



Q8] consider the following values  $y$  and  $\hat{y}$  calculate Categorical Cross-entropy loss.

Ans:

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\hat{y} = \begin{bmatrix} 0.1 & 0.2 & 0.5 & 0.2 & 0 \\ 0.5 & 0.1 & 0.2 & 0.2 & 0 \\ 0.3 & 0.2 & 0.1 & 0.2 & 0.2 \\ 0.1 & 0.4 & 0.4 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 & 0 \end{bmatrix}$$

Categorical cross-entropy loss

$$= -\frac{1}{n} \sum_{i=1}^n \sum_{j=1}^m y_i \log \hat{y}_{ij}$$

Lets take First training sample:-

$$y_1 = [0 \quad 0 \quad 1 \quad 0 \quad 0]$$

$$\hat{y}_1 = [0.1 \quad 0.2 \quad 0.5 \quad 0.2 \quad 0]$$

$$L_1 = \sum_{j=1}^m [0 \times \log(0.1) + 0 \times \log(0.2) + 1 \times \log(0.5) + 0 \times \log(0.2) + 0 \times \log(0)]$$

$$= [0 + 0 + -0.301 + 0 + 0]$$

$$L_1 = -0.301$$

Let's consider second sample.

$$L_2 = 1$$

$$Y_2 = [0 \ 1 \ 0 \ 0 \ 0]$$

$$\hat{Y}_2 = [0.5 \ 0.1 \ 0.2 \ 0.2 \ 0]$$

$$L_2 = 0 \times \log(0.5) + 1 \times \log(0.1) + 0 \times \log(0.2) + 0 \times \log(0.2) + 0 \times \log(0)$$

$$L_2 = -1$$

$$Y_3 = [0 \ 0 \ 0 \ 1 \ 0]$$

$$\hat{Y}_3 = [0.3 \ 0.2 \ 0.1 \ 0.2 \ 0.2]$$

$$L_3 = 0 \times \log(0.3) + 0 \times \log(0.2) + 0 \times \log(0.1) + 1 \times \log(0.2) + 0 \times \log(0.2)$$

$$L_3 = -0.6989$$

$$Y_4 = [0 \ 0 \ 0 \ 1 \ 0]$$

$$\hat{Y}_4 = [0.1 \ 0.4 \ 0.4 \ 0 \ 0]$$

$$L_4 = 0 \times \log(0.1) + 0 \times \log(0.4) + 0 \times \log(0.4) + 1 \times \log(0) + 0 \times \log(0)$$

$$L_4 = 0$$

$$Y_5 = [1 \ 0 \ 0 \ 0 \ 0]$$

$$\hat{Y}_5 = [0.5 \ 0.5 \ 0 \ 0 \ 0]$$

$$= 1 \times \log(0.5) + 0 + 0 + 0 + 0$$

$$= -0.301$$

$$\begin{aligned}\text{Loss} &= -\frac{1}{n} [-0.301 + (-1) - 0.6989 + 0 - 0.301] \\ &= -\frac{1}{5} [-2.3009]\end{aligned}$$

$$\text{Loss} = 0.46018.$$

Entropy  
Categorical loss of given samples is  
0.46018.