Structural Equation Modeling with Latent Variables using R

Muhammad Yaseen

The consists of a that specifies the relation of observed to latent variables and a that shows the influence of latent variables on each other.

The first component of the structural equations is the latent variable model:

In , (eta), the vector of latent endogenous random variables, is ; (xi) the latent exogenous random variables, is ; is the coefficient matrix showing the influence of the latent endogenous variables on each other; (Gamma) is the coefficient matrix for the effects of on . The matrix is non-singular. (zeta) is the disturbance vector that is assumed to have an expected value of zero and which is uncorrelated with .

The second component of the general system is the measurement model:

The and the vectors are observed variables, (Lambda) and are the coefficient matrices that show the relation of to and to respectively, and (epsilon) and (delta) are the errors of measurement for and , respectively. The errors of measurement are assumed to be uncorrelated with , and and with each other.

Also (Phi), (Psi), (Theta), (Theta) and (Sigma).

**Latent Variables:** Variables of Interest But Not Directly Measureable

**Common in Sciences:** Intelligence, Worker Productivity, Diseases, Happiness, Value of House, Carrying Capacity, “Free” Market, Disturbance Variables, etc

Each Model

Unique values for parameters?

If then

# Linear Model

## Introduction

The model relating the normal dependent variable with the explanatory variables is

![Figure 1 Path Diagram of Linear Model](data:application/pdf;base64,)

Figure 1 Path Diagram of Linear Model

## Linear Modeling Approach

Linear Model (LM) can only measure direct effects

![Figure 2 Path Diagram of Regression](data:application/pdf;base64,)

Figure 2 Path Diagram of Regression

load("Income.RData")  
# Income  
str(Income)

'data.frame': 25 obs. of 5 variables:  
 $ Income : num 40.3 39.6 40.8 40.3 40 38.1 40.4 40.7 40.8 37.1 ...  
 $ Value : int 180 121 161 161 179 99 114 202 184 90 ...  
 $ Education : int 14 15 14 14 14 14 15 14 13 14 ...  
 $ Age : int 53 49 44 39 53 46 42 49 37 43 ...  
 $ MortgagePayment: int 230 370 397 181 387 304 285 551 370 135 ...

Income.lm.fm1 <-  
 lm(  
 formula = Income ~ Value + Education + Age +  
 MortgagePayment  
 , data = Income  
 # , subset  
 # , weights  
 # , na.action  
 , method = "qr"  
 , model = TRUE  
 , x = FALSE  
 , y = FALSE  
 , qr = TRUE  
 , singular.ok = TRUE  
 , contrasts = NULL  
 # , offset  
 # , ...  
 )  
  
summary(Income.lm.fm1)

Call:  
lm(formula = Income ~ Value + Education + Age + MortgagePayment,   
 data = Income, method = "qr", model = TRUE, x = FALSE, y = FALSE,   
 qr = TRUE, singular.ok = TRUE, contrasts = NULL)  
  
Residuals:  
 Min 1Q Median 3Q Max   
-1.05615 -0.37792 -0.05208 0.47685 1.20372   
  
Coefficients:  
 Estimate Std. Error t value Pr(>|t|)   
(Intercept) 28.438187 3.442601 8.261 7.08e-08 \*\*\*  
Value 0.032302 0.005712 5.656 1.55e-05 \*\*\*  
Education 0.607859 0.277795 2.188 0.0407 \*   
Age -0.037207 0.035780 -1.040 0.3108   
MortgagePayment -0.001345 0.001449 -0.928 0.3644   
---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
  
Residual standard error: 0.685 on 20 degrees of freedom  
Multiple R-squared: 0.6463, Adjusted R-squared: 0.5756   
F-statistic: 9.137 on 4 and 20 DF, p-value: 0.0002286

summary(Income.lm.fm1)$coef

Estimate Std. Error t value Pr(>|t|)  
(Intercept) 28.438187019 3.442600555 8.2606700 7.077863e-08  
Value 0.032301977 0.005711513 5.6555905 1.553422e-05  
Education 0.607858710 0.277794920 2.1881563 4.069801e-02  
Age -0.037207429 0.035780364 -1.0398840 3.108026e-01  
MortgagePayment -0.001345319 0.001449331 -0.9282344 3.643539e-01

![Figure 3 Path Diagram of Regression with estimates of coefficient obtained from ](data:application/pdf;base64,)

Figure 3 Path Diagram of Regression with estimates of coefficient obtained from

## Strucural Equation Modeling Approach

Strucural Equation Model (SEM) can measure direct as well as indirect effects

![Figure 4 Path Diagram of Regression](data:application/pdf;base64,)

Figure 4 Path Diagram of Regression

library(lavaan)  
  
Income.sem.m1 <- '  
 # Regressions  
 Income ~ gamma1 \* Value + gamma2 \* Education +  
 gamma3 \* Age + gamma4 \* MortgagePayment  
'  
  
Income.sem.fm1 <-  
 lavaan::sem(  
 model = Income.sem.m1  
 , data = Income  
 , ordered = NULL  
 , sampling.weights = NULL  
 , sample.cov = NULL  
 , sample.mean = NULL  
 , sample.nobs = NULL  
 , group = NULL  
 , cluster = NULL  
 # , constraints = ""  
 , WLS.V = NULL  
 , NACOV = NULL  
 # , ...  
 )  
  
# anova(Income.sem.fm1)  
# coef(Income.sem.fm1)  
parameterEstimates(Income.sem.fm1, standardized = TRUE)

lhs op rhs label est se  
1 Income ~ Value gamma1 0.032 0.005  
2 Income ~ Education gamma2 0.608 0.248  
3 Income ~ Age gamma3 -0.037 0.032  
4 Income ~ MortgagePayment gamma4 -0.001 0.001  
5 Income ~~ Income 0.375 0.106  
6 Value ~~ Value 773.526 0.000  
7 Value ~~ Education -2.619 0.000  
8 Value ~~ Age 30.408 0.000  
9 Value ~~ MortgagePayment 1087.181 0.000  
10 Education ~~ Education 0.458 0.000  
11 Education ~~ Age 2.216 0.000  
12 Education ~~ MortgagePayment -14.822 0.000  
13 Age ~~ Age 27.840 0.000  
14 Age ~~ MortgagePayment -18.584 0.000  
15 MortgagePayment ~~ MortgagePayment 10739.898 0.000  
 z pvalue ci.lower ci.upper std.lv std.all std.nox  
1 6.323 0.000 0.022 0.042 0.032 0.872 0.031  
2 2.446 0.014 0.121 1.095 0.608 0.399 0.590  
3 -1.163 0.245 -0.100 0.026 -0.037 -0.191 -0.036  
4 -1.038 0.299 -0.004 0.001 -0.001 -0.135 -0.001  
5 3.536 0.000 0.167 0.583 0.375 0.354 0.354  
6 NA NA 773.526 773.526 773.526 1.000 773.526  
7 NA NA -2.619 -2.619 -2.619 -0.139 -2.619  
8 NA NA 30.408 30.408 30.408 0.207 30.408  
9 NA NA 1087.181 1087.181 1087.181 0.377 1087.181  
10 NA NA 0.458 0.458 0.458 1.000 0.458  
11 NA NA 2.216 2.216 2.216 0.621 2.216  
12 NA NA -14.822 -14.822 -14.822 -0.211 -14.822  
13 NA NA 27.840 27.840 27.840 1.000 27.840  
14 NA NA -18.584 -18.584 -18.584 -0.034 -18.584  
15 NA NA 10739.898 10739.898 10739.898 1.000 10739.898

# fitmeasures(Income.sem.fm1)  
fitted(Income.sem.fm1)$cov

Income Value Eductn Age   
Income 1.061   
Value 20.800 773.526   
Education 0.131 -2.619 0.458   
Age 1.318 30.408 2.216 27.840  
MortgagePayment 12.351 1087.181 -14.822 -18.584  
 MrtggP   
Income   
Value   
Education   
Age   
MortgagePayment 10739.898

# residuals(Income.sem.fm1, type = "cor")  
# modificationIndices(Income.sem.fm1)  
var(Income)

Income Value Education Age  
Income 1.105433 21.667000 0.1365000 1.373333  
Value 21.667000 805.756667 -2.7283333 31.675000  
Education 0.136500 -2.728333 0.4766667 2.308333  
Age 1.373333 31.675000 2.3083333 29.000000  
MortgagePayment 12.865667 1132.480000 -15.4400000 -19.358333  
 MortgagePayment  
Income 12.86567  
Value 1132.48000  
Education -15.44000  
Age -19.35833  
MortgagePayment 11187.39333

cor(Income)

Income Value Education Age  
Income 1.0000000 0.7259899 0.1880438 0.24255525  
Value 0.7259899 1.0000000 -0.1392157 0.20721237  
Education 0.1880438 -0.1392157 1.0000000 0.62085779  
Age 0.2425553 0.2072124 0.6208578 1.00000000  
MortgagePayment 0.1156915 0.3771934 -0.2114343 -0.03398635  
 MortgagePayment  
Income 0.11569153  
Value 0.37719344  
Education -0.21143430  
Age -0.03398635  
MortgagePayment 1.00000000

# vcov(Income.sem.fm1)  
  
  
summary(  
 object = Income.sem.fm1  
 , header = TRUE  
 , fit.measures = FALSE  
 , estimates = TRUE  
 , ci = FALSE  
 , fmi = FALSE  
 , standardized = TRUE  
 , rsquare = TRUE  
 , std.nox = FALSE  
 , modindices = FALSE  
 , nd = 3L  
)

lavaan 0.6-3 ended normally after 20 iterations  
  
 Optimization method NLMINB  
 Number of free parameters 5  
  
 Number of observations 25  
  
 Estimator ML  
 Model Fit Test Statistic 0.000  
 Degrees of freedom 0  
  
Parameter Estimates:  
  
 Information Expected  
 Information saturated (h1) model Structured  
 Standard Errors Standard  
  
Regressions:  
 Estimate Std.Err z-value P(>|z|)  
 Income ~   
 Value (gmm1) 0.032 0.005 6.323 0.000  
 Educatn (gmm2) 0.608 0.248 2.446 0.014  
 Age (gmm3) -0.037 0.032 -1.163 0.245  
 MrtggPy (gmm4) -0.001 0.001 -1.038 0.299  
 Std.lv Std.all  
   
 0.032 0.872  
 0.608 0.399  
 -0.037 -0.191  
 -0.001 -0.135  
  
Variances:  
 Estimate Std.Err z-value P(>|z|)  
 .Income 0.375 0.106 3.536 0.000  
 Std.lv Std.all  
 0.375 0.354  
  
R-Square:  
 Estimate   
 Income 0.646

![Figure 5 Path Diagram of Regression with estimates of coefficient obtained from ](data:application/pdf;base64,)

Figure 5 Path Diagram of Regression with estimates of coefficient obtained from

![Figure 6 Path Diagram of Regression](data:application/pdf;base64,)

Figure 6 Path Diagram of Regression

library(lavaan)  
Income.sem.m2 <- '  
 # Regressions  
 Income ~ gamma1 \* Value + gamma2 \* Education +  
 gamma3 \* Age + gamma4 \* MortgagePayment  
  
 # Variances and Covariances  
 Value ~~ r12 \* Education  
 Value ~~ r13 \* Age  
 Value ~~ r14 \* MortgagePayment  
 Education ~~ r23 \* Age  
 Education ~~ r24 \* MortgagePayment  
 Age ~~ r34 \* MortgagePayment  
 Income ~~ sigma2I \* Income  
'  
  
Income.sem.fm2 <-  
 lavaan::sem(  
 model = Income.sem.m2  
 , data = Income  
 , ordered = NULL  
 , sampling.weights = NULL  
 , sample.cov = NULL  
 , sample.mean = NULL  
 , sample.nobs = NULL  
 , group = NULL  
 , cluster = NULL  
 # , constraints = ""  
 , WLS.V = NULL  
 , NACOV = NULL  
 # , ...  
 )  
  
# anova(Income.sem.fm2)  
# coef(Income.sem.fm2)  
parameterEstimates(Income.sem.fm2, standardized = TRUE)

lhs op rhs label est se  
1 Income ~ Value gamma1 0.032 0.005  
2 Income ~ Education gamma2 0.608 0.248  
3 Income ~ Age gamma3 -0.037 0.032  
4 Income ~ MortgagePayment gamma4 -0.001 0.001  
5 Value ~~ Education r12 -2.619 3.799  
6 Value ~~ Age r13 30.408 29.973  
7 Value ~~ MortgagePayment r14 1087.181 616.102  
8 Education ~~ Age r23 2.216 0.840  
9 Education ~~ MortgagePayment r24 -14.822 14.331  
10 Age ~~ MortgagePayment r34 -18.584 109.425  
11 Income ~~ Income sigma2I 0.375 0.106  
12 Value ~~ Value 773.526 218.786  
13 Education ~~ Education 0.458 0.129  
14 Age ~~ Age 27.840 7.874  
15 MortgagePayment ~~ MortgagePayment 10739.898 3037.702  
 z pvalue ci.lower ci.upper std.lv std.all std.nox  
1 6.323 0.000 0.022 0.042 0.032 0.872 0.872  
2 2.446 0.014 0.121 1.095 0.608 0.399 0.399  
3 -1.163 0.245 -0.100 0.026 -0.037 -0.191 -0.191  
4 -1.038 0.299 -0.004 0.001 -0.001 -0.135 -0.135  
5 -0.689 0.491 -10.065 4.827 -2.619 -0.139 -0.139  
6 1.015 0.310 -28.338 89.154 30.408 0.207 0.207  
7 1.765 0.078 -120.358 2294.719 1087.181 0.377 0.377  
8 2.637 0.008 0.569 3.863 2.216 0.621 0.621  
9 -1.034 0.301 -42.910 13.265 -14.822 -0.211 -0.211  
10 -0.170 0.865 -233.052 195.884 -18.584 -0.034 -0.034  
11 3.536 0.000 0.167 0.583 0.375 0.354 0.354  
12 3.536 0.000 344.713 1202.340 773.526 1.000 1.000  
13 3.536 0.000 0.204 0.711 0.458 1.000 1.000  
14 3.536 0.000 12.407 43.273 27.840 1.000 1.000  
15 3.536 0.000 4786.112 16693.684 10739.898 1.000 1.000

# fitmeasures(Income.sem.fm2)  
fitted(Income.sem.fm2)$cov

Income Value Eductn Age   
Income 1.061   
Value 20.800 773.526   
Education 0.131 -2.619 0.458   
Age 1.318 30.408 2.216 27.840  
MortgagePayment 12.351 1087.181 -14.822 -18.584  
 MrtggP   
Income   
Value   
Education   
Age   
MortgagePayment 10739.898

# residuals(Income.sem.fm2, type = "cor")  
# modificationIndices(Income.sem.fm2)  
var(Income)

Income Value Education Age  
Income 1.105433 21.667000 0.1365000 1.373333  
Value 21.667000 805.756667 -2.7283333 31.675000  
Education 0.136500 -2.728333 0.4766667 2.308333  
Age 1.373333 31.675000 2.3083333 29.000000  
MortgagePayment 12.865667 1132.480000 -15.4400000 -19.358333  
 MortgagePayment  
Income 12.86567  
Value 1132.48000  
Education -15.44000  
Age -19.35833  
MortgagePayment 11187.39333

cor(Income)

Income Value Education Age  
Income 1.0000000 0.7259899 0.1880438 0.24255525  
Value 0.7259899 1.0000000 -0.1392157 0.20721237  
Education 0.1880438 -0.1392157 1.0000000 0.62085779  
Age 0.2425553 0.2072124 0.6208578 1.00000000  
MortgagePayment 0.1156915 0.3771934 -0.2114343 -0.03398635  
 MortgagePayment  
Income 0.11569153  
Value 0.37719344  
Education -0.21143430  
Age -0.03398635  
MortgagePayment 1.00000000

# vcov(Income.sem.fm2)  
  
  
  
summary(  
 object = Income.sem.fm2  
 , header = TRUE  
 , fit.measures = TRUE  
 , estimates = TRUE  
 , ci = FALSE  
 , fmi = FALSE  
 , standardized = TRUE  
 , rsquare = TRUE  
 , std.nox = FALSE  
 , modindices = FALSE  
 , nd = 3L  
)

lavaan 0.6-3 ended normally after 114 iterations  
  
 Optimization method NLMINB  
 Number of free parameters 15  
  
 Number of observations 25  
  
 Estimator ML  
 Model Fit Test Statistic 0.000  
 Degrees of freedom 0  
 Minimum Function Value 0.0000000000000  
  
Model test baseline model:  
  
 Minimum Function Test Statistic 47.107  
 Degrees of freedom 10  
 P-value 0.000  
  
User model versus baseline model:  
  
 Comparative Fit Index (CFI) 1.000  
 Tucker-Lewis Index (TLI) 1.000  
  
Loglikelihood and Information Criteria:  
  
 Loglikelihood user model (H0) -385.524  
 Loglikelihood unrestricted model (H1) -385.524  
  
 Number of free parameters 15  
 Akaike (AIC) 801.048  
 Bayesian (BIC) 819.331  
 Sample-size adjusted Bayesian (BIC) 772.815  
  
Root Mean Square Error of Approximation:  
  
 RMSEA 0.000  
 90 Percent Confidence Interval 0.000 0.000  
 P-value RMSEA <= 0.05 NA  
  
Standardized Root Mean Square Residual:  
  
 SRMR 0.000  
  
Parameter Estimates:  
  
 Information Expected  
 Information saturated (h1) model Structured  
 Standard Errors Standard  
  
Regressions:  
 Estimate Std.Err z-value P(>|z|)  
 Income ~   
 Value (gmm1) 0.032 0.005 6.323 0.000  
 Educatn (gmm2) 0.608 0.248 2.446 0.014  
 Age (gmm3) -0.037 0.032 -1.163 0.245  
 MrtggPy (gmm4) -0.001 0.001 -1.038 0.299  
 Std.lv Std.all  
   
 0.032 0.872  
 0.608 0.399  
 -0.037 -0.191  
 -0.001 -0.135  
  
Covariances:  
 Estimate Std.Err z-value P(>|z|)  
 Value ~~   
 Educatin (r12) -2.619 3.799 -0.689 0.491  
 Age (r13) 30.408 29.973 1.015 0.310  
 MrtggPym (r14) 1087.181 616.102 1.765 0.078  
 Education ~~   
 Age (r23) 2.216 0.840 2.637 0.008  
 MrtggPym (r24) -14.822 14.331 -1.034 0.301  
 Age ~~   
 MrtggPym (r34) -18.584 109.425 -0.170 0.865  
 Std.lv Std.all  
   
 -2.619 -0.139  
 30.408 0.207  
 1087.181 0.377  
   
 2.216 0.621  
 -14.822 -0.211  
   
 -18.584 -0.034  
  
Variances:  
 Estimate Std.Err z-value P(>|z|)  
 .Income (sg2I) 0.375 0.106 3.536 0.000  
 Value 773.526 218.786 3.536 0.000  
 Educatn 0.458 0.129 3.536 0.000  
 Age 27.840 7.874 3.536 0.000  
 MrtggPy 10739.898 3037.702 3.536 0.000  
 Std.lv Std.all  
 0.375 0.354  
 773.526 1.000  
 0.458 1.000  
 27.840 1.000  
 10739.898 1.000  
  
R-Square:  
 Estimate   
 Income 0.646

![Figure 7 Path Diagram of Regression with estimates of coefficient](data:application/pdf;base64,)

Figure 7 Path Diagram of Regression with estimates of coefficient

# Multivariate Linear Model

## Introduction

The model relating the normal dependent variables with the explanatory variables is

In matrix form the multivariate linear model may be written as

Therefore, the multivariate linear model for the -th observation is

The and the vectors are observed variables, (Gamma) is the coefficient matrix for the effects of on . (epsilon) is the disturbance vector that is assumed to have an expected value of zero .

![Figure 8 Path Diagram for Multivariate Linear Model](data:application/pdf;base64,)

Figure 8 Path Diagram for Multivariate Linear Model

# Structural Equation Modeling with Observed Variables

## Introduction

The general model of structural equations with observed variables is

In matrix form the general model of structural equations with observed variables may be written as

Therefore, the general model of structural equations with observed variables for the -th observation is

The and the vectors are observed variables, is the coefficient matrix showing the influence of the endogenous variables on each other; (Gamma) is the coefficient matrix for the effects of on . The matrix is non-singular. (zeta) is the disturbance vector that is assumed to have an expected value of zero .

![Figure 9 Path Diagram for Structural Equations Model with Observed Variables](data:application/pdf;base64,)

Figure 9 Path Diagram for Structural Equations Model with Observed Variables

![Figure 10 Path Diagram with Indirecet Effects](data:application/pdf;base64,)

Figure 10 Path Diagram with Indirecet Effects

load("Income.RData")  
# Income  
str(Income)

'data.frame': 25 obs. of 5 variables:  
 $ Income : num 40.3 39.6 40.8 40.3 40 38.1 40.4 40.7 40.8 37.1 ...  
 $ Value : int 180 121 161 161 179 99 114 202 184 90 ...  
 $ Education : int 14 15 14 14 14 14 15 14 13 14 ...  
 $ Age : int 53 49 44 39 53 46 42 49 37 43 ...  
 $ MortgagePayment: int 230 370 397 181 387 304 285 551 370 135 ...

library(lavaan)  
  
Income.sem.m3 <- '  
 # Regressions  
 Income ~ beta12 \* Education + gamma11 \* Value +  
 gamma12 \* MortgagePayment + gamma13 \* Age  
 Education ~ gamma22 \* Age  
  
 # Variances and Covariances  
 Value ~~ r12 \* MortgagePayment  
 Value ~~ r13 \* Age  
 Value ~~ r14 \* Education  
 MortgagePayment ~~ r23 \* Age  
 MortgagePayment ~~ r24 \*Education  
 Education ~~ sigma2E \* Education  
 Income ~~ sigma2I \* Income  
  
# Indirect Effects  
 IndEf := gamma22\*beta12  
  
# Total Effects (Direct + Indirect Effets)  
 TotEf := gamma13 + (gamma22\*beta12)  
'  
  
Income.sem.fm3 <-  
 lavaan::sem(  
 model = Income.sem.m3  
 , data = Income  
 , ordered = NULL  
 , sampling.weights = NULL  
 , sample.cov = NULL  
 , sample.mean = NULL  
 , sample.nobs = NULL  
 , group = NULL  
 , cluster = NULL  
 # , constraints = ""  
 , WLS.V = NULL  
 , NACOV = NULL  
 # , ...  
 )  
  
# anova(Income.sem.fm3)  
# coef(Income.sem.fm3)  
parameterEstimates(Income.sem.fm3, standardized = TRUE)

lhs op rhs label est  
1 Income ~ Education beta12 0.608  
2 Income ~ Value gamma11 0.032  
3 Income ~ MortgagePayment gamma12 -0.001  
4 Income ~ Age gamma13 -0.037  
5 Education ~ Age gamma22 0.080  
6 Value ~~ MortgagePayment r12 1087.181  
7 Value ~~ Age r13 30.408  
8 Education ~~ Value r14 -5.040  
9 MortgagePayment ~~ Age r23 -18.584  
10 Education ~~ MortgagePayment r24 -13.343  
11 Education ~~ Education sigma2E 0.281  
12 Income ~~ Income sigma2I 0.375  
13 Value ~~ Value 773.526  
14 MortgagePayment ~~ MortgagePayment 10739.898  
15 Age ~~ Age 27.840  
16 IndEf := gamma22\*beta12 IndEf 0.048  
17 TotEf := gamma13+(gamma22\*beta12) TotEf 0.011  
 se z pvalue ci.lower ci.upper std.lv std.all  
1 0.248 2.446 0.014 0.121 1.095 0.608 0.399  
2 0.005 6.323 0.000 0.022 0.042 0.032 0.872  
3 0.001 -1.038 0.299 -0.004 0.001 -0.001 -0.135  
4 0.032 -1.163 0.245 -0.100 0.026 -0.037 -0.191  
5 0.020 3.960 0.000 0.040 0.119 0.080 0.621  
6 616.102 1.765 0.078 -120.358 2294.719 1087.181 0.377  
7 29.973 1.015 0.310 -28.338 89.154 30.408 0.207  
8 3.057 -1.649 0.099 -11.031 0.951 -5.040 -0.342  
9 109.425 -0.170 0.865 -233.052 195.884 -18.584 -0.034  
10 11.304 -1.180 0.238 -35.499 8.813 -13.343 -0.243  
11 0.080 3.536 0.000 0.125 0.437 0.281 0.615  
12 0.106 3.536 0.000 0.167 0.583 0.375 0.354  
13 218.786 3.536 0.000 344.713 1202.340 773.526 1.000  
14 3037.702 3.536 0.000 4786.111 16693.684 10739.898 1.000  
15 7.874 3.536 0.000 12.407 43.273 27.840 1.000  
16 0.023 2.081 0.037 0.003 0.094 0.048 0.248  
17 0.027 0.415 0.678 -0.042 0.064 0.011 0.057  
 std.nox  
1 0.399  
2 0.872  
3 -0.135  
4 -0.191  
5 0.621  
6 0.377  
7 0.207  
8 -0.342  
9 -0.034  
10 -0.243  
11 0.615  
12 0.354  
13 1.000  
14 1.000  
15 1.000  
16 0.248  
17 0.057

# fitmeasures(Income.sem.fm3)  
fitted(Income.sem.fm3)$cov

Income Eductn Value MrtggP   
Income 1.061   
Education 0.131 0.458   
Value 20.800 -2.619 773.526   
MortgagePayment 12.351 -14.822 1087.181 10739.898  
Age 1.318 2.216 30.408 -18.584  
 Age   
Income   
Education   
Value   
MortgagePayment   
Age 27.840

# residuals(Income.sem.fm3, type = "cor")  
# modificationIndices(Income.sem.fm3)  
var(Income)

Income Value Education Age  
Income 1.105433 21.667000 0.1365000 1.373333  
Value 21.667000 805.756667 -2.7283333 31.675000  
Education 0.136500 -2.728333 0.4766667 2.308333  
Age 1.373333 31.675000 2.3083333 29.000000  
MortgagePayment 12.865667 1132.480000 -15.4400000 -19.358333  
 MortgagePayment  
Income 12.86567  
Value 1132.48000  
Education -15.44000  
Age -19.35833  
MortgagePayment 11187.39333

# vcov(Income.sem.fm3)  
  
  
summary(  
 object = Income.sem.fm3  
 , header = TRUE  
 , fit.measures = FALSE  
 , estimates = TRUE  
 , ci = FALSE  
 , fmi = FALSE  
 , standardized = TRUE  
 , rsquare = TRUE  
 , std.nox = FALSE  
 , modindices = FALSE  
 , nd = 3L  
)

lavaan 0.6-3 ended normally after 112 iterations  
  
 Optimization method NLMINB  
 Number of free parameters 15  
  
 Number of observations 25  
  
 Estimator ML  
 Model Fit Test Statistic 0.000  
 Degrees of freedom 0  
 Minimum Function Value 0.0000000000000  
  
Parameter Estimates:  
  
 Information Expected  
 Information saturated (h1) model Structured  
 Standard Errors Standard  
  
Regressions:  
 Estimate Std.Err z-value P(>|z|)  
 Income ~   
 Educatn (bt12) 0.608 0.248 2.446 0.014  
 Value (gm11) 0.032 0.005 6.323 0.000  
 MrtggPy (gm12) -0.001 0.001 -1.038 0.299  
 Age (gm13) -0.037 0.032 -1.163 0.245  
 Education ~   
 Age (gm22) 0.080 0.020 3.960 0.000  
 Std.lv Std.all  
   
 0.608 0.399  
 0.032 0.872  
 -0.001 -0.135  
 -0.037 -0.191  
   
 0.080 0.621  
  
Covariances:  
 Estimate Std.Err z-value P(>|z|)  
 Value ~~   
 MrtggPym (r12) 1087.181 616.102 1.765 0.078  
 Age (r13) 30.408 29.973 1.015 0.310  
 .Education ~~   
 Value (r14) -5.040 3.057 -1.649 0.099  
 MortgagePayment ~~   
 Age (r23) -18.584 109.425 -0.170 0.865  
 .Education ~~   
 MrtggPym (r24) -13.343 11.304 -1.180 0.238  
 Std.lv Std.all  
   
 1087.181 0.377  
 30.408 0.207  
   
 -5.040 -0.342  
   
 -18.584 -0.034  
   
 -13.343 -0.243  
  
Variances:  
 Estimate Std.Err z-value P(>|z|)  
 .Educatn (sg2E) 0.281 0.080 3.536 0.000  
 .Income (sg2I) 0.375 0.106 3.536 0.000  
 Value 773.526 218.786 3.536 0.000  
 MrtggPy 10739.898 3037.702 3.536 0.000  
 Age 27.840 7.874 3.536 0.000  
 Std.lv Std.all  
 0.281 0.615  
 0.375 0.354  
 773.526 1.000  
 10739.898 1.000  
 27.840 1.000  
  
R-Square:  
 Estimate   
 Education 0.385  
 Income 0.646  
  
Defined Parameters:  
 Estimate Std.Err z-value P(>|z|)  
 IndEf 0.048 0.023 2.081 0.037  
 TotEf 0.011 0.027 0.415 0.678  
 Std.lv Std.all  
 0.048 0.248  
 0.011 0.057

![Figure 11 Path Diagram with estimates of coefficient including Indirecet Effects](data:application/pdf;base64,)

Figure 11 Path Diagram with estimates of coefficient including Indirecet Effects

![Figure 12 Path Diagram for job performance of farm managers](data:application/pdf;base64,)

Figure 12 Path Diagram for job performance of farm managers

library(lavaan)  
  
Warren5V <-  
 matrix(  
 data =  
 c(  
 0.0209, 0.0177, 0.0245, 0.0046,  
 0.0177, 0.0520, 0.0280, 0.0044,  
 0.0245, 0.0280, 0.1212, -0.0063,  
 0.0046, 0.0044, -0.0063, 0.0901  
 )  
 , nrow = 4  
 , ncol = 4  
 , byrow = TRUE  
 , dimnames = list(c("Performance", "Knowledge",  
 "Value", "Satisfaction")  
 , c("Performance", "Knowledge",  
 "Value", "Satisfaction"))  
 )  
  
Warren5V

Performance Knowledge Value Satisfaction  
Performance 0.0209 0.0177 0.0245 0.0046  
Knowledge 0.0177 0.0520 0.0280 0.0044  
Value 0.0245 0.0280 0.1212 -0.0063  
Satisfaction 0.0046 0.0044 -0.0063 0.0901

Warren5V.sem.m1 <- '  
 # Regressions  
 Performance ~ gamma11 \* Knowledge + gamma12 \* Value +  
 gamma13 \* Satisfaction  
  
 # Variances and Covariances  
 Knowledge ~~ r12 \* Value  
 Knowledge ~~ r13 \* Satisfaction  
 Value ~~ r23 \* Satisfaction  
 Performance ~~ sigma2P \* Performance  
'  
  
Warren5V.sem.fm1 <-  
 lavaan::sem(  
 model = Warren5V.sem.m1  
 # , data  
 , ordered = NULL  
 , sampling.weights = NULL  
 , sample.cov = Warren5V  
 , sample.mean = NULL  
 , sample.nobs = 98  
 , group = NULL  
 , cluster = NULL  
 # , constraints = ""  
 , WLS.V = NULL  
 , NACOV = NULL  
 # , ...  
 )  
  
# methods(class = class(Warren5V.sem.fm1))  
# anova(Warren5V.sem.fm1)  
# coef(Warren5V.sem.fm1)  
parameterEstimates(Warren5V.sem.fm1, standardized = TRUE)

lhs op rhs label est se z  
1 Performance ~ Knowledge gamma11 0.258 0.053 4.847  
2 Performance ~ Value gamma12 0.145 0.035 4.158  
3 Performance ~ Satisfaction gamma13 0.049 0.038 1.281  
4 Knowledge ~~ Value r12 0.028 0.008 3.293  
5 Knowledge ~~ Satisfaction r13 0.004 0.007 0.635  
6 Value ~~ Satisfaction r23 -0.006 0.010 -0.596  
7 Performance ~~ Performance sigma2P 0.012 0.002 7.000  
8 Knowledge ~~ Knowledge 0.051 0.007 7.000  
9 Value ~~ Value 0.120 0.017 7.000  
10 Satisfaction ~~ Satisfaction 0.089 0.013 7.000  
 pvalue ci.lower ci.upper std.lv std.all std.nox  
1 0.000 0.154 0.363 0.258 0.407 0.407  
2 0.000 0.077 0.213 0.145 0.349 0.349  
3 0.200 -0.026 0.123 0.049 0.101 0.101  
4 0.001 0.011 0.044 0.028 0.353 0.353  
5 0.525 -0.009 0.018 0.004 0.064 0.064  
6 0.551 -0.027 0.014 -0.006 -0.060 -0.060  
7 0.000 0.009 0.016 0.012 0.601 0.601  
8 0.000 0.037 0.066 0.051 1.000 1.000  
9 0.000 0.086 0.154 0.120 1.000 1.000  
10 0.000 0.064 0.114 0.089 1.000 1.000

# fitmeasures(Warren5V.sem.fm1)  
fitted(Warren5V.sem.fm1)$cov

Prfrmn Knwldg Value Stsfct  
Performance 0.021   
Knowledge 0.018 0.051   
Value 0.024 0.028 0.120   
Satisfaction 0.005 0.004 -0.006 0.089

# residuals(Warren5V.sem.fm1, type = "cor")  
# modificationIndices(Warren5V.sem.fm1)  
Warren5V

Performance Knowledge Value Satisfaction  
Performance 0.0209 0.0177 0.0245 0.0046  
Knowledge 0.0177 0.0520 0.0280 0.0044  
Value 0.0245 0.0280 0.1212 -0.0063  
Satisfaction 0.0046 0.0044 -0.0063 0.0901

# vcov(Warren5V.sem.fm1)  
  
  
summary(  
 object = Warren5V.sem.fm1  
 , header = TRUE  
 , fit.measures = FALSE  
 , estimates = TRUE  
 , ci = FALSE  
 , fmi = FALSE  
 , standardized = TRUE  
 , rsquare = TRUE  
 , std.nox = FALSE  
 , modindices = FALSE  
 , nd = 3L  
)

lavaan 0.6-3 ended normally after 46 iterations  
  
 Optimization method NLMINB  
 Number of free parameters 10  
  
 Number of observations 98  
  
 Estimator ML  
 Model Fit Test Statistic 0.000  
 Degrees of freedom 0  
  
Parameter Estimates:  
  
 Information Expected  
 Information saturated (h1) model Structured  
 Standard Errors Standard  
  
Regressions:  
 Estimate Std.Err z-value P(>|z|) Std.lv  
 Performance ~   
 Knowldg (gm11) 0.258 0.053 4.847 0.000 0.258  
 Value (gm12) 0.145 0.035 4.158 0.000 0.145  
 Stsfctn (gm13) 0.049 0.038 1.281 0.200 0.049  
 Std.all  
   
 0.407  
 0.349  
 0.101  
  
Covariances:  
 Estimate Std.Err z-value P(>|z|) Std.lv  
 Knowledge ~~   
 Value (r12) 0.028 0.008 3.293 0.001 0.028  
 Satsfctn (r13) 0.004 0.007 0.635 0.525 0.004  
 Value ~~   
 Satsfctn (r23) -0.006 0.010 -0.596 0.551 -0.006  
 Std.all  
   
 0.353  
 0.064  
   
 -0.060  
  
Variances:  
 Estimate Std.Err z-value P(>|z|) Std.lv  
 .Prfrmnc (sg2P) 0.012 0.002 7.000 0.000 0.012  
 Knowldg 0.051 0.007 7.000 0.000 0.051  
 Value 0.120 0.017 7.000 0.000 0.120  
 Stsfctn 0.089 0.013 7.000 0.000 0.089  
 Std.all  
 0.601  
 1.000  
 1.000  
 1.000  
  
R-Square:  
 Estimate  
 Performance 0.399

![Figure 13 Path Diagram with estimates of coefficient for job performance of farm managers](data:application/pdf;base64,)

Figure 13 Path Diagram with estimates of coefficient for job performance of farm managers

# Factor Analysis

## Introduction

The general model for confirmatory factor analysis is

The and the vectors are observed variables, (Lambda) and are the coefficient matrices that show the relation of to and to respectively, and (epsilon) and (delta) are the errors of measurement for and , respectively. The errors of measurement are assumed to be uncorrelated with and and with each other.

![Figure 14 Path Diagram for Aptitude](data:application/pdf;base64,)

Figure 14 Path Diagram for Aptitude

![Figure 15 Path Diagram for Holzinger and Swineford (1939)](data:application/pdf;base64,)

Figure 15 Path Diagram for Holzinger and Swineford (1939)

library(lavaan)  
  
load("HSF.RData")  
# HSF  
str(HSF)

'data.frame': 73 obs. of 6 variables:  
 $ VisualPerc: num 33 30 36 28 30 20 17 33 30 36 ...  
 $ Cubes : num 22 25 33 25 25 25 21 31 22 28 ...  
 $ Lozenges : num 17 20 36 9 11 6 6 30 20 22 ...  
 $ Paragrap : num 8 10 17 10 11 9 5 11 8 13 ...  
 $ Sentence : num 17 23 25 18 21 21 10 23 17 24 ...  
 $ WordMean : num 10 18 41 11 8 16 10 18 20 36 ...

HSF.sem.m1 <- '  
 # Latent Variables  
 Spatial =~ lambda1 \* VisualPerc + lambda2 \* Cubes +  
 lambda3 \* Lozenges  
 Verbal =~ lambda4 \* Paragrap + lambda5 \* Sentence +  
 lambda6 \* WordMean  
  
 # Variances and Covariances  
 Spatial ~~ r12 \* Verbal  
'  
  
HSF.sem.fm1 <-  
 lavaan::sem(  
 model = HSF.sem.m1  
 , data = HSF  
 , ordered = NULL  
 , sampling.weights = NULL  
 , sample.cov = NULL  
 , sample.mean = NULL  
 , sample.nobs = NULL  
 , group = NULL  
 , cluster = NULL  
 # , constraints = ""  
 , WLS.V = NULL  
 , NACOV = NULL  
 # , ...  
 )  
  
# methods(class = class(HSF.sem.fm1))  
# anova(HSF.sem.fm1)  
# coef(HSF.sem.fm1)  
parameterEstimates(HSF.sem.fm1, standardized = TRUE)

lhs op rhs label est se z pvalue  
1 Spatial =~ VisualPerc lambda1 1.000 0.000 NA NA  
2 Spatial =~ Cubes lambda2 0.610 0.142 4.279 0.000  
3 Spatial =~ Lozenges lambda3 1.198 0.270 4.436 0.000  
4 Verbal =~ Paragrap lambda4 1.000 0.000 NA NA  
5 Verbal =~ Sentence lambda5 1.334 0.159 8.379 0.000  
6 Verbal =~ WordMean lambda6 2.234 0.262 8.541 0.000  
7 Spatial ~~ Verbal r12 7.315 2.553 2.865 0.004  
8 VisualPerc ~~ VisualPerc 23.873 5.945 4.016 0.000  
9 Cubes ~~ Cubes 11.602 2.566 4.521 0.000  
10 Lozenges ~~ Lozenges 28.275 7.837 3.608 0.000  
11 Paragrap ~~ Paragrap 2.834 0.863 3.286 0.001  
12 Sentence ~~ Sentence 7.967 1.856 4.292 0.000  
13 WordMean ~~ WordMean 19.925 4.917 4.052 0.000  
14 Spatial ~~ Spatial 23.302 8.068 2.888 0.004  
15 Verbal ~~ Verbal 9.682 2.144 4.516 0.000  
 ci.lower ci.upper std.lv std.all std.nox  
1 1.000 1.000 4.827 0.703 0.703  
2 0.330 0.889 2.943 0.654 0.654  
3 0.669 1.728 5.784 0.736 0.736  
4 1.000 1.000 3.112 0.880 0.880  
5 1.022 1.646 4.151 0.827 0.827  
6 1.722 2.747 6.952 0.841 0.841  
7 2.311 12.319 0.487 0.487 0.487  
8 12.221 35.525 23.873 0.506 0.506  
9 6.572 16.631 11.602 0.572 0.572  
10 12.914 43.636 28.275 0.458 0.458  
11 1.143 4.524 2.834 0.226 0.226  
12 4.329 11.605 7.967 0.316 0.316  
13 10.288 29.563 19.925 0.292 0.292  
14 7.490 39.114 1.000 1.000 1.000  
15 5.481 13.884 1.000 1.000 1.000

# fitmeasures(HSF.sem.fm1)  
fitted(HSF.sem.fm1)$cov

VslPrc Cubes Lozngs Pargrp Sentnc WordMn  
VisualPerc 47.175   
Cubes 14.209 20.265   
Lozenges 27.919 17.024 61.726   
Paragrap 7.315 4.461 8.765 12.516   
Sentence 9.759 5.950 11.692 12.916 25.197   
WordMean 16.344 9.966 19.583 21.633 28.859 68.260

# residuals(HSF.sem.fm1, type = "cor")  
# modificationIndices(HSF.sem.fm1)  
var(HSF)

VisualPerc Cubes Lozenges Paragrap Sentence  
VisualPerc 47.829909 15.137938 26.899734 8.450723 12.820396  
Cubes 15.137938 20.546804 17.658105 3.402207 4.092085  
Lozenges 26.899734 17.658105 62.583714 9.181507 13.411339  
Paragrap 8.450723 3.402207 9.181507 12.689878 13.042237  
Sentence 12.820396 4.092085 13.411339 13.042237 25.546804  
WordMean 13.217846 6.934741 24.280061 22.019597 29.245814  
 WordMean  
VisualPerc 13.217846  
Cubes 6.934741  
Lozenges 24.280061  
Paragrap 22.019597  
Sentence 29.245814  
WordMean 69.208143

# vcov(HSF.sem.fm1)  
  
  
summary(  
 object = HSF.sem.fm1  
 , header = TRUE  
 , fit.measures = FALSE  
 , estimates = TRUE  
 , ci = FALSE  
 , fmi = FALSE  
 , standardized = TRUE  
 , rsquare = TRUE  
 , std.nox = FALSE  
 , modindices = FALSE  
 , nd = 3L  
)

lavaan 0.6-3 ended normally after 64 iterations  
  
 Optimization method NLMINB  
 Number of free parameters 13  
  
 Number of observations 73  
  
 Estimator ML  
 Model Fit Test Statistic 7.962  
 Degrees of freedom 8  
 P-value (Chi-square) 0.437  
  
Parameter Estimates:  
  
 Information Expected  
 Information saturated (h1) model Structured  
 Standard Errors Standard  
  
Latent Variables:  
 Estimate Std.Err z-value P(>|z|) Std.lv  
 Spatial =~   
 VislPrc (lmb1) 1.000 4.827  
 Cubes (lmb2) 0.610 0.142 4.279 0.000 2.943  
 Lozengs (lmb3) 1.198 0.270 4.436 0.000 5.784  
 Verbal =~   
 Paragrp (lmb4) 1.000 3.112  
 Sentenc (lmb5) 1.334 0.159 8.379 0.000 4.151  
 WordMen (lmb6) 2.234 0.262 8.541 0.000 6.952  
 Std.all  
   
 0.703  
 0.654  
 0.736  
   
 0.880  
 0.827  
 0.841  
  
Covariances:  
 Estimate Std.Err z-value P(>|z|) Std.lv  
 Spatial ~~   
 Verbal (r12) 7.315 2.553 2.865 0.004 0.487  
 Std.all  
   
 0.487  
  
Variances:  
 Estimate Std.Err z-value P(>|z|) Std.lv  
 .VisualPerc 23.873 5.945 4.016 0.000 23.873  
 .Cubes 11.602 2.566 4.521 0.000 11.602  
 .Lozenges 28.275 7.837 3.608 0.000 28.275  
 .Paragrap 2.834 0.863 3.286 0.001 2.834  
 .Sentence 7.967 1.856 4.292 0.000 7.967  
 .WordMean 19.925 4.917 4.052 0.000 19.925  
 Spatial 23.302 8.068 2.888 0.004 1.000  
 Verbal 9.682 2.144 4.516 0.000 1.000  
 Std.all  
 0.506  
 0.572  
 0.458  
 0.226  
 0.316  
 0.292  
 1.000  
 1.000  
  
R-Square:  
 Estimate  
 VisualPerc 0.494  
 Cubes 0.428  
 Lozenges 0.542  
 Paragrap 0.774  
 Sentence 0.684  
 WordMean 0.708

![Figure 16 Path Diagram with estimates of coefficients](data:application/pdf;base64,)

Figure 16 Path Diagram with estimates of coefficients

# Strucural Equation Modeling with Latent Variables

## Introduction

The consists of a that specifies the relation of observed to latent variables and a that shows the influence of latent variables on each other.

The first component of the structural equations is the latent variable model:

In , (eta), the vector of latent endogenous random variables, is ; (xi) the latent exogenous random variables, is ; is the coefficient matrix showing the influence of the latent endogenous variables on each other; (Gamma) is the coefficient matrix for the effects of on . The matrix is non-singular. (zeta) is the disturbance vector that is assumed to have an expected value of zero and which is uncorrelated with .

The second component of the general system is the measurement model:

The and the vectors are observed variables, (Lambda) and are the coefficient matrices that show the relation of to and to respectively, and (epsilon) and (delta) are the errors of measurement for and , respectively. The errors of measurement are assumed to be uncorrelated with , and and with each other.

Also (Phi), (Psi), (Theta), (Theta) and (Sigma).

![Figure 17 Path Diagram for job performance of farm managers](data:application/pdf;base64,)

Figure 17 Path Diagram for job performance of farm managers

library(lavaan)  
  
Warren8V <-  
 matrix(  
 data =  
c(  
0.0271, 0.0172, 0.0219, 0.0164, 0.0284, 0.0217, 0.0083, 0.0074,  
0.0172, 0.0222, 0.0193, 0.0130, 0.0294, 0.0185, 0.0011, 0.0015,  
0.0219, 0.0193, 0.0876, 0.0317, 0.0383, 0.0356, -0.0001, 0.0035,  
0.0164, 0.0130, 0.0317, 0.0568, 0.0151, 0.0230, 0.0055, 0.0089,  
0.0284, 0.0294, 0.0383, 0.0151, 0.1826, 0.0774, -0.0087, -0.0007,  
0.0217, 0.0185, 0.0356, 0.0230, 0.0774, 0.1473, -0.0069, -0.0088,  
0.0083, 0.0011, -0.0001, 0.0055, -0.0087, -0.0069, 0.1137, 0.0722,  
0.0074, 0.0015, 0.0035, 0.0089, -0.0007, -0.0088, 0.0722, 0.1024  
)  
 , nrow = 8  
 , ncol = 8  
 , byrow = TRUE  
 , dimnames = list(c("Performance1", "Performance2",  
 "Knowledge1", "Knowledge2",  
 "Value1", "Value2",  
 "Satisfaction1", "Satisfaction2")  
 , c("Performance1", "Performance2",  
 "Knowledge1", "Knowledge2",  
 "Value1", "Value2",  
 "Satisfaction1", "Satisfaction2")  
 )  
  
 )  
  
Warren8V

Performance1 Performance2 Knowledge1 Knowledge2  
Performance1 0.0271 0.0172 0.0219 0.0164  
Performance2 0.0172 0.0222 0.0193 0.0130  
Knowledge1 0.0219 0.0193 0.0876 0.0317  
Knowledge2 0.0164 0.0130 0.0317 0.0568  
Value1 0.0284 0.0294 0.0383 0.0151  
Value2 0.0217 0.0185 0.0356 0.0230  
Satisfaction1 0.0083 0.0011 -0.0001 0.0055  
Satisfaction2 0.0074 0.0015 0.0035 0.0089  
 Value1 Value2 Satisfaction1 Satisfaction2  
Performance1 0.0284 0.0217 0.0083 0.0074  
Performance2 0.0294 0.0185 0.0011 0.0015  
Knowledge1 0.0383 0.0356 -0.0001 0.0035  
Knowledge2 0.0151 0.0230 0.0055 0.0089  
Value1 0.1826 0.0774 -0.0087 -0.0007  
Value2 0.0774 0.1473 -0.0069 -0.0088  
Satisfaction1 -0.0087 -0.0069 0.1137 0.0722  
Satisfaction2 -0.0007 -0.0088 0.0722 0.1024

Warren8V.sem.m1 <- '  
 # Latent Variables  
 Performance =~ lambda1Y \* Performance1 + lambda2Y \* Performance2  
 Knowledge =~ lambda1X \* Knowledge1 + lambda2X \* Knowledge2  
 Value =~ lambda3X \* Value1 + lambda4X \* Value2  
 Satisfaction =~ lambda5X \* Satisfaction1 + lambda6X \* Satisfaction2  
  
 # Regressions  
 Performance ~ gamma11 \* Knowledge + gamma12 \*Value +  
 gamma13 \*Satisfaction  
  
 # Variances and Covariances  
 Knowledge ~~ r12 \* Value  
 Knowledge ~~ r13 \* Satisfaction  
 Value ~~ r23 \* Satisfaction  
 Performance ~~ sigma2P \* Performance  
'  
  
Warren8V.sem.fm1 <-  
 lavaan::sem(  
 model = Warren8V.sem.m1  
 # , data  
 , ordered = NULL  
 , sampling.weights = NULL  
 , sample.cov = Warren8V  
 , sample.mean = NULL  
 , sample.nobs = 98  
 , group = NULL  
 , cluster = NULL  
 # , constraints = ""  
 , WLS.V = NULL  
 , NACOV = NULL  
 # , ...  
 )  
  
# methods(class = class(Warren8V.sem.fm1))  
# anova(Warren8V.sem.fm1)  
# coef(Warren8V.sem.fm1)  
parameterEstimates(Warren8V.sem.fm1, standardized = TRUE)

lhs op rhs label est se z  
1 Performance =~ Performance1 lambda1Y 1.000 0.000 NA  
2 Performance =~ Performance2 lambda2Y 0.867 0.116 7.489  
3 Knowledge =~ Knowledge1 lambda1X 1.000 0.000 NA  
4 Knowledge =~ Knowledge2 lambda2X 0.683 0.160 4.274  
5 Value =~ Value1 lambda3X 1.000 0.000 NA  
6 Value =~ Value2 lambda4X 0.763 0.184 4.149  
7 Satisfaction =~ Satisfaction1 lambda5X 1.000 0.000 NA  
8 Satisfaction =~ Satisfaction2 lambda6X 0.792 0.436 1.816  
9 Performance ~ Knowledge gamma11 0.337 0.124 2.711  
10 Performance ~ Value gamma12 0.176 0.079 2.237  
11 Performance ~ Satisfaction gamma13 0.061 0.054 1.132  
12 Knowledge ~~ Value r12 0.037 0.012 3.052  
13 Knowledge ~~ Satisfaction r13 0.004 0.009 0.464  
14 Value ~~ Satisfaction r23 -0.008 0.013 -0.614  
15 Performance ~~ Performance sigma2P 0.007 0.003 2.590  
16 Performance1 ~~ Performance1 0.007 0.002 3.126  
17 Performance2 ~~ Performance2 0.007 0.002 3.891  
18 Knowledge1 ~~ Knowledge1 0.041 0.011 3.629  
19 Knowledge2 ~~ Knowledge2 0.035 0.007 5.193  
20 Value1 ~~ Value1 0.080 0.025 3.266  
21 Value2 ~~ Value2 0.087 0.018 4.916  
22 Satisfaction1 ~~ Satisfaction1 0.022 0.049 0.453  
23 Satisfaction2 ~~ Satisfaction2 0.045 0.031 1.428  
24 Knowledge ~~ Knowledge 0.046 0.015 3.154  
25 Value ~~ Value 0.100 0.032 3.164  
26 Satisfaction ~~ Satisfaction 0.090 0.051 1.754  
 pvalue ci.lower ci.upper std.lv std.all std.nox  
1 NA 1.000 1.000 0.140 0.856 0.856  
2 0.000 0.640 1.093 0.121 0.819 0.819  
3 NA 1.000 1.000 0.214 0.728 0.728  
4 0.000 0.370 0.997 0.146 0.618 0.618  
5 NA 1.000 1.000 0.317 0.745 0.745  
6 0.000 0.402 1.123 0.242 0.633 0.633  
7 NA 1.000 1.000 0.300 0.896 0.896  
8 0.069 -0.063 1.646 0.238 0.747 0.747  
9 0.007 0.093 0.581 0.516 0.516 0.516  
10 0.025 0.022 0.330 0.398 0.398 0.398  
11 0.257 -0.044 0.166 0.130 0.130 0.130  
12 0.002 0.013 0.060 0.542 0.542 0.542  
13 0.643 -0.013 0.022 0.064 0.064 0.064  
14 0.539 -0.033 0.018 -0.084 -0.084 -0.084  
15 0.010 0.002 0.012 0.337 0.337 0.337  
16 0.002 0.003 0.012 0.007 0.268 0.268  
17 0.000 0.004 0.011 0.007 0.329 0.329  
18 0.000 0.019 0.063 0.041 0.471 0.471  
19 0.000 0.022 0.048 0.035 0.619 0.619  
20 0.001 0.032 0.128 0.080 0.444 0.444  
21 0.000 0.053 0.122 0.087 0.599 0.599  
22 0.650 -0.074 0.119 0.022 0.198 0.198  
23 0.153 -0.017 0.106 0.045 0.442 0.442  
24 0.002 0.017 0.074 1.000 1.000 1.000  
25 0.002 0.038 0.163 1.000 1.000 1.000  
26 0.080 -0.011 0.191 1.000 1.000 1.000

# fitmeasures(Warren8V.sem.fm1)  
fitted(Warren8V.sem.fm1)$cov

Prfrm1 Prfrm2 Knwld1 Knwld2 Value1 Value2 Stsfc1  
Performance1 0.027   
Performance2 0.017 0.022   
Knowledge1 0.022 0.019 0.087   
Knowledge2 0.015 0.013 0.031 0.056   
Value1 0.030 0.026 0.037 0.025 0.181   
Value2 0.023 0.020 0.028 0.019 0.077 0.146   
Satisfaction1 0.005 0.005 0.004 0.003 -0.008 -0.006 0.113  
Satisfaction2 0.004 0.004 0.003 0.002 -0.006 -0.005 0.071  
 Stsfc2  
Performance1   
Performance2   
Knowledge1   
Knowledge2   
Value1   
Value2   
Satisfaction1   
Satisfaction2 0.101

# residuals(Warren8V.sem.fm1, type = "cor")  
# modificationIndices(Warren8V.sem.fm1)  
Warren8V

Performance1 Performance2 Knowledge1 Knowledge2  
Performance1 0.0271 0.0172 0.0219 0.0164  
Performance2 0.0172 0.0222 0.0193 0.0130  
Knowledge1 0.0219 0.0193 0.0876 0.0317  
Knowledge2 0.0164 0.0130 0.0317 0.0568  
Value1 0.0284 0.0294 0.0383 0.0151  
Value2 0.0217 0.0185 0.0356 0.0230  
Satisfaction1 0.0083 0.0011 -0.0001 0.0055  
Satisfaction2 0.0074 0.0015 0.0035 0.0089  
 Value1 Value2 Satisfaction1 Satisfaction2  
Performance1 0.0284 0.0217 0.0083 0.0074  
Performance2 0.0294 0.0185 0.0011 0.0015  
Knowledge1 0.0383 0.0356 -0.0001 0.0035  
Knowledge2 0.0151 0.0230 0.0055 0.0089  
Value1 0.1826 0.0774 -0.0087 -0.0007  
Value2 0.0774 0.1473 -0.0069 -0.0088  
Satisfaction1 -0.0087 -0.0069 0.1137 0.0722  
Satisfaction2 -0.0007 -0.0088 0.0722 0.1024

# vcov(Warren8V.sem.fm1)  
  
  
  
summary(  
 object = Warren8V.sem.fm1  
 , header = TRUE  
 , fit.measures = FALSE  
 , estimates = TRUE  
 , ci = FALSE  
 , fmi = FALSE  
 , standardized = TRUE  
 , rsquare = TRUE  
 , std.nox = FALSE  
 , modindices = FALSE  
 , nd = 3L  
)

lavaan 0.6-3 ended normally after 75 iterations  
  
 Optimization method NLMINB  
 Number of free parameters 22  
  
 Number of observations 98  
  
 Estimator ML  
 Model Fit Test Statistic 10.441  
 Degrees of freedom 14  
 P-value (Chi-square) 0.729  
  
Parameter Estimates:  
  
 Information Expected  
 Information saturated (h1) model Structured  
 Standard Errors Standard  
  
Latent Variables:  
 Estimate Std.Err z-value P(>|z|) Std.lv  
 Performance =~   
 Prfrmn1 (lm1Y) 1.000 0.140  
 Prfrmn2 (lm2Y) 0.867 0.116 7.489 0.000 0.121  
 Knowledge =~   
 Knwldg1 (lm1X) 1.000 0.214  
 Knwldg2 (lm2X) 0.683 0.160 4.274 0.000 0.146  
 Value =~   
 Value1 (lm3X) 1.000 0.317  
 Value2 (lm4X) 0.763 0.184 4.149 0.000 0.242  
 Satisfaction =~   
 Stsfct1 (lm5X) 1.000 0.300  
 Stsfct2 (lm6X) 0.792 0.436 1.816 0.069 0.238  
 Std.all  
   
 0.856  
 0.819  
   
 0.728  
 0.618  
   
 0.745  
 0.633  
   
 0.896  
 0.747  
  
Regressions:  
 Estimate Std.Err z-value P(>|z|) Std.lv  
 Performance ~   
 Knowldg (gm11) 0.337 0.124 2.711 0.007 0.516  
 Value (gm12) 0.176 0.079 2.237 0.025 0.398  
 Stsfctn (gm13) 0.061 0.054 1.132 0.257 0.130  
 Std.all  
   
 0.516  
 0.398  
 0.130  
  
Covariances:  
 Estimate Std.Err z-value P(>|z|) Std.lv  
 Knowledge ~~   
 Value (r12) 0.037 0.012 3.052 0.002 0.542  
 Satsfctn (r13) 0.004 0.009 0.464 0.643 0.064  
 Value ~~   
 Satsfctn (r23) -0.008 0.013 -0.614 0.539 -0.084  
 Std.all  
   
 0.542  
 0.064  
   
 -0.084  
  
Variances:  
 Estimate Std.Err z-value P(>|z|) Std.lv  
 .Prfrmnc (sg2P) 0.007 0.003 2.590 0.010 0.337  
 .Prfrmn1 0.007 0.002 3.126 0.002 0.007  
 .Prfrmn2 0.007 0.002 3.891 0.000 0.007  
 .Knwldg1 0.041 0.011 3.629 0.000 0.041  
 .Knwldg2 0.035 0.007 5.193 0.000 0.035  
 .Value1 0.080 0.025 3.266 0.001 0.080  
 .Value2 0.087 0.018 4.916 0.000 0.087  
 .Stsfct1 0.022 0.049 0.453 0.650 0.022  
 .Stsfct2 0.045 0.031 1.428 0.153 0.045  
 Knowldg 0.046 0.015 3.154 0.002 1.000  
 Value 0.100 0.032 3.164 0.002 1.000  
 Stsfctn 0.090 0.051 1.754 0.080 1.000  
 Std.all  
 0.337  
 0.268  
 0.329  
 0.471  
 0.619  
 0.444  
 0.599  
 0.198  
 0.442  
 1.000  
 1.000  
 1.000  
  
R-Square:  
 Estimate  
 Performance 0.663  
 Performance1 0.732  
 Performance2 0.671  
 Knowledge1 0.529  
 Knowledge2 0.381  
 Value1 0.556  
 Value2 0.401  
 Satisfaction1 0.802  
 Satisfaction2 0.558