

# Direct Fourier Reconstruction Reference Manual<sup>1</sup>

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## Chapter 1

# CentralSlice

CT Image Reconstruction using Central Slice Theorem. Refer to the project homepage  
<http://code.google.com/p/centralslice/>





## Chapter 2

# File Index

### 2.1 File List

Here is a list of all documented files with brief descriptions:

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## Chapter 3

# File Documentation

### 3.1 `add_noise.m` File Reference

Add noise to the image.

#### Functions

- ret `add_noise` (type image, type SNRdB)  
*Add gaussian noise to the image.*

#### 3.1.1 Detailed Description

Add noise to the image.

Definition in file `add_noise.m`.

#### 3.1.2 Function Documentation

##### 3.1.2.1 `ret add_noise ( type image, type SNRdB )`

Add gaussian noise to the image.

Assumes the exposure is long enough that central limit theorem is valid. (Short exposure is currently not supported)

Signal-to-noise ratio is estimated by the ratio ( $\frac{\mu}{\sigma}$ ), where  $\mu$  is the expected (perfect) measurement result,  $\sigma$  is the variance of the noise. In dB scale it is  $20\log_{10}(\frac{A_{\text{noise}}}{A_{\text{noise}}})$

#### Parameters

<i>image</i>	matrix of the image
<i>SNRdB</i>	signal to noise ratio (1~inf)

**Return values**

<code>new_image</code>	new image loaded with noise
------------------------	-----------------------------

Definition at line 18 of file [add\\_noise.m](#).

**3.2 add\_noise.m**

```

1  %%
2  %! @file
3  % Add noise to the image.
4  %
5
6  %! Add gaussian noise to the image. Assumes the exposure is long enough
   that central limit theorem is valid. (Short exposure is currently
   not supported)
7
8  % Signal-to-noise ratio is estimated by the ratio (  $\frac{\mu}{\sigma}$  ), where  $\mu$  is the expected (perfect)
   measurement result,  $\sigma$  is the variance of the noise. In
   dB scale it is  $20\log_{10}(\frac{A_{\text{term}}{\text{noise}}}{A_{\text{term}}{\text{noise}}})$ 
9
10 % @param image matrix of the image
11 % @param SNRdB signal to noise ratio (1-inf)
12 % @retval new_image new image loaded with noise
13
14 function new_image = add_noise(image, SNRdB)
15
16 [height width] = size(image);
17
18 % estimate the variance of noise in each measurement of each sensor.
19 SNR = 10^(SNRdB/20);
20 signal_amplitude = mean(mean(image));
21 variance = signal_amplitude / SNR;
22
23 noise = randn(height,width) * variance;
24
25 new_image = image + noise;

```

**3.3 apply\_fft1.m File Reference**

Apply Fast Fourier transform to the Radon image.

**Functions**

- `rets apply_fft1` (type Radon, type `DEBUG`)

*Apply FFT to each columns of the matrix, then shifts the DC to the DFT centre.*

### 3.3.1 Detailed Description

Apply Fast Fourier transform to the Radon image.

Definition in file [apply\\_fft1.m](#).

### 3.3.2 Function Documentation

#### 3.3.2.1 `rets apply_fft1 ( type Radon, type DEBUG )`

Apply FFT to each columns of the matrix, then shifts the DC to the DFT centre.

The value of `axis_omega_s` in each row is defined by the formula  $\text{axis\_omega\_s} = x * (2 * \pi / dx)$  where  $dx=1$ .

#### Parameters

<i>Radon</i>	Radon image. Number of rows must be the power of 2 for FFT to work.
<i>DEBUG</i>	Debug mode. Save the Fourier_Radon image in real part and imaginary part.

#### Return values

<i>Fourier_Radon</i>	Radon image in Fourier Space
<i>axis_omega_s</i>	value of <code>omega_s</code> in each row

Definition at line 16 of file [apply\\_fft1.m](#).

## 3.4 apply\_fft1.m

```

1  %%
2  %! @file
3  % Apply Fast Fourier transform to the Radon image
4  %
5
6  %%
7  %! Apply FFT to each columns of the matrix, then shifts the DC to the
      DFT centre. The value of axis_omega_s in each row is defined by the
      formula axis_omega_s = x * (2*pi / dx) where dx=1.
8  % @param Radon Radon image. Number of rows must be the power of 2 for
      FFT to work.
9  % @param DEBUG Debug mode. Save the Fourier_Radon image in real part
      and imaginary part.
10 % @retval Fourier_Radon Radon image in Fourier Space
11 % @retval axis_omega_s value of omega_s in each row
12 function [Fourier_Radon axis_omega_s] = apply_fft1(Radon,DEBUG)
13
14 % Apply FFT to each column of the radon image
15 Fourier_Radon = fft(ifftshift(Radon,1));
16
17 % Label the axis_omega_s,theta axes;
18 [size_omega_s size_theta] = size(Fourier_Radon);
19 dx=1;

```

```

20 % d_omega = 2*pi / Period; where dx = Period / N
21 d_omega = 2*pi / (size_omega_s*dx);
22 axis_omega_s = [0:(size_omega_s/2-1) (-size_omega_s/2):-1] * (d_omega
    / dx);
23
24 % Shift the DC to the DFT centre
25 axis_omega_s = fftshift(axis_omega_s);
26 Fourier_Radon = fftshift(Fourier_Radon,1);
27
28 if(DEBUG)
29     idx = 1:size_theta; %we do not need to know the angles in this function
30
31     save_image(idx,axis_omega_s, real(Fourier_Radon),...
32         'Fourier transform of Radon Space, Real Part',...
33         'slice index','omega_s'); % Save the radon image (real part)
34     save_image(idx,axis_omega_s, imag(Fourier_Radon),...
35         'Fourier transform of Radon Space, Imaginary Part',...
36         'slice index','omega_s'); % Save the radon image (imaginary part)
37
38     stem(axis_omega_s, abs(Fourier_Radon(:,1)));
39     axis tight;
40     title('Slice at angle theta=0 in Fourier Space')
41     xlabel('omega_s'),ylabel('Absolute Value'); % save the slice at 0
42     print -dpng Slice_at_angle_theta_0_in_Fourier_Space.png
43 end

```

## 3.5 damage\_sensors.m File Reference

Disable some X-ray detectors.

### Functions

- ret [damage\\_sensors](#) (type Radon, type damage\_ratio)  
*Disable X-ray detectors in the CT machine.*

### 3.5.1 Detailed Description

Disable some X-ray detectors.

Definition in file [damage\\_sensors.m](#).

### 3.5.2 Function Documentation

#### 3.5.2.1 ret damage\_sensors ( type Radon, type damage\_ratio )

Disable X-ray detectors in the CT machine.

Sensors are chosen at random and fed with null signal during the CAT scanning.

**Parameters**

<i>Radon</i>	Radon projection image when all sensors works normally.
<i>damage_ratio</i>	fraction of sensors damaged. =0 none; =1, all.

**Return values**

<i>damage_radon</i>	new Radon projection image with damaged sensors.
---------------------	--

Definition at line 16 of file [damage\\_sensors.m](#).

**3.6 damage\_sensors.m**

```

1  %%
2  %! @file
3  % Disable some X-ray detectors
4  %
5
6  %%
7  %! Disable X-ray detectors in the CT machine. Sensors are chosen at
8  %   random and fed with null signal during the CAT scanning.
9  % @param Radon Radon projection image when all sensors works normally.
10 % @param damage_ratio fraction of sensors damaged. =0 none; =1, all.
11 % @retval damage_radon new Radon projection image with damaged sensors.
12 %
13 function damage_radon = damage_sensors(Radon, damage_ratio)
14 damage_radon = Radon; %copy the Radon image
15
16 if(damage_ratio ~= 0) % use this function only when necessary
17
18     [numb_sensor scan_angle] = size(Radon); %find the size of the Radon
19     image
20     total_damage = round((numb_sensor - 1)*damage_ratio) + 1; %total number
21     of sensor damage
22
23     sensor_index = round(1 + (numb_sensor-1).*rand(1, total_damage));
24     damage_radon(sensor_index,:) = 0; %specify which sensors need to be
25     nullified
26
27 end

```

**3.7 image\_crop.m File Reference**

Crop the image to specified range.

**Functions**

- [rets image\\_crop](#) (type image, type axis\_xy, type xy\_min, type xy\_max, type [DEBUG](#))

### 3.7.1 Detailed Description

Crop the image to specified range.

Definition in file [image\\_crop.m](#).

### 3.7.2 Function Documentation

**3.7.2.1** `rets image_crop ( type image, type axis_xy, type xy_min, type xy_max, type DEBUG )`

#### Parameters

<i>image</i>	Image to be cropped
<i>axis_xy</i>	original range of axes in the image
<i>xy_min</i>	top left hand corner of the crop box
<i>xy_max</i>	bottom right hand corner of the crop box
<i>DEBUG</i>	Debug mode. If DEBUG=1, save the preview image.

#### Return values

<i>new_image</i>	cropped image
<i>new_axis_xy</i>	new axes range

Definition at line 18 of file [image\\_crop.m](#).

## 3.8 image\_crop.m

```

1  %%
2  %! @file
3  % Crop the image to specified range.
4  %
5
6  %%
7  %! @param image Image to be cropped
8  % @param axis_xy original range of axes in the image
9  % @param xy_min top left hand corner of the crop box
10 % @param xy_max bottom right hand corner of the crop box
11 % @param DEBUG Debug mode. If DEBUG=1, save the preview image.
12 % @retval new_image cropped image
13 % @retval new_axis_xy new axes range
14 function [new_image new_axis_xy] = image_crop(image,axis_xy,xy_min,
15         xy_max,DEBUG)
16
17 % Make axis_xy a row vector
18 if( size(axis_xy,2) ==1)
19     axis_xy = axis_xy';
20 end
21
22 % define crop box
23 [row_idx col_idx val] = find( axis_xy>xy_min & axis_xy<xy_max );
24 idx_begin = col_idx(1);
25 idx_end = col_idx(length(row_idx));

```

```

26 new_image = image(idx_begin:idx_end,idx_begin:idx_end);
27 new_axis_xy = axis_xy(idx_begin:idx_end);
28
29 if (DEBUG)
30 figure
31 imagesc(axis_xy,axis_xy,real(image)),colormap(gray),colorbar
32 xlim([xy_min xy_max]),ylim([xy_min xy_max])
33 title('Cropbox preview'),xlabel('x'),ylabel('y')
34 print -dpng 'cropbox_preview.png'
35 end

```

## 3.9 inverse\_Fourier\_2D.m File Reference

Apply inverse Fourier 2D transform to the image.

### Functions

- rets [inverse\\_Fourier\\_2D](#) (type *Fourier\_2D*, type *omega\_xy*, type *DEBUG*)

### 3.9.1 Detailed Description

Apply inverse Fourier 2D transform to the image.

Definition in file [inverse\\_Fourier\\_2D.m](#).

### 3.9.2 Function Documentation

**3.9.2.1** rets *inverse\_Fourier\_2D* ( type *Fourier\_2D*, type *omega\_xy*, type *DEBUG* )

#### Parameters

<i>Fourier_2D</i>	matrix of the interpolated 2D Fourier space
<i>omega_xy</i>	value of <i>omega_x</i> (or <i>omega_y</i> ) in each column (or row) of matrix <i>Fourier_2D</i> .
<i>DEBUG</i>	Debug mode. If <i>DEBUG</i> =1, save the image of the reconstructed image in imaginary part.

#### Return values

<i>Final_image</i>	Inverse Fourier transform of matrix <i>Fourier_2D</i> .
<i>axis_xy</i>	value of <i>x</i> (or <i>y</i> ) in each column (or row) of <i>Final_image</i>

Definition at line 16 of file [inverse\\_Fourier\\_2D.m](#).

## 3.10 inverse\_Fourier\_2D.m

```

1  %%
2  %! @file
3  % Apply inverse Fourier 2D transform to the image
4  %
5
6  %%
7  %! @param Fourier_2D matrix of the interpolated 2D Fourier space
8  % @param omega_xy value of omega_x (or omega_y) in each column (or row)
9  %   of matrix \c Fourier_2D.
10 % @param DEBUG Debug mode. If DEBUG=1, save the image of the
11 %   reconstructed image in imaginary part.
12 % @retval Final_image Inverse Fourier transform of matrix \c Fourier_2D
13 %   .
14 % @retval axis_xy value of x (or y) in each column (or row) of \c
15 %   Final_image
16 function [Final_image,axis_xy] = inverse_Fourier_2D(Fourier_2D,omega_xy
17 ,DEBUG)
18
19 % Shift the DC to the left top corner
20 shifted_Fourier_2D = ifftshift(Fourier_2D);
21
22 % Apply inverse 2D Fourier transform
23 shifted_Final_image = ifft2(shifted_Fourier_2D);
24
25 % Shift the DC back to the centre
26 Final_image = fftshift(shifted_Final_image);
27
28 %Label the axes x and y
29 size_omega = length(omega_xy);
30 d_omega = mean(diff(omega_xy));
31 dx = 2*pi/(d_omega*size_omega);
32 N_image = 65;
33 axis_xy = omega_xy * (dx / d_omega);
34
35 if(DEBUG)
36     save_image(axis_xy,axis_xy,imag(Final_image),...
37         'Reconstructed Image, imaginary part','x','y');
38 end

```

### 3.11 main.m File Reference

Main process of the simulation.

#### Functions

- void `main` (type `shape`, type `N_image`, type `N_theta`, type `SNRdB`, type `interp_m`, type `oversampling_ratio`, type `damage_ratio`, type `DEBUG`)

*Main process of the simulation.*

#### 3.11.1 Detailed Description

Main process of the simulation.



Definition in file [main.m](#).

### 3.11.2 Function Documentation

**3.11.2.1** `void main ( type shape, type N_image, type N_theta, type SNRdB, type interp_m, type oversampling_ratio, type damage_ratio, type DEBUG )`

Main process of the simulation.

This script generates a radon projection image from a selected phantom. Then 1D Fourier transform is applied to each projection angle. The result is then interpolated onto the cartesian plane according to Central slice theorem. Lastly inverse 2D Fourier transform is applied to reproduce the image.

#### Parameters

<i>shape</i>	shape of the phantom. Can be 'Shepp-Logan', 'Modified Shepp-Logan', 'dot', 'square', or 'stripe'
<i>N_image</i>	mininum size of the phantom image (in pixels)
<i>N_theta</i>	Number of slices in Radon scan from 0deg to 180deg (excluding 180deg)
<i>SNRdB</i>	Signal to Noise Ratio in log scale.
<i>interp_m</i>	method of interpolation. Can be 'nearest', 'linear' or 'cubic'
<i>oversampling_ratio</i>	oversampling ratio. Increase the Nyquist frequency to reduce aliasing. =1, none; >1 oversampling.
<i>damage_ratio</i>	fraction of sensors damaged. =0, none; =1, all damaged.
<i>DEBUG</i>	mode. If set to 1, many more figures are printed out for debugging process.

Definition at line 33 of file [main.m](#).

## 3.12 main.m

```

1  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
2  %! @mainpage CentralSlice
3  % CT Image Reconstruction using Central Slice Theorem.
4  %
5  % Refer to the project homepage http://code.google.com/p/centralslice/
6
7  %! @file
8  % Main process of the simulation.
9
10 %! @example start_simulation.m
11 %
12
13 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
14 %! Main process of the simulation.
15 % This script generates a radon projection image from a selected
    phantom.
16 % Then 1D Fourier transform is applied to each projection angle. The
    result is then interpolated onto the cartesian plane according to

```

```

        Central slice theorem. Lastly inverse 2D Fourier transform is
        applied to reproduce the image.
17 % @param shape shape of the phantom. Can be 'Shepp-Logan', 'Modified
    Shepp-Logan', 'dot', 'square', or 'stripe'
18 % @param N_image mininum size of the phantom image (in pixels)
19 % @param N_theta Number of slices in Radon scan from 0deg to 180deg (
    excluding 180deg)
20 % @param SNRdB Signal to Noise Ratio in log scale.
21 % @param interp_m method of interpolation. Can be 'nearest','linear' or
    'cubic'
22 % @param oversampling_ratio oversampling ratio. Increase the Nyquist
    frequency to reduce aliasing. =1, none; >1 oversampling.
23 % @param damage_ratio fraction of sensors damaged. =0, none; =1, all
    damaged.
24 % @param DEBUG mode. If set to 1, many more figures are printed out for
    debugging process.
25 function main(shape,N_image,N_theta,SNRdB,interp_m,oversampling_ratio,
    damage_ratio,DEBUG)
26 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
27 %% MAKE A PHANTOM AND APPLY RADON TRANSFORMATION
28
29 Phantom = make_phantom(shape,floor(N_image/sqrt(2))); % Make a
    phantom.
30
31 axis_xy = linspace(-N_image/2,N_image/2,N_image);
32 save_image(axis_xy,axis_xy,Phantom,...
33     'Phantom','x','y'); % Save the phantom image
34
35 % Angles for Radon Projection.
36 % It should be from 0deg to 180deg. The last angular sample normally is
    smaller than 180deg.
37 d_theta = 180 / N_theta;
38 THETA = linspace(0,180-d_theta,N_theta);
39
40 % Workaround a bug in Matlab function RADON, which assumes the y-axis
    points downwards instead of pointing upward
41 Phantom_flipy = flipud(Phantom);
42 Radon = radon(Phantom_flipy,THETA); % Apply Radon transform.
43
44 no_of_sensors = size(Radon,1)
45
46 Radon = add_noise(Radon,SNRdB); % Add noise to the image
47
48 %% Sensor damage: nullify some sensors
49 damage_radon = damage_sensors(Radon, damage_ratio);
50
51 %% Zeropadding: expand the matrix to power of 2 before doing FFT
52 [Radon2 axis_s] = zeropad(damage_radon);
53
54 save_image(THETA,axis_s,Radon2,...
55     'Radon Projection','theta','s'); % Save the radon image
56
57 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
58 %% 1D FOURIER TRANSFORM
59 [Fourier_Radon omega_s] = apply_fft1(Radon2,DEBUG);
60
61 save_image(THETA, omega_s, abs(Fourier_Radon),...
62     'Fourier transform of Radon Space, Absolute Value',...
63     'theta','omega_s');
64
65 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
66

```

```

67 %% INTERPOLATION: Map slices from polar coordinates to rectangular
    coordinates
68 [Fourier_2D omega_xy] = polar_to_rect (THETA, omega_s, Fourier_Radon,
    N_image*oversampling_ratio, interp_m, DEBUG);
69
70 save_image (omega_xy, omega_xy, log(abs(Fourier_2D)), 'Interpolated Fourier
    Space (log scale)', 'omega_x', 'omega_y')
71
72 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
73 %% INVERSE 2D FOURIER TRANSFORM
74 [Reconstructed_image axis_xy_2] = inverse_Fourier_2D(Fourier_2D,
    omega_xy, DEBUG);
75
76 % Crop image
77 xy_min = axis_xy(1);
78 xy_max = axis_xy(length(axis_xy));
79 [Crop_image new_axis_xy] = image_crop(Reconstructed_image, axis_xy_2,
    xy_min, xy_max, DEBUG);
80
81 save_image (new_axis_xy, new_axis_xy, real(Crop_image), ...
82     'Reconstructed Image', 'x', 'y');

```

## 3.13 make\_phantom.m File Reference

Make a phantom.

### Functions

- ret [make\\_phantom](#) (type *shape*, type *N*)  
Construct a matrix of a selected phantom.

### 3.13.1 Detailed Description

Make a phantom.

Definition in file [make\\_phantom.m](#).

### 3.13.2 Function Documentation

#### 3.13.2.1 ret make\_phantom ( type *shape*, type *N* )

Construct a matrix of a selected phantom.

#### Parameters

<i>shape</i>	Type of the phantom. Can be 'Shepp-Logan', 'Modified Shepp-Logan', 'dot', 'square', 'stripe' or 'offcentre dot'
<i>N</i>	Size of the matrix

**Return values**

$P$	Matrix of the phantom image
-----	-----------------------------

Definition at line 16 of file [make\\_phantom.m](#).

**3.14 make\_phantom.m**

```

1  %%
2  %! @file
3  % Make a phantom.
4  %
5
6  %%
7  %! Construct a matrix of a selected phantom.
8  % @param shape Type of the phantom. Can be 'Shepp-Logan', 'Modified
9  %   Shepp-Logan', 'dot', 'square', 'stripe' or 'offcentre dot'
10 % @param N Size of the matrix
11 % @retval P Matrix of the phantom image
12 %
13 function P = make_phantom(shape,N)
14
15     % T width of the square pulse or the stripe. T must be an even
16     % number.
17     T=round(N/4)*2;
18     R=T/2;
19
20 switch shape
21 case {'Shepp-Logan','Modified Shepp-Logan'}
22     % Modified Shepp-Logan' gives better visual perception than 'Shepp-
23     % Logan'
24     P = phantom(shape,N);
25     P = flipud(P);
26 case {'dot'}
27     R=4;
28     x=linspace(-N/2,N/2,N); y=x; [X, Y]=meshgrid(x,y); P=(X.^2 +Y.^2 ≤
29     R^2);
30 case {'square'}
31     P=[zeros(N, (N-T)/2) ones(N, T) zeros(N, (N-T)/2)];
32     P=P'*P;
33 case {'stripe'}
34     P=[zeros(N, (N-T)/2) ones(N, T) zeros(N, (N-T)/2)];
35 case {'circle'}
36     x=linspace(-N/2,N/2,N); y=x; [X, Y]=meshgrid(x,y); P=(X.^2 +Y.^2 ≤
37     R^2);
38 case {'offcentre dot'}
39     % make a off-centre dot
40     P=zeros(N);
41     idx = round(N/4);
42     P(idx:idx+1,idx:idx+1)=1;
43 end

```

**3.15 polar\_to\_rect.m File Reference**

Map polar coordinates to rectangular coordinates.

## Functions

- `rets polar_to_rect` (type *theta*, type *omega\_s*, type *Fourier\_Radon*, type *N\_image*, type *interp\_m*, type *DEBUG*)

### 3.15.1 Detailed Description

Map polar coordinates to rectangular coordinates.

Definition in file [polar\\_to\\_rect.m](#).

### 3.15.2 Function Documentation

**3.15.2.1** `rets polar_to_rect ( type theta, type omega_s, type Fourier_Radon, type N_image, type interp_m, type DEBUG )`

#### Parameters

<i>theta</i>	angles of Radon transform. Values of <i>theta</i> in each columns of <i>Fourier_Radon</i>
<i>omega_s</i>	values of <i>omega_s</i> in each rows of <i>Fourier_Radon</i>
<i>Fourier_Radon</i>	Matrix of Fourier transformed Radon image
<i>N_image</i>	minimum size of the image
<i>interp_m</i>	method of interpolation, can be 'nearest', 'linear' or 'cubic'
<i>DEBUG</i>	Debug mode. If <i>DEBUG</i> =1, surface of <i>Fourier_Radon</i> in polar coordinates and in rectangular coordinates will be saved.

#### Return values

<i>Fourier_2D</i>	Matrix of the mapped Fourier space. By central slice theorem, this is equivalent to the 2D Fourier transform of the original image.
<i>axis_omega_xy</i>	values of <i>omega_x</i> (or <i>omega_y</i> ) in the columns (or rows) of <i>Fourier_2D</i> .

Definition at line 20 of file [polar\\_to\\_rect.m](#).

## 3.16 polar\_to\_rect.m

```

1 %%
2 %! @file
3 % Map polar coordinates to rectangular coordinates
4 %
5
6 %%
7 %!
8 % @param theta angles of Radon transform. Values of theta in each
9 %   columns of Fourier_Radon
10 % @param omega_s values of omega_s in each rows of Fourier_Radon

```

```

10 % @param Fourier_Radon Matrix of Fourier transformed Radon image
11 % @param N_image minimum size of the image
12 % @param interp_m method of interpolation, can be 'nearest','linear' or
    'cubic'
13 % @param DEBUG Debug mode. If DEBUG=1, surface of Fourier_Radon in
    polar coordinates and in rectangular coordinates will be saved.
14 % @retval Fourier_2D Matrix of the mapped Fourier space. By central
    slice theorem, this is equivalent to the 2D Fourier transform of
    the original image.
15 % @retval axis_omega_xy values of omega_x (or omega_y) in the columns (
    or rows) of Fourier_2D.
16 function [Fourier_2D axis_omega_xy] = polar_to_rect(theta,omega_s,
    Fourier_Radon,N_image,interp_m,DEBUG)
17 %% Check correctness of input data
18 [size_omega_s size_theta] = size(Fourier_Radon);
19 length_theta = length(theta);
20 length_omega_s = length(omega_s);
21
22 if(length_theta ~= size_theta)
23     error('size of theta does not match with the size of Fourier_Radon!')
24 elseif(length_omega_s ~= size_omega_s)
25     error('size of omega_s does not match with the size of Fourier_Radon!'
    )
26 end
27
28 %% Preparations
29 % extend the range of Fourier Radon space so that value at theta=0 and
    theta =180 can be interpolated
30 % Disabled so that the effect of scan range could be investigated
31 %Extended_Fourier_Radon = horzcat( Fourier_Radon, Fourier_Radon(:,
    size_theta) );
32 %theta = [theta 180];
33
34 % Label each elements in the matrix Fourier_Radon with the
    corresponding theta and omega_s:
35 [THETA OMEGA_S] = meshgrid(theta,omega_s);
36
37 %Define the desired scale of the rectangular coordinates
38 x = linspace(-N_image/2,N_image/2,N_image);
39 y = x;
40 dx=1;
41 d_omega = 2*pi/N_image;
42 omega_x = x * (d_omega / dx);
43 omega_y=omega_x;
44
45 % Label each (omega_x, omega_y) to (omega_s, theta)
46 [OMEGA_X OMEGA_Y] = meshgrid(omega_x, omega_y);
47 [THETA_I OMEGA_SI] = cart2pol(OMEGA_X,OMEGA_Y);
48
49 % map from {theta,omega_s : [-pi,pi],[0,inf]} to {theta,omega_s : [0,
    180],[-inf,inf]}
50 OMEGA_SI = OMEGA_SI .* sign(THETA_I);
51 THETA_I = mod( THETA_I * (180/pi), 180);
52
53 %% Apply interpolation
54 % disable extrapolation: everything outside the defined space is set to
    zero.
55 Fourier_2D = interp2(THETA,OMEGA_S,Fourier_Radon,...
    THETA_I,OMEGA_SI,interp_m,0);
56 axis_omega_xy = omega_x;
57
58
59 %% DEBUG: Print surface of Fourier_Radon before and after interpolation

```

```

60 if (DEBUG)
61 [WX WY] = pol2cart (THETA, OMEGA_S);
62 figure
63 surf (WX, WY, abs (Fourier_Radon), ...
64       'edgecolor', 'none')
65 colormap (jet), colorbar
66 title ('Surface of Fourier transformed Radon space, before interpolation
        ')
67 xlabel ('omega_x'), ylabel ('omega_y')
68 print -dpng '
        Surface_of_Fourier_transformed_Radon_space_before_interpolation.png
        '
69
70 figure
71 surf (omega_x, omega_y, abs (Fourier_2D), ...
72       'edgecolor', 'none')
73 colormap (jet), colorbar
74 title ('Surface of Fourier transformed Radon space, after interpolation'
        ')
75 xlabel ('omega_x'), ylabel ('omega_y')
76 print -dpng '
        Surface_of_Fourier_transformed_Radon_space_after_interpolation.png'
77 end

```

## 3.17 save\_image.m File Reference

Save the gray-scale representation of the image.

### Functions

- void [save\\_image](#) (type x, type y, type Z, type Title, type Xlabel, type Ylabel)  
*Save the gray-scale representation of the image.*

### 3.17.1 Detailed Description

Save the gray-scale representation of the image.

Definition in file [save\\_image.m](#).

### 3.17.2 Function Documentation

#### 3.17.2.1 void save\_image ( type x, type y, type Z, type Title, type Xlabel, type Ylabel )

Save the gray-scale representation of the image.

#### Parameters

<i>x</i>	value of x in each column
<i>x</i>	value of y in each row

<i>Z</i>	matrix of the image
<i>Title</i>	title of the graph
<i>Xlabel</i>	label of the x-axis
<i>Ylabel</i>	label of the y-axis

Definition at line 17 of file [save\\_image.m](#).

### 3.18 save\_image.m

```

1  %%
2  %! @file
3  % Save the gray-scale representation of the image.
4
5  %! Save the gray-scale representation of the image.
6  % @param x value of x in each column
7  % @param y value of y in each row
8  % @param Z matrix of the image
9  % @param Title title of the graph
10 % @param Xlabel label of the x-axis
11 % @param Ylabel label of the y-axis
12
13 function save_image(x,y,Z,Title,Xlabel,Ylabel)
14 figure
15 imagesc(x,y,Z)
16 % flip the y-axis to pointing upward
17 set(gca,'YDir','normal')
18 colormap(gray),colorbar
19 title(Title)
20 if(nargin > 2)
21     xlabel(Xlabel),ylabel(Ylabel)
22 end
23 print('-dpng',strcat(strrep(Title,' ','_'),'.png'))

```

### 3.19 start\_simulation.m File Reference

Program Initiation.

#### Variables

- `DEBUG = 1`  
*Debug the program.*
- `shape`  
*Shape of the phantom.*



### 3.19.1 Detailed Description

Program Initiation. Defines several important variables before starting the main process.

Definition in file [start\\_simulation.m](#).

### 3.19.2 Variable Documentation

#### 3.19.2.1 DEBUG = 1

Debug the program.

When DEBUG=1, extra figures will be saved in current directory which can be used for debugging process.

Definition at line 15 of file [start\\_simulation.m](#).

#### 3.19.2.2 shape

**Initial value:**

'Modified Shepp-Logan'

Shape of the phantom.

Can be 'Shepp-Logan', 'Modified Shepp-Logan', 'dot', 'square', 'stripe' or 'offcentre dot'.

Definition at line 23 of file [start\\_simulation.m](#).

## 3.20 start\_simulation.m

```

1 %%
2 %! @file
3 % Program Initiation.
4 % Defines several important variables before starting the main process.
5
6 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
7 %% DEBUG MODE
8
9 %! Debug the program.
10 % When DEBUG=1, extra figures will be saved in current directory which
    can be used for debugging process.
11 DEBUG = 1;
12
13 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
14 %% Parameters
15
16 %! Shape of the phantom. Can be 'Shepp-Logan', 'Modified Shepp-Logan',
    'dot', 'square', 'stripe' or 'offcentre dot'.
```

```

17 shape='Modified Shepp-Logan'
18
19 %! Size of the phantom.
20 % This specifies the number of rows and columns in the matrix of
   Phantom.
21 % N_image is suggested to be an odd number.
22 N_image = 371
23
24 %! Number of slices in Radon scan from 0deg to 180deg (excluding 180deg
   )
25 N_theta = 180
26
27 %number of sensors damaged, damage_ratio varies from 0 to 1
28 damage_ratio = 0
29
30 %! Signal to noise ratio, in deciBell (dB)
31 SNRdB = inf
32
33 %! Interpolation method
34 interp_m = 'linear'
35
36 %! Oversampling_ratio oversampling ratio. Increase the Nyquist
   frequency to reduce aliasing. =1, none; >1 oversampling.
37 oversampling_ratio=1
38
39 %! Zeropadding ratio. Avoid overlapping of artefacts to the phantom
   after applying inverse Fourier transform. 1 -> minimal; >1 ->
   zeropadding.
40 %zeropadding_ratio=1;
41
42 main(shape,N_image,N_theta,SNRdB,interp_m,oversampling_ratio,
   damage_ratio,DEBUG);

```

## 3.21 zeropad.m File Reference

Expand the matrix to power of 2 before doing FFT.

### Functions

- `rets zeropad` (type Radon)

*Zeropad each column to prepare for FFT.*

### 3.21.1 Detailed Description

Expand the matrix to power of 2 before doing FFT.

Definition in file `zeropad.m`.

### 3.21.2 Function Documentation

#### 3.21.2.1 `rets zeropad ( type Radon )`

Zeropad each column to prepare for FFT.

Expand the length of each column to the power of 2. The value of *s* in each row is also computed.

#### Parameters

<i>Radon</i>	matrix of Radon image
--------------	-----------------------

#### Return values

<i>Radon2</i>	expaned matrix of Radon image
<i>axis_s</i>	value of <i>s</i> in each row

Definition at line 15 of file `zeropad.m`.

## 3.22 zeropad.m

```

1 %%
2 %! @file
3 % Expand the matrix to power of 2 before doing FFT.
4 %
5
6 %%
7 %! Zeropad each column to preprare for FFT. Expand the length of each
8 %   column to the power of 2. The value of s in each row is also
9 %   computed.
10 % @param Radon matrix of Radon image
11 % @retval Radon2 expaned matrix of Radon image
12 % @retval axis_s value of s in each row
13 function [Radon2 axis_s] = zeropad(Radon)
14
15 [size_s size_theta] = size(Radon);
16 next_power_of_2 = pow2(nextpow2(size_s));
17
18 % Shift the DC to the left
19 shifted_Radon = ifftshift(Radon,1);
20
21 % Estimate the size of zeropad required
22 size_zeropad = next_power_of_2 - size_s;
23 zeropad = zeros(size_zeropad,size_theta);
24
25 if(size_s & 2) % if length is an odd number, the 'middle' is between (
26     size_s + 1)/2 and (size_s + 1)/2+1
27     mid_position = (size_s + 1)/2;
28 else % if length is an even number, the 'middle' is between size_s/2
29     and size_s/2+1
30     mid_position = size_s / 2;
31 end
32
33 % Add zeros to the middle of the shifted signal

```

```
30 shifted_Radon2 = vertcat(shifted_Radon(1:mid_position,:), zeropad,  
    shifted_Radon((mid_position+1):size_s,:));  
31  
32 % Shift the DC back to the centre  
33 Radon2 = fftshift(shifted_Radon2,1);  
34  
35 % label the new s axis  
36 axis_s = linspace(-next_power_of_2/2,next_power_of_2/2,next_power_of_2)  
    ;
```

## Chapter 4

# Example Documentation

### 4.1 start\_simulation.m

Refer to Section 3.19 on page 20 for detailed explanation.

```
1 %%
2 %! @file
3 % Program Initiation.
4 % Defines several important variables before starting the main process.
5
6 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
7 %% DEBUG MODE
8
9 %! Debug the program.
10 % When DEBUG=1, extra figures will be saved in current directory which
    can be used for debugging process.
11 DEBUG = 1;
12
13 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
14 %% Parameters
15
16 %! Shape of the phantom. Can be 'Shepp-Logan', 'Modified Shepp-Logan',
    'dot', 'square', 'stripe' or 'offcentre dot'.
17 shape='Modified Shepp-Logan'
18
19 %! Size of the phantom.
20 % This specifies the number of rows and columns in the matrix of
    Phantom.
21 % N_image is suggested to be an odd number.
22 N_image = 371
23
24 %! Number of slices in Radon scan from 0deg to 180deg (excluding 180deg
    )
25 N_theta = 180
26
27 %number of sensors damaged, damage_ratio varies from 0 to 1
28 damage_ratio = 0
29
30 %! Signal to noise ratio, in deciBell (dB)
31 SNRdB = inf
```

```
32
33     %! Interpolation method
34     interp_m = 'linear'
35
36     %! Oversampling_ratio oversampling ratio. Increase the Nyquist
37       frequency to reduce aliasing. =1, none; >1 oversampling.
38     oversampling_ratio=1
39
40     %! Zeropadding ratio. Avoid overlapping of artefacts to the phantom
41       after applying inverse Fourier transform. 1 -> minimal; >1 ->
42       zeropadding.
43     %zeropadding_ratio=1;
44
45     main(shape,N_image,N_theta,SNRdB,interp_m,oversampling_ratio,
46         damage_ratio,DEBUG);
```

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