

Plane Regions in Polar Coordinates

Worcester Polytechnic Institute
Department of Mathematical Sciences

Calculus Labs

Area Example 1

Using polar coordinate inequalities describe the region in the first quadrant of the xy -plane that lies inside of the graph of

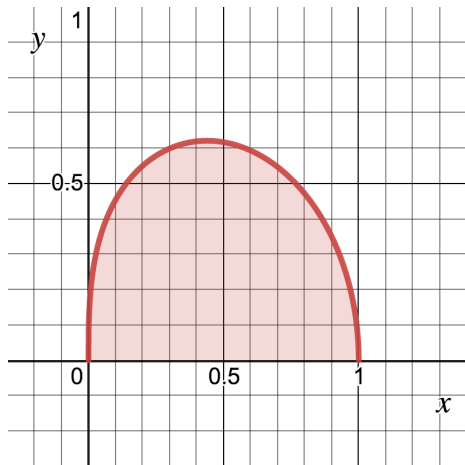
$$r = \sqrt{\cos \theta}. \quad (1)$$

We plot this region using www.desmos.com/calculator

Area Depiction

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Area Example 2

Using polar coordinate inequalities describe the region in the xy -plane that lies inside of the graph of

$$r = 3 \sin \theta \quad (2)$$

and outside of the graph of

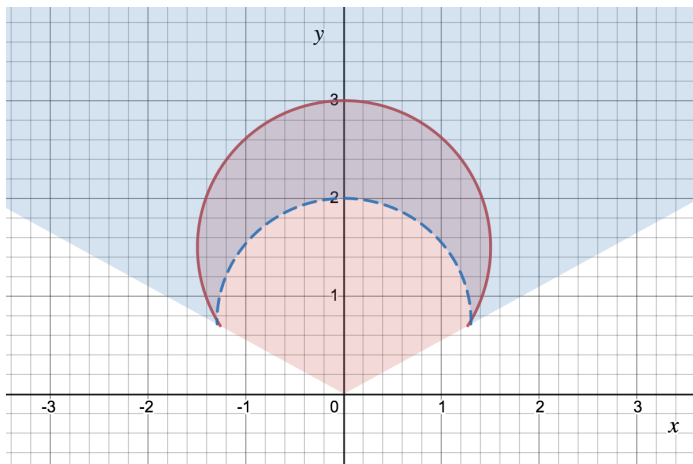
$$r = 1 + \sin \theta. \quad (3)$$

We plot this region using www.desmos.com/calculator

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Analytic Solution

We find the intersection of

$$r = 3 \sin \theta \quad (4)$$

and

$$r = 1 + \sin \theta. \quad (5)$$

$$3 \sin \theta = 1 + \sin \theta$$

Polar Region

The region in the the xy -plane that lies inside of the graph of

$$r = 3 \sin \theta \quad (6)$$

and outside of the graph of

$$r = 1 + \sin \theta. \quad (7)$$

can be described by

$$1 + \sin \theta \leq r \leq 3 \sin \theta \text{ and } \frac{\pi}{6} \leq \theta \leq \frac{5\pi}{6} \quad (8)$$