Markow Chain:

$$P(X_1=j) = \sum_{k=1}^{r} P(X_1=j \mid X_0=k) = \sum_{k=1}^{r} P_{kj}\, P(X_0=k)$$

$$\pi^{(1)} = \pi^{(0)} P$$

$$\pi^{(n+1)} = \pi^{(n)} \qquad n=0,1,2\ldots$$

$$\pi^{(n)} = \pi^{(0)} P^{n} \qquad n=0,1,2\ldots$$

$$\pi^{(3)} = \pi^{(2)} P = \pi^{0} P^{3}$$

$$\pi^{(2)} = \pi^{(1)} P = \pi^{(0)} P^{2}$$

$$\pi^{(1)} = \pi^{(0)} P$$

$n \qquad\qquad n+1$

$$i \xrightarrow{P_{ij}} j$$

$m \qquad\qquad m+n$

$$i \text{ ---- } \bigcirc \cdots \bigcirc \cdots j$$

want to get from i to j in n steps

$P_{ij}^{(n)} \leftarrow r_{ij}(n)$ textbook

Use total probability.

$$P_{ij}^{(2)} = P(X_2=j \mid X_0=i) = \sum_{k\le s} P(X_2=j \mid X_1=k, X_0=i)\, P(X_1=k \mid X_0=i)$$

$$= \sum P_{kj}\, P_{ik}$$

$$\frac{1}{3} + \frac{2}{9} = \frac{5}{9}$$

$$\frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{3}$$

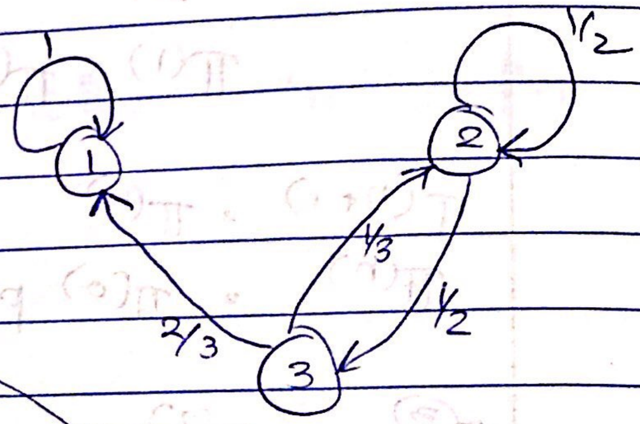
$$\frac{1}{6} + \frac{1}{9} = \frac{\cancel{15}\ 5}{\cancel{54}\ 18}$$

$$\therefore P(X_2 = j \mid X_1 = k) = P_{kj}$$

$$P_{ij}^{(2)} = \sum_{k \in S} P_{kj} \, P_{ik}$$

Going from i to j in 2 steps

$$P = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \hline \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 2/3 & 1/3 & 0 \end{bmatrix} \end{array}$$

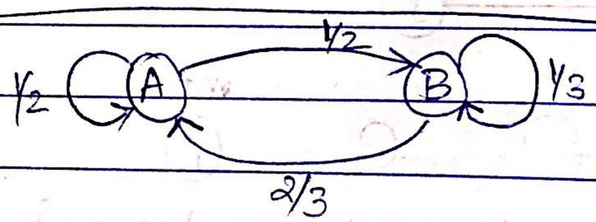


$$\pi^{(1)} = \begin{bmatrix} 1/3 & 1/3 & 1/3 \end{bmatrix}$$

Value grows

$$\pi^{(2)} = \begin{bmatrix} 5/9 & 5/18 & 1/6 \end{bmatrix}$$

Q.



$$\pi^{(0)} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$P = \begin{array}{c|c} & \begin{matrix} A & B \end{matrix} \\ \hline \begin{matrix} A \\ B \end{matrix} & \begin{bmatrix} 1/2 & 1/2 \\ 2/3 & 1/3 \end{bmatrix} \end{array}$$

What is $P(X_3 = B) = ?$

$$\therefore \pi^{(3)} = \pi^{(2)} P = \pi^{(0)} P^3$$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 \\ 2/3 & 1/3 \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 \\ 2/3 & 1/3 \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 \\ 2/3 & 1/3 \end{bmatrix}$$

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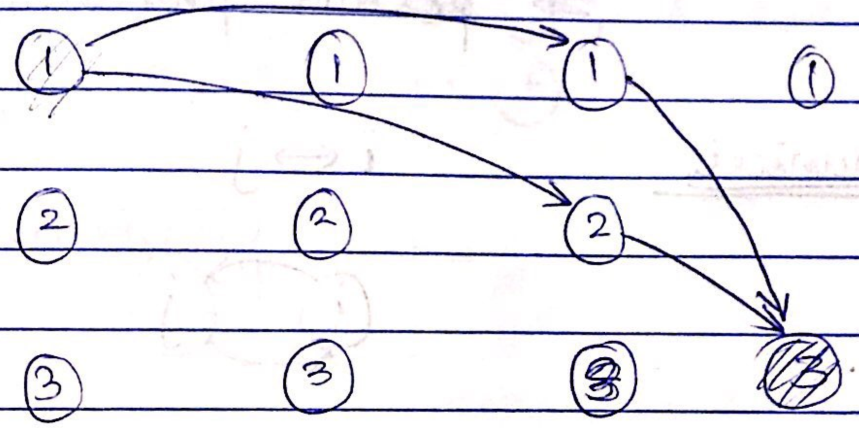
Chapman-Kolmogorov Equation

$$P_{ij}^{(m+n)} = P(X_{m+n} = j \mid X_0 = i) = \sum_{k \in S} P_{ik}^{(m)} P_{kj}^{(n)}$$

X_1 - - - - - - - - - - - - X_4



$$P_{13}^{(3)} = \sum_{k=1}^{3} P_{k3} P_{1k}^{(2)}$$

writing 3 step in 2 step probability

$4 - 1 + 3$

$2 + 2$

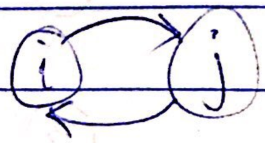
Classification of States

accesible j is accessible i

⇒ if theres a probability to go

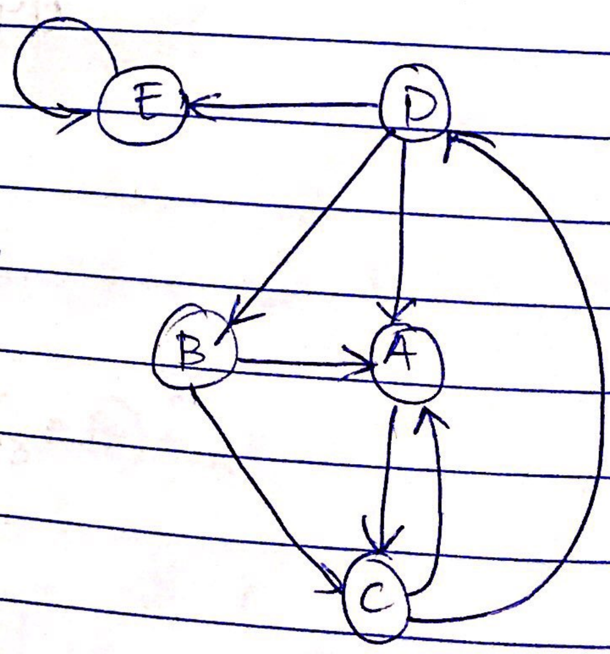
⇒ positive probability

communicate $i \leftrightarrow j$



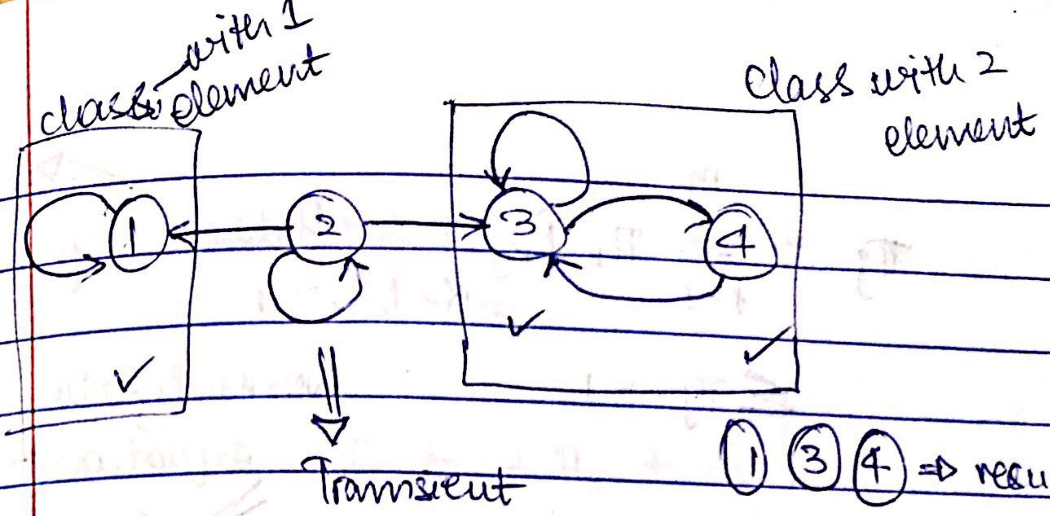
recurrent if you will visit that state again

transient once visited, you wont visit



D is transient

B is reccurrent



①③④ ⇒ recurrent

If I start from ①, I can end up at ① or ③ or ④

Doesn't it depend on initial state??

Periodicity

7.3 Steady state Behaviour

$$\pi_j^{(n)} \simeq P(X_n = j) \quad \text{if } n \to \infty$$

or large number

⇓

Steady state Behaviour

Steady state convergence Theorem:

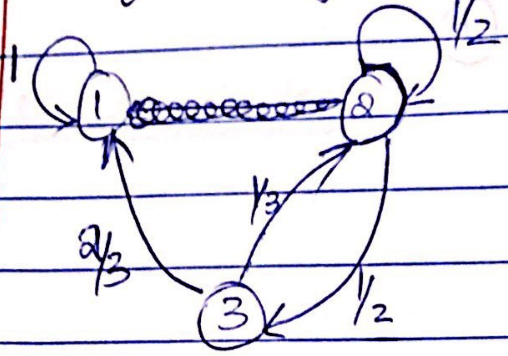
a) $\lim_{n\to\infty} P_{ij}(n) = \pi_j$

b) $\pi_j = \sum_{K=1}^{m} \pi_K P_{Kj}$ Balance Equation $\Rightarrow$ gives (r-1) equation

$K = 1, 2, \cdots r$

$\sum \pi_j = 1$ Normalization equation - 1 $\Rightarrow$ 1 independant equation

$S = \{1 \cdots\cdots r\}$



$\pi_1 = P_{11}\pi_1 + P_{21}\pi_2 + P_{31}\pi_3$

$\pi_2 = P_{12}\pi_1 + P_{22}\pi_2 + P_{32}\pi_3$

We dont write π_3 as its dependent on π_2

Only π_1 and π_2 are independent

$\pi_1 + \pi_2 + \pi_3 = 1$

$\pi_1 = 1 \cdot \pi_1 + 0 \cdot \pi_2 + \frac{2}{3}\pi_3 = \pi_1 + \frac{2}{3}\pi_3$

$\Rightarrow$

$$\pi_1 = \pi_1 + \frac{2}{3}\pi_3$$

$$\Rightarrow \pi_3 = 0$$

$$\pi_2 = 0 \cdot \pi_1 + \frac{1}{2}\pi_2 + \frac{1}{3}\pi_3$$

$$\frac{1}{2}\pi_2 = \frac{1}{3}\pi_3 \qquad \because \pi_3 = 0$$

$$\therefore \pi_2 = 0$$

$$\therefore \pi_1 + \pi_2 + \pi_3 = 0$$

$$\Rightarrow \pi_1 = 1$$

* when we start at 1 we end up at 1 when $n \to \infty$??

No.

We will always start at 1

Example 7.6

$P(\text{rain}) = p$

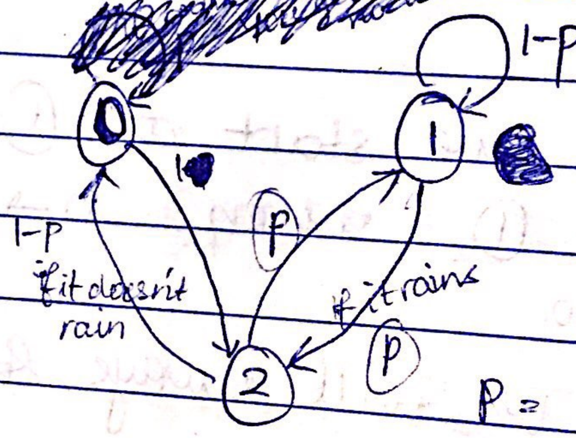
Take the umbrella only when it rains.

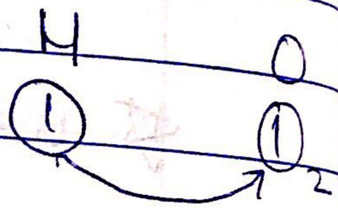
What defines the Markov chain?

\# of umbrellas.

$X \rightarrow$ No. of umbrellas I have ~~with~~ with me

$S = \{0, 1, 2\}$





$$P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1-p & p \\ 1-p & p & 0 \end{bmatrix}$$

Regardless of where I am, if it ~~are~~ rains I carry my umb. from 1 umb. location to other.

$P(\text{get wet})$

If it rains & I have no umb.

$$p * \underbrace{P(X_n = 0)}_{\pi_0}$$

$$\pi_0 = P_{00}\pi_0 + P_{10}\pi_1 + P_{20}\pi_2$$

$$\pi_1 = P_{01}\pi_0 + P_{11}\pi_1 + P_{21}\pi_2$$

Why isn't it also independent?

$$\pi_0 = 0.\pi_0 + 0.\pi_1 + (1-p)\pi_2$$

It is
You can take it

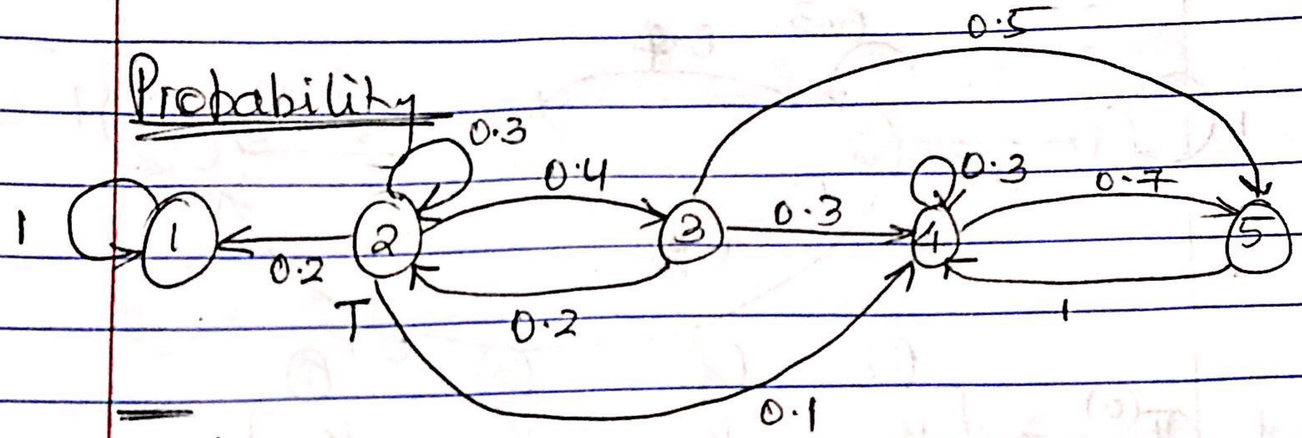
$$\pi_0 = (1-p)\pi_2$$

$$\pi_1 = 0.\pi_0 + (1-p)\pi_1 + p.\pi_2$$

$$\pi_0 = \frac{1-p}{3-p}$$

This is for the long term

Probability



class 1

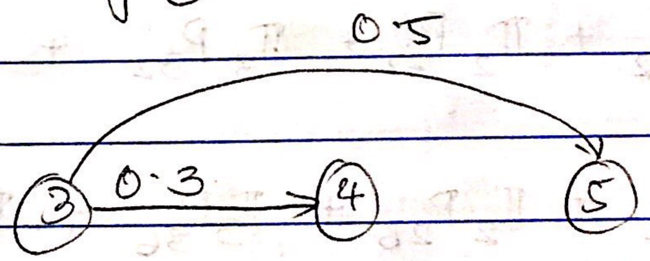
class 2

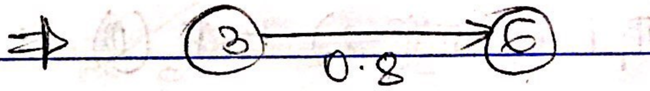
(2) Transient.
(1) Recurrent.
(3) Transient.
(4) Recurrent
(5) Recurrent.

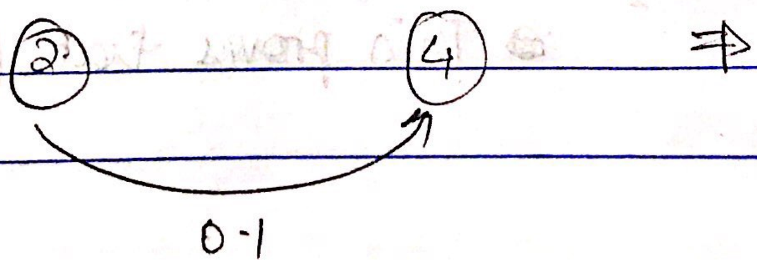
☆ either end up at (1) or (4)&(5) ⇒ class 1 Or class 2

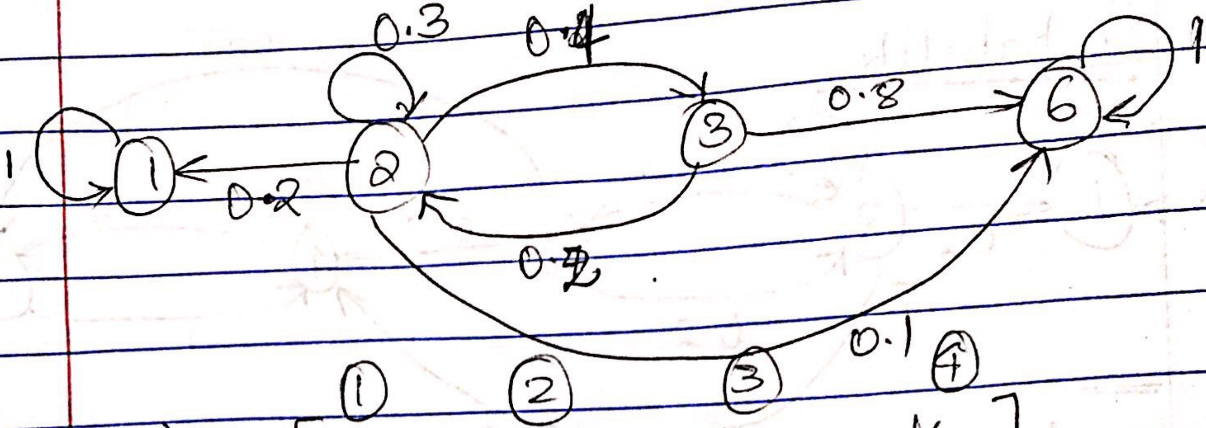
class: 2 or more states communicat to each other.

Replacing (4) and (5) with (6)









$$\begin{array}{ccccc} & 1 & 2 & 3 & 4 \\ \pi^{(0)} = & [\,1/4 & 1/4 & 1/4 & 1/4\,] \end{array}$$

Initial state

Steady state Behavior of the System

Steady State convergence Theorem

$$\pi_j = \sum_{k=1}^{m} \pi_k P_{kj} = (m-1) \text{ independent equations.}$$

$$\sum_{k=1}^{m} \pi_j = 1$$

$$\pi_1 = \pi_1 P_{11} + \pi_2 P_{21} + \pi_3 P_{31} + \pi_6 P_{61}$$

$$\pi_2 = \pi_1 P_{12} + \pi_2 P_{22} + \pi_3 P_{32} + \pi_6 P_{62}$$

$$\pi_6 = \pi_1 P_{16} + \pi_2 P_{26} + \pi_3 P_{36} + \pi_6 P_{66}$$

$$\Rightarrow \cancel{\pi_1} = \cancel{\pi_1 \cdot 1} + \pi_2(0.2) + \cancel{\pi_3(0)} + \cancel{\pi_6(0)}$$

$$\pi_2 = 0$$

This proves that 2 is Transient

$$\pi_2 = \pi_1(0) + \underbrace{\pi_2(0.3)}_{0} + \pi_3(0.2) + \pi_6(0)$$

$$\pi_2 = \pi_3(0.2)$$

$\therefore \pi_3 = 0 \quad \because \pi_2 = 0$

③ is transient.

~~$\pi_6 = \pi_1(0) + \pi_2(0.1)$~~

Now we know,

$$\pi_1 + \underbrace{\pi_2 + \pi_3}_{0} + \pi_6 = 1$$

$$\therefore \pi_1 = 1 - \pi_6$$

$$\pi_6 = \pi_1(0) + \pi_2(0.1) + \pi_3(0.8) + \pi_6(1)$$

$$\pi_6 = \pi_6$$

Can we ~~do~~ solve for π_1 or π_6

No.

$$\pi_1 = 1 - \pi_6$$

This proves that if started at ② we will end up at ① or ⑥

Short-term Probability that you will end up in 1 or 6

equally likely% Long-term You will end up in either 1 or 6

Let $a_i = P(X_n$ eventually become equal to the absorbing state $s \mid X_0 = i)$

2 different absorbing states 1 and 6

$a_s = 1 \Rightarrow$ start at 6 and end up at 6

$a_i = \phi \Rightarrow$ start at 1 and end up at any another state

Transient states $a_i = \sum_{j=1}^{M} P_{ij} a_j$ Transient state

Probability

$a_1 = 0$ (1 becoming 6) $s = 6$

$a_6 = 1$ (6 becoming 6)

$a_2 = P_{21} a_1 + P_{23} a_3 + P_{22} a_2 + P_{26} a_6$

$a_3 = P_{31} a_1 + P_{32} a_2 + P_{33} a_3 + P_{36} a_6$

$P_{21} = P_{31} = 0$

Previous 'π' P_{ji}

Here $a_i \rightarrow P_{ij}$

$\therefore \cancel{a_2} = (0.7)a_2 - 0.4a_3 = 0.1$

$-(0.2)a_2 + a_3 = 0.8$

$\cancel{a_2}$ $\underbrace{a_2 = \frac{21}{31}}_{\text{start at (2) and end at (6)}}$ $\underbrace{a_3 = \frac{29}{31}}_{\text{start at (3) and end at (6)}}$

What we calculated was $\pi_1 = 1 - \pi_6$

Probability that you started at i and ended up at 6

Probability : $\cancel{a_2 = 1 - \frac{21}{31}}$

$\therefore \pi_1 = 1 - \pi_6$

$a_2 = 1 - \frac{21}{31}$ start at 2 end at 1

$a_3 = 1 - \frac{29}{31}$ start at 3 end at 1

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Expected time that start at i and end up at s

NOW,

μ_i = E[# of transition until absorption, starting from i]

μ_i if i is recurrent.

$$\mu_1 = 0$$
$$\mu_6 = 0$$

μ_i if i is transient

$$\mu_i = \underbrace{1}_{\text{first you have to go to } j \Rightarrow 1 \text{ step}} + \sum_{j=1}^{n} P_{ij}\,\mu_j$$

$\therefore \mu_1 = \mu_6 = 0$

$$\mu_2 = 1 + \underbrace{\cancel{P_{21}\mu_1}}_{0} + P_{22}\mu_2 + P_{23}\mu_3 + \underbrace{\cancel{P_{26}\mu_6}}_{0}$$

$$\mu_3 = 1 + \cancelto{0}{P_{31}\mu_1} + P_{32}\mu_2 + P_{33}\mu_3 + \cancelto{0}{P_{36}\mu_6}$$

$$\mu_2 = 1 + (0.3)\mu_2 + (0.4)\mu_3$$

$$(0.7)\mu_2 - (0.4)\mu_3 = 1$$

$$\mu_3 = 1 + (0.2)\mu_2 + (0)\mu_3$$

$$\mu_3 - (0.2)\mu_2 = 1$$

$$\therefore \mu_2 = \frac{1.4}{0.62} \Big\} \text{Avergge time from 2 to \$}$$

$$\mu_3 = 1 + \frac{0.28}{0.62} \Big\} \text{3 to \$}$$

☆☆ E[time from 2 to 1]
= E[time from 2 to 6]

☆☆ E[time from 3 to 1]
= E[time from 3 to 6]

Project

1000 people.

watch movies

derive 2 kinds of information

F_1 is subjective measure

F_2 is NOT subjective measure

F1 and F2 are independent

make it normalised

F1 ~~and~~ F2 ~~are~~ is normal distribution

Get μ and σ^2

Data of 100 people - given to learn this distribution.

Task: Design a classifier for any new data. Basically build a model.

Five Classes C_1 C_2 ..... C_5

$i = \operatorname{argmax} P(C_i \mid x)$

x → feature

for example if you use f_1 as X

$\rightarrow i = \operatorname{argmax} P(C_i \mid F_1)$

HINTS:

1) Only use F1
2) Only use F2
3) Use F1 and F2

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$C_1 : 100 \rightarrow m_1, \sigma_1^2$ $\quad P(F_1|C_1) = N(m_1, \sigma_1^2)$

$C_2 : 100 \rightarrow m_2, \sigma_2^2$ $\quad P(F_1|C_2) = N(m_2, \sigma_2^2)$

$C_3 : 100 \rightarrow m_3, \sigma_3^2$ $\quad \vdots$

$C_4 : 100 \rightarrow m_4, \sigma_4^2$

$C_5 : 100 \rightarrow m_5, \sigma_5^2$ $\quad P(F|C_5) = N(m_5, \sigma_5^2)$

Calculate $\rightarrow$ Use Z score make it normalised

$$P(C_1|F_1) = \frac{P(F_1|C_1)\,P(C_1)}{P(F_1)}$$

$$P(C_2|F_1)$$

for comparison remove $P(F_1)$

$$P(C_1|F_1) = P(F_1|C_1)\cancel{P(C_1)} \rightarrow N(m_1, \sigma_1^2)$$

person f_1 Actual

	C_1	C_2	C_3	C_4	C_5
Predicted C_1	50				
C_2					
C_3					
C_4					
C_5					

CLT . $50 \pm \longrightarrow$

$$P(X|C_1) = N\left(\begin{bmatrix} m_1 \\ m_2 \end{bmatrix} \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}\right)$$