Taylor Polynomials
Arctangent

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Calculus Labs

Maclaurin Series for arctan u

$$\frac{d\arctan(u)}{du} = \frac{1}{1+u^2}$$

$$\frac{1}{1+u^2} = \frac{a}{1-r}$$ - Geometric series sum

$$\frac{1}{1--u^2} = \sum_{n=0}^{\infty}[-u^2]^n = \sum_{n=0}^{\infty}(-1)^n u^{2n}$$

$1 - u^2 + u^4 - u^6 + \dots$ for $|u| < 1$.

Maclaurin Series for arctan u

$$\arctan u = \int \frac{d \arctan u}{du} du = \int \frac{1}{1 - -u^2} \text{ du}$$

$$= \int \sum_{n=0}^{\infty} (-1)^n u^{2n} du = \sum_{n=0}^{\infty} (-1)^n \frac{u^{2n+1}}{2n+1} + C$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{u^{2n+1}}{2n+1} + C$$

$$= C + u - \frac{u^3}{3} + \frac{u^5}{5} - \frac{u^7}{7} + \dots \text{ for } |u| < 1$$

$$\arctan u = C + u - \frac{u^3}{3} + \frac{u^5}{5} - \frac{u^7}{7} + \dots \text{ for } |u| < 1 \qquad (1)$$

Since $\arctan 0 = 0$, $\arctan 0 = C + 0 - \frac{0^3}{3} + \frac{0^5}{5} - \frac{0^7}{7} + \dots = 0$ and $C = 0$.

Maclaurin Series for arctan u

$$\arctan u = \sum_{n=0}^{\infty} (-1)^n \frac{u^{2n+1}}{2n+1} = u - \frac{u^3}{3} + \frac{u^5}{5} - \frac{u^7}{7} + \ldots \text{ for } |u| < 1 \tag{2}$$

Maclaurin Polynomials for arctan u

$$T_1(u) = u$$

$$T_3(u) = u - \frac{u^3}{3}$$

$$T_5(u) = u - \frac{u^3}{3} + \frac{u^5}{5}$$

$$T_7(u) = u - \frac{u^3}{3} + \frac{u^5}{5} - \frac{u^7}{7}$$

Graph in Desmos: https://www.desmos.coms

Substitution

$$\arctan u = \sum_{n=0}^{\infty} (-1)^n \frac{u^{2n+1}}{2n+1} = u - \frac{u^3}{3} + \frac{u^5}{5} - \frac{u^7}{7} + \dots \text{ for } |u| < 1 \tag{3}$$

Substitute $u = bx$ into above:

$$\arctan(bx) = \sum_{n=0}^{\infty} (-1)^n \frac{(bx)^{2n+1}}{2n+1} = \sum_{n=0}^{\infty} (-1)^n \frac{b^{2n+1} x^{2n+1}}{2n+1}$$

$$= bx - \frac{b^3 x^3}{3} + \frac{b^5 x^5}{5} - \frac{b^7 x^7}{7} + \dots \text{ for } |bx| < 1 \text{ or } |x| < \frac{1}{|b|}$$

Assignment

Lab assignment works with:

$$f(u) = \ln(1+u) \tag{4}$$

$$\frac{d\ln(1+u)}{du} = \frac{1}{1+u} \tag{5}$$