| CODE   | COURSE NAME         | CATEGORY | L | T | Р | CREDIT |
|--------|---------------------|----------|---|---|---|--------|
| MET201 | MECHANICS OF SOLIDS | PCC      | 3 | 1 | 0 | 4      |

#### Preamble:

This course helps the students to understand the concept of stress and strain in different types of structure/machine under various loading conditions. The course also covers simple and compound stresses due to forces, stresses and deflection in beams due to bending, torsion in circular section, strain energy, different theories of failure, stress in thin cylinder thick cylinder and spheres due to external and internal pressure.

Prerequisite: EST100 ENGINEERING MECHANICS

## **Course Outcomes:**

After the completion of the course the student will be able to

| CO 1 | Determine the stresses, strains and displacements of structures by tensorial and graphical (Mohr's circle) approaches |
|------|---|
| CO 2 | Analyse the strength of materials using stress-strain relationships for structural and thermal loading                |
| CO 3 | Perform basic design of shafts subjected to torsional loading and analyse beams subjected to bending moments          |
| CO 4 | Determine the deformation of structures subjected to various loading conditions using strain energy methods           |
| CO 5 | Analyse column buckling and appreciate the theories of failures and its relevance in engineering design               |

# Mapping of course outcomes with program outcomes

|      | PO 1 | PO 2 | PO 3 | PO 4 | PO 5 | PO 6 | PO 7 | PO 8 | PO 9 | РО | РО | РО |
|------|------|------|------|------|------|------|------|------|------|----|----|----|
|      |      |      |      |      |      |      |      |      | 1100 | 10 | 11 | 12 |
| CO 1 | 3    | 3    | 2    |      | D.   | 2017 |      | 00 0 | 100  |    |    | 1  |
| CO 2 | 3    | 3    | 2    |      | 100  |      | 100  |      |      |    |    | 1  |
| CO 3 | 3    | 3    | 1    |      |      |      |      |      |      |    |    | 2  |
| CO 4 | 3    | 3    | 1    |      |      |      |      |      |      |    |    | 1  |
| CO 5 | 3    | 3    | 1    |      |      |      |      | -    |      |    |    | 1  |

#### **Assessment Pattern**

| Bloom's    |     | nuous<br>ent Tests | End Semester |  |  |
|------------|-----|--------------------|--------------|--|--|
| Category   | 1 2 |                    | Examination  |  |  |
| Remember   | 10  | 10                 | 20           |  |  |
| Understand | 20  | 20                 | 30           |  |  |
| Apply      | 20  | 20                 | 50           |  |  |
| Analyse    | API | ABI                |              |  |  |
| Evaluate   | rry | TIA                | IOIC         |  |  |
| Create     | 1-1 | HIN                | V ( )   (    |  |  |

#### Mark distribution

| Total Marks | CIE | ESE | ESE<br>Duration |
|-------------|-----|-----|-----------------|
| 150         | 50  | 100 | 3 hours         |

## **Continuous Internal Evaluation Pattern:**

Attendance : 10 marks
Continuous Assessment Test (2 numbers) : 25 marks
Assignment/Quiz/Course project : 15 marks

#### **End Semester Examination Pattern:**

There will be two parts; Part A and Part B. Part A contain 10 questions with 2 questions from each module and having 3 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which student should answer any one. Each question carries 14 marks and can have a maximum of 2 subdivisions.

# **COURSE LEVEL ASSESSMENT QUESTIONS**

## Course Outcome 1 (CO1):

- 1. Determine the resultant traction at a point in a plane using the stress tensor.
- 2. Evaluate the principal stresses, principal strains and their directions from a given state of stress or strain.
- 3. Write the stress tensor and strain tensor.

## Course Outcome 2 (CO2)

- 1. Write the generalized Hooke's law for stress-strain relations.
- 2. Estimate the state of strain from a given state of stress.
- 3. Analyse the strength of a structure subjected to thermal loading.

## Course Outcome 3(CO3):

- 1. Design a shaft to transmit power and torque.
- 2. Draw the shear force and bending moment diagrams.
- 3. Determine the bending stress on a beam subjected to pure bending.

#### Course Outcome 4 (CO4):

- 1. Apply strain energy method to estimate the deformation of a structure.
- 2. Use strain energy method to calculate deformations for multiple loads.
- 3. Use strain energy method to estimate the loads acting on a structure for a maximum deflection.

#### Course Outcome 5 (CO5):

- 1. Analyse a column for buckling load.
- 2. Use Rankine formula to determine the crippling load of columns.
- 3. A bolt is subjected to a direct tensile load of 20 kN and a shear load of 15 kN. Suggest suitable size of this bolt according to various theories of elastic failure, if the yield stress in simple tension is 360 MPa. A factor of safety 2 should be used. Assume Poisson's ratio as 0.3.

## **SYLLABUS**

#### Module 1

Deformation behaviour of elastic solids in equilibrium under the action of a system of forces, method of sections. Stress vectors on Cartesian coordinate planes passing through a point, stress at a point in the form of a matrix. Equality of cross shear, Cauchy's equation. Displacement, gradient of displacement, Cartesian strain matrix, strain- displacement relations (small-strain only), Simple problems to find strain matrix. Stress tensor and strain tensor for plane stress and plane strain conditions. Principal planes and principal stress, meaning of stress invariants, maximum shear stress. Mohr's circle for 2D case.

#### Module 2

Stress-strain diagram, Stress-Strain curves of Ductile and Brittle Materials, Poisson's ratio.

Constitutive equations-generalized Hooke's law, equations for linear elastic isotropic solids in terms of Young's Modulus and Poisson's ratio, Hooke's law for Plane stress and plane strain conditions Relations between elastic constants E, G, v and K(derivation not required).

Calculation of stress, strain and change in length in axially loaded members with single and composite materials, Effects of thermal loading – thermal stress and thermal strain. Thermal stress on a prismatic bar held between fixed supports.

#### Module 3

Torsional deformation of circular shafts, assumptions for shafts subjected to torsion within elastic deformation range, derivation of torsion formula Torsional rigidity, Polar moment of inertia, basic design of transmission shafts. Simple problems to estimate the stress in solid and hollow shafts.

Shear force and bending moment diagrams for cantilever and simply supported beams. Differential equations between load, shear force and bending moment.

Normal and shear stress in beams: Derivation of flexural formula, section modulus, flexural rigidity, numerical problems to evaluate bending stress, economic sections.

Shear stress formula for beams: (Derivation not required), shear stress distribution for a rectangular section.

#### Module 4

Deflection of beams using Macauley's method

Elastic strain energy and Complementary strain energy. Elastic strain energy for axial loading, transverse shear, bending and torsional loads. Expressions for strain energy in terms of load, geometry and material properties of the body for axial, shearing, bending and torsional loads. Castigliano's second theorem, reciprocal relation(Proof not required for Castigliano's second theorem, reciprocal relation).

Simple problems to find the deflections using Castigliano's theorem.

## Module 5

Fundamentals of bucking and stability, critical load, equilibrium diagram for buckling of an idealized structure. Buckling of columns with pinned ends, Euler's buckling theory for long columns. Critical stress, slenderness ratio, Rankine's formula for short columns.

Introduction to Theories of Failure, Rankine's theory for maximum normal stress, Guest's theory for maximum shear stress, Saint-Venant's theory for maximum normal strain, Hencky-von Mises theory for maximum distortion energy, Haigh's theory for maximum strain energy

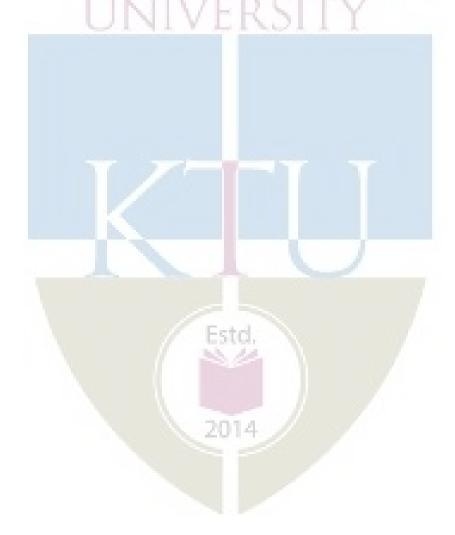
#### **Text Books**

- 1. Mechanics of materials in S.I. Units, R.C. Hibbeler, Pearson Higher Education 2018
- 2. Advanced Mechanics of Solids, L. S. Srinath, McGraw Hill Education

3. Design of Machine Elements, V. B Bhandari, McGraw Hill Education

## **Reference Books**

- 1. Engineering Mechanics of Solids, Popov E., PHI 2002
- 2. Mechanics of Materials S. I. units, Beer, Johnston, Dewolf, McGraw Hills 2017
- 3. Mechanics of Materials, Pytel A. and Kiusalaas J. Cengage Learning India Private Limited,  $2^{nd}$ Edition, 2015
- 4. Strength of Materials, Rattan, McGraw Hills 2011
- 5. Strength of Materials, Surendra Singh, S. K. Kataria& Sons



# **COURSE PLAN**

| No     | Topic   | No of lectures |
|--------|---|----------------|
| 1      | Module 1: Stress and Strain Analysis  | 9 hours        |
| 1.1    | Describe the deformation behaviour of elastic solids in equilibrium under the action of a system of forces. Describe method of sections to illustrate stress as resisting force per unit area. Stress vectors on Cartesian coordinate planes passing through a point and writing stress at a point in the form of a matrix. | 2 hr           |
| 1.2    | Equality of cross shear (Derivation not required). Write Cauchy's equation (Derivation not required), Find resultant stress, Normal and shear stress on a plane given stress tensor and direction cosines (no questions for finding direction cosines).   | 2 hr           |
| 1.3    | Displacement, gradient of displacement, Cartesian strain matrix, Write strain-displacement relations (small-strain only), Simple problems to find strain matrix given displacement field (2D and 3D), write stress tensor and strain tensor for Plane stress and plane strain conditions.                                   | 1 hr           |
| 1.4    | Concepts of principal planes and principal stress, characteristic equation of stress matrix and evaluation of principal stresses and principal planes as an eigen value problem, meaning of stress invariants, maximum shear stress   | 2 hrs          |
| 11.5   | Mohr's circle for 2D case: find principal stress, <mark>pl</mark> anes, stress on an arbitrary plane, maximum shear stress graphically using Mohr's circle  | 2 hrs          |
| 2      | Module 2: Stress - Strain Relationships   | 9 hours        |
| 12 1   | Stress-strain diagram, Stress—Strain curves of Ductile and Brittle Materials, Poisson's ratio   | 1 hr           |
| 2.2    | Constitutive equations-generalized Hooke's law, equations for linear elastic isotropic solids in in terms of Young's Modulus and Poisson's ratio (3D).  Hooke's law for Plane stress and plane strain conditions  Relations between elastic constants E, G, v and K(derivation not required),  Numerical problems           | 2 hrs          |
| 2.3    | Calculation of stress, strain and change in length in axially loaded members with single and composite materials, Effects of thermal loading – thermal stress and thermal strain. Thermal stress on a prismatic bar held between fixed supports.  | 2 hrs          |
|        |   | 4 hrs          |
| 3      | Module 3: Torsion of circular shafts, Shear Force-Bending Moment Diagrams and Pure bending  | 9 hours        |
| .D. II | Torsional deformation of circular shafts, assumptions for shafts subjected to torsion within elastic deformation range, derivation of torsion formula   | 1 hr           |
| ٥.۷    | shaft. Simple problems to estimate the stress in solid and hollow shafts  | 1 hr           |
| 3.3    | Numerical problems for basic design of circular shafts subjected to externally applied torques  | 1 hr           |
| 3.4    | Shear force and bending moment diagrams for cantilever and simply   | 2 hrs          |

|     | WEST IN THE E  | 10111111 |
|-----|--|----------|
|     | supported beams subjected to point load, moment, UDL and linearly varying  |          |
|     | load   |          |
| 3.5 | Differential equations between load, shear force and bending moment.   | 1 hr     |
|     | Normal and shear stress in beams: Derivation of flexural formula, section modulus, flexural rigidity, numerical problems to evaluate bending stress, economic sections  Shear stress formula for beams: (Derivation not required), numerical problem to find shear stress distribution for rectangular section | 3 hrs    |
| 4   | Module 4: Deflection of beams, Strain energy   | 8 hours  |
| 4.1 | Deflection of cantilever and simply supported beams subjected to point load, moment and UDL using Macauley's method (procedure and problems with multiple loads)   | 2 hrs    |
| 4.2 | Linear elastic loading, elastic strain energy and Complementary strain energy.<br>Elastic strain energy for axial loading, transverse shear, bending and torsional<br>loads (short derivations in terms of loads and deflections).   | 2 hr     |
|     | Expressions for strain energy in terms of load, geometry and material properties of the body for axial, shearing, bending and torsional loads. Simple problems to solve elastic deformations   | 2 hrs    |
| 4.4 | Castigliano's second theorem to find displacements, reciprocal relation, (Proof not required for Castigliano's second theorem and reciprocal relation).  | 1 hr     |
| 4.5 | Simple problems to find the deflections using Castigliano's theorem  | 1 hr     |
| 5   | Module 5: Buckling of Columns, Theories of F <mark>ail</mark> ure  | 8 hours  |
| 5.1 | Fundamentals of bucking and stability, critical <mark>lo</mark> ad, Euler's formula for long columns, assumptions and limitations, effect of end conditions(derivation only for pinned ends), equivalent length  | 2 hr     |
| 5.2 | Critical stress, slenderness ratio, Rankine's formula for short columns, Problems  | 3 hr     |
|     | Introduction to Theories of Failure. Rankine's theory for maximum normal stress, Guest's theory for maximum shear stress, Saint-Venant's theory for maximum normal strain  | 2 hr     |
| 5.4 | Hencky-von Mises theory for maximum distortion energy, Haigh's theory for maximum strain energy  | 1 hr     |

## **MODEL QUESTION PAPER**

## **APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**

#### THIRD SEMESTER B.TECH DEGREE EXAMINATION

**Course Code: MET201** 

**Course Name: MECHANICS OF SOLIDS** 

Max. Marks: 100 Duration: 3 Hours

## PART - A

### (ANSWER ALL QUESTIONS, EACH QUESTION CARRIES 3 MARKS)

- 1. Express the stress invariants in terms of Cartesian components of stress and principal stress.
- 2. Write down the Cauchy's strain displacement relationships.
- 3. Distinguish between the states of plane stress and plane strain.
- 4. Explain the generalized Hooke's law for a Linear elastic isotropic material.
- 5. List any three important assumptions in the theory of torsion.
- 6. Write the significance of flexural rigidity and section modulus in the analysis of beams.
- 7. Discuss reciprocal relation for multiple loads on a structure.
- 8. Express the strain energy for a cantilever beam subjected to a transverse point load at free end.
- 9. Discuss Saint-Venant's theory of failure.
- 10. Explain the term 'critical load' with reference to the buckling of slender columns.

#### PART - B

## (ANSWER ONE FULL QUESTION FROM EACH MODULE)

## MODULE - 1

- 11. a) The state of stress at a point is given by  $\sigma_{xx}$  = 12.31 MPa,  $\sigma_{yy}$  = 8.96 MPa,  $\sigma_{zz}$  = 4.34 MPa,  $\tau_{xy}$  = 4.2 MPa,  $\tau_{yz}$  = 5.27 MPa,  $\tau_{xz}$  = 0.84 MPa. Determine the principal stresses. (7 marks)
  - b) The displacement field for a body is given by  $\mathbf{u} = (x^2 + y)\mathbf{i} + (3 + z)\mathbf{j} + (x^2 + 2y)\mathbf{k}$ . What is the deformed position of a point originally at (3,1,-2)? Write the strain tensor at the point (-3,-1,2).

(7 marks)

OR

12. a) The state of plane stress at a point is given by  $\sigma_{xx}$  = 40 MPa,  $\sigma_{yy}$  = 20 MPa and  $\tau_{xy}$  = 16 MPa. Using Mohr's circle determine the i) principal stresses and principal planes and ii) maximum shear stress. (7 marks)

b) The state of stress at a point is given below. Find the resultant stress vector acting on a plane with direction cosines  $n_x$ =0.47,  $n_y$ =0.82 and  $n_z$ =0.33. Find the normal and tangential stresses acting on this plane. (7 marks)

$$\sigma_{ij} = \begin{bmatrix} 10 & 5 & -10 \\ 5 & 20 & -15 \\ -10 & -15 & -10 \end{bmatrix} MPa |$$

### MODULE - 2

- 13. a) Calculate Modulus of Rigidity and Young's Modulus of a cylindrical bar of diameter 30 mm and of 1.5 m length if the longitudinal strain in a bar during a tensile stress is four times the lateral strain. Find the change in volume when the bar is subjected to a hydrostatic pressure of 100  $\text{N/mm}^2$ . Take E =  $10^5$  N/mm (9 marks)
  - b) A straight bar 450 mm long is 40 mm in diameter for the first 250 mm length and 20 mm diameter for the remaining length. If the bar is subjected to an axial pull of 15 kNfind the maximum axial stress produced and the total extension of the bar. Take  $E = 2x10^5 \text{ N/mm}^2$

(5 marks)

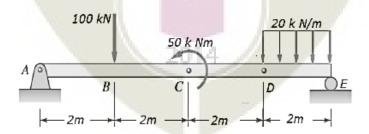
OR

- 14. a) A brass bar 20mm diameter is enclosed in a steel tube of 25mm internal diameter and 50mm external diameter. Both bar and tube is of same length and fastened rigidly at their ends. The composite bar is free of stress at 20°C. To what temperature the assembly must be heated to generate a compressive stress of 48MPa in brass bar? Also determine the stress in steel tube.  $E_{steel} = 200$ GPa and  $E_{brass} = 84$ GPa,  $\alpha_{steel} = 12 \times 10^{-6}$ /°C and  $\alpha_{brass} = 18 \times 10^{-6}$ /°C. (9 marks)
  - b) Draw the stress-strain diagram for a ductile material and explain the salient points.

(5 marks)

#### MODULE - 3

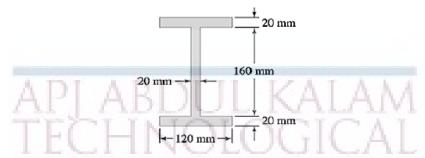
15. a) Draw shear force and bending moment diagram for the beam given in the figure. (9 marks)



b) Compare the strength of a hollow shaft of diameter ratio 0.75 to that of a solid shaft by considering the permissible shear stress. Both the shafts are of same material, of same length and weight.

(5 marks)

16. a) A simply supported beam of span of 10 m carries a UDL of 40 kN/m. The cross section is of I shape as given below. Calculatethe maximum stress produced due to bending and plot thebending stress distribution. (9 marks)



b) The shear stress of a solid shaft is not to exceed 40 N/mm<sup>2</sup> when the power transmitted is 20 kW at 200 rpm. Determine the minimum diameter of the shaft. (5 marks)

#### MODULE - 4

- 17. a) A horizontal girder of steel having uniform section is 14 m long and is simply supported at its ends. It carries concentrated loads of 120 kN and 80 kN at two points 3 m and 4.5 m from the two ends respectively. Moment of inertia for the section of the girder is  $16 \times 10^8$  mm<sup>4</sup> and  $E_s = 210$  kN/mm<sup>2</sup>. Calculate the deflection of the girder at points under the two loads and maximum deflection using Macaulay's method. (8 marks)
  - b) Derive the expressions for elastic strain energy in terms of applied load/moment and material property for the cases of a) Axial force b) Bending moment. (6 marks)

OR

18. a) Calculate the displacement in the direction of load P applied at a distance of L/3 from the left end for a simply supported beam of span L as shown in the figure.



(10 marks)

b) State Castigliano's second theorem and explain its significance.

(4 marks)

## **MODULE - 5**

19. a) Find the crippling load for a hollow steel column 50mm internal diameter and 5mm thick. The column is 5m long with one end fixed and other end hinged. Use Rankine's formula and Rankine's constant as 1/7500 and  $\sigma_c = 335$  N/mm<sup>2</sup>. Compare this load by crippling load given by Euler's formula. Take E = 110 GPa. (8 marks)

b) Explain the maximum normal stress theory, maximum strain energy theory and maximum shear stress theory of failure. (6 marks)

OR

20. a) The principal stresses at a point in an elastic material are 22 N/mm²(tensile), 110 N/mm² (tensile) and 55 N/mm² (compressive). If the elastic limit in simple tension is 210 N/mm², then determine whether the failure of material will occur or not according to Maximum principal stress theory, Maximum shear stress theory and maximum distortion energy theory.

(9 marks)

b) Derive Euler's formula for a column with both ends hinged.

(5 marks)

