DATA624-HW5-ExpoSmoothing

 $FPP-Hyndman\ exercises\ 7.1,\ 7.5,\ 7.6,\ 7.7,\ 7.8,\ 7.9$

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```
library(fable)
## Loading required package: fabletools
library(forecast)
## Registered S3 method overwritten by 'quantmod':
    method
                      from
##
    as.zoo.data.frame zoo
##
## Attaching package: 'forecast'
## The following objects are masked from 'package:fabletools':
##
       GeomForecast, StatForecast
##
library(fpp2)
## Loading required package: ggplot2
## Loading required package: fma
## Loading required package: expsmooth
library(kableExtra)
```

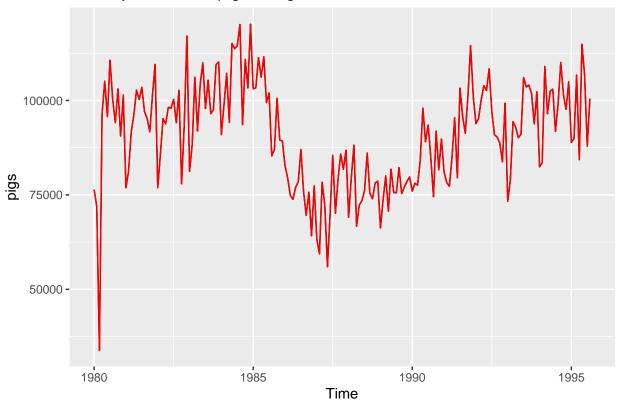
Homework 5 - Exponential Smoothing

Do exercises 7.1, 7.5, 7.6, 7.7, 7.8 and 7.9 in Hyndman. Please submit both your Rpubs link as well as attach the .rmd file with your code.

- 7.1 Consider the pigs series the number of pigs slaughtered in Victoria each month.
- a) Use the ses() function in R to find the optimal values of α and ℓ_0 , and generate forecasts for the next four months.

```
# Monthly total number of pigs slaughtered in Victoria, Australia (Jan 1980 - Aug 1995)
pigs.title <- "Monthly number of pigs slaughtered in Victoria, Australia"
autoplot(pigs) + ggtitle(pigs.title) + geom_line(color="red")
```

Monthly number of pigs slaughtered in Victoria, Australia



```
pigs.ses_forecast <- ses(pigs, h=4)
summary(pigs.ses_forecast)</pre>
```

```
##
## Forecast method: Simple exponential smoothing
##
## Model Information:
## Simple exponential smoothing
##
## Call:
## ses(y = pigs, h = 4)
##
## Smoothing parameters:
## alpha = 0.2971
```

```
##
##
     Initial states:
##
       1 = 77260.0561
##
##
     sigma: 10308.58
##
                 AICc
                           BIC
        AIC
## 4462.955 4463.086 4472.665
##
## Error measures:
                       ME
                             RMSE
                                        MAE
                                                   MPE
                                                           MAPE
                                                                      MASE
                                                                                  ACF1
## Training set 385.8721 10253.6 7961.383 -0.922652 9.274016 0.7966249 0.01282239
##
## Forecasts:
##
            Point Forecast
                                Lo 80
                                         Hi 80
                                                   Lo 95
                                                            Hi 95
## Sep 1995
                   98816.41 85605.43 112027.4 78611.97 119020.8
                   98816.41 85034.52 112598.3 77738.83 119894.0
## Oct 1995
## Nov 1995
                   98816.41 84486.34 113146.5 76900.46 120732.4
## Dec 1995
                   98816.41 83958.37 113674.4 76092.99 121539.8
pigs.params <- pigs.ses_forecast$model$fit$par</pre>
pigs.alpha <- pigs.params[1]</pre>
pigs.l_0 <-pigs.params[2]</pre>
```

The optimal value of α is 0.2971488 and the optimal value of ℓ_0 is 77260.0561459.

b) Compute a 95% prediction interval for the first forecast using $\hat{y} \pm 1.96s$ where s is the standard deviation of the residuals.

```
# Compute the first forecast, and the standard deviation
pigs.ses_stdev <- sd(pigs.ses_forecast$residuals)
pigs_ses_forecast_1 <- pigs.ses_forecast$mean[1]

# Compute the prediction interval
pigs.my_pred95 <- c(
    my.Lower.95 = pigs_ses_forecast_1 - 1.96 * pigs.ses_stdev,
    my.Upper.95 = pigs_ses_forecast_1 + 1.96 * pigs.ses_stdev
)
# 95% prediction interval for the first forecast - calculated
pigs.my_pred95

## my.Lower.95 my.Upper.95
## 78679.97 118952.84</pre>
```

```
pigs.R_pred95 <- c(
   R.Lower = pigs.ses_forecast$lower[1,"95%"],
   R.Upper = pigs.ses_forecast$upper[1,"95%"])
# 95% prediction interval for the first forecast - as produced by R
pigs.R_pred95</pre>
```

Compare your interval with the interval produced by R.

```
## R.Lower.95% R.Upper.95%
## 78611.97 119020.84
```

The interval computed by R is slightly wider than the interval computed manually:

```
## Lower.95 Upper.95
## pigs.my_pred95 78679.97 118952.8
## pigs.R_pred95 78611.97 119020.8
```

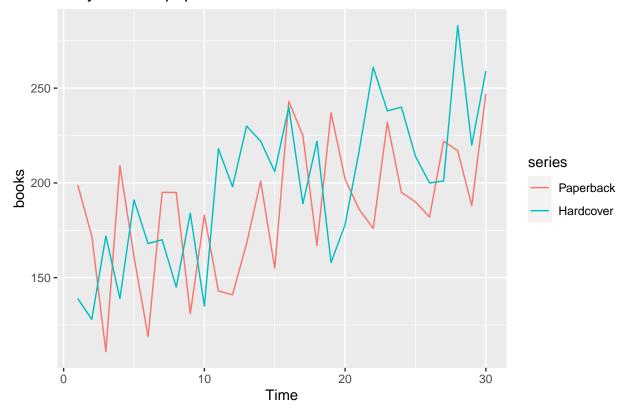
- 7.5 Data set books contains the daily sales of paperback and hardcover books at the same store. The task is to forecast the next four days' sales for paperback and hardcover books.
- a) Plot the series and discuss the main features of the data.

```
# Daily sales of paperback and hardcover books at the same store.
summary(books)
```

```
##
      Paperback
                       Hardcover
##
            :111.0
                     Min.
                             :128.0
    1st Qu.:167.2
##
                     1st Qu.:170.5
    Median :189.0
                     Median :200.5
    Mean
            :186.4
                     Mean
                             :198.8
    3rd Qu.:207.2
                     3rd Qu.:222.0
##
    Max.
            :247.0
                     Max.
                             :283.0
```

autoplot(books)+ggtitle("Daily sales of paperback and hardcover books at the same store")

Daily sales of paperback and hardcover books at the same store



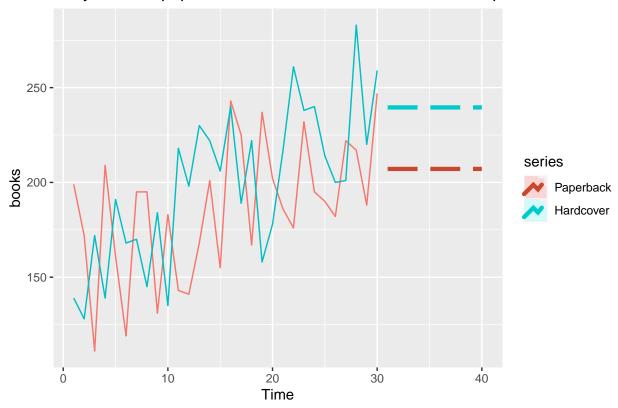
The dataset contains the daily sales of paperback and hardcover books over a period of 30 days. Both series exhibit similarly upward trends over the month, but no seasonality (e.g., during the course of each "week" is evident.

The dataset is not labeled in a way which could enable identification of days of the week (e.g., weekdays vs. weekends.))

b) Use the ses() function to forecast each series, and plot the forecasts.

```
# SES forecasts of books data
hardcover_forecast_ses <- ses(books[,"Hardcover"])
paperback_forecast_ses <- ses(books[,"Paperback"])
autoplot(books) + ggtitle("Daily sales of paperback and hardcover books, with SES predictions") +
   autolayer(hardcover_forecast_ses, series="Hardcover", PI=FALSE,size=1.5,linetype=5) +
   autolayer(paperback_forecast_ses, series="Paperback", PI=FALSE,size=1.5,linetype=5)</pre>
```

Daily sales of paperback and hardcover books, with SES predictions



c) Compute the RMSE values for the training data in each case.

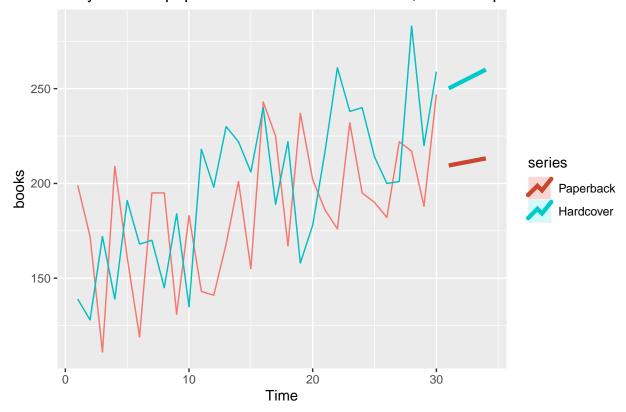
```
## Hardcover PaperBack
## 31.93101 33.63769
```

Under the SES model, the	hardcover RMSE is 31.93	31015 and the paperbac	ck RMSE is 33.6376868.

- 7.6 We will continue with the daily sales of paperback and hardcover books in data set books.
- a) Apply Holt's linear method to the paperback and hardback series and compute four-day forecasts in each case.

```
# Holt predictions for books
hardcover_forecast_holt <- holt(books[,"Hardcover"], h=4)
paperback_forecast_holt <- holt(books[,"Paperback"], h=4)
autoplot(books) + ggtitle("Daily sales of paperback and hardcover books, with Holt predictions") +
   autolayer(hardcover_forecast_holt, series="Hardcover", PI=FALSE,size=1.5,linetype=1) +
   autolayer(paperback_forecast_holt, series="Paperback", PI=FALSE,size=1.5,linetype=1)</pre>
```

Daily sales of paperback and hardcover books, with Holt predictions



b) Compare the RMSE measures of Holt's method for the two series to those of simple exponential smoothing in the previous question. (Remember that Holt's method is using one more parameter than SES.)

```
# Hardcover
hardcover_RMSE_holt <- sqrt(hardcover_forecast_holt$model$mse)
# Paperback
paperback_RMSE_holt <- sqrt(paperback_forecast_holt$model$mse)
# Holt RMSE</pre>
```

```
## Hardcover PaperBack
## 27.19358 31.13692
```

Under the HOLT model, the hardcover RMSE is 27.193578 and the paperback RMSE is 31.136923.

```
## Hardcover PaperBack
## books_RMSE_ses 31.93101 33.63769
## books_RMSE_holt 27.19358 31.13692
```

The RMSE for Holt is lower than that for SES.

Discuss the merits of the two forecasting methods for these data sets. The SES method provides a simple, straightline forecast, while the Holt forecasting method incorporates the trend, which is increasing.

c) Compare the forecasts for the two series using both methods. Which do you think is best?

```
## SES Holt
## [1,] 239.5601 250.1739
## [2,] 239.5601 253.4765
## [3,] 239.5601 256.7791
## [4,] 239.5601 260.0817
```

The Holt forecast appears better, as it incorporates the increasing trend, and is thus always greater than the SES forecast.

d) Calculate a 95% prediction interval for the first forecast for each series, using the RMSE values and assuming normal errors.

Compute hardcover Holt prediction intervals

Table 1: Hardcover Prediction Intervals (Holt

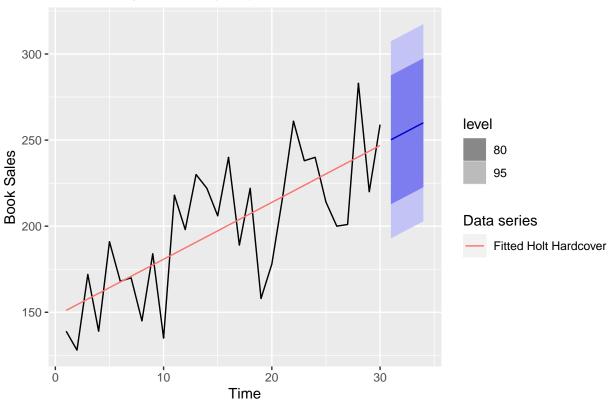
	Lower.95%	Upper. 95%
my_hardcover_holt_95	196.8745	303.4733
R_hardcover_holt_95	192.9222	307.4256

	Lower.95%	Upper.95%
my_hardcover_holt_95	196.8745	303.4733
R_hardcover_holt_95	192.9222	307.4256

Compare hardcover Holt prediction intervals

The interval calculated by R is wider.

Hardcover prediction (Holt)



Compute paperback Holt prediction intervals

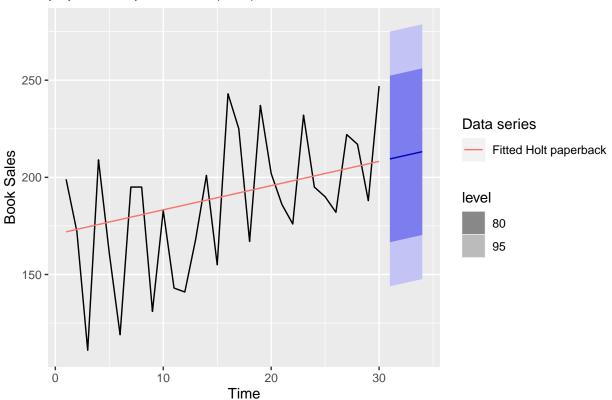
Compare paperback Holt prediction intervals

The interval calculated by R is wider.

Table 2: Paperback Prediction Intervals (Holt)

	Lower.95%	Upper.95%
my_paperback_holt_95	148.4384	270.4951
R_paperback_holt_95	143.9130	275.0205

paperback prediction (Holt)



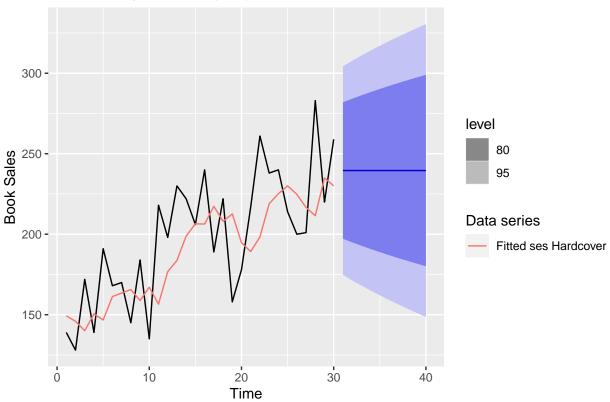
Compare your intervals with those produced using ses and holt. Compute hardcover SES prediction intervals

Table 3: Hardcover Prediction Intervals (SES)

	Lower.95%	Upper.95%
my_hardcover_ses_95	176.9753	302.1449
R_hardcover_ses_95	174.7799	304.3403

Compare hardcover SES prediction intervals

Hardcover prediction (ses)



Compute paperback SES prediction intervals

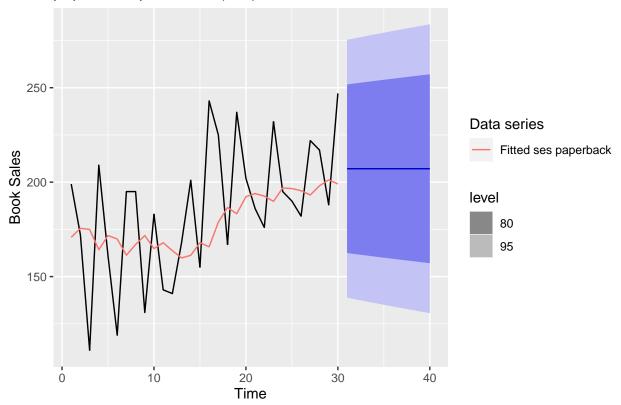
Compare paperback SES prediction intervals

The intervals computed by R are wider.

Table 4: Paperback Prediction Intervals (SES)

	Lower.95%	Upper.95%
my_paperback_ses_95	141.1798	273.0395
R_paperback_ses_95	138.8670	275.3523

paperback prediction (ses)



The intervals computed by R are wider than those computed using the RMSE.

7.7 For this exercise use data set eggs, the price of a dozen eggs in the United States from 1900–1993.

Experiment with the various options in the holt() function to see how much the forecasts change with damped trend, or with a Box-Cox transformation. Try to develop an intuition of what each argument is doing to the forecasts.

[Hint: use h=100 when calling holt() so you can clearly see the differences between the various options when plotting the forecasts.]

```
# Best model:
fc <- holt(eggs, h=100, lambda=0.04, damped=FALSE)
accuracy(fc)

## ME RMSE MAE MPE MAPE MASE
## Training set 0.7425725 26.35977 19.05123 -1.171722 9.687307 0.9397685
## ACF1
## Training set 0.02850751
```

```
indexes=1:1000
n=length(indexes)
lambda_grid = rep(0,n)
ME_grid = rep(0,n)
RMSE_grid = rep(0,n)
MAE_grid = rep(0,n)
MPE_grid = rep(0,n)
MAPE_grid = rep(0,n)
MASE_grid = rep(0,n)
ACF1_grid = rep(0,n)
mse grid = rep(0,n)
amse_grid = rep(0,n)
meanresid2 = rep(0,n)
sqrtmeanresid2 = rep(0,n)
for (i in indexes) {
  lambda_grid[i] = i/1000
  result = holt(eggs, h=100, lambda=lambda_grid[i], damped=FALSE)
  result.acc = accuracy(result)
  #print(c(lambda_grid[i],result.acc))
  ME_grid[i] = result.acc[1,"ME"]
  RMSE_grid[i] = result.acc[1,"RMSE"]
  MAE_grid[i] = result.acc[1,"MAE"]
  MPE_grid[i] = result.acc[1,"MPE"]
  MAPE_grid[i] = result.acc[1,"MAPE"]
  MASE_grid[i] = result.acc[1,"MASE"]
  ACF1_grid[i] = result.acc[1,"ACF1"]
  mse_grid[i] = result$model$mse
  amse grid[i] = result$model$amse
  meanresid2[i] = mean(result$residuals^2)
  sqrtmeanresid2[i] = sqrt(mean(result$residuals^2))
```

```
}
biggrid <- cbind(lambda=lambda_grid,</pre>
     ME=ME_grid,
      RMSE=RMSE_grid,
     MAE=MAE_grid,
     MPE=MPE_grid,
     MAPE=MAPE_grid,
     MASE=MASE_grid,
      ACF1=ACF1_grid,
      mse=mse_grid,
      amse=amse_grid,
      meanresid2=meanresid2,
      sqrtmeanresid2=sqrtmeanresid2)
minRMSE = min(RMSE_grid)
whichlambda = which(RMSE_grid==minRMSE)
print(paste("The minimum RMSE = ", minRMSE," occurs when lambda = ", lambda_grid[whichlambda]))
Which model gives the best RMSE?
## [1] "The minimum RMSE = 26.3597707115837 occurs when lambda = 0.04"
The minimum RMSE = 26.3597707 occurs when lambda = 0.04
```

7.8 Recall your retail time series data (from Exercise 3 in Section 2.10).

a) Why is multiplicative seasonality necessary for this series?

```
mycode <- "A3349396W"

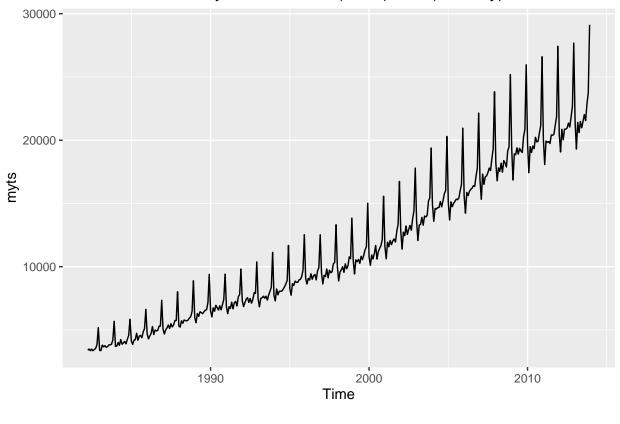
mytitle <- "Monthly Turnover; Total(State); Total(Industry)"

mymain <- paste(mycode, mytitle)

myts <- readxl::read_excel("retail.xlsx", skip=1)[,mycode] %>%

   ts(frequency=12, start=c(1982,4))
autoplot(myts, main=mymain)
```

A3349396W Monthly Turnover; Total(State); Total(Industry)



The seasonal variation increases with the level of the series. Therefore, we need to use multiplicative seasonality.

b) Apply Holt-Winters' multiplicative method to the data.

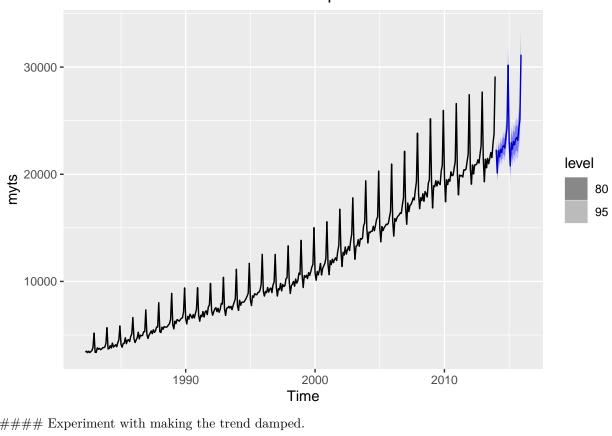
```
HoltWintersMult1 <- hw(myts, seasonal='multiplicative', damped=FALSE)
HoltWintersMult1$mean</pre>
```

```
##
             Jan
                      Feb
                               Mar
                                         Apr
                                                  May
                                                           Jun
                                                                     Jul
                                                                              Aug
## 2014 22297.15 20124.85 22208.45 21632.11 22323.80 22023.66 22553.45 22687.24
## 2015 23039.88 20793.37 22944.16 22346.76 23059.27 22747.26 23292.44 23428.59
##
                      Oct
                                        Dec
             Sep
                               Nov
```

```
## 2014 22440.53 23527.65 24298.79 30185.18
## 2015 23171.84 24292.31 25086.38 31160.95
```

autoplot(HoltWintersMult1)

Forecasts from Holt-Winters' multiplicative method

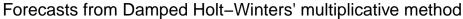


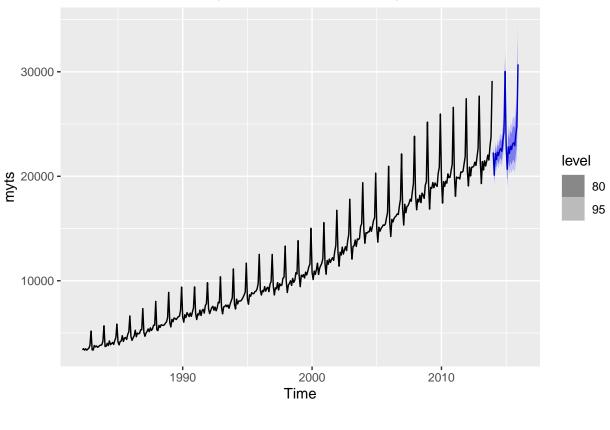
Experiment with making the trend damped.

HoltWintersMultDamped <- hw(myts, seasonal='multiplicative', damped=T)</pre> HoltWintersMultDamped\$mean

```
Feb
                               Mar
                                         Apr
                                                           Jun
                                                                    Jul
                                                  May
                                                                              Aug
## 2014 22276.11 20108.13 22190.12 21606.22 22296.49 21992.63 22515.73 22649.45
## 2015 22918.44 20674.87 22801.46 22188.11 22883.52 22558.73 23082.38 23206.80
##
             Sep
                      Oct
                               Nov
## 2014 22383.95 23452.83 24209.06 30039.58
## 2015 22922.55 24004.66 24766.10 30715.56
```

autoplot(HoltWintersMultDamped)





c) Compare the RMSE of the one-step forecasts from the two methods. Which do you prefer?

```
print("Holt-Winters Multiplicative (not damped):")
## [1] "Holt-Winters Multiplicative (not damped):"
accuracy(HoltWintersMult1)
##
                      ME
                             RMSE
                                        MAE
                                                   MPE
                                                           MAPE
                                                                      MASE
## Training set 8.069135 221.5921 173.1308 0.05953008 1.726817 0.2859523
##
## Training set -0.1631927
print("Holt-Winters Multiplicative Damped:")
## [1] "Holt-Winters Multiplicative Damped:"
accuracy(HoltWintersMultDamped)
##
                      ME
                              RMSE
                                        MAE
                                                  MPE
                                                          MAPE
                                                                     MASE
                                                                                ACF1
```

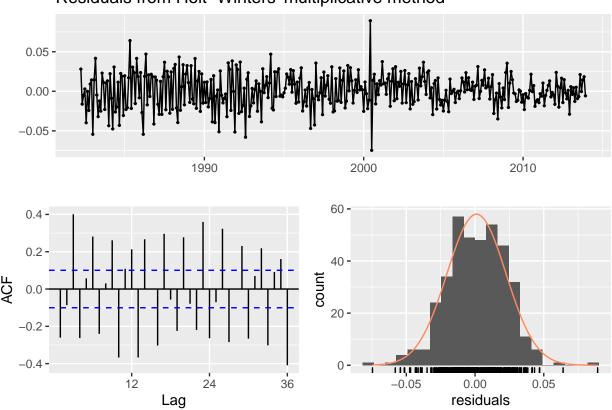
The RMSE is slightly lower on the non-damped model, which I would prefer as the trend is upward.

Training set 30.12733 224.6438 175.4635 0.2717607 1.762675 0.2898051 -0.1748363

d) Check that the residuals from the best method look like white noise.

checkresiduals(HoltWintersMult1)

Residuals from Holt-Winters' multiplicative method



```
##
## Ljung-Box test
##
## data: Residuals from Holt-Winters' multiplicative method
## Q* = 584.69, df = 8, p-value < 0.00000000000000022
##
## Model df: 16. Total lags used: 24</pre>
```

There are significant correlations in the residuals.

There appears to be a quarterly pattern, as sales may be affected by whether one is in the beginning or the end of each quarter, with substantial reversion.

Therefore, these residuals do not look like white noise.

e) Now find the test set RMSE, while training the model to the end of 2010.

```
myts %>%
window(end=c(2010,12)) %>%
```

```
hw(seasonal='multiplicative', damped=FALSE) -> myresults
```

```
Point Forecast
                              Lo 80
                                       Hi 80
                                                 Lo 95
## Jan 2011
                  20349.84 19754.77 20944.92 19439.75 21259.94
## Feb 2011
                  18202.63 17648.89 18756.37 17355.76 19049.50
## Mar 2011
                  20151.23 19514.46 20787.99 19177.38 21125.07
## Apr 2011
                  19756.25 19108.67 20403.82 18765.87 20746.62
## May 2011
                  20332.60 19642.14 21023.05 19276.64 21388.56
## Jun 2011
                  20020.07 19316.57 20723.56 18944.17 21095.97
## Jul 2011
                  20609.79 19861.19 21358.39 19464.90 21754.68
## Aug 2011
                  20447.17 19680.24 21214.10 19274.25 21620.09
## Sep 2011
                  20381.22 19592.56 21169.88 19175.07 21587.37
## Oct 2011
                  21440.46 20585.29 22295.62 20132.60 22748.32
## Nov 2011
                  22050.76 21144.95 22956.57 20665.45 23436.08
## Dec 2011
                  27761.87 26588.26 28935.47 25966.99 29556.74
## Jan 2012
                  21127.55 20167.35 22087.76 19659.05 22596.06
## Feb 2012
                  18896.07 18015.53 19776.61 17549.41 20242.74
## Mar 2012
                  20916.48 19917.60 21915.36 19388.83 22444.14
                  20504.14 19501.12 21507.16 18970.16 22038.12
## Apr 2012
## May 2012
                  21099.90 20043.11 22156.69 19483.67 22716.12
## Jun 2012
                  20773.21 19708.42 21838.00 19144.75 22401.66
## Jul 2012
                  21382.70 20261.48 22503.91 19667.94 23097.45
## Aug 2012
                  21211.60 20074.26 22348.93 19472.19 22951.00
## Sep 2012
                  21140.82 19982.16 22299.48 19368.81 22912.84
## Oct 2012
                  22237.06 20991.80 23482.33 20332.59 24141.53
                  22867.52 21559.56 24175.48 20867.17 24867.88
## Nov 2012
## Dec 2012
                  28787.01 27105.85 30468.17 26215.90 31358.12
```

accuracy(myresults,x=myts)

```
## Training set 10.07947 215.6308 168.7072 0.06360609 1.805918 0.2804746
## Test set -222.58896 353.0499 271.9102 -1.00551589 1.249486 0.4520490
## Training set -0.1743335 NA
## Test set -0.1375517 0.1647206
```

Can you beat the seasonal naïve approach from Exercise 8 in Section 3.7? The test set RMSE is 353.0499 compared to 1389.337 for the seasonal naïve method (Homework 2, final problem.) So, on this dataset, the Holt-Winters method is much better that the seasonal naïve method.

7.9 For the same retail data, try an STL decomposition applied to the Box-Cox transformed series, followed by ETS on the seasonally adjusted data.

```
myts %>%
  window(end=c(2010,12)) %>%
  stlf(lambda=0) -> mySTLdecomposition
mySTLdecomposition
```

```
##
            Point Forecast
                              Lo 80
                                       Hi 80
                                                Lo 95
                                                         Hi 95
                 20313.41 19798.92 20841.27 19531.86 21126.24
## Jan 2011
## Feb 2011
                 18256.07 17778.98 18745.96 17531.49 19010.59
## Mar 2011
                 20155.53 19611.66 20714.49 19329.72 21016.63
## Apr 2011
                 19762.16 19211.22 20328.89 18925.82 20635.45
## May 2011
                 20324.80 19739.14 20927.83 19435.98 21254.26
## Jun 2011
                 20045.40 19448.20 20660.93 19139.30 20994.39
## Jul 2011
                 20590.38 19956.01 21244.91 19628.15 21599.78
## Aug 2011
                 20523.25 19869.30 21198.71 19531.60 21565.23
## Sep 2011
                 20469.28 19794.72 21166.82 19446.68 21545.65
## Oct 2011
                 21547.18 20812.84 22307.43 20434.29 22720.68
## Nov 2011
                 22162.94 21381.91 22972.49 20979.66 23412.95
## Dec 2011
                 27832.05 26818.06 28884.38 26296.33 29457.46
## Jan 2012
                 21099.11 20304.61 21924.70 19896.21 22374.74
## Feb 2012
                 18962.19 18224.39 19729.86 17845.51 20148.74
## Mar 2012
                 20935.12 20093.71 21811.77 19662.07 22290.60
## Apr 2012
                 20526.53 19674.65 21415.29 19238.10 21901.25
## May 2012
                 21110.93 20206.57 22055.76 19743.63 22572.93
## Jun 2012
                 20820.72 19900.42 21783.59 19429.83 22311.19
## Jul 2012
                 21386.79 20411.79 22408.37 19913.78 22968.76
## Aug 2012
                 21317.06 20315.13 22368.39 19803.95 22945.77
## Sep 2012
                 21261.00 20231.20 22343.23 19706.40 22938.25
## Oct 2012
                 22380.60 21263.93 23555.91 20695.54 24202.86
## Nov 2012
                  23020.17 21837.53 24266.86 21236.27 24953.92
## Dec 2012
                  28908.56 27380.01 30522.44 26603.84 31412.94
```

```
accuracy(mySTLdecomposition,x=myts)
```

```
## Training set -0.07696934 NA

ME RMSE MAE MPE MAPE MAPE MASE

## Training set -0.02333538 0.1839252
```

How does that compare with your best previous forecasts on the test set?

Here the RMSE is 390.4325, which is worse than the 353.0499 obtained from Holt-Winters.