

## ★ Fuzzy Logic:-

- Fuzzy Logic is a mathematical Logic which tries to solve problems with an open and imprecise spectrum of data.
- Fuzzy Logic makes easy to obtain an array of precise conclusion.
- Fuzzy Logic is basically design to achieve the best possible solution to complex problems from all the available data and input data.
- Fuzzy Logic are considered as best solution finder.
- Fuzzy Logic make decision based on imprecise information.
- In crisp logic or boolean logic we have only possible two outcomes i.e. yes, or, no. Crisp Logic have strict boundary that is that may be either true or false. In this logic do not know degree of the problem/solution, and it is hardcoded. When uncertainty comes then this logic is fail to deal uncertainty we require Fuzzy Logic.
- In Fuzzy Logic deals with we can handle uncertainty with the use of degree of belief and membership function.
- degree of belief means if we talking about the student then some students are good and some student are bad. then in a crisp logic there is may be only possible two outcomes i.e good or bad.



but in Fuzzy Logic tells us how much student is bad or how much student is good. degree of belief always <sup>ranges</sup> between 0 and 1.

- Fuzzy Logic is designed to consider the best possible decision by considering all available information.
- Fuzzy Logic is derived from Fuzzy Set theory and finds the appropriate rather than definite or precise pattern.
- Fuzzy Logic is used to represent uncertainty.
- Fuzzy Logic represents the belongingness of member of crisp set to fuzzy set.

Example:- Let's take an example of a car speed.

our membership function:- in boolean logic.

if  $x \geq 40$ , then speed is fast  
we need to break.  
else  $x < 40$  then speed is slow  
we need to be fast.

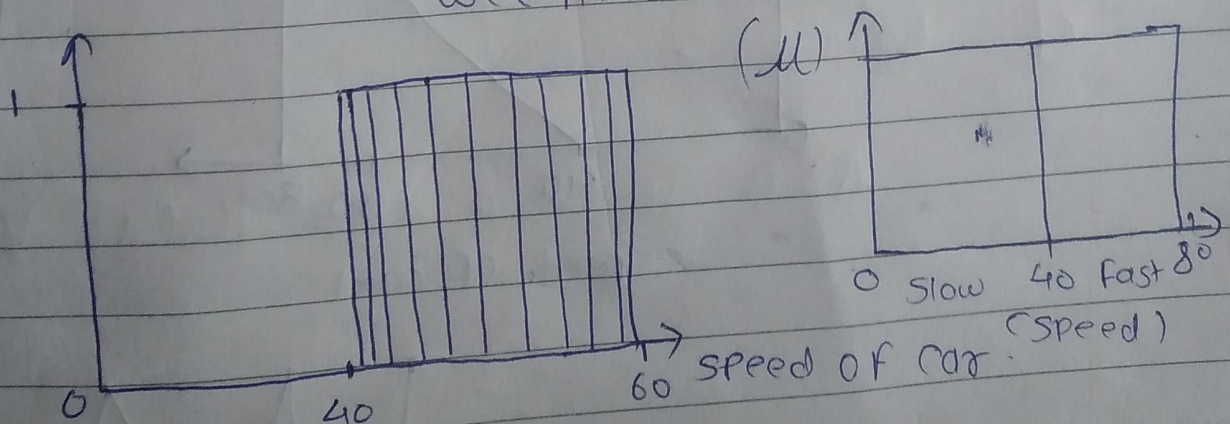


Fig:- Output graph of a boolean Logic/



Fuzzy Logic is a combination of a  
Relational algebra + Boolean Logic + predicate Logic = Fuzzy Logic

If we want to represent the go output with degree of belief then we want to represent the problem in Fuzzy Logic.

Let's

ex: check the degree of fastness of car.

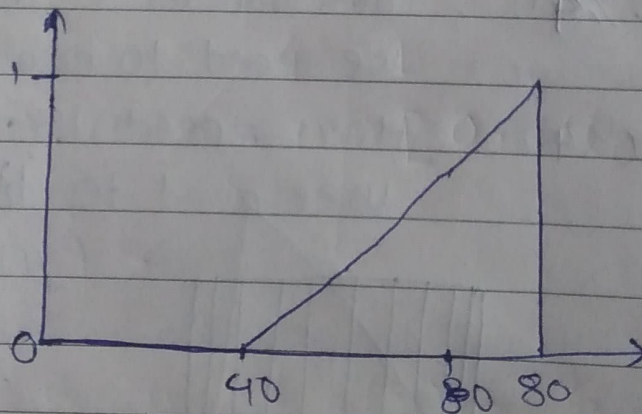
membership function.

0, if  $\text{Speed}(x) \leq 40$

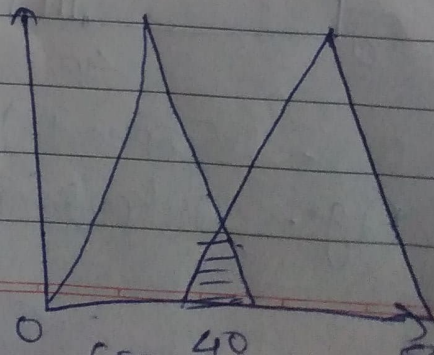
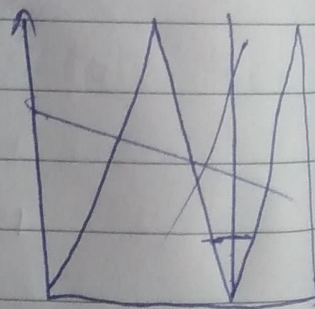
$\frac{\text{Speed}(x) - 40}{10}$ , if  $40 < \text{Speed} \leq 50$

1, if  $\text{Speed}(x) > 50$

the output graph of this Fuzzy Logic could be like this:-



(Speed of car)





\* Boolean Logic representations:

$$U = \{1, 2, 3, 4, 5\}$$

$$S = \{1, 2\}$$

membership function we want to know that the sample space  $(S)$  should be contain in universal set  $(U)$

then we represent like below:-

$$= \{ \overset{x, \mu}{(1, 1)}, (2, 1), (3, 0), (4, 0), (5, 0) \}$$

\* check the degree of fastness in fuzzy logic:-

$$0, \text{ if Speed}(x) \leq 40$$

$$\frac{\text{Speed}(x) - 40}{10}, \text{ if } 40 < \text{Speed}(x) < 50$$

$$1, \text{ if Speed}(x) \geq 50$$

$$U = \{30, 60, 42\}$$

$$\text{Fuzzy Set} = (x, \mu(x))$$

$$= \{ (30, 0), (60, 1), (42, 0.2) \}$$

\* operation of Fuzzy Logic:-

1] Union:-

$$\mu_{A \cup B} = \max(\mu_A(x), \mu_B(x)), x \in U$$

2] Intersection:-

$$\mu_{A \cap B} = \min(\mu_A(x), \mu_B(x)), x \in U$$

3] Complement:-

$$\mu_{\bar{A}}(x) = [1 - \mu_A(x)], x \in U$$

4] Bold union:-

$$\mu_{A \oplus B} = \min[1, \mu_A(x) + \mu_B(x)], x \in U$$

5] Bold intersection:-

$$\mu_{A \odot B}(x) = \max[0, \mu_A(x) + \mu_B(x) - 1]$$

6] Equality:-

$$A = B \text{ if } \mu_A(x) = \mu_B(x) \quad \forall x \in S$$



perform following operation on fuzzy set :-

$$U = \{ 5, 10, 20, 25, 30, 40 \}$$

$$A = \{ (10, 0.2), (20, 0.4), (25, 0.7), (5, 0.2), (30, 0.9), (40, 1) \}$$

$$B = \{ (10, 0.4), (20, 0.1), (25, 0.9), (30, 0.2), (40, 0.6) \}$$

- 1) Union      2) Intersection, 3) Complement of  $\bar{A}$   
3) Bold union      4) Bold intersection      5) Equality.

① Union :-

$$\mu_{A \cup B} = \{ (10, 0.4), (20, 0.4), (25, 0.9), (5, 0.2), (30, 0.9), (40, 1) \}$$

2) Intersection :-

$$\mu_{A \cap B} = \{ (10, 0.2), (20, 0.1), (25, 0.7), (30, 0.2), (40, 0.6) \}$$

3) complement :-

$$\mu_{\bar{A}}(x) = \{ (10, 1-0.2), (20, 1-0.4), (25, 1-0.7), (5, 1-0.2), (30, 1-0.9), (40, 1-1) \}$$

$$= \{ (10, 0.8), (20, 0.6), (25, 0.3), (5, 0.8), (30, 0.1), (40, 0) \}$$



Bold union :-

$$\mu_{A \oplus B} = \min[1, \mu_A(x) + \mu_B(x)], x \in U$$
$$= \{ (10, 0.6), (20, 0.5), (25, 1), (30, 1), (40, 1) \}$$

$$\mu_{A \oplus B} = \{ \min(1, 0.2 + 0.4), \min(1, 0.4 + 0.1), \min(1, 0.7 + 0.9), \min(1, 0.9 + 0.2), \min(1, 1 + 0.6) \}$$

$$= \{ (10, 0.6), (20, 0.5), (25, 1), (30, 1), (40, 1) \}$$

Bold Intersection:-

$$\mu_{A \cdot B} = \max[0, \mu_A(x) + \mu_B(x) - 1], x \in U$$

$$\mu_{A \cdot B} = \{ (10, 0.6), (20, 0.5), (25, 0.6), (30, 0.1), (40, 0.6) \}$$

$$(10, 0.2) \wedge (10, 0.4) = 0.2 + 0.4 - 1$$

$$= 0.6 - 1$$

$$= -0.4$$

$$= \max(0, -0.4)$$

$$= 0$$



\* Crisp Relation representations:-

$$A = \{1, 2, 3\}$$

$$B = \{4, 7, 8\}$$

$$A \times B = \left\{ \begin{array}{l} (1, 4), (1, 7), (1, 8) \\ (2, 4), (2, 7), (2, 8) \\ (3, 4), (3, 7), (3, 8) \end{array} \right\}$$

$$R_1 = \{ (a, b) \mid a < b, (a, b) \in A \times B \}$$

$$= \left\{ \begin{array}{l} (1, 4), (1, 7), (1, 8) \\ (2, 4), (2, 7), (2, 8) \\ (3, 4), (3, 7), (3, 8) \end{array} \right\}$$

	4	7	8
1	1	1	1
2	1	1	1
3	1	1	1

$$R_2 = \{ (a, b) \mid a > b, (a, b) \in A \times B \}$$

$$R_2 = \{ \emptyset \}$$

	4	7	8
1	0	0	0
2	0	0	0
3	0	0	0