

Taylor Polynomials Arctangent

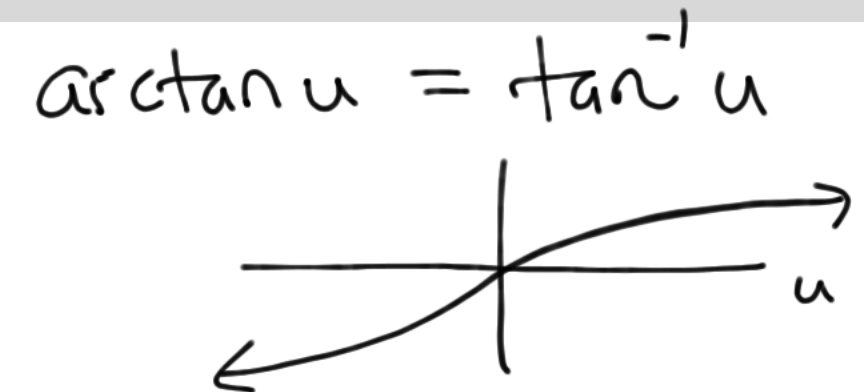
Worcester Polytechnic Institute
Department of Mathematical Sciences

Calculus Labs

Maclaurin Series for $\arctan u$

$$a = 0$$

$$\frac{d \arctan(u)}{du} = \frac{1}{1 + u^2}$$



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Lab
Assignment

$$\frac{\textcircled{1}}{1 + u^2} = \frac{\textcircled{a}}{1 - \textcircled{r}}$$

$(-u^2)$ ↗

- Geometric series sum

$$\frac{1}{1 - \underbrace{-u^2}_r} = \sum_{n=0}^{\infty} [-u^2]^n = \sum_{n=0}^{\infty} (-1)^n u^{2n}$$

$$[(-1)u^2]^n$$

$$1 - u^2 + u^4 - u^6 + \dots \text{ for } |u| < 1.$$

$$\begin{aligned} r &= -u^2 \\ |r| &= |-u^2| < 1 \\ |u^2| &< 1 \\ |u| &< 1 \end{aligned}$$

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$$\arctan u = \int \frac{d \arctan u}{du} du = \int \frac{1}{1 - (-u^2)} du$$

$$= \int \sum_{n=0}^{\infty} (-1)^n u^{2n} du = \sum_{n=0}^{\infty} (-1)^n \frac{u^{2n+1}}{2n+1} + C$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{u^{2n+1}}{2n+1} + C$$

$$= C + u - \frac{u^3}{3} + \frac{u^5}{5} - \frac{u^7}{7} + \dots \text{ for } |u| < 1$$

$$\arctan u = C + u - \frac{u^3}{3} + \frac{u^5}{5} - \frac{u^7}{7} + \dots \text{ for } |u| < 1 \quad (1)$$

Since $\arctan 0 = 0$, $\arctan 0 = C + 0 - \frac{0^3}{3} + \frac{0^5}{5} - \frac{0^7}{7} + \dots = 0$
and $C = 0$.



Maclaurin Series for $\arctan u$

$$T_3(u) = u - \frac{u^3}{3}$$

$$\arctan u = \sum_{n=0}^{\infty} (-1)^n \frac{u^{2n+1}}{2n+1} = u - \frac{u^3}{3} + \frac{u^5}{5} - \frac{u^7}{7} + \dots \quad \text{for } |u| < 1 \quad (2)$$

$$\uparrow$$
$$T_1(u) = u$$

Maclaurin Polynomials for $\arctan u$

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$$T_1(x) = u$$

$$T_3(x) = u - \frac{u^3}{3}$$

$$T_5(x) = u - \frac{u^3}{3} + \frac{u^5}{5}$$

$$T_7(x) = u - \frac{u^3}{3} + \frac{u^5}{5} - \frac{u^7}{7}$$

Graph in Desmos: <https://www.desmos.com>

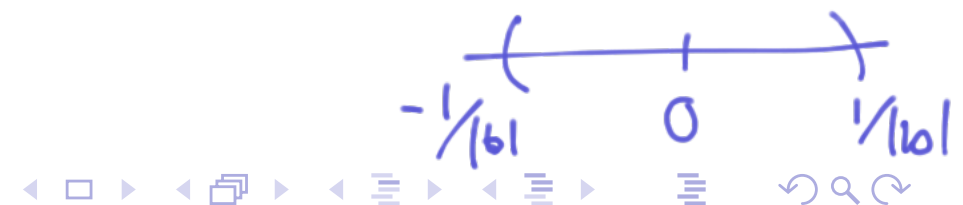
Substitution

$$\arctan u = \sum_{n=0}^{\infty} (-1)^n \frac{u^{2n+1}}{2n+1} = u - \frac{u^3}{3} + \frac{u^5}{5} - \frac{u^7}{7} + \dots \text{ for } |u| < 1 \quad (3)$$

Substitute $u = bx$ into above:

$$\arctan(bx) = \sum_{n=0}^{\infty} (-1)^n \frac{(bx)^{2n+1}}{2n+1} = \sum_{n=0}^{\infty} (-1)^n \frac{b^{2n+1} x^{2n+1}}{2n+1}$$

$$= bx - \frac{b^3 x^3}{3} + \frac{b^5 x^5}{5} - \frac{b^7 x^7}{7} + \dots \text{ for } \underbrace{|bx|}_{|u| < 1} < 1 \text{ or } |x| < \frac{1}{|b|}$$



Assignment

Lab assignment works with:

$$f(u) = \ln(1 + u) \quad (4)$$

$$\frac{d \ln(1 + u)}{du} = \frac{1}{1 + u} = \frac{1}{1 - -u} \quad (5)$$