4t} = \alpha_{41,1}\, dx_{1t-1} + \alpha_{42,1}\, dx_{2t-1} + \alpha_{43,1}\, dx_{3t-1} + \alpha_{44,1}\, dx_{4t-1} + \alpha_{45,1}\, dx_{5t-1} + \varepsilon_{4t}$$

$$dx_{5t} = \alpha_{51,1}\, dx_{1t-1} + \alpha_{52,1}\, dx_{2t-1} + \alpha_{53,1}\, dx_{3t-1} + \alpha_{54,1}\, dx_{4t-1} + \alpha_{55,1}\, dx_{5t-1} + \varepsilon_{5t}$$

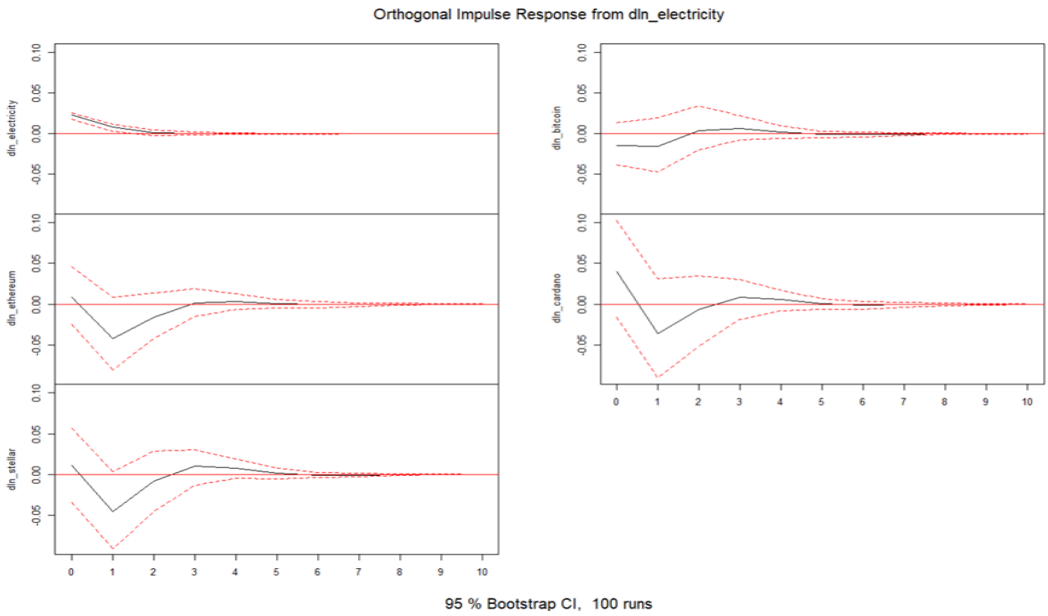

Figure 9: Orthogonal Impulse Response from dln_electricity

Figure 9 displays the estimated impulse responses from each of the five variables in the 5-dim VAR(1) over the first 11 periods (Period from 0 to 10) to a one standard deviation orthogonalized shock to dln_electricity in period 0. The red lines are 95% confidence intervals. Notice that it is evident from Figure 5 that the orthogonalized shock to dln_electricity in period 0 has impact effect on all 5 variables. That is, it can affect all 5 variables in period 0. This is not surprising since it is the first variable in the current VAR ordering. In general, a shock to the variable x_j has no impact effect on any variable that precedes x_j in the VAR ordering. We can also observe that the effects on all variables of this shock gradually converge to zero, suggesting that the 5-dim VAR(1) model is stationary. This conclusion is supported by the stationarity test, with the full analysis provided in Appendix 03. The results confirm that the model is stationary, which explains why the impact of the shock eventually die out over time.

Table 2: Impulse response coeffs arising from one std. dev. orthogonalised shock to dln_electricity

Period	*dln_electricity*	*dln_bitcoin*	*dln_ethereum*	*dln_cardano*	*dln_stellar*
1	0.02256	-0.01496	0.00874	0.03944	0.01116
2	0.00785	-0.01570	-0.04250	-0.03598	-0.04603
3	0.00138	0.00390	-0.01584	-0.00684	-0.00838
4	-0.00016	0.00662	0.00171	0.00815	0.01027
5	-0.00012	0.00222	0.00319	0.00533	0.00763
6	0.00002	-0.00056	0.00048	0.00042	0.00174
7	0.00003	-0.00089	-0.00077	-0.00118	-0.00063
8	0.00000	-0.00042	-0.00061	-0.00081	-0.00061
9	-0.00001	-0.00009	-0.00021	-0.00025	-0.00016
10	-0.00001	0.00000	-0.00002	-0.00002	0.00003
11	0.00000	0.00000	0.00001	0.00001	0.00003

Note: All coefficients are rounded to 5 decimal places and raw data can be found in the Appendix 04 or R code section 2.2.

The columns in Table 2 (excluding the Period column) contain the estimated impulse responses of dln_electricity, dln_bitcoin, dln_ethereum, dln_cardano, and dln_stellar over the first 11 periods to a one standard deviation orthogonalized shock to dln_electricity in period 1, respectively. For example, the effect on dln_electricity in period 1 of a one standard deviation orthogonalized shock to dln_electricity in period 1 is 0. 02256. The effect on dln_cardano in period 3 of a one standard deviation orthogonalized shock to dln_electricity in period 1 is -0.00684.

As explained before, this 5-dim VAR(1) model is stationary, so the effect of this shock will eventually die out. Table 2 shows that by period 11 or 12, the effects are minimal. This shock also has impact impact on all five variables, since dln_electricity is the first variable in the current VAR ordering and we will explain in detail the effect of different VAR ordering on the impact effect in the subsequent report section 2.3. In general, a shock to a variable x_j has no impact effect on any variable that precedes x_j in the VAR ordering. What's more, we can also calculate the cumulative effect on variables of a one standard deviation orthogonalized shock to dln_electricity in period 1, if interested. For example, by the end of period 3, the cumulative effect of a one standard deviation orthogonalized shock to dln_electricity in period 1 on dln_stellar is 0.01116 – 0.04603 – 0.00838 = -0.04325

iii)

The columns in Table 3 (excluding the Period column) below contain the estimated impulse responses of dln_bitcoin, dln_ethereum, dln_cardano, dln_stellar, and dln_electricity respectively over the first 11 periods to a one standard deviation orthogonalized shock to dln_electricity in period 1 when we change the VAR ordering.(We put dln_electricity in the last place instead of the first one)

Note that the original VAR ordering for Table 2 is: $x_t' = (dx_{1t}, dx_{2t}, dx_{3t}, dx_{4t}, dx_{5t})'$

The new VAR ordering for Table 3 is: $x_t' = (dx_{2t}, dx_{3t}, dx_{4t}, dx_{5t}, dx_{1t})'$

$$dx_{1t} = dlnelectricity_t \; ; \; dx_{2t} = dlnbitcoin_t \; ; \; dx_{3t} = dlnethereum_t$$

$$dx_{4t} = dlncardano_t; \; dx_{5t} = dlnstellar_t$$

Period	*dln_bitcoin*	*dln_ethereum*	*dln_cardano*	*dln_stellar*	*dln_electricity*
1	0	0	0	0	0.02153
2	-0.00412	-0.04305	-0.03464	-0.03647	0.00701
3	0.00964	-0.01085	0.00137	0.00442	0.00089
4	0.00711	0.00386	0.01077	0.01548	-0.00026
5	0.00107	0.00268	0.00426	0.00772	-0.00010
6	-0.00138	-0.00048	-0.00095	0.00080	0.00003
7	-0.00116	-0.00125	-0.00179	-0.00109	0.00001
8	-0.00045	-0.00072	-0.00093	-0.00066	-0.00002
9	-0.00009	-0.00021	-0.00024	-0.00012	-0.00002

10	-0.00001	-0.00002	-0.00002	0.00004	-0.00001
11	-0.00002	0.00000	-0.00001	0.00002	0.00000

Table 3: Impulse response coeffs arising from one std. dev. orthogonalised shock to dln_electricity for new VAR ordering

Note: All coefficients are rounded to 5 decimal places and the whole process can be found in the Appendix 05 or R code section 2.3.

From table 3 above, it is evident that a one standard deviation orthogonalized shock to dln_electricity in period 1 now has no impact effect on the remaining cryptocurrencies variables. That is, it cannot affect the 4 cryptocurrencies variables in period 1 since dln_electricity now comes after the 4 cryptocurrencies variables in the new VAR ordering shown above. (Refer to Figure 10 in Appendix 06 for visual inspection of this observation above). In contrast, the orthogonalized shock to dln_electricity in period 1 has impact effect on all 4 cryptocurrencies variables in period 1 as shown in Table 2 since it is the first variable in the original VAR ordering. In general, changing the order of the VAR variables would change which variables are preceding the orthogonalized shocked variable and, thus, would lead to different impulse response results.

The dependence of the orthogonalized impulses on the ordering of the variables in the VAR is a serious limitation of orthogonalized impulse response analysis, as the choice of ordering is often arbitrary. The matrix P which is used to orthogonalize is not unique and different P matrices will lead to different impulse response matrices. It can be shown that choosing a particular P matrix is equivalent to imposing a particular ordering or ranking on the variables in the VAR. There are two circumstances when orthogonalized impulse response analysis is useful:(Refer to the detailed explanation in Appendix 07 if interested) 1) When the pattern of the impulse responses is not very sensitive to the ordering of the variables. 2) When economic theory or common sense strongly indicates that a particular ordering of the variables is appropriate.

iv)

Note that we are using the same VAR ordering as 2.2 for this question

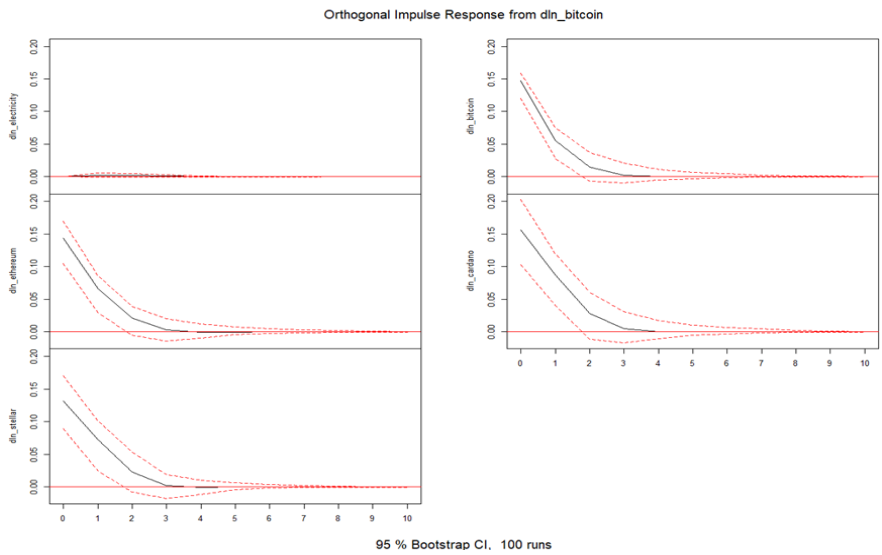

Figure 11: Orthogonal Impulse Response from dln_bitcoin

Figure 11 shows the impulse responses of the five variables in the VAR(1) model to a one standard deviation orthogonalized shock to dln_bitcoin at period 0. The red lines indicate 95 percent confidence intervals. The shock has no impact effect on dln_electricity but affects the other cryptocurrencies variables immediately in Period 0. This outcome is due to the VAR ordering, where dln_bitcoin is placed after dln_electricity and before the remaining cryptocurrencies variables, meaning it cannot contemporaneously influence variables that precede it. As discussed in earlier sections, since the VAR(1) model is stationary, the impact of the shock gradually dies out over time. This pattern is clearly shown in the plot above.

Table 4: Impulse response coeffs arising from one std. dev. orthogonalised shock to dln_bitcoin

Period	*dln_electricity*	*dln_ethereum*	*dln_cardano*	*dln_stellar*
1	0	0.14387	0.15634	0.13164
2	0.00257	0.06650	0.08667	0.07200
3	0.00219	0.02133	0.02866	0.02257
4	0.00120	0.00302	0.00498	0.00234
5	0.00045	-0.00059	0.00038	-0.00070
6	0.00012	-0.00012	0.00060	0.00031
7	0.00002	0.00025	0.00065	0.00068
8	0.00001	0.00017	0.00030	0.00038
9	0.00001	0.00002	0.00003	0.00008
10	0.00000	-0.00004	-0.00005	-0.00002
11	0.00000	-0.00003	-0.00003	-0.00002

Note: All coefficients are rounded to 5 decimal places and raw data can be found in the Appendix 08 or R code section 2.4.

The columns in Table 4 (excluding the Period column) contain the estimated impulse responses of dln_electricity, dln_ethereum, dln_cardano, and dln_stellar over the first 11 periods to a one standard deviation orthogonalized shock to dln_bitcoin in period 1, respectively. For example, the effect on dln_electricity in period 1 of a one standard deviation orthogonalized shock to dln_bitcoin in period 1 is 0. The effect on dln_cardano in period 3 of a one standard deviation orthogonalized shock to dln_bitcoin in period 1 is 0.02866.

As explained before, this VAR model is stationary, so the effect of this shock will eventually die out. Table 4 shows that by period 11 or 12, the effects are minimal. This shock also has no impact effect on dln_electricity but it has impact effects on other cryptocurrencies variables. This is because dln_bitcoin, the shock variable, is positioned after dln_electricity but before the other cryptocurrency variables in the VAR ordering. As a result, it cannot contemporaneously influence variables that came before it in period 1, but it can have impact effect on those that came after it. What' more, we can also calculate the cumulative effect on variables of a one standard deviation orthogonalized shock to dln_bitcoin in period 1, if interested. For example, by the end of period 2, the cumulative effect of a one standard deviation orthogonalized shock to dln_bitcoin in period 1 on dln_ethereum is 0.14387 + 0.06650 = 0.21037.

v) In this section, we apply OLS to estimate each equation individually to support our bi-directional Granger-causality analysis. In theory, a VAR model is estimated by OLS, equation by equation. Therefore, the coefficient estimates obtained from the single-equation OLS estimation are identical to those produced by the VAR() function, since each equation in the VAR model can be estimated independently using the same set of lagged regressors. (refer to Appendix 09)

In general, bi-directional Granger-Causality exists between two variables when each one helps predict the future values of the other. In other words, X Granger-causes Y, and at the same time, Y Granger-causes X.

Table 5: Cardano ↔ Bitcoin Bi-directional Granger-Causality Test

Test No.	**Null Hypothesis**	**LR calc**	**LR crit**	**Reject Null?**
1	Cardano does not Granger-cause Bitcoin	0.717	3.841	No
2	Bitcoin does not Granger-cause Cardano	11.152	3.841	Yes

Note: Refer to Appendix 10 for the complete Granger-Causality analysis procedure. Refer to R code in R file Section 2.5.2.1

Overall, no bi-directional Granger-Causality exists between cardano and bitcoin at 5% significance level since cardano does not Granger-cause bitcoin.

Table 6: Cardano ↔ Ethereum Bi-directional Granger-Causality Test

Test No.	**Null Hypothesis**	**LR calc**	**LR crit**	**Reject Null?**
3	Cardano does not Granger-cause Ethereum	8.809	3.841	Yes
4	Ethereum does not Granger-cause Cardano	4.909	3.841	Yes

Note: Refer to Appendix 11 for the complete Granger-Causality analysis procedure .Refer to R code in R file Section 2.5.2.2

Overall, the bi-directional Granger-Causality exists between cardano and ethereum at 5% significance level since cardano Granger-causes ethereum and, at the same time, ethereum Granger-causes cardano.

Table 7: Stellar ↔ Bitcoin Bi-directional Granger-Causality Test

Test No.	**Null Hypothesis**	**LR calc**	**LR crit**	**Reject Null?**
5	Stellar does not Granger-cause Bitcoin	5.929	3.841	Yes
6	Bitcoin does not Granger-cause Stellar	15.837	3.841	Yes

Note: Refer to Appendix 12 for the complete Granger-Causality analysis procedure. Refer to R code in R file Section 2.5.2.3

Overall, the bi-directional Granger-Causality exists between stellar and bitcoin at 5% significance level since stellar Granger-causes bitcoin and, at the same time, bitcoin Granger-causes stellar.

Table 8: Stellar ↔ Ethereum Bi-directional Granger-Causality Test

Test No.	**Null Hypothesis**	**LR calc**	**LR crit**	**Reject Null?**
7	Stellar does not Granger-cause Ethereum	4.782	3.841	Yes
8	Ethereum does not Granger-cause Stellar	15.121	3.841	Yes

Note: Refer to Appendix 13 for the complete Granger-Causality analysis procedure. Refer to R code in R file Section 2.5.2.4

Overall, the bi-directional Granger-Causality exists between stellar and ethereum since stellar Granger-causes ethereum and, at the same time, ethereum Granger-causes stellar.

vi)

In this part, we began by considering whether the five I(1) variables might be cointegrated, based on the co-movements observed earlier. To guide model selection, we referred to the Granger Representation Theorem, which links cointegration to the appropriate model structure. Since the rank of the Π matrix was found to be zero, indicating no cointegration, we concluded that a VAR model in first differences is the most suitable choice for our first differenced logged data. The final VAR model for our log first-differenced data is a 5-dimensional VAR(1), in which the residuals behave like white noise.

Next, using this model, we analyzed the impulse response coefficients to understand how a one standard deviation orthogonalized shock to one variable affects the others over time. The results highlight the importance of VAR ordering in impulse response analysis. We also observed that the effects of the shocks gradually die out over time, which aligns with the behavior of a stationary VAR(1) model. This was later confirmed through a formal stationarity test.

Lastly, we conducted 8 Granger-Causality tests to examine whether bi-directional Granger-causality exists between each pair of clean and dirty cryptocurrencies. The results show that bi-directional causality exists between Cardano and Ethereum, Stellar and Ethereum, and Stellar and Bitcoin. However, no bi-directional causality was found between Cardano and Bitcoin. It is important to note that this analysis based on a 5-dimensional VAR(1) model, and the Granger causality results may be sensitive to both the choice of variables and the selected lag order. Thus, conclusions should be interpreted with caution, as adjustments to the information set or lag structure could affect the outcomes.

Part 3

i)

Note that we are using the logged variables in their level form [i.e., the dataframe “df” as introduced in Part 2(i)] for the VECM analyses conducted in Parts 3 and 4. The rationale for applying the log transformation is as follows. First, it standardizes the scale across variables with different units. Second, it allows us to interpret the cointegrating vector (CIV) or long run relationship in a log-log form, allowing for elasticity analysis. Third,

it is a common econometric practice that helps stabilize variance and improve model interpretability.

Based on the analysis in Section 1.3.2, all 5 variables are I(1) processes containing only stochastic trends. Their level plot (Figure 2) also shows some degree of co-movement, especially among the cryptocurrency variables. Therefore, in this section, we formally test for cointegration among the five I(1) variables using the Johansen methodology. Before we can test for cointegration, we must determine the number of lags to include in our VECM. The number of lags in the VECM will be one less than the number of lags in the VAR in level form. So, we must determine the VAR order first.

```
> VARselect(df, lag.max = 10, type = "none")
$selection
AIC(n)  HQ(n)  SC(n) FPE(n)
     2      2      1      2
```

Figure 12: Output of VARselect function

Notice that the data frame for VARselect() is the data frame “df” and not “ddf” in Part 2, since we are conducting cointegration analysis. Since none of our variable appears to have a linear deterministic trend according to our analysis in report section 1.3.2, we do not wish to include a constant in the VAR. Therefore, we set type = "none". From Figure 12, we can see that SC (Schwarz) criteria suggest a VAR(1) while other 3 information criteria suggest a VAR(2).

We initially estimate a VAR(1) model, as suggested by SC. Then we use serial.test() function to test whether the errors in this VAR(1) model is vector white noise. If this VAR(1) model has serially correlated errors, we will add lags incrementally until the serial correlation is eliminated. The null and alternative is that:

$$H_0: \rho_1 = \rho_2 = \cdots = \rho_{16} = 0; \epsilon_t \text{ is a vector white noise}$$

$$H_1: \text{at least one } \rho_j \neq 0; \epsilon_t \text{ is not a vector white noise}$$

Since the p-value = 0.0009 < 0.05 (Refer to Appendix 14), we reject the null and conclude that errors in VAR(1) are not vector white noise, therefore mode lags need to be included in the VAR specification. Next, we estimate a VAR (2) and test the errors again in the VAR(2). Since the p-value = 0.1399 > 0.05 (Refer to Appendix 15), we reject the null and conclude that errors in VAR(2) are vector white noise at 5% significance level. Therefore, the final VAR order is 2. Since the VAR order is 2, the number of lags in our VECM is 1.

Based on the above, we end up selecting a VAR of the form:

$$\underset{(5\times1)}{x_t} = \underset{(5\times5)}{A_1}\underset{(5\times1)}{x_{t-1}} + \underset{(5\times5)}{A_2}\underset{(5\times1)}{x_{t-2}} + \underset{(5\times1)}{\varepsilon_t}$$

$$x_t' = (x_{1t}, x_{2t}, x_{3t}, x_{4t}, x_{5t})'; x_{1t} = \text{lnelectricity}_t\ ;\ x_{2t} = \text{lnbitcoin}_t\ ;\ x_{3t} = \text{lnethereum}_t$$

$$x_{4t} = \text{lncardano}_t;\ x_{5t} = \text{lnstellar}_t$$

Implying an VECM of the form:

$$\underset{(5\times1)}{dx_t} = \underset{(5\times5)}{\Pi}\underset{(5\times1)}{x_{t-1}} + \underset{(5\times5)}{\Gamma_1}\underset{(5\times1)}{dx_{t-1}} + \underset{(5\times1)}{\varepsilon_t}$$

$$dx_t' = (dx_{1t}, dx_{2t}, dx_{3t}, dx_{4t}, dx_{5t})'$$; d represents first differencing

$$dx_{1t} = dlnelectricity_t \; ; dx_{2t} = dlnbitcoin_t \; ; dx_{3t} = dlnethereum_t$$

$$dx_{4t} = dlncardano_t; \; dx_{5t} = dlnstellar_t$$

ii)

```
# Estimate a VECM model using the VECM() function.
fit_vecm <- VECM(df, lag = 1, include = "none", estim = "ML", LRinclude = "const")
summary(fit_vecm)
```

Figure 13: VECM estimation

We specify lag = 1 based on our lag selection analysis in Part 3(i). The estim = "ML" argument applies maximum likelihood estimation. The include and LRinclude settings determine the inclusion of deterministic components in the VECM. As all five logged variables exhibit non-zero means (Appendix 16), we include an intercept in the cointegrating equation by setting LRinclude = "const". Also, since no variable shows evidence of a deterministic trend as indicated in report section 1.3.2, we exclude intercepts from the VECM equations (include = "none"). This specification corresponds to the M2 representation (see Appendix 17 for details). The VECM results are presented in the table below.(see Appendix 18 for R output)

Table 9: VECM Estimation Results

Full sample size:89	End sample size: 87	
Number of variables: 5	Number of estimated slope parameters:30	
AIC -2003.85	BIC -1920.009	SSR 13.72464

(a) Cointegrating vector (estimated by ML)

	ln_electricity	*ln_bitcoin*	*ln_ethereum*	*ln_cardano*	*ln_stellar*	*Constant*
r1	1	0.0616	-1.1694	1.8297	-0.7649	5.1563

(b) Adjustment Coefficients (Error Correction Term)

	ECT Coefficient
Equation ln_electricity	-0.0127(0.0080)
Equation ln_bitcoin	0.0538(0.0529)
Equation ln_ethereum	0.1834(0.0650)**
Equation ln_cardano	-0.0492(0.0917)
Equation ln_stellar	0.0645(0.0784)

(c) Short-Run Dynamics (Lag 1 coefficients)

	dln_electricity	dln_bitcoin	dln_ethereum	dln_cardano	dln_stellar
Equation ln_electricity	0.3234 (0.1036)**	-0.0191 (0.0218)	0.0381 (0.0192).	0.0108 (0.0165)	-0.0148 (0.0184)
Equation ln_bitcoin	-0.1811 (0.6863)	0.6987 (0.1445)***	-0.1693 (0.1269)	0.0328 (0.1093)	-0.2113 (0.1217).
Equation ln_ethereum	-1.9648 (0.8431)*	0.5400 (0.1775)**	-0.1891 (0.1559)	0.2074 (0.1342)	-0.1217 (0.1495)

Equation ln_cardano	-1.6180 (1.1892)	0.8157 (0.2504)**	-0.4699 (0.2199)*	0.5828 (0.1893)**	-0.4380 (0.2109)*
Equation ln_stellar	-1.6818 (1.0161)	0.8780 (0.2140)***	-0.7477 (0.1879)***	0.2137 (0.1618)	0.1336 (0.1802)

Note: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

From Table 9 (b), we can see that not all adjustments coefficients are statistically significant, suggesting there may not true cointegration relations among variables.

iii)

We conduct the trace and maximum eigenvalue tests over a sequence from 0 to 4 to determine how many estimated eigenvalues are statistically different from zero. The cointegrating rank r is equal to the number of non-zero eigenvalues of Π. The sequence of tests terminates as soon as we fail to reject the null hypothesis. For the trace test, the initial hypothesis test is given below:

$H_0: r = 0$ $\qquad$ $H_1: r > 0$

For the maximum eigenvalue test, the initial hypothesis test is given below:

$H_0: r = 0$ $\qquad$ $H_1: r = 1$

Table 10: trace and maximum eigenvalue tests

r	*trace*	*trace_pval*	*trace_pval_T*	*eigen*	*eigen_pval*
0	62.80728	0.3692	0.4479	28.88073	0.2226
1	33.92655	0.7719	0.8059	19.60728	0.4573
2	14.31927	0.9604	0.9657	6.425967	0.9834
3	7.893305	0.8298	0.8364	5.662753	0.819
4	2.230553	0.7315	<0.001	2.230553	0.7304

Note that: r being tested for the values 0 to 4. The maximum value of r is always m-1. Since m = 5 in our case (5 variables), therefore the maximum value of r being tested for is 4. If r = 5, it would mean that all 5 variables are I(0) processes to begin with.

We can see that the p-value for the initial test (at r = 0) for both the Trace and Maximum Eigenvalue tests are greater than 0.05 (Trace:0.4479 > 0.05; MX:0.2226 > 0.05) meaning we do not reject either of the null hypotheses at 5% significance level and conclude that r = 0. Both the Trace and Maximum Eigenvalue tests lead to the conclusion that there is no cointegrating vector (CIV) or more accurately, that the cointegrating rank is 0 or Π is a null matrix.

iv)

Although both the trace and maximum eigenvalue tests suggest that there are no cointegrating vectors, we proceed by estimating a VECM with a forced cointegration rank of 2 for the purposes of this assignment. The two cointegrating vectors are extracted from the model output and rounded to three decimal places, as shown below. (Refer to Appendix 19 for procedures above in detail)

The first CIV normalized with respect to ln_electricity is that:

$$\beta_1' = (1.000, 0.000, 0.324, -0.522, 0.470, -4.435)$$

The Second CIV normalized with respect to ln_bitcoin is that:

$$\beta_2' = (0.000, 1.000, -24.233, 29.406, -20.035, 155.666)$$

v)

Using the two normalized CIVs obtained in Part 3(iv), we form the linear combinations of variables for the following two cointegration equations, normalized with respect to *ln(electricity*) and *ln(bitcoin),* respectively.

$$Z_1 \equiv \beta_1' X = -4.435 + \ln(electricity) + 0.324\ln(\text{ethereum}) - 0.522\ln(\text{cardano}) + 0.470\ln(\text{stellar}) \sim I(0)$$

$$Z_2 \equiv \beta_2' X = 155.666 + \ln(bitcoin) - 24.233\ln(\text{ethereum}) + 29.406\ln(\text{cardano}) - 20.035\ln(\text{stellar}) \sim I(0)$$

Note: ln(bitcoin) and ln(electricity) are excluded from the Z_1 and Z_2 ,respectively, as their β coefficients are approximately zero.

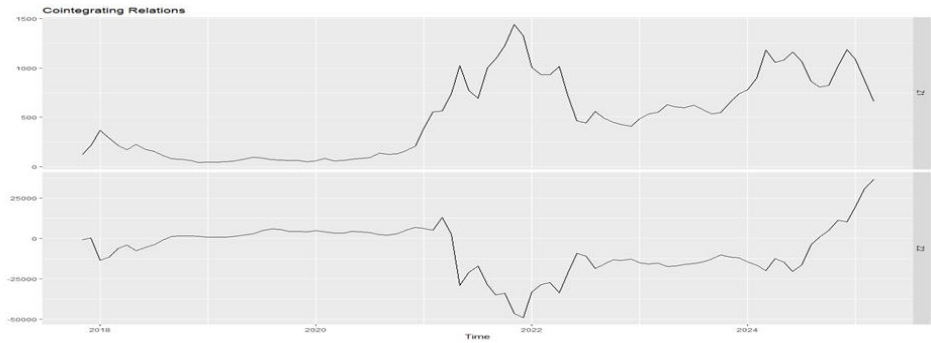

Figure 14: Visualization for 2 Cointegration equations

Figure 14 displays Z_1 and Z_2, both of which are clearly non-stationary and exhibit stochastic trends, indicating they are not I(0) processes. While the linear combinations of cointegrating relations are theoretically expected to be I(0), the cointegration rank r = 2 was manually imposed in the VECM estimation for Part 3 iv). In contrast, the trace and maximum eigenvalue tests in Part 3 iii) suggested r = 0, meaning no significant cointegration exists in the dataset. That's why the resulting linear combinations do not behave like I(0) processes.

vi)

$$\underset{(m\times1)}{dx_t} = \underset{(m\times1)}{A_0} + \underset{(m\times m)}{\Pi}\underset{(m\times1)}{x_{t-1}} + \sum_{j=1}^{p-1}\underset{(m\times m)}{\Gamma_j}\underset{(m\times1)}{dx_{t-1}} + \underset{(m\times1)}{\varepsilon_t} \quad (5.29)$$

The Granger Representation Theorem states that the presence and number of cointegrating relationships among a set of I(1) variables can be determined by the rank of the Π matrix, denoted by r, in the VECM, where $0 \leq r \leq m$. The 3 cases are stated below.

Case 1: If r = 0, then $x_t \sim I(1)$, Π is null matrix, and there are no CIVs

Case 2: If r = m (full rank), then $x_t \sim I(0)$,

Case 3: If 0 < r < m, then $x_t \sim I(1)$, and there exist r linearly independent CIVs

According to the Granger Representation Theorem and the findings in Part 3(iii), the rank of the Π is 0 for our data. This places us in Case 1, where Π is null matrix, indicating no CIVs exist among the 5 I(1) variables or in other words, they are not cointegrated. When rank (Π) = 0, the equation (5.29) reduced to:

$$\underset{(m\times1)}{dx_t} = \underset{(m\times1)}{A_0} + \sum_{j=1}^{p-1} \underset{(m\times m)}{\Gamma_j} \underset{(m\times1)}{dx_{t-1}} + \underset{(m\times1)}{\varepsilon_t} \quad (5.30)$$

Therefore, a VAR in first difference is the appropriate model for our data.

Part 4

i)&ii)&iii)

Note: For the sake of coherence and smoother explanation, we combine the answers to the first three questions of Part 4 in the following discussion.

```
# Estimate VECM aussuming a cointegration rank of 2
fit_vecm2 <- VECM(df, lag = 1, r = 2, include = "none", estim = "ML", LRinclude = "const" )
```

Figure 15: VECM estimation (forced r = 2)

As shown in Figure 15, we imposed a cointegration rank of 2, even though our earlier analysis in Part 3(iii) suggests that the true rank is 0. Details on the other arguments used are provided in Part 3(ii) and are not repeated here.The general formula for VECM (M2) model we are using is given below:

$$\underset{(m\times1)}{dx_t} = \underset{(m\times r)}{\alpha} (\underset{(r\times1)}{\beta_0'} + \underset{(r\times1)}{\beta' x_{t-1}}) + \sum_{j=1}^{p-1} \underset{(m\times m)}{\Gamma_j} \underset{(m\times1)}{dx_{t-j}} + \underset{(m\times1)}{\varepsilon_t} \quad (M2)$$

For our case,

$$\underset{(5\times1)}{dx_t} = \underset{(5\times2)}{\alpha} (\underset{(2\times1)}{\beta_0'} + \underset{(2\times1)}{\beta' x_{t-1}}) + \underset{(5\times5)}{\Gamma_1} \underset{(5\times1)}{dx_{t-1}} + \underset{(5\times1)}{\varepsilon_t}$$

Refer to the details of dx_t and x_t provided at the end of Part 3 (i).

The estimated VECM equations are that: (Refer to Appendix 20)

$$d\ln(\widehat{elec})_t = -\underset{(0.0488)}{0.1713}\hat{u}_{1,t-1} - \underset{(0.0010)}{0.0038}\hat{u}_{2,t-1} + lags(d\ln(elec_t, bit_t, eth_t, card_t, stel))$$

$$d\ln(\widehat{bit})_t = -\underset{(0.3433)}{0.1787}\hat{u}_{1,t-1} - \underset{(0.0073)}{0.0011}\hat{u}_{2,t-1} + lags(d\ln(elec, bit, eth, card, stel))$$

$$d\ln(\widehat{eth})_t = -\underset{(0.4144)}{0.5649}\hat{u}_{1,t-1} - \underset{(0.0088)}{0.0030}\hat{u}_{2,t-1} + lags(d\ln(elec, bit, eth, card, stel))$$

$$d\ln(\widehat{card})_t = -\underset{(0.5824)}{1.1901}\hat{u}_{1,t-1} - \underset{(0.0123)}{0.0248}\hat{u}_{2,t-1} + lags(d\ln(elec, bit, eth, card, stel))$$

$$d\ln(\widehat{stel})_t = -\underset{(0.4874)}{1.2521}\hat{u}_{1,t-1} - \underset{(0.0103)}{0.0212}\hat{u}_{2,t-1} + lags(d\ln(elec, bit, eth, card, stel))$$

where:

$\hat{u}_{1,t-1} \equiv \beta_1' X_{t-1} = \ln(elec)_{t-1} - 4.435 + +0.324\ln(\mathrm{eth})_{t-1} - 0.522\ln(\mathrm{card})_{t-1} + 0.470\ln(\mathrm{stel})_{t-1}$

$\hat{u}_{2,t-1} \equiv \beta_2' X_{t-1} = \ln(bit)_{t-1} + 155.666 - 24.234\ln(\mathrm{eth})_{t-1} + 29.406\ln(\mathrm{card})_{t-1} - 20.035\ln(\mathrm{stel})_{t-1}$

Note: ln(bitcoin) and ln(electricity) are excluded from the $\hat{u}_{1,t-1}$ and $\hat{u}_{2,t-1}$, respectively, as their β coefficients are approximately zero.

From the VECM equations above, we can see that the change in monthly price of electricity, bitcoin, ethereum, cardano, and stellar in period *t* is partly driven by the $u_{1,t-1}$ and $u_{2,t-1}$ which are the equilibrium error or the deviations from the long run relationship in period *t – 1*. However, since no true cointegrating relationship was found (i.e. r = 0), these terms were imposed manually and should be interpreted with caution.

Table 10: VECM Estimation Results (Assume r = 2) (see Appendix 20 for R output)

Full sample size:89	End sample size: 87	
Number of variables: 5	Number of estimated slope parameters:35	
AIC -2009.457	BIC -1908.355	SSR 13.01943

(a) Cointegrating vector (estimated by ML) (rounded up to 3 decimal places)

	ln_electricity	*ln_bitcoin*	*ln_ethereum*	*ln_cardano*	*ln_stellar*	*Constant*
r1	1.000	0.000	0.324	-0.522	0.470	-4.435
r2	0.000	1.000	-24.233	29.406	-20.035	155.666

(b) Adjustment Coefficients (Error Correction Term)

	ECT1	ECT2
Equation ln_electricity	-0.1713(0.0488)***	-0.0038(0.0010)***
Equation ln_bitcoin	-0.1787(0.3433)	-0.0011(0.0073)
Equation ln_ethereum	-0.5649(0.4144)	-0.0030(0.0088)
Equation ln_cardano	-1.1901(0.5824)*	-0.0248(0.0123)*
Equation ln_stellar	-1.2521(0.4874)*	-0.0212(0.0103)*

(c) Short-Run Dynamics (Lag 1 coefficients)

	dln_electricity	dln_bitcoin	dln_ethereum	dln_cardano	dln_stellar
Equation ln_electricity	0.3894 (0.0999)***	-0.0170 (0.0206)	0.0183 (0.0191)	0.0111 (0.0156)	-0.0052 (0.0176)
Equation ln_bitcoin	-0.0842 (0.7029)	0.7019 (0.1451) ***	-0.1984 (0.1342)	0.0331 (0.1096)	-0.1972 (0.1238)
Equation ln_ethereum	-1.6532 (0.8485).	0.5502 (0.1751) **	-0.2828 (0.1620).	0.2085 (0.1323)	-0.0764 (0.1495)
Equation ln_cardano	-1.1428 (1.1926)	0.8312 (0.2461) **	-0.6128 (0.2277)**	0.5845 (0.1860)**	-0.3689 (0.2101).
Equation ln_stellar	-1.1334 (0.9982)	0.8959 (0.2060) ***	-0.9125 (0.1906)***	0.2157 (0.1557)	0.2133 (0.1758)

Note: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

From Table 10 (b), we can see that not all adjustments coefficients of ECT1 and ECT2 are statistically significant. This is congruent to our conclusion in Part 3 iii) that there is no cointegrating relationship for this data (r = 0). Since we forced it to have a cointegration rank of 2 so it is not surprising not all ECTs are statistically significant. Some may be significant due to spurious regression. In terms of short-run dynamics, we can observe: 1) A marginal increase in the change in either electricity, bitcoin, or cardano in period *t-1* has a significant positive effect on their own values in period *t*, ceteris paribus. 2) A marginal increase in the change in bitcoin in period *t-1* has a significant positive effect on the values of ethereum, cardano, & stellar in period *t*, ceteris paribus. 3) A marginal increase in the change in Ethereum in period *t-1* has a significant negative effect on the value of cardano in period *t*, ceteris paribus.

iv)

In addition to the Trace and Maximum Eigenvalue tests, cointegration in a VECM can also be assessed by examining the individual significance of adjustment coefficients (i.e., the ECT columns in the R output). If all coefficients within the same ECT column are statistically significant, it suggests a valid cointegration relationship. However, if only some are significant, the result may due to spurious regression since they are all I(1) variables. For example, in our case, as shown in Table 10b, not all coefficients in ECT1 or ECT2 are significant. This is expected, as we forced a cointegration rank of 2, even though Part 3(iii) indicates that the true rank is 0.

v)

Given the 2 normalized CIVs shown in Table 10, the estimated long-run equilibrium relationships are as follows: (note that we put normalized variables left hand side)

$$\ln(elec) = 4.435 - 0.324\ln(\text{eth}) + 0.522\ln(\text{card}) - 0.470\ln(\text{stel}) \quad (1)$$

$$\ln(bit) = -155.666 + 24.233\ln(\text{eth}) - 29.406\ln(\text{card}) + 20.035\ln(\text{stel}) \quad (2)$$

Note: ln(bitcoin) and ln(electricity) are excluded from the first and second equation, respectively, as their β coefficients are approximately zero.

The first equation shows that a 1% increase in the monthly price of ethereum, cardano, and stellar is associated with a 0.324% decrease, 0.522% increase, and 0.470% decrease in the monthly price of electricity, respectively, on average, ceteris paribus, in the long run. The second equation shows that a 1% increase in the monthly price of ethereum, cardano, and stellar corresponds to a 24.233% increase, 29.406% decrease, and 20.035% increase in the monthly price of bitcoin, respectively, on average, ceteris paribus, in the long run. It is important to note that the relationships obtained above are not reliable since they are based on an incorrect cointegration rank and should be interpreted with caution.

vi)

In summary, in part 3, we selected a VAR(2) model based on residual diagnostics, which implies using one lag in the VECM. The Trace and Maximum Eigenvalue tests both indicate that the cointegration rank is 0, meaning these 5 I(1) variables are not cointegrated. Based on this result and the Granger Representation Theorem, a VAR model in first differences is the appropriate model for our data. In Part 4, we forced the VECM to have a cointegration rank of 2. However, the VECM results show that not all adjustment coefficients in ECT1 and ECT2 columns are statistically significant, reinforcing our earlier conclusion that the true cointegration rank is 0. Some are significant may be due to spurious regression since all 5 variables are I(1) processes. The lack of overall significance suggests no valid cointegrating relationship. Although we derived two long-run equations from the estimated 2 CIVs, the long run relationships obtained are not reliable, as they are based on an incorrect cointegration rank and should be interpreted with caution.

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Appendix

01

Although the empirical evidence shows that the unit roots tests have a low power, we still apply the Augmented Dickey-Fuller (ADF) test to each series in both levels and first differencing forms to formally verify the presence of unit root and establish the order of integration for each variable. We begin by applying the Augmented Dickey-Fuller (ADF) test to each variable in levels to formally test for the presence of unit roots. Following this, we conduct a second round of ADF tests on the first-differenced series to verify whether the variables are indeed I(1) process, as suggested by the visual inspection.

As discussed in section 1.3.2 of the report, these 5 variables appear to be I(1) processes, each containing only a stochastic trend. And they also appear to have non-zero means. Therefore, when conducting the first Augmented Dickey-Fuller (ADF) tests on the series in levels, we specify the argument type = “drift” to account for the presence of a non-zero mean without a deterministic trend. The general form of regression equation for first ADF test for each variable is that:

$$dy_t = a + \gamma y_{t-1} + \alpha_1 dy_{t-1} + \cdots + \alpha_{p-1} dy_{t-(p-1)} + e_t$$

$$\text{where } e_t \sim \text{i.i.d.}(0, \sigma^2)$$

The null and alternative hypothesis is that:

$$H_0: \gamma = 0 \quad \text{i.e} \quad y_t \text{ has at least one unit root}$$

$$H_1: \gamma < 0 \quad \text{i.e} \quad y_t \text{ does not have a unit root and it is I(0)}$$

To formally test whether all variables are indeed I(1) process as suggested by the visual inspection, we then conduct the ADF tests to first-differenced series of each variable. As discussed in report section 1.3.2 of the report, these 5 variables appear to have approximately zero mean after first differencing. Therefore, when conducting the second Augmented Dickey-Fuller (ADF) tests to the first differenced series of each variable, we specify the argument type = “none”. The general form of regression equation for second ADF test for each variable is that:

$$dy_t' = \gamma y_{t-1}' + \alpha_1 dy_{t-1}' + \cdots + \alpha_{p-1} dy_{t-(p-1)}' + e_t$$

$$\text{where } e_t \sim \text{i.i.d.}(0, \sigma^2); \; y_t' = y_t - y_{t-1}$$

The null and alternative hypothesis is that:

$$H_0: \gamma = 0 \quad \text{i.e} \quad y_t' \text{ has at least one unit root}$$

$$H_1: \gamma < 0 \quad \text{i.e} \quad y_t' \text{ does not have a unit root and it is I(0)}$$

The overall logic of building the ADF regression model for each variable:

Step 1: Choose lag truncation parameter using AIC

```
# Choose lag truncation parameter using AIC
adf_selectP_stellar <- ur.df(stellar, type = "drift", lags = 20, selectlags = "AIC")
```

We first use AIC to help us choose the lag truncation number, as shown in the above picture.

Step 2: Re-estimate with fixed lag

```
> # re estimate with fixed lag
> adf_stellar <- ur.df(stellar, lags = 4, selectlags = "Fixed", type = "drift")
```

Then, we re-estimate the ADF regression with fixed lags as indicated by lag truncation number in Step 1.

Step 3: Ljung-Box test on residuals and re-identification if necessary

```
> # Ljung-Box test on residuals
> LjungBox(adf_residuals_stellar, lags = seq(10, 32, 2), fitdf = 6)
```

The null and alternative hypothesis of LjungBox test is shown below.

$$H_0: \rho_1 = \rho_2 = \cdots = \rho_p = 0$$

$$H_1: \exists\, j \in \{1, \ldots, 10\} \text{ such that } \rho_j \neq 0$$

Next, we conduct the Ljung–Box test on the residuals obtained from the ADF model estimated in Step 2 over a specified range of lags. If the residuals resemble white noise over a specified range of lags, we consider the test statistics (t-statistic and F-statistic) from the ADF model in Step 2 to be reliable. However, if the Ljung–Box test indicates significant autocorrelation in the residuals over multiple lags, we re-specify the ADF model by gradually increasing the number of lagged difference terms, adding them one at a time, until the residuals approximate a white noise process over the chosen lag sequence.

Step 4: Extract the test statistics and critical values

```
# Test statistics and critical values
adf_stellar@teststat
adf_stellar@cval
```

Finally, we extract the test statistics and critical values from the model that passed the Step 3 residual diagnostic. And we conduct the sequential test based on the test statistics and critical values obtained

Please note that in the following two ADF tests for each variable (one for the level series and one for the first-differenced series), we omitted Seps 1 to 3 because the construction of each ADF model is just a repetition of the above process. If the reader is interested, please refer to the R code Section 1.3.4.1 to 1.3.4.5

ADF tests for electricity

First ADF test for electricity:

```
> # Test statistics and critical values
> adf_electricity_final@teststat
                tau2     phi1
statistic 0.06956823 1.248599
> adf_electricity_final@cval
      1pct  5pct 10pct
tau2 -3.51 -2.89 -2.58
phi1  6.70  4.71  3.86
```

S1

The ADF regression equation for electricity is:

$$dy_t = a + \gamma y_{t-1} + \alpha_1 dy_{t-1} + \cdots + \alpha_{p-1} dy_{t-(p-1)} + e_t \quad (1.1)$$

$$\text{where } e_t \sim \text{i.i.d.}(0, \sigma^2)$$

The estimated ADF equation is that:(Refer to R code Section 1.3.4.1)

$$dy_t = \underset{(0.38804)}{0.04079} + \underset{(0.03449)}{0.00240} y_{t-1} + \underset{(0.15811)}{0.00323} dy_{t-1} + \cdots + \underset{(0.16725)}{0.12796} dy_{t-22} + e_t$$

```
> LjungBox(adf_electricity_final@res, lags = seq(36, 52, 2), fitdf = 24)
 lags statistic df    p-value
   36  20.95479 12 0.05104643
   38  21.00200 14 0.10158137
   40  26.75660 16 0.04426906
   42  28.68207 18 0.05240582
   44  30.70676 20 0.05918267
   46  32.52126 22 0.06893322
   48  33.29790 24 0.09794226
   50  33.50139 26 0.14804391
   52  34.95816 28 0.17108849
```

The LjungBox results suggest that there is no autocorrelation between residuals
Therefore, the t stat and F stat generated from the ADF models above is reliable

S2

$$H_0: \gamma = 0 \quad \text{i.e. electricity has at least one unit root}$$

$$H_1: \gamma < 0 \quad \text{i.e. electricity does not have a unit root}$$

Since $t_{calc} = 0.07 > t_{crit,0.05} = -2.89$, we fail to reject the null hypothesis that electricity has at least one unit root at 5% significance level.

S3

Failure to reject the null of a unit root at Step 2 may be due to a loss of power arising from erroneously including a constant in (1.1). Therefore, we test for the presence of the constant, given the presence of a unit root.

$$H_0: a = \gamma = 0$$

The unrestricted model is given by (1.1)

The restricted model is given below (i.e. when true):

$$dy_t = \gamma y_{t-1} + \alpha_1 dy_{t-1} + \cdots + \alpha_{p-1} dy_{t-(p-1)} + e_t$$

Since $F1_{calc} = 1.25 < F1_{crit,0.05} = 4.71$,we fail to reject the null hypothesis that at 5% significance level. This implies that constant term is not significant in (1.1). This implies that we used the correct critical value for S2 (since the actual and estimated DGPs are different) when we failed to reject the null of at least one unit root. Hence, we conclude that electricity has at least one unit root.

Second ADF test for delectricity:

When we tested for unit roots in electricity above, we were unable to reject null hypothesis. This suggests that electricity has at least one unit root. That is, electricity is not an I(0) process. To formally test whether electricity is I(1) we can perform an ADF test on delectricity since if electricity is I(1) then delectricity should I(0). Note that delectricity is obtained by taking a first differencing for electricity.

```
> # Test statistics and critical values
> adf_delectricity@teststat
               tau1
statistic -5.291002
> adf_delectricity@cval
     1pct  5pct 10pct
tau1 -2.6 -1.95 -1.61
```

S1

The ADF regression equation for delectricity is:

$$dy_t' = \gamma y_{t-1}' + \alpha_1 dy_{t-1}' + \cdots + \alpha_{p-1} dy_{t-(p-1)}' + e_t \ (1.2)$$

$$\text{where } e_t \sim \text{i.i.d.}(0,\sigma^2);\ y_t' = y_t - y_{t-1}$$

The estimated ADF equation is that:(Refer to R code Section 1.3.4.1)

$$dy_t' = \underset{(0.3566)}{-1.8868} y_{t-1}' + \underset{(0.3102)}{1.1436} dy_{t-1}' + \cdots + \underset{(0.1215)}{0.1775} dy_{t-8}' + e_t$$

```
> LjungBox(adf_residuals_delectricity, lags = seq(16, 28, 2), fitdf = 9)
 lags statistic df    p-value
   16  10.79783  7 0.14768408
   18  14.22156  9 0.11465918
   20  18.77796 11 0.06519743
   22  20.47047 13 0.08408412
   24  26.79467 15 0.03045448
   26  29.28801 17 0.03196095
   28  29.48034 19 0.05879557
```

The LjungBox results suggest that there is no autocorrelation between residuals
Therefore, the t stat and F stat generated from the ADF models above is reliable

S2

$$H_0: \gamma = 0 \quad \text{i.e. delectricity has at least one unit root}$$

$$H_1: \gamma < 0 \quad \text{i.e. delectricity is I(0), therefore electricity is I(1)}$$

Since $t_{calc} = -5.29 < t_{crit,0.05} = -1.95$, we reject the null hypothesis that delectricity has at least one unit root at 5% significance level and conclude that electricity is indeed a I(1) process since differencing it once make it stationary. Overall, electricity is I(1) process containing only stochastic trend.

ADF tests for bitcoin

First ADF test for bitcoin:

```
> # Test statistics and critical values
> adf_bitcoin@teststat
                tau2      phi1
statistic -0.6403125 0.8040377
> adf_bitcoin@cval
      1pct  5pct 10pct
tau2 -3.51 -2.89 -2.58
phi1  6.70  4.71  3.86
```

S1

The ADF regression equation for bitcoin is:

$$dy_t = a + \gamma y_{t-1} + \alpha_1 dy_{t-1} + \cdots + \alpha_{p-1} dy_{t-(p-1)} + e_t \quad (1.1)$$

where $e_t \sim \text{i.i.d.}(0, \sigma^2)$

The estimated ADF equation is that:(Refer to R code Section 1.3.4.2)

$$dy_t = \underset{(825.33)}{979.13} - \underset{(0.02343)}{0.01500} y_{t-1} + \underset{(0.10744)}{0.62502} dy_{t-1} - \underset{(0.11293)}{0.26026} dy_{t-2} + e_t$$

```
> LjungBox(adf_residuals_bitcoin, lags = seq(8, 32, 2), fitdf = 4)
 lags statistic df    p-value
    8  9.309041  4 0.05382242
   10 10.265140  6 0.11392134
   12 11.013138  8 0.20095591
   14 12.460729 10 0.25540379
   16 14.402568 12 0.27574294
   18 14.867454 14 0.38727504
   20 14.997884 16 0.52479354
   22 16.947896 18 0.52669010
   24 17.198574 20 0.64004429
   26 17.571605 22 0.73106419
   28 18.368144 24 0.78479338
   30 24.644602 26 0.53915676
   32 27.049970 28 0.51552713
```

The LjungBox results suggest that there is no autocorrelation between residuals
Therefore, the t stat and F stat generated from the ADF models above is reliable

S2

$H_0: \gamma = 0$ i.e. bitcoin has at least one unit root

$H_1: \gamma < 0$ i.e. bitcoin does not have a unit root

Since $t_{calc} = -0.64 > t_{crit,0.05} = -2.89$, we fail to reject the null hypothesis that bitcoin has at least one unit root at 5% significance level.

S3

Failure to reject the null of a unit root at Step 2 may be due to a loss of power arising from erroneously including a constant in (1.1). Therefore, we test for the presence of the constant, given the presence of a unit root.

$$H_0: a = \gamma = 0$$

The unrestricted model is given by (1.1)

The restricted model is given below (i.e. when true):

$$dy_t = \gamma y_{t-1} + \alpha_1 dy_{t-1} + \cdots + \alpha_{p-1} dy_{t-(p-1)} + e_t$$

Since $F1_{calc} = 0.80 < F1_{crit,0.05} = 4.71$,we fail to reject the null hypothesis that at 5% significance level. This implies that constant term is not significant in (1.1). This implies that we used the correct critical value for S2 (since the actual and estimated DGPs are different) when we failed to reject the null of at least one unit root. Hence, we conclude that bitcoin has at least one unit root.

Second ADF test for dbitcoin:

When we tested for unit roots in bitcoin above, we were unable to reject null hypothesis. This suggests that bitcoin has at least one unit root. That is, bitcoin is not an I(0) process. To formally test whether bitcoin is I(1) we can perform an ADF test on dbitcoin since if bitcoin is I(1) then dbitcoin should I(0). Note that dbitcoin is obtained by taking a first differencing for bitcoin.

```
> # Test statistics and critical values
> adf_dbitcoin@teststat
               tau1
statistic -5.923109
> adf_dbitcoin@cval
     1pct  5pct 10pct
tau1 -2.6 -1.95 -1.61
```

S1

The ADF regression equation for dbitcoin is:

$$dy_t' = \gamma y_{t-1}' + \alpha_1 dy_{t-1}' + \cdots + \alpha_{p-1} dy_{t-(p-1)}' + e_t \ (1.2)$$

$$\text{where } e_t \sim \text{i.i.d.}(0, \sigma^2);\ y_t' = y_t - y_{t-1}$$

The estimated ADF equation is that:(Refer to R code Section 1.3.4.2)

$$dy_t' = \underset{(0.1085)}{-0.6428} y_{t-1}' + \underset{(0.1050)}{0.2693} dy_{t-1}' + e_t$$

```
> LjungBox(adf_residuals_dbitcoin, lags = seq(8, 28, 2), fitdf = 2)
 lags statistic df   p-value
    8  9.890749  6 0.1293288
   10 10.921511  8 0.2061863
   12 11.455991 10 0.3231120
   14 13.091739 12 0.3624081
   16 15.221711 14 0.3631687
   18 15.691293 16 0.4747061
   20 15.810963 18 0.6057384
   22 17.728957 20 0.6052579
   24 17.960514 22 0.7083269
   26 18.273652 24 0.7895353
   28 19.156605 26 0.8297502
```

The LjungBox results suggest that there is no autocorrelation between residuals
Therefore, the t stat and F stat generated from the ADF models above is reliable

S2

$H_0: \gamma = 0$ i.e. dbitcoin has at least one unit root

$H_1: \gamma < 0$ i.e. dbitcoin is I(0), therefore bitcoin is I(1)

Since $t_{calc} = -5.92 < t_{crit,0.05} = -1.95$,we reject the null hypothesis that dbitcoin has at least one unit root at 5% significance level and conclude that bitcoin is indeed a I(1) process since differencing it once make it stationary. Overall, bitcoin is I(1) process containing only stochastic trend.

ADF tests for Ethereum

First ADF test for ethereum:

```
> # Test statistics and critical values
> adf_ethereum@teststat
               tau2     phi1
statistic -1.701264 1.485943
> adf_ethereum@cval
      1pct  5pct 10pct
tau2 -3.51 -2.89 -2.58
phi1  6.70  4.71  3.86
```

S1

The ADF regression equation for ethereum is:

$$dy_t = a + \gamma y_{t-1} + \alpha_1 dy_{t-1} + \cdots + \alpha_{p-1} dy_{t-(p-1)} + e_t \quad (1.1)$$

$$\text{where } e_t \sim \text{i.i.d.}(0, \sigma^2)$$

The estimated ADF equation is that:(Refer to R code Section 1.3.4.3)

$$dy_t = \underset{(56.83988)}{85.65385} - \underset{(0.02977)}{0.05065} y_{t-1} + \underset{(0.11029)}{0.45037} dy_{t-1} - \underset{(0.11070)}{0.34481} dy_{t-2} + \underset{(0.11215)}{0.23470} dy_{t-3} + e_t$$

```
> # Ljung-Box test on residuals
> LjungBox(adf_residuals_ethereum, lags = seq(10, 32, 2), fitdf = 5)
 lags statistic df   p-value
   10  9.042703  5 0.1073729
   12 10.607533  7 0.1566780
   14 13.446293  9 0.1434297
   16 14.744594 11 0.1944907
   18 15.002084 13 0.3072227
   20 15.007256 15 0.4508947
   22 16.334951 17 0.5002263
   24 19.514809 19 0.4242867
   26 19.522379 21 0.5516680
   28 21.178554 23 0.5701843
   30 22.087293 25 0.6306984
   32 24.276756 27 0.6149240
```

The LjungBox results suggest that there is no autocorrelation between residuals
Therefore, the t stat and F stat generated from the ADF models above is reliable

S2

$H_0: \gamma = 0$ i.e. ethereum has at least one unit root

$H_1: \gamma < 0$ i.e. ethereum does not have a unit root

Since $t_{calc} = -1.70 > t_{crit,0.05} = -2.89$,we fail to reject the null hypothesis that ethereum has at least one unit root at 5% significance level.

S3

Failure to reject the null of a unit root at Step 2 may be due to a loss of power arising from erroneously including a constant in (1.1). Therefore, we test for the presence of the constant, given the presence of a unit root.

$$H_0: a = \gamma = 0$$

The unrestricted model is given by (1.1)

The restricted model is given below (i.e. when true):

$$dy_t = \gamma y_{t-1} + \alpha_1 dy_{t-1} + \cdots + \alpha_{p-1} dy_{t-(p-1)} + e_t$$

Since $F1_{calc} = 1.49 < F1_{crit,0.05} = 4.71$,we fail to reject the null hypothesis that at 5% significance level. This implies that constant term is not significant in (1.1). This implies that we used the correct critical value for S2 (since the actual and estimated DGPs are different) when we failed to reject the null of at least one unit root. Hence, we conclude that ethereum has at least one unit root.

Second ADF test for dethereum:

When we tested for unit roots in ethereum above, we were unable to reject null hypothesis. This suggests that ethereum has at least one unit root. That is, ethereum is not an I(0) process. To formally test whether ethereum is I(1) we can perform an ADF test on dethereum since if ethereum is I(1) then dethereum should I(0). Note that dethereum is obtained by taking a first differencing for ethereum.

```
> # Test statistics and critical values
> adf_dethereum@teststat
               tau1
statistic -4.372489
> adf_dethereum@cval
     1pct  5pct 10pct
tau1 -2.6 -1.95 -1.61
```

S1

The ADF regression equation for dethereum is:

$$dy_t' = \gamma y_{t-1}' + \alpha_1 dy_{t-1}' + \cdots + \alpha_{p-1} dy_{t-(p-1)}' + e_t \ (1.2)$$

where $e_t \sim$ i.i.d.$(0, \sigma^2)$; $y_t' = y_t - y_{t-1}$

The estimated ADF equation is that:(Refer to R code Section 1.3.4.3)

$$dy_t' = \underset{(0.1704)}{-0.7453} y_{t-1}' + \underset{(0.1297)}{0.1679} dy_{t-1}' - \underset{(0.1104)}{0.2048} dy_{t-2}' + e_t$$

```
> # Ljung-Box test on residuals
> LjungBox(adf_residuals_dethereum, lags = seq(8, 28, 2), fitdf = 3)
 lags statistic df   p-value
    8  7.491683  5 0.1865648
   10  9.263291  7 0.2343016
   12 10.610094  9 0.3033841
   14 14.382741 11 0.2125335
   16 16.033852 13 0.2473067
   18 16.374770 15 0.3575909
   20 16.395000 17 0.4960257
   22 17.893650 19 0.5295525
   24 20.830795 21 0.4693185
   26 20.841700 23 0.5907542
   28 22.893039 25 0.5838092
```

The LjungBox results suggest that there is no autocorrelation between residuals

Therefore, the t stat and F stat generated from the ADF models above is reliable

S2

$$H_0: \gamma = 0 \quad \text{i.e. dethereum has at least one unit root}$$
$$H_1: \gamma < 0 \quad \text{i.e. dethereum is I(0), therefore ethereum is I(1)}$$

Since $t_{calc} = -4.37 < t_{crit,0.05} = -1.95$, we reject the null hypothesis that dethereum has at least one unit root at 5% significance level and conclude that ethereum is indeed a I(1) process since differencing it once make it stationary. Overall, ethereum is I(1) process containing only stochastic trend.

ADF tests for cardano

First ADF test for cardano:

```
> # Test statistics and critical values
> adf_cardano@teststat
              tau2     phi1
statistic -2.43185 3.003064
> adf_cardano@cval
      1pct  5pct 10pct
tau2 -3.51 -2.89 -2.58
phi1  6.70  4.71  3.86
```

S1

The ADF regression equation for cardano is:

$$dy_t = a + \gamma y_{t-1} + \alpha_1 dy_{t-1} + \cdots + \alpha_{p-1} dy_{t-(p-1)} + e_t \quad (1.1)$$

$$\text{where } e_t \sim \text{i.i.d.}(0, \sigma^2)$$

The estimated ADF equation is that:(Refer to R code Section 1.3.4.4)

$$dy_t = \underset{(0.02436)}{0.04714} - \underset{(0.03590)}{0.08731} y_{t-1} + \underset{(0.10902)}{0.47613} dy_{t-1} + \cdots + \underset{(0.10488)}{0.23725} dy_{t-5} + e_t$$

```
> LjungBox(adf_residuals_cardano, lags = seq(12, 32, 2), fitdf = 7)
 lags statistic df   p-value
   12  6.358078  5 0.2729186
   14  7.705850  7 0.3592449
   16  7.771715  9 0.5573056
   18  7.798795 11 0.7312061
   20  8.363272 13 0.8191551
   22  9.987552 15 0.8205222
   24 12.769228 17 0.7514925
   26 13.081463 19 0.8343748
   28 13.295818 21 0.8979095
   30 13.581916 23 0.9382215
   32 15.086450 25 0.9393385
```

The LjungBox results suggest that there is no autocorrelation between residuals
Therefore, the t stat and F stat generated from the ADF models above is reliable

S2

$$H_0: \gamma = 0 \quad \text{i.e. cardano has at least one unit root}$$

$$H_1: \gamma < 0 \quad \text{i.e. cardano does not have a unit root}$$

Since $t_{calc} = -2.43 > t_{crit,0.05} = -2.89$, we fail to reject the null hypothesis that cardano has at least one unit root at 5% significance level.

S3

Failure to reject the null of a unit root at Step 2 may be due to a loss of power arising from erroneously including a constant in (1.1). Therefore, we test for the presence of the constant, given the presence of a unit root.

$$H_0: a = \gamma = 0$$

The unrestricted model is given by (1.1)

The restricted model is given below (i.e. when true):

$$dy_t = \gamma y_{t-1} + \alpha_1 dy_{t-1} + \cdots + \alpha_{p-1} dy_{t-(p-1)} + e_t$$

Since $F1_{calc} = 3.00 < F1_{crit,0.05} = 4.71$, we fail to reject the null hypothesis that at 5% significance level. This implies that constant term is not significant in (1.1). This implies that we used the correct critical value for S2 (since the actual and estimated DGPs are different) when we failed to reject the null of at least one unit root. Hence, we conclude that cardano has at least one unit root.

Second ADF test for cardano:

When we tested for unit roots in cardano above, we were unable to reject null hypothesis. This suggests that cardano has at least one unit root. That is, cardano is not an I(0) process. To formally test whether cardano is I(1) we can perform an ADF test on dcardano since if cardano is I(1) then dcardano should I(0). Note that dcardano is obtained by taking a first differencing for cardano.

```
> # Test statistics and critical values
> adf_dcardano@teststat
               tau1
statistic -3.190118
> adf_dcardano@cval
     1pct  5pct 10pct
tau1 -2.6 -1.95 -1.61
```

S1

The ADF regression equation for dcardano is:

$$dy_t' = \gamma y_{t-1}' + \alpha_1 dy_{t-1}' + \cdots + \alpha_{p-1} dy_{t-(p-1)}' + e_t \quad (1.2)$$

$$\text{where } e_t \sim \text{i.i.d.}(0, \sigma^2);\ y_t' = y_t - y_{t-1}$$

The estimated ADF equation is that:(Refer to R code Section 1.3.4.4)

$$dy_t' = \underset{(0.18790)}{-0.59942} y_{t-1}' + \underset{(0.16971)}{0.04659} dy_{t-1}' + \cdots - \underset{(0.10531)}{0.18607} dy_{t-4}' + e_t$$

```
> # Ljung-Box test on residuals
> LjungBox(adf_residuals_dcardano, lags = seq(10, 28, 2), fitdf = 5)
 lags statistic df   p-value
   10  7.181649  5 0.2074776
   12  8.176330  7 0.3173026
   14  9.594448  9 0.3843026
   16  9.787499 11 0.5495972
   18  9.839291 13 0.7070204
   20 10.462673 15 0.7896722
   22 11.536235 17 0.8273705
   24 14.066540 19 0.7797986
   26 14.113417 21 0.8646784
   28 14.715564 23 0.9045373
```

The LjungBox results suggest that there is no autocorrelation between residuals Therefore, the t stat and F stat generated from the ADF models above is reliable

S2

$H_0: \gamma = 0$ i.e. dcardano has at least one unit root

$H_1: \gamma < 0$ i.e. dcardano is I(0), therefore cardano is I(1)

Since $t_{calc} = -3.19 < t_{crit,0.05} = -1.95$, we reject the null hypothesis that dcardano has at least one unit root at 5% significance level and conclude that cardano is indeed a I(1) process since differencing it once make it stationary. Overall, cardano is I(1) process containing only stochastic trend.

ADF tests for stellar

First ADF test for stellar:

```
> # Test statistics and critical values
> adf_stellar@teststat
               tau2     phi1
statistic -2.548184 3.247506
> adf_stellar@cval
      1pct  5pct 10pct
tau2 -3.51 -2.89 -2.58
phi1  6.70  4.71  3.86
```

S1

The ADF regression equation for stellar is:

$$dy_t = a + \gamma y_{t-1} + \alpha_1 dy_{t-1} + \cdots + \alpha_{p-1} dy_{t-(p-1)} + e_t \quad (1.1)$$

$$\text{where } e_t \sim \text{i.i.d.}(0, \sigma^2)$$

The estimated ADF equation is that:(Refer to R code Section 1.3.4.5)

$$dy_t = \underset{(0.01050)}{0.02284} - \underset{(0.05308)}{0.13527} y_{t-1} + \underset{(0.11070)}{0.38786} dy_{t-1} + \cdots + \underset{(0.08284)}{0.00813} dy_{t-4} + e_t$$

```
> LjungBox(adf_residuals_stellar, lags = seq(10, 32, 2), fitdf = 6)
 lags statistic df    p-value
   10  9.083637  4 0.05904262
   12  9.127185  6 0.16655062
   14 10.185747  8 0.25223027
   16 10.496692 10 0.39804844
   18 10.903563 12 0.53720098
   20 13.302049 14 0.50289002
   22 13.980851 16 0.60014050
   24 15.581289 18 0.62174356
   26 17.651742 20 0.61033775
   28 17.998798 22 0.70605957
   30 18.048308 24 0.80065870
   32 22.672681 26 0.65139173
```

The LjungBox results suggest that there is no autocorrelation between residuals

Therefore, the t stat and F stat generated from the ADF models above is reliable

S2

$H_0: \gamma = 0$ i.e. stellar has at least one unit root

$H_1: \gamma < 0$ i.e. stellar does not have a unit root

Since $t_{calc} = -2.55 > t_{crit,0.05} = -2.89$, we fail to reject the null hypothesis that stellar has at least one unit root at 5% significance level.

S3

Failure to reject the null of a unit root at Step 2 may be due to a loss of power arising from erroneously including a constant in (1.1). Therefore, we test for the presence of the constant, given the presence of a unit root.

$$H_0: a = \gamma = 0$$

The unrestricted model is given by (1.1)

The restricted model is given below (i.e. when true):

$$dy_t = \gamma y_{t-1} + \alpha_1 dy_{t-1} + \cdots + \alpha_{p-1} dy_{t-(p-1)} + e_t$$

Since $F1_{calc} = 3.25 < F1_{crit,0.05} = 4.71$, we fail to reject the null hypothesis that at 5% significance level. This implies that constant term is not significant in (1.1). This implies that we used the correct critical value for S2 (since the actual and estimated DGPs are different) when we failed to reject the null of at least one unit root. Hence, we conclude that stellar has at least one unit root.

Second ADF test for stellar:

When we tested for unit roots in stellar above, we were unable to reject null hypothesis. This suggests that stellar has at least one unit root. That is, stellar is not an I(0) process. To formally test whether stellar is I(1) we can perform an ADF test on dstellar since if stellar is I(1) then dstellar should I(0). Note that dstellar is obtained by taking a first differencing for stellar.

```
> # Test statistics and critical values
> adf_dstellar@teststat
              tau1
statistic -4.872774
> adf_dstellar@cval
     1pct  5pct 10pct
tau1 -2.6 -1.95 -1.61
```

S1

The ADF regression equation for dstellar is:

$$dy_t' = \gamma y_{t-1}' + \alpha_1 dy_{t-1}' + \cdots + \alpha_{p-1} dy_{t-(p-1)}' + e_t \quad (1.2)$$

$$\text{where } e_t \sim \text{i.i.d.}(0, \sigma^2);\ y_t' = y_t - y_{t-1}$$

The estimated ADF equation is that:(Refer to R code Section 1.3.4.5)

$$dy_t' = \underset{(0.18463)}{-0.89966} y_{t-1}' + \underset{(0.16090)}{0.23259} dy_{t-1}' - \underset{(0.10484)}{0.02743} dy_{t-2}' + \underset{(0.08121)}{0.05516} dy_{t-3}' + e_t$$

```
> # Ljung-Box test on residuals
> LjungBox(adf_residuals_dstellar, lags = seq(10, 28, 2), fitdf = 4)
 lags statistic df   p-value
   10  8.165758  6 0.2262093
   12  8.221598  8 0.4121287
   14  9.057735 10 0.5266332
   16  9.229324 12 0.6832283
   18  9.501769 14 0.7976281
   20 11.022102 16 0.8081190
   22 11.506897 18 0.8716229
   24 12.872447 20 0.8827894
   26 14.313291 22 0.8899879
   28 14.807925 24 0.9262531
```

The LjungBox results suggest that there is no autocorrelation between residuals
Therefore, the t stat and F stat generated from the ADF models above is reliable

S2

$$H_0: \gamma = 0 \quad \text{i.e. dstellar has at least one unit root}$$
$$H_1: \gamma < 0 \quad \text{i.e. dstellar is I(0), therefore stellar is I(1)}$$

Since $t_{calc} = -4.87 < t_{crit,0.05} = -1.95$,we reject the null hypothesis that dstellar has at least one unit root at 5% significance level and conclude that stellar is indeed a I(1) process since differencing it once make it stationary. Overall, stellar is I(1) process containing only stochastic trend.

02

```
> ## Section 1.3.3: Extreme Observations Analysis (spikes & outliers)
> tsoutliers(electricity)
$index
[1] 40

$replacements
[1] 10.59894

> tsoutliers(bitcoin)
$index
[1] 41 42 49 86 87

$replacements
[1] 46983.42 48000.45 54021.78 89490.66 92194.50

> tsoutliers(ethereum)
$index
 [1]  3 43 45 48 49 50 54 56 57 77 80 83 86 87

$replacements
 [1]  720.3918 2352.8436 2787.5858 3158.6744 3175.3480 3082.0003 2632.5861 2069.7142 1984.5845
[10] 2848.6633 3247.0279 2634.1339 2830.9215 2661.2044

> tsoutliers(cardano)
$index
 [1]  1  3 39 45 46 47 48 49 86 87

$replacements
 [1] 0.3392734 0.3190370 0.4917745 1.4930519 1.4835563 1.4462126 1.4123770 1.4172407 0.6784673
[10] 0.7334165

> tsoutliers(stellar)
$index
 [1]  1  2  3  4 40 42 43 45 84 86 87

$replacements
 [1] 0.2824833 0.2573481 0.2644455 0.2882156 0.3491254 0.3921530 0.3539810 0.3377589 0.1720212
[10] 0.2635042 0.2893566
```

03

Write this 5-dim VAR (1) model in matrix form:

$$X_t = A_1 X_{t-1} + U_t$$

Rewriting the equation using the lag operator L:

$$(I_5 - A_1 L)X_t = U_t$$

Therefore, the autoregressive matrix polynomial:

$$\Phi(L) = I_5 - A_1 L$$

The roots of Φ(L) satisfies:

$$\Phi(r) = I_5 - A_1 r = 0$$

To find the roots, we compute the determinant of the matrix polynomial:

$$det[\Phi(r)] = 0$$

```
> # Check the stationarity of this 5-dim VAR(1) model
> engien <- roots(var1)
> roots <- 1/engien
> mod_r <- Mod(roots)
> mod_r
[1] 2.284599 2.284599 2.328739 2.328739 5.202096
```

3.1 The VAR(P) process given by (3.6) is stationary if the roots of $\Phi(L)$ have modulus greater than one. That is,

$$\text{mod}(r_i) > 1$$

for any r_i that satisfies the equation

$$|I_m - A_1 r_i - A_2 r_i^2 - .... - A_p r_i^p| = 0.$$

According to the theorem 3.1 above, this 5-dim VAR (1) is a stationary process since all the roots of $\Phi(L)$ have modulus greater than 1.

04

```
> ## Print irf of one standard deviation orthogonalised shock to electricity on all 5 variables
> print(irf_var1$irf$dln_electricity)
      dln_electricity   dln_bitcoin  dln_ethereum   dln_cardano   dln_stellar
 [1,]    2.255999e-02 -1.496038e-02  8.736782e-03  3.943803e-02  1.115999e-02
 [2,]    7.849302e-03 -1.569512e-02 -4.250459e-02 -3.598217e-02 -4.602769e-02
 [3,]    1.380276e-03  3.904409e-03 -1.584404e-02 -6.841683e-03 -8.376037e-03
 [4,]   -1.627558e-04  6.623916e-03  1.713319e-03  8.153739e-03  1.027481e-02
 [5,]   -1.206607e-04  2.222297e-03  3.189700e-03  5.333859e-03  7.633690e-03
 [6,]    2.196588e-05 -5.607906e-04  4.750252e-04  4.177388e-04  1.740050e-03
 [7,]    2.884752e-05 -8.901862e-04 -7.654004e-04 -1.180124e-03 -6.346720e-04
 [8,]   -2.585223e-06 -4.176008e-04 -6.125927e-04 -8.088770e-04 -6.131560e-04
 [9,]   -1.480822e-05 -8.913401e-05 -2.132872e-04 -2.472667e-04 -1.644643e-04
[10,]   -1.068312e-05 -1.257825e-07 -2.115551e-05 -1.759297e-05  2.739106e-05
[11,]   -4.298623e-06 -3.292346e-06  9.908983e-06  7.025920e-06  3.075149e-05
```

05

```
> # Print irf of shock to electricity on all 5 variables for new VAR ordering
> print(irf_var1_new$irf$dln_electricity)
        dln_bitcoin  dln_ethereum   dln_cardano   dln_stellar dln_electricity
 [1,]  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00    2.153177e-02
 [2,] -4.116791e-03 -4.304800e-02 -3.463918e-02 -3.647285e-02    7.014692e-03
 [3,]  9.641538e-03 -1.085086e-02  1.373900e-03  4.418554e-03    8.880850e-04
 [4,]  7.112086e-03  3.855580e-03  1.076914e-02  1.547518e-02   -2.637663e-04
 [5,]  1.065731e-03  2.678325e-03  4.263323e-03  7.715882e-03   -9.657923e-05
 [6,] -1.381125e-03 -4.752367e-04 -9.459285e-04  7.984027e-04    2.946127e-05
 [7,] -1.159832e-03 -1.246551e-03 -1.787809e-03 -1.086430e-03    1.232021e-05
 [8,] -4.506355e-04 -7.215134e-04 -9.280608e-04 -6.580559e-04   -1.992000e-05
 [9,] -8.856450e-05 -2.076795e-04 -2.415713e-04 -1.185389e-04   -2.373230e-05
[10,] -1.061845e-05 -1.573650e-05 -2.025733e-05  4.361365e-05   -1.350296e-05
[11,] -1.530055e-05  3.658475e-06 -7.486190e-06  2.292255e-05   -4.894374e-06
```

06

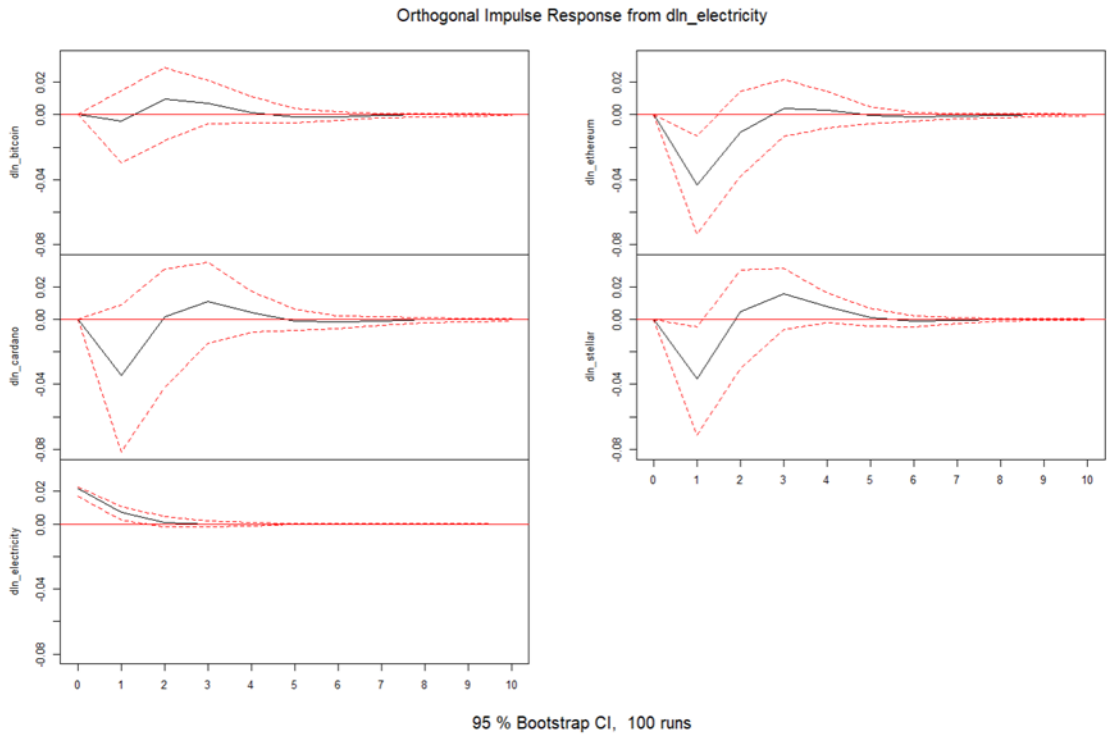

Figure 10: Orthogonal Impulse Response from dln_electricity for new VAR ordering

Notice that it is evident from Figure 6 that the orthogonalized shock to dln_electricity in period 0 has no impact effect on cryptocurrencies variables (i.e. period 0 in the graphs). That is, it cannot affect cryptocurrencies variables in period 0 (i.e. the estimated impulse response coefficients in period 0 are zero for all cryptocurrencies variables).

07

- There are two circumstances when orthogonalized impulse response analysis is useful:
  - When the pattern of the impulse responses is not very sensitive to the ordering of the variables.

.9 Impulse Response Analysis XXII

.2 Orthogonalized impulse responses

- When economic theory or common sense strongly indicates that a particular ordering of the variables is appropriate.
- For example, assume that we have a trivariate VAR with exchange rates, prices and wages as the variables. Since prices and wages tend to be "sticky" in the very short run, and the exchange rate responds almost instantaneously to shocks, the following ordering might be reasonable,

$$X = (\text{wages, prices, exchange rate})' \tag{3.76}$$

The ordering in (3.76) permits wages and price shocks to have an immediate effect on the exchange rate, but does not allow an exchange rate shock to have an immediate impact on prices and wages.

08

```
> ### Section 2.4
> # Table 4
> irf_result <- irf(var1, impulse = "dln_bitcoin", response = c("dln_electricity", "dln_ethereum","dln_cardano","dln_stellar"))
> irf_result

Impulse response coefficients
$dln_bitcoin
      dln_electricity  dln_ethereum   dln_cardano   dln_stellar
 [1,]    0.000000e+00  1.438705e-01  1.563414e-01  1.316433e-01
 [2,]    2.568809e-03  6.649615e-02  8.666699e-02  7.199919e-02
 [3,]    2.188525e-03  2.133087e-02  2.866496e-02  2.257218e-02
 [4,]    1.197131e-03  3.018968e-03  4.978868e-03  2.343160e-03
 [5,]    4.548494e-04 -5.853489e-04  3.796298e-04 -6.968239e-04
 [6,]    1.176360e-04 -1.248313e-04  5.984264e-04  3.113923e-04
 [7,]    2.204008e-05  2.518804e-04  6.489957e-04  6.750046e-04
 [8,]    7.670281e-06  1.654626e-04  2.956174e-04  3.772991e-04
 [9,]    5.496653e-06  1.620727e-05  2.819778e-05  8.236185e-05
[10,]    2.420194e-06 -3.578028e-05 -4.661811e-05 -2.147771e-05
[11,]    8.747205e-08 -2.636915e-05 -3.178883e-05 -2.103009e-05
```

09

Estimated coefficients from summary(var1)

```
> ## Section 2.5.1 justify the use of single equation OLS estimation to conduct Granger-Causality test
>
> summary(var1)

VAR Estimation Results:
=========================
Endogenous variables: dln_electricity, dln_bitcoin, dln_ethereum, dln_cardano, dln_stellar
Deterministic variables: none
Sample size: 87
Log Likelihood: 404.246
Roots of the characteristic polynomial:
0.4377 0.4377 0.4294 0.4294 0.1922
Call:
VAR(y = ddf, p = 1, type = "none")


Estimation results for equation dln_electricity:
================================================
dln_electricity = dln_electricity.l1 + dln_bitcoin.l1 + dln_ethereum.l1 + dln_cardano.l1 + dln_stellar.l1

                    Estimate Std. Error t value Pr(>|t|)
dln_electricity.l1  0.3257834  0.1045440   3.116  0.00253 **
dln_bitcoin.l1     -0.0153849  0.0218895  -0.703  0.48414
dln_ethereum.l1     0.0363064  0.0193021   1.881  0.06353 .
dln_cardano.l1     -0.0005688  0.0149897  -0.038  0.96982
dln_stellar.l1     -0.0022673  0.0167466  -0.135  0.89264
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1


Residual standard error: 0.02256 on 82 degrees of freedom
Multiple R-Squared: 0.1735,     Adjusted R-squared: 0.1231
F-statistic: 3.442 on 5 and 82 DF,  p-value: 0.007136


Estimation results for equation dln_bitcoin:
============================================
dln_bitcoin = dln_electricity.l1 + dln_bitcoin.l1 + dln_ethereum.l1 + dln_cardano.l1 + dln_stellar.l1

                   Estimate Std. Error t value Pr(>|t|)
dln_electricity.l1 -0.19120    0.68641  -0.279   0.7813
dln_bitcoin.l1      0.68293    0.14372   4.752 8.48e-06 ***
dln_ethereum.l1    -0.16167    0.12673  -1.276   0.2057
dln_cardano.l1      0.08110    0.09842   0.824   0.4123
dln_stellar.l1     -0.26441    0.10995  -2.405   0.0184 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1


Residual standard error: 0.1481 on 82 degrees of freedom
Multiple R-Squared: 0.2693,     Adjusted R-squared: 0.2247
F-statistic: 6.044 on 5 and 82 DF,  p-value: 8.229e-05

Estimation results for equation dln_ethereum:
=============================================
dln_ethereum = dln_electricity.l1 + dln_bitcoin.l1 + dln_ethereum.l1 + dln_cardano.l1 + dln_stellar.l1

                   Estimate Std. Error t value Pr(>|t|)
dln_electricity.l1  -1.9993     0.8780  -2.277  0.02539 *
dln_bitcoin.l1       0.4862     0.1838   2.644  0.00980 **
dln_ethereum.l1     -0.1631     0.1621  -1.006  0.31721
dln_cardano.l1       0.3721     0.1259   2.956  0.00407 **
dln_stellar.l1      -0.3028     0.1407  -2.153  0.03430 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1


Residual standard error: 0.1895 on 82 degrees of freedom
Multiple R-Squared: 0.3127,     Adjusted R-squared: 0.2708
F-statistic:  7.46 on 5 and 82 DF,  p-value: 8.227e-06
```

```
Estimation results for equation dln_cardano:
============================================
dln_cardano = dln_electricity.l1 + dln_bitcoin.l1 + dln_ethereum.l1 + dln_cardano.l1 + dln_stellar.l1

                  Estimate Std. Error t value Pr(>|t|)
dln_electricity.l1  -1.6087     1.1839  -1.359  0.17793
dln_bitcoin.l1       0.8302     0.2479   3.349  0.00123 **
dln_ethereum.l1     -0.4769     0.2186  -2.182  0.03199 *
dln_cardano.l1       0.5387     0.1698   3.173  0.00212 **
dln_stellar.l1      -0.3895     0.1897  -2.054  0.04319 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1


Residual standard error: 0.2555 on 82 degrees of freedom
Multiple R-Squared: 0.3174,     Adjusted R-squared: 0.2758
F-statistic: 7.627 on 5 and 82 DF,  p-value: 6.318e-06


Estimation results for equation dln_stellar:
============================================
dln_stellar = dln_electricity.l1 + dln_bitcoin.l1 + dln_ethereum.l1 + dln_cardano.l1 + dln_stellar.l1

                  Estimate Std. Error t value Pr(>|t|)
dln_electricity.l1 -1.69391    1.01401  -1.671 0.098633 .
dln_bitcoin.l1      0.85906    0.21231   4.046 0.000117 ***
dln_ethereum.l1    -0.73862    0.18722  -3.945 0.000167 ***
dln_cardano.l1      0.27160    0.14539   1.868 0.065323 .
dln_stellar.l1      0.06994    0.16243   0.431 0.667915
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1


Residual standard error: 0.2188 on 82 degrees of freedom
Multiple R-Squared: 0.3674,     Adjusted R-squared: 0.3289
F-statistic: 9.527 on 5 and 82 DF,  p-value: 3.422e-07
```

Covariance matrix of residuals:

	dln_electricity	dln_bitcoin	dln_ethereum	dln_cardano	dln_stellar
dln_electricity	0.0005055	-0.0003524	0.0001912	0.0009044	0.0002707
dln_bitcoin	-0.0003524	0.0218760	0.0210452	0.0225129	0.0193150
dln_ethereum	0.0001912	0.0210452	0.0358913	0.0342698	0.0284153
dln_cardano	0.0009044	0.0225129	0.0342698	0.0652101	0.0436975
dln_stellar	0.0002707	0.0193150	0.0284153	0.0436975	0.0477765

Correlation matrix of residuals:

	dln_electricity	dln_bitcoin	dln_ethereum	dln_cardano	dln_stellar
dln_electricity	1.00000	-0.1060	0.04489	0.1575	0.05508
dln_bitcoin	-0.10596	1.0000	0.75106	0.5961	0.59745
dln_ethereum	0.04489	0.7511	1.00000	0.7084	0.68620
dln_cardano	0.15751	0.5961	0.70837	1.0000	0.78287
dln_stellar	0.05508	0.5975	0.68620	0.7829	1.00000

OLS estimation, equation by equation

```
> # OLS: dln_electricity as dependent variable
> dlnelectricity_OLS <- dynlm(dln_electricity  ~ L(dln_electricity,1)+ L(d
(dln_stellar, 1)- 1)
> summary(dlnelectricity_OLS)

Time series regression with "ts" data:
Start = 2018(1), End = 2025(3)

Call:
dynlm(formula = dln_electricity ~ L(dln_electricity, 1) + L(dln_bitcoin,
    1) + L(dln_ethereum, 1) + L(dln_cardano, 1) + L(dln_stellar,
    1) - 1)

Residuals:
      Min        1Q    Median        3Q       Max
-0.062607 -0.011353  0.000179  0.014639  0.068774

Coefficients:
                        Estimate Std. Error t value Pr(>|t|)
L(dln_electricity, 1)  0.3257834  0.1045440   3.116  0.00253 **
L(dln_bitcoin, 1)     -0.0153849  0.0218895  -0.703  0.48414
L(dln_ethereum, 1)     0.0363064  0.0193021   1.881  0.06353 .
L(dln_cardano, 1)     -0.0005688  0.0149897  -0.038  0.96982
L(dln_stellar, 1)     -0.0022673  0.0167466  -0.135  0.89264
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.02256 on 82 degrees of freedom
Multiple R-squared:  0.1735,    Adjusted R-squared:  0.1231
F-statistic: 3.442 on 5 and 82 DF,  p-value: 0.007136
```

```
> # OLS: dln_bitcoin as dependent variable
> dlnbitcoin_OLS <- dynlm(dln_bitcoin ~ L(dln_electricity,1)+ L(dln_b
ar, 1)- 1)
> summary(dlnbitcoin_OLS)

Time series regression with "ts" data:
Start = 2018(1), End = 2025(3)

Call:
dynlm(formula = dln_bitcoin ~ L(dln_electricity, 1) + L(dln_bitcoin,
    1) + L(dln_ethereum, 1) + L(dln_cardano, 1) + L(dln_stellar,
    1) - 1)

Residuals:
     Min       1Q   Median       3Q      Max
-0.32235 -0.09761  0.00057  0.10781  0.41321

Coefficients:
                       Estimate Std. Error t value Pr(>|t|)
L(dln_electricity, 1) -0.19120    0.68641  -0.279   0.7813
L(dln_bitcoin, 1)      0.68293    0.14372   4.752 8.48e-06 ***
L(dln_ethereum, 1)    -0.16167    0.12673  -1.276   0.2057
L(dln_cardano, 1)      0.08110    0.09842   0.824   0.4123
L(dln_stellar, 1)     -0.26441    0.10995  -2.405   0.0184 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1481 on 82 degrees of freedom
Multiple R-squared:  0.2693,    Adjusted R-squared:  0.2247
F-statistic: 6.044 on 5 and 82 DF,  p-value: 8.229e-05
```

```
> # OLS: dln_ethereum as dependent variable
> dlnethereum_OLS <- dynlm(dln_ethereum ~ L(dln_electricity,1)+ L(dln
llar, 1)- 1)
> summary(dlnethereum_OLS)

Time series regression with "ts" data:
Start = 2018(1), End = 2025(3)

Call:
dynlm(formula = dln_ethereum ~ L(dln_electricity, 1) + L(dln_bitcoin,
    1) + L(dln_ethereum, 1) + L(dln_cardano, 1) + L(dln_stellar,
    1) - 1)

Residuals:
     Min       1Q   Median       3Q      Max
-0.43563 -0.09307  0.00209  0.10312  0.57707

Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
L(dln_electricity, 1)  -1.9993     0.8780  -2.277  0.02539 *
L(dln_bitcoin, 1)       0.4862     0.1838   2.644  0.00980 **
L(dln_ethereum, 1)     -0.1631     0.1621  -1.006  0.31721
L(dln_cardano, 1)       0.3721     0.1259   2.956  0.00407 **
L(dln_stellar, 1)      -0.3028     0.1407  -2.153  0.03430 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1895 on 82 degrees of freedom
Multiple R-squared:  0.3127,    Adjusted R-squared:  0.2708
F-statistic:  7.46 on 5 and 82 DF,  p-value: 8.227e-06
```

```
> # OLS: dln_cardano as dependent variable
> dlncardano_OLS <- dynlm(dln_cardano ~ L(dln_electricity,1)+ L(dln_b
llar, 1)- 1)
> summary(dlncardano_OLS)

Time series regression with "ts" data:
Start = 2018(1), End = 2025(3)

Call:
dynlm(formula = dln_cardano ~ L(dln_electricity, 1) + L(dln_bitcoin,
    1) + L(dln_ethereum, 1) + L(dln_cardano, 1) + L(dln_stellar,
    1) - 1)

Residuals:
     Min       1Q   Median       3Q      Max
-0.53572 -0.18527  0.00673  0.14690  0.72023

Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
L(dln_electricity, 1)  -1.6087     1.1839  -1.359  0.17793
L(dln_bitcoin, 1)       0.8302     0.2479   3.349  0.00123 **
L(dln_ethereum, 1)     -0.4769     0.2186  -2.182  0.03199 *
L(dln_cardano, 1)       0.5387     0.1698   3.173  0.00212 **
L(dln_stellar, 1)      -0.3895     0.1897  -2.054  0.04319 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2555 on 82 degrees of freedom
Multiple R-squared:  0.3174,    Adjusted R-squared:  0.2758
F-statistic: 7.627 on 5 and 82 DF,  p-value: 6.318e-06
```

```
> # OLS: dln_stellar as dependent variable
> dlnstellar_OLS <- dynlm(dln_stellar ~ L(dln_electricity,1)+ L(dln_b
ar, 1)- 1)
> summary(dlnstellar_OLS)

Time series regression with "ts" data:
Start = 2018(1), End = 2025(3)

Call:
dynlm(formula = dln_stellar ~ L(dln_electricity, 1) + L(dln_bitcoin,
    1) + L(dln_ethereum, 1) + L(dln_cardano, 1) + L(dln_stellar,
    1) - 1)

Residuals:
     Min       1Q   Median       3Q      Max
-0.54686 -0.14637 -0.02536  0.11021  0.86665

Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
L(dln_electricity, 1) -1.69391    1.01401  -1.671 0.098633 .
L(dln_bitcoin, 1)      0.85906    0.21231   4.046 0.000117 ***
L(dln_ethereum, 1)    -0.73862    0.18722  -3.945 0.000167 ***
L(dln_cardano, 1)      0.27160    0.14539   1.868 0.065323 .
L(dln_stellar, 1)      0.06994    0.16243   0.431 0.667915
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.2188 on 82 degrees of freedom
Multiple R-squared:  0.3674,    Adjusted R-squared:  0.3289
F-statistic: 9.527 on 5 and 82 DF,  p-value: 3.422e-07
```

10

Test whether bi-directional Granger-Causality exists between CARDANO and BITCOIN

A bi-directional Granger-Causality exists between cardano and bitcoin if and only if cardano Granger-causes bitcoin and, at the same time, bitcoin Granger-causes cardano. Therefore, we need to do the following 2 tests.

Test 1: Test the null hypothesis that cardano does not Granger-cause bitcoin

The unrestricted model for the test is given by:

$$dx_{2t} = \alpha_{21,1}\, dx_{1t-1} + \alpha_{22,1}\, dx_{2t-1} + \alpha_{23,1}\, dx_{3t-1} + \alpha_{24,1}\, dx_{4t-1} + \alpha_{25,1}\, dx_{5t-1} + \varepsilon_{2t}$$

$$x_t' = (dx_{1t}, dx_{2t}, dx_{3t}, dx_{4t}, dx_{5t})'$$

Note that all variables are in log first-difference form

$$dx_{1t} = dlnelectricity_t\ ;\ dx_{2t} = dlnbitcoin_t\ ;\ dx_{3t} = dlnethereum_t$$

$$dx_{4t} = dlncardano_t;\ dx_{5t} = dlnstellar_t$$

The null and alternative hypothesis is that:

$H_0: \alpha_{24,1} = 0$ i.e. cardano does not Granger-cause bitcoin

$H_1: \alpha_{24,1} \neq 0.$ i.e. cardano Granger-causes bitcoin

Therefore, the restricted model for the test is (when H_0 true) :

$$dx_{2t} = \alpha_{21,1}\, dx_{1t-1} + \alpha_{22,1}\, dx_{2t-1} + \alpha_{23,1}\, dx_{3t-1} + \alpha_{25,1}\, dx_{5t-1} + \varepsilon_{2t}$$

Under the null hypothesis,

$$LR = 2(L_u - L_r) \overset{asy}{\sim} \chi_1^2$$

```
> ## Section 2.5.2 Granger-Causality test
>
> # Section 2.5.2.1-----Test whether bi-directional Granger-Causality exists between CARDANO and BITCOIN
>
> # ----------Test the null hypothesis that dln_cardano does not Granger-cause dln_bitcoin
> # unrestricted model for the test
> UR_1 <- dynlm(dln_bitcoin ~ L(dln_electricity,1)+ L(dln_bitcoin,1) + L(dln_ethereum,1) + L(dln_cardano, 1)+ L(dln_stellar, 1)- 1)
> LU_1 = logLik(UR_1)
> LU_1
'log Lik.' 45.27191 (df=6)
>
> # restricted model for the test
> R_1 <- dynlm(dln_bitcoin ~ L(dln_electricity,1)+ L(dln_bitcoin,1) + L(dln_ethereum,1) + L(dln_stellar, 1)- 1)
> LR_1 <- logLik(R_1)
> LR_1
'log Lik.' 44.91317 (df=5)
>
> # LR calc
> 2 * (LU_1 - LR_1)
'log Lik.' 0.7174799 (df=6)
>
> # critiacal value
> qchisq(0.95, 1)
[1] 3.841459
>
> # Since 0.717 < 3.841 we fail to reject the null and conclude that dln_cardano does not Granger-cause dln_bitcoin
```

Since $LR_{\text{calc}} = 0.717 < LR_{crit,0.05} = 3.841$,we fail to reject the null hypothesis at 5% significance level and conclude that cardano does not Granger-cause bitcoin.

Test 2: Test the null hypothesis that bitcoin does not Granger-cause cardano

The unrestricted model for the test is given by:

$$dx_{4t} = \alpha_{41,1}\, dx_{1t-1} + \alpha_{42,1}\, dx_{2t-1} + \alpha_{43,1}\, dx_{3t-1} + \alpha_{44,1}\, dx_{4t-1} + \alpha_{45,1}\, dx_{5t-1} + \varepsilon_{4t}$$

$$x_t' = (dx_{1t}, dx_{2t}, dx_{3t}, dx_{4t}, dx_{5t})'$$

Note that all variables are in log first-difference form

$$dx_{1t} = dlnelectricity_t\ ;\ dx_{2t} = dlnbitcoin_t\ ;\ dx_{3t} = dlnethereum_t$$

$$dx_{4t} = dlncardano_t;\ dx_{5t} = dlnstellar_t$$

The null and alternative hypothesis is that:

$H_0: \alpha_{42,1} = 0$ i.e. bitcoin does not Granger-cause cardano

$H_1: \alpha_{42,1} \neq 0.$ i.e. bitcoin Granger-causes cardano

Therefore, the restricted model for the test is (when H_0 true) :

$$dx_{4t} = \alpha_{41,1}\, dx_{1t-1} + \alpha_{43,1}\, dx_{3t-1} + \alpha_{44,1}\, dx_{4t-1} + \alpha_{45,1}\, dx_{5t-1} + \varepsilon_{4t}$$

Under the null hypothesis,

$$LR = 2(L_u - L_r) \overset{\text{asy}}{\sim} \chi_1^2$$

```
> # ----------Test the null hypothesis that dln_bitcoin does not Granger-cause dln_cardano
> # unrestricted model for the test
> UR_2 <- dynlm(dln_cardano ~ L(dln_electricity,1)+ L(dln_bitcoin,1) + L(dln_ethereum,1) + L(dln_cardano, 1)+ L(dln_stellar, 1)- 1)
> LU_2 = logLik(UR_2)
> LU_2
'log Lik.' -2.153597 (df=6)
>
> # restricted model for the test
> R_2 <- dynlm(dln_cardano ~ L(dln_electricity,1) + L(dln_ethereum,1) + L(dln_cardano, 1)+ L(dln_stellar, 1)- 1)
> LR_2 <- logLik(R_2)
> LR_2
'log Lik.' -7.729749 (df=5)
>
> # LR calc
> 2 * (LU_2 - LR_2)
'log Lik.' 11.1523 (df=6)
>
> # critiacal value
> qchisq(0.95, 1)
[1] 3.841459
>
> # Since 11.152 > 3.841 we reject the null and conclude that dln_bitcoin Granger-cause dln_cardano
> # Overall, no bi-directional Granger-Causality exists between CARDANO and BITCOIN
```

Since $LR_{\text{calc}} = 11.152 > LR_{crit,0.05} = 3.841$,we reject the null hypothesis at 5% significance level and conclude that bitcoin Granger-causes cardano.

Overall, no bi-directional Granger-Causality exists between cardano and bitcoin at 5% significance level since cardano does not Granger-cause bitcoin.

11

Test whether bi-directional Granger-Causality exists between CARDANO and ETHEREUM

A bi-directional Granger-Causality exists between cardano and ethereum if and only if cardano Granger-causes ethereum and, at the same time, ethereum Granger-causes cardano. Therefore, we need to do the following 2 tests.

Test 3: Test the null that cardano does not Granger-cause ethereum

The unrestricted model for the test is given by:

$$dx_{3t} = \alpha_{31,1}\, dx_{1t-1} + \alpha_{32,1}\, dx_{2t-1} + \alpha_{33,1}\, dx_{3t-1} + \alpha_{34,1}\, dx_{4t-1} + \alpha_{35,1}\, dx_{5t-1} + \varepsilon_{3t}$$

$$x_t' = (dx_{1t}, dx_{2t}, dx_{3t}, dx_{4t}, dx_{5t})'$$

Note that all variables are in log first-difference form

$$dx_{1t} = dlnelectricity_t\ ;\ dx_{2t} = dlnbitcoin_t\ ;\ dx_{3t} = dlnethereum_t$$

$$dx_{4t} = dlncardano_t;\ dx_{5t} = dlnstellar_t$$

The null and alternative hypothesis is that:

$H_0: \alpha_{34,1} = 0$ i.e. cardano does not Granger-cause ethereum

$H_1: \alpha_{34,1} \neq 0.$ i.e. cardano Granger-causes ethereum

Therefore, the restricted model for the test is (when H_0 true) :

$$dx_{3t} = \alpha_{31,1}\, dx_{1t-1} + \alpha_{32,1}\, dx_{2t-1} + \alpha_{33,1}\, dx_{3t-1} + \alpha_{35,1}\, dx_{5t-1} + \varepsilon_{3t}$$

Under the null hypothesis,

$$LR = 2(L_u - L_r) \overset{asy}{\sim} \chi_1^2$$

```
> # Section 2.5.2.2-----Test whether bi-directional Granger-Causality exists between CARDANO and ETHEREUM
>
> # ----------Test the null hypothesis that dln_cardano does not Granger-cause dln_ethereum
> # unrestricted model for the test
> UR_3 <- dynlm(dln_ethereum ~ L(dln_electricity,1)+ L(dln_bitcoin,1) + L(dln_ethereum,1) + L(dln_cardano, 1)+ L(dln_stellar, 1)- 1)
> LU_3 = logLik(UR_3)
> LU_3
'log Lik.' 23.85053 (df=6)
>
> # restricted model for the test
> R_3 <- dynlm(dln_ethereum ~ L(dln_electricity,1)+ L(dln_bitcoin,1) + L(dln_ethereum,1) + L(dln_stellar, 1)- 1)
> LR_3 <- logLik(R_3)
> LR_3
'log Lik.' 19.4461 (df=5)
>
> # LR calc
> 2 * (LU_3 - LR_3)
'log Lik.' 8.808854 (df=6)
>
> # critiacal value
> qchisq(0.95, 1)
[1] 3.841459
>
> # Since 8.809 > 3.841 we reject the null and conclude that dln_cardano Granger-cause dln_ethereum
```

Since $LR_{\text{calc}} = 8.809 > LR_{crit,0.05} = 3.841$,we reject the null hypothesis at 5% significance level and conclude that cardano Granger-causes ethereum.

Test 4: Test the null that ethereum does not Granger-cause cardano

The unrestricted model for the test is given by:

$$dx_{4t} = \alpha_{41,1}\, dx_{1t-1} + \alpha_{42,1}\, dx_{2t-1} + \alpha_{43,1}\, dx_{3t-1} + \alpha_{44,1}\, dx_{4t-1} + \alpha_{45,1}\, dx_{5t-1} + \varepsilon_{4t}$$

$$x_t' = (dx_{1t}, dx_{2t}, dx_{3t}, dx_{4t}, dx_{5t})'$$

Note that all variables are in log first-difference form

$$dx_{1t} = dlnelectricity_t\ ;\ dx_{2t} = dlnbitcoin_t\ ;\ dx_{3t} = dlnethereum_t$$

$$dx_{4t} = dlncardano_t;\ dx_{5t} = dlnstellar_t$$

The null and alternative hypothesis is that:

$H_0: \alpha_{43,1} = 0$ i.e. ethereum does not Granger-cause cardano

$H_1: \alpha_{43,1} \neq 0.$ i.e. ethereum Granger-causes cardano

Therefore, the restricted model for the test is (when H_0 true) :

$$dx_{4t} = \alpha_{41,1}\, dx_{1t-1} + \alpha_{42,1}\, dx_{2t-1} + \alpha_{44,1}\, dx_{4t-1} + \alpha_{45,1}\, dx_{5t-1} + \varepsilon_{4t}$$

Under the null hypothesis,

$$LR = 2(L_u - L_r) \overset{asy}{\sim} \chi_1^2$$

```
> # ----------Test the null hypothesis that dln_ethereum does not Granger-cause dln_cardano
> # unrestricted model for the test
> UR_4 <- dynlm(dln_cardano ~ L(dln_electricity,1)+ L(dln_bitcoin,1) + L(dln_ethereum,1) + L(dln_cardano, 1)+ L(dln_stellar, 1)- 1)
> LU_4 = logLik(UR_4)
> LU_4
'log Lik.' -2.153597 (df=6)
>
> # restricted model for the test
> R_4 <- dynlm(dln_cardano ~ L(dln_electricity,1)+ L(dln_bitcoin,1)  + L(dln_cardano, 1)+ L(dln_stellar, 1)- 1)
> LR_4 <- logLik(R_4)
> LR_4
'log Lik.' -4.608058 (df=5)
>
> # LR calc
> 2 * (LU_4 - LR_4)
'log Lik.' 4.908921 (df=6)
>
> # critiacal value
> qchisq(0.95, 1)
[1] 3.841459
>
> # Since 4.909 > 3.841 we reject the null and conclude that dln_ethereum Granger-cause dln_cardano
> # Overall,the bi-directional Granger-Causality exists between CARDANO and ETHERUM
```

Since $LR_{\text{calc}} = 4.909 > LR_{crit,0.05} = 3.841$,we reject the null hypothesis at 5% significance level and conclude that ethereum Granger-causes cardano.

Overall, the bi-directional Granger-Causality exists between cardano and ethereum at 5% significance level since cardano Granger-causes ethereum and, at the same time, ethereum Granger-causes cardano.

12

Test whether bi-directional Granger-Causality exists between STELLAR and BITCOIN

A bi-directional Granger-Causality exists between stellar and bitcoin if and only if stellar Granger-causes bitcoin and, at the same time, bitcoin Granger-causes stellar. Therefore, we need to do the following 2 tests.

Test 5: Test the null hypothesis that stellar does not Granger-cause bitcoin

The unrestricted model for the test is given by:

$$dx_{2t} = \alpha_{21,1}\, dx_{1t-1} + \alpha_{22,1}\, dx_{2t-1} + \alpha_{23,1}\, dx_{3t-1} + \alpha_{24,1}\, dx_{4t-1} + \alpha_{25,1}\, dx_{5t-1} + \varepsilon_{2t}$$

$$x_t' = (dx_{1t}, dx_{2t}, dx_{3t}, dx_{4t}, dx_{5t})'$$

Note that all variables are in log first-difference form

$$dx_{1t} = dlnelectricity_t\ ;\ dx_{2t} = dlnbitcoin_t\ ;\ dx_{3t} = dlnethereum_t$$

$$dx_{4t} = dlncardano_t;\ dx_{5t} = dlnstellar_t$$

The null and alternative hypothesis is that:

$$H_0: \alpha_{25,1} = 0 \text{ i.e. stellar does not Granger-cause bitcoin}$$

$$H_1: \alpha_{25,1} \neq 0. \text{ i.e. stellar Granger-causes bitcoin}$$

Therefore, the restricted model for the test is (when H_0 true) :

$$dx_{2t} = \alpha_{21,1}\, dx_{1t-1} + \alpha_{22,1}\, dx_{2t-1} + \alpha_{23,1}\, dx_{3t-1} + \alpha_{24,1}\, dx_{4t-1} + \varepsilon_{2t}$$

Under the null hypothesis,

$$LR = 2(L_u - L_r) \overset{asy}{\sim} \chi_1^2$$

```
> # Section 2.5.2.3-----Test whether bi-directional Granger-Causality exists between STELLAR and BITCOIN
>
> # ----------Test the null hypothesis that dln_stellar does not Granger-cause dln_bitcoin
> # unrestricted model for the test
> UR_5 <- dynlm(dln_bitcoin ~ L(dln_electricity,1)+ L(dln_bitcoin,1) + L(dln_ethereum,1) + L(dln_cardano, 1)+ L(dln_stellar, 1)- 1)
> LU_5 = logLik(UR_5)
> LU_5
'log Lik.' 45.27191 (df=6)
>
> # restricted model for the test
> R_5 <- dynlm(dln_bitcoin ~ L(dln_electricity,1)+ L(dln_bitcoin,1) + L(dln_ethereum,1) + L(dln_cardano, 1)- 1)
> LR_5 <- logLik(R_5)
> LR_5
'log Lik.' 42.30758 (df=5)
>
> # LR calc
> 2 * (LU_5 - LR_5)
'log Lik.' 5.928676 (df=6)
>
> # critiacal value
> qchisq(0.95, 1)
[1] 3.841459
>
> # Since 5.929 > 3.841 we reject the null and conclude that dln_stellar Granger-cause dln_bitcoin
```

Since $LR_{\text{calc}} = 5.929 > LR_{crit,0.05} = 3.841$,we reject the null hypothesis at 5% significance level and conclude that stellar Granger-causes bitcoin.

Test 6: Test the null hypothesis that bitcoin does not Granger-cause stellar

The unrestricted model for the test is given by:

$$dx_{5t} = \alpha_{51,1}\, dx_{1t-1} + \alpha_{52,1}\, dx_{2t-1} + \alpha_{53,1}\, dx_{3t-1} + \alpha_{54,1}\, dx_{4t-1} + \alpha_{55,1}\, dx_{5t-1} + \varepsilon_{5t}$$

$$x_t' = (dx_{1t}, dx_{2t}, dx_{3t}, dx_{4t}, dx_{5t})'$$

Note that all variables are in log first-difference form

$$dx_{1t} = dlnelectricity_t\ ;\ dx_{2t} = dlnbitcoin_t\ ;\ dx_{3t} = dlnethereum_t$$

$$dx_{4t} = dlncardano_t;\ dx_{5t} = dlnstellar_t$$

The null and alternative hypothesis is that:

$H_0: \alpha_{52,1} = 0$ i.e. bitcoin does not Granger-cause stellar

$H_1: \alpha_{52,1} \neq 0.$ i.e. bitcoin Granger-causes stellar

Therefore, the restricted model for the test is (when H_0 true) :

$$dx_{5t} = \alpha_{51,1}\, dx_{1t-1} + \alpha_{53,1}\, dx_{3t-1} + \alpha_{54,1}\, dx_{4t-1} + \alpha_{55,1}\, dx_{5t-1} + \varepsilon_{5t}$$

Under the null hypothesis,

$$LR = 2(L_u - L_r) \overset{asy}{\sim} \chi_1^2$$

```
> # ----------Test the null hypothesis that dln_bitcoin does not Granger-cause dln_stellar
> # unrestricted model for the test
> UR_6 <- dynlm(dln_stellar ~ L(dln_electricity,1)+ L(dln_bitcoin,1) + L(dln_ethereum,1) + L(dln_cardano, 1)+ L(dln_stellar, 1)- 1)
> LU_6 = logLik(UR_6)
> LU_6
'log Lik.' 11.32493 (df=6)
>
> # restricted model for the test
> R_6 <- dynlm(dln_stellar ~ L(dln_electricity,1) + L(dln_ethereum,1) + L(dln_cardano, 1)+ L(dln_stellar, 1)- 1)
> LR_6 <- logLik(R_6)
> LR_6
'log Lik.' 3.406589 (df=5)
>
> # LR calc
> 2 * (LU_6 - LR_6)
'log Lik.' 15.83668 (df=6)
>
> # critiacal value
> qchisq(0.95, 1)
[1] 3.841459
>
> # Since 15.837 > 3.841 we reject the null and conclude that dln_bitcoin Granger-cause dln_stellar
> # Overall,the bi-directional Granger-Causality exists between STELLAR and BITCOIN
```

Since $LR_{\text{calc}} = 15.837 > LR_{crit,0.05} = 3.841$,we reject the null hypothesis at 5% significance level and conclude that bitcoin Granger-causes stellar.

Overall, the bi-directional Granger-Causality exists between stellar and bitcoin at 5% significance level since stellar Granger-causes bitcoin and, at the same time, bitcoin Granger-causes stellar.

13

Test whether bi-directional Granger-Causality exists between STELLAR and ETHEREUM

A bi-directional Granger-Causality exists between stellar and ethereum if and only if stellar Granger-causes ethereum and, at the same time, ethereum Granger-causes stellar. Therefore, we need to do the following 2 tests.

Test 7: Test the null hypothesis that stellar does not Granger-cause ethereum

The unrestricted model for the test is given by:

$$dx_{3t} = \alpha_{31,1}\, dx_{1t-1} + \alpha_{32,1}\, dx_{2t-1} + \alpha_{33,1}\, dx_{3t-1} + \alpha_{34,1}\, dx_{4t-1} + \alpha_{35,1}\, dx_{5t-1} + \varepsilon_{3t}$$

$$x_t' = (dx_{1t}, dx_{2t}, dx_{3t}, dx_{4t}, dx_{5t})'$$

Note that all variables are in log first-difference form

$$dx_{1t} = dlnelectricity_t\ ;\ dx_{2t} = dlnbitcoin_t\ ;\ dx_{3t} = dlnethereum_t$$

$$dx_{4t} = dlncardano_t;\ dx_{5t} = dlnstellar_t$$

The null and alternative hypothesis is that:

$H_0: \alpha_{35,1} = 0$ i.e. stellar does not Granger-cause ethereum

$H_1: \alpha_{35,1} \neq 0.$ i.e. stellar Granger-causes ethereum

Therefore, the restricted model for the test is (when H_0 true) :

$$dx_{3t} = \alpha_{31,1}\, dx_{1t-1} + \alpha_{32,1}\, dx_{2t-1} + \alpha_{33,1}\, dx_{3t-1} + \alpha_{34,1}\, dx_{4t-1} + \varepsilon_{3t}$$

Under the null hypothesis,

$$LR = 2(L_u - L_r) \overset{asy}{\sim} \chi_1^2$$

```
> # Section 2.5.2.4-----Test whether bi-directional Granger-Causality exists between STELLAR and ETHEREUM
>
> # ----------Test the null hypothesis that dln_stellar does not Granger-cause dln_ethereum
> # unrestricted model for the test
> UR_7 <- dynlm(dln_ethereum ~ L(dln_electricity,1)+ L(dln_bitcoin,1) + L(dln_ethereum,1) + L(dln_cardano, 1)+ L(dln_stellar, 1)- 1)
> LU_7 = logLik(UR_7)
> LU_7
'log Lik.' 23.85053 (df=6)
>
> # restricted model for the test
> R_7 <- dynlm(dln_ethereum ~ L(dln_electricity,1)+ L(dln_bitcoin,1) + L(dln_ethereum,1) + L(dln_cardano, 1)- 1)
> LR_7 <- logLik(R_7)
> LR_7
'log Lik.' 21.45952 (df=5)
>
> # LR calc
> 2 * (LU_7 - LR_7)
'log Lik.' 4.782013 (df=6)
>
> # critiacal value
> qchisq(0.95, 1)
[1] 3.841459
>
> # Since 4.782 > 3.841 we reject the null and conclude that dln_stellar Granger-cause dln_ethereum
```

Since $LR_{\text{calc}} = 4.782 > LR_{crit,0.05} = 3.841$,we reject the null hypothesis at 5% significance level and conclude that stellar Granger-causes ethereum.

Test 8: Test the null hypothesis that ethereum does not Granger-cause stellar

The unrestricted model for the test is given by:

$$dx_{5t} = \alpha_{51,1}\, dx_{1t-1} + \alpha_{52,1}\, dx_{2t-1} + \alpha_{53,1}\, dx_{3t-1} + \alpha_{54,1}\, dx_{4t-1} + \alpha_{55,1}\, dx_{5t-1} + \varepsilon_{5t}$$

$$x_t' = (dx_{1t}, dx_{2t}, dx_{3t}, dx_{4t}, dx_{5t})'$$

Note that all variables are in log first-difference form

$$dx_{1t} = dlnelectricity_t\ ;\ dx_{2t} = dlnbitcoin_t\ ;\ dx_{3t} = dlnethereum_t$$

$$dx_{4t} = dlncardano_t;\ dx_{5t} = dlnstellar_t$$

The null and alternative hypothesis is that:

$H_0: \alpha_{53,1} = 0$ i.e. ethereum does not Granger-cause stellar

$H_1: \alpha_{53,1} \neq 0.$ i.e. ethereum Granger-causes stellar

Therefore, the restricted model for the test is (when H_0 true) :

$$dx_{5t} = \alpha_{51,1}\, dx_{1t-1} + \alpha_{52,1}\, dx_{2t-1} + \alpha_{54,1}\, dx_{4t-1} + \alpha_{55,1}\, dx_{5t-1} + \varepsilon_{5t}$$

Under the null hypothesis,

$$LR = 2(L_u - L_r) \overset{\text{asy}}{\sim} \chi_1^2$$

```
> # ----------Test the null hypothesis that dln_ethereum does not Granger-cause dln_stellar
> # unrestricted model for the test
> UR_8 <- dynlm(dln_stellar ~ L(dln_electricity,1)+ L(dln_bitcoin,1) + L(dln_ethereum,1) + L(dln_cardano, 1)+ L(dln_stellar, 1)- 1)
> LU_8 = logLik(UR_8)
> LU_8
'log Lik.' 11.32493 (df=6)
>
> # restricted model for the test
> R_8 <- dynlm(dln_stellar ~ L(dln_electricity,1)+ L(dln_bitcoin,1)  + L(dln_cardano, 1)+ L(dln_stellar, 1)- 1)
> LR_8 <- logLik(R_8)
> LR_8
'log Lik.' 3.764636 (df=5)
>
> # LR calc
> 2 * (LU_8 - LR_8)
'log Lik.' 15.12059 (df=6)
>
> # critiacal value
> qchisq(0.95, 1)
[1] 3.841459
>
> # Since 15.121 > 3.841 we reject the null and conclude that dln_ethereum Granger-cause dln_stellar
> # Overall,the bi-directional Granger-Causality exists between STELLAR and ETHEREUM
```

Since $LR_{\text{calc}} = 15.121 > LR_{crit,0.05} = 3.841$,we reject the null hypothesis at 5% significance level and conclude that ethereum Granger-causes stellar.

Overall, the bi-directional Granger-Causality exists between stellar and ethereum since stellar Granger-causes ethereum and, at the same time, ethereum Granger-causes stellar.

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```
> #Estimate VAR(1) model
> var1_part3 <- VAR(df, p = 1, type = "none")
> #Test for serial correlation in residuals
> serial.test(var1_part3, lags.pt = 16, type = "PT.asymptotic")

        Portmanteau Test (asymptotic)

data:  Residuals of VAR object var1_part3
Chi-squared = 466.32, df = 375, p-value = 0.0009024

> #Since p value = 0.0009 < 0.05 there is serial correaltions for VAR(1)
```

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```
> #Estimate VAR(2) model
> var2 <- VAR(df, p = 2, type = "none")
> #Test for serial correlation in residuals
> serial.test(var2, lags.pt = 16, type = "PT.asymptotic")

        Portmanteau Test (asymptotic)

data:  Residuals of VAR object var2
Chi-squared = 378.69, df = 350, p-value = 0.1399

> # since p value = 0.1399, the residuals from VAR(2) is vector white nosie
> # Since the VAR order is 2 then the the numer of lags for VECM is 1
```

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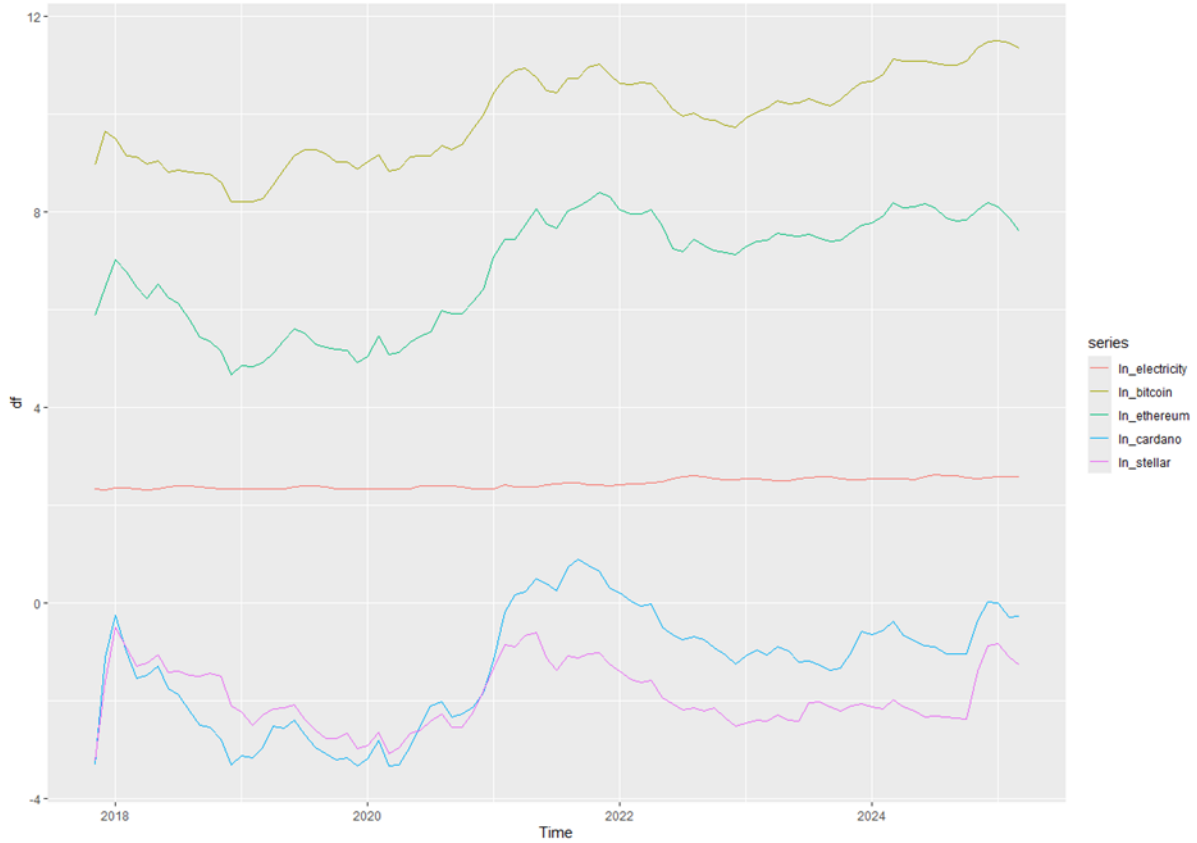
```
> summary(df)
 ln_electricity    ln_bitcoin       ln_ethereum      ln_cardano        ln_stellar
 Min.   :2.319   Min.   : 8.206   Min.   :4.675   Min.   :-3.3474   Min.   :-3.2306
 1st Qu.:2.342   1st Qu.: 9.129   1st Qu.:5.612   1st Qu.:-2.4088   1st Qu.:-2.3752
 Median :2.412   Median :10.021   Median :7.291   Median :-1.0658   Median :-2.1302
 Mean   :2.441   Mean   : 9.931   Mean   :6.830   Mean   :-1.3254   Mean   :-1.9360
 3rd Qu.:2.538   3rd Qu.:10.727   3rd Qu.:7.811   3rd Qu.:-0.5019   3rd Qu.:-1.4022
 Max.   :2.617   Max.   :11.513   Max.   :8.399   Max.   : 0.8984   Max.   :-0.5073
```



We can see that all 5 logged variables appear to have a non-zero mean

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- It can be shown that a VECM with deterministic terms can be generalized as

$$\underset{(mx1)}{dx_t} = \underset{(mx1)}{\mu_0} + \underset{(mx1)}{\mu_1 t} + \underset{(mxr)}{\alpha}\left(\underset{(rx1)}{\beta_0'} + \underset{(rx1)}{\beta_1' t} + \underset{(rx1)}{\beta' x_{t-1}}\right) + \sum_{j=1}^{p-1} \underset{(mxm)}{\Gamma_j}\,\underset{(mx1)}{dx_{t-j}} + \underset{(mx1)}{\varepsilon_t} \tag{5.67}$$

- The role of each of the deterministic terms in (5.67) is as follows:
  - μ_0 imparts a linear deterministic trend to x_t.
  - $\mu_1 t$ imparts a quadratic deterministic trend to x_t.
  - β_0 contributes a constant to each of the long run equilibrium relationships.
  - $\beta_1 t$ contributes a linear deterministic trend to each of the long run equilibrium relationships.

Case 2

$$\underset{(mx1)}{dx_t} = \underset{(mxr)}{\alpha}\left(\underset{(rx1)}{\beta_0'} + \underset{(rx1)}{\beta' x_{t-1}}\right) + \sum_{j=1}^{p-1} \underset{(mxm)}{\Gamma_j}\,\underset{(mx1)}{dx_{t-j}} + \underset{(mx1)}{\varepsilon_t}. \tag{M2}$$

- M2 allows only for intercepts in the long run equilibrium relationships and is obtained by imposing the restrictions

$$\mu_0 = \mu_1 = \beta_1 = 0.$$

- β_0' is referred to as a **restricted constant**.
- For M2, all the series in x_t are $I(1)$ **without a drift** (i.e. no series has a drift), and the cointegrating relations $\beta' x_{t-1}$ have **a non-zero mean**.

We summarize the characteristics of the data and the cointegrating relations implied by the 5 models above:

	x_t		
	Mean	**Trend**	**Cointegrating relation, $\beta' x_{t-1}$**
Case 1	Zero	None	Zero-mean
Case 2	Non-zero	None	Non-zero mean
Case 3	Non-zero	Linear	Non-zero mean
Case 4	Non-zero	Linear	Linear trend
Case 5	Non-zero	Quadratic	Linear trend

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```
> fit_vecm <- VECM(df, lag = 1, include = "none", estim = "ML", LRinclude = "const")
> summary(fit_vecm)
#############
###Model VECM
#############
Full sample size: 89    End sample size: 87
Number of variables: 5  Number of estimated slope parameters 30
AIC -2003.85    BIC -1920.009   SSR 13.72464
Cointegrating vector (estimated by ML):
   ln_electricity ln_bitcoin ln_ethereum ln_cardano ln_stellar    const
r1              1 0.06161652   -1.169444   1.289732  -0.764893 5.156314


                       ECT                ln_electricity -1   ln_bitcoin -1       ln_ethereum -1      ln_cardano -1
Equation ln_electricity -0.0127(0.0080)    0.3234(0.1036)**    -0.0191(0.0218)     0.0381(0.0192).     0.0108(0.0165)
Equation ln_bitcoin     0.0538(0.0529)     -0.1811(0.6863)     0.6987(0.1445)***   -0.1693(0.1269)     0.0328(0.1093)
Equation ln_ethereum    0.1834(0.0650)**   -1.9648(0.8431)*    0.5400(0.1775)**    -0.1891(0.1559)     0.2074(0.1342)
Equation ln_cardano     -0.0492(0.0917)    -1.6180(1.1892)     0.8157(0.2504)**    -0.4699(0.2199)*    0.5828(0.1893)**
Equation ln_stellar     0.0645(0.0784)     -1.6818(1.0161)     0.8780(0.2140)***   -0.7477(0.1879)***  0.2137(0.1618)
                       ln_stellar -1
Equation ln_electricity -0.0148(0.0184)
Equation ln_bitcoin     -0.2113(0.1217).
Equation ln_ethereum    -0.1217(0.1495)
Equation ln_cardano     -0.4380(0.2109)*
Equation ln_stellar     0.1336(0.1802)
```

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```
> # Estimate VECM aussuming a cointegration rank of 2
> fit_vecm2 <- VECM(df, lag = 1, r = 2, include = "none", estim = "ML", LRinclude = "const" )
> # Extract cointegrating vector
> fit_vecm2$model.specific$coint
                         r1        r2
ln_electricity  1.000000e+00   0.00000
ln_bitcoin      3.469447e-18   1.00000
ln_ethereum     3.237396e-01 -24.23350
ln_cardano     -5.221693e-01  29.40609
ln_stellar      4.695786e-01 -20.03475
const          -4.435273e+00 155.66582
> #Extract the cointegrating vectors separately
> beta1 <- fit_vecm2$model.specific$coint[,1] #1st cointegrating vector
> # Round to 3 decimal places
> round(beta1, 3)
ln_electricity     ln_bitcoin    ln_ethereum     ln_cardano     ln_stellar          const
         1.000          0.000          0.324         -0.522          0.470         -4.435
> beta2 <- fit_vecm2$model.specific$coint[,2] #2nd cointegrating vector
> # Round to 3 decimal places
> round(beta2, 3)
ln_electricity     ln_bitcoin    ln_ethereum     ln_cardano     ln_stellar          const
         0.000          1.000        -24.233         29.406        -20.035        155.666
```

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```
> fit_vecm2 <- VECM(df, lag = 1, r = 2, include = "none", estim = "ML", LRinclude = "const" )
> summary(fit_vecm2)
#############
###Model VECM
#############
Full sample size: 89    End sample size: 87
Number of variables: 5  Number of estimated slope parameters 35
AIC -2009.457   BIC -1908.355   SSR 13.01943
Cointegrating vector (estimated by ML):
   ln_electricity   ln_bitcoin ln_ethereum ln_cardano  ln_stellar      const
r1              1 3.469447e-18   0.3237396 -0.5221693   0.4695786  -4.435273
r2              0 1.000000e+00 -24.2334968 29.4060942 -20.0347499 155.665825


                       ECT1                ECT2                ln_electricity -1   ln_bitcoin -1
Equation ln_electricity -0.1713(0.0488)***  -0.0038(0.0010)***  0.3894(0.0999)***   -0.0170(0.0206)
Equation ln_bitcoin     -0.1787(0.3433)     -0.0011(0.0073)     -0.0842(0.7029)     0.7019(0.1451)***
Equation ln_ethereum    -0.5649(0.4144)     -0.0030(0.0088)     -1.6532(0.8485).    0.5502(0.1751)**
Equation ln_cardano     -1.1901(0.5824)*    -0.0248(0.0123)*    -1.1428(1.1926)     0.8312(0.2461)**
Equation ln_stellar     -1.2521(0.4874)*    -0.0212(0.0103)*    -1.1334(0.9982)     0.8959(0.2060)***
                       ln_ethereum -1      ln_cardano -1       ln_stellar -1
Equation ln_electricity 0.0183(0.0191)      0.0111(0.0156)      -0.0052(0.0176)
Equation ln_bitcoin     -0.1984(0.1342)     0.0331(0.1096)      -0.1972(0.1238)
Equation ln_ethereum    -0.2828(0.1620).    0.2085(0.1323)      -0.0764(0.1495)
Equation ln_cardano     -0.6128(0.2277)**   0.5845(0.1860)**    -0.3689(0.2101).
Equation ln_stellar     -0.9125(0.1906)***  0.2157(0.1557)      0.2133(0.1758)
```