

MAT 102	VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND TRANSFORMS	CATEGORY	L	T	P	CREDIT	Year of Introduction
		BSC	3	1	0	4	2019

**Preamble:** This course introduces the concepts and applications of differentiation and integration of vector valued functions, differential equations, Laplace and Fourier Transforms. The objective of this course is to familiarize the prospective engineers with some advanced concepts and methods in Mathematics which include the Calculus of vector valued functions, ordinary differential equations and basic transforms such as Laplace and Fourier Transforms which are invaluable for any engineer's mathematical tool box. The topics treated in this course have applications in all branches of engineering.

**Prerequisite:** Calculus of single and multi variable functions.

**Course Outcomes:** After the completion of the course the student will be able to

CO 1	Compute the derivatives and line integrals of vector functions and learn their applications
CO 2	Evaluate surface and volume integrals and learn their inter-relations and applications.
CO 3	Solve homogeneous and non-homogeneous linear differential equation with constant coefficients
CO 4	Compute Laplace transform and apply them to solve ODEs arising in engineering
CO 5	Determine the Fourier transforms of functions and apply them to solve problems arising in engineering

#### Mapping of course outcomes with program outcomes

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO 1	3	3	3	3	2	1			1	2		2
CO 2	3	3	3	3	2	1			1	2		2
CO 3	3	3	3	3	2	1			1	2		2
CO 4	3	3	3	3	2	1			1	2		2
CO 5	3	3	3	3	2	1			1	2		2

#### Assessment Pattern

Bloom's Category	Continuous Assessment Tests		End Semester Examination (Marks)
	Test 1 (Marks)	Test 2 (Marks)	
Remember	10	10	20
Understand	20	20	40
Apply	20	20	40
Analyse			
Evaluate			

Create			
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### Mark distribution

Total Marks	CIE (Marks)	ESE (Marks)	ESE Duration
150	50	100	3 hours

### Continuous Internal Evaluation Pattern:

Attendance : 10 marks

Continuous Assessment Test (2 numbers) : 25 marks

Assignment/Quiz/Course project : 15 marks

**Assignments:** Assignment should include specific problems highlighting the applications of the methods introduced in this course in science and engineering.

**End Semester Examination Pattern:** There will be two parts; Part A and Part B. Part A contain 10 questions with 2 questions from each module, having 3 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which student should answer any one. Each question can have maximum 2 sub-divisions and carry 14 marks.

### Course Level Assessment Questions

**Course Outcome 1 (CO1):** Compute the derivatives and line integrals of vector functions and learn their applications

1. How would you calculate the speed, velocity and acceleration at any instant of a particle moving in space whose position vector at time  $t$  is  $\mathbf{r}(t)$ ?
2. Find the work done by the force field  $F = (e^x - y^3)\mathbf{i} + (\cos y + x^3)\mathbf{j}$  on a particle that travels once around the unit circle centred at origin having radius 1.
3. When do you say that a vector field is conservative? What are the implications if a vector field is conservative?

**Course Outcome 2 (CO2):** Evaluate surface and volume integrals and learn their inter-relations and applications

1. Write any one application each of line integral, double integral and surface integral.
2. Use the divergence theorem to find the outward flux of the vector field  $F(x, y, z) = z\mathbf{k}$  across the

$$x^2 + y^2 + z^2 = a^2$$

3. State Greens theorem. Use Green's theorem to express the area of a plane region bounded by a curve as a line integral.

**Course Outcome 3 (CO3):** Solve homogeneous and non-homogeneous linear differential equation with constant coefficients

1. If  $y_1(x)$  and  $y_2(x)$  are solutions of  $y'' + py' + qy = 0$ , where  $p, q$  are constants, show that

$y_1(x) + y_2(x)$  is also a solution.

2. Solve the differential equation  $y'' + y = 0.001x^2$  using method of undetermined coefficient.

3. Solve the differential equation of  $y''' - 3y'' + 3y' - y = e^x - x - 1$ .

**Course Outcome 4 (CO4):** Compute Laplace transform and apply them to solve ODEs arising in engineering

1. What is the inverse Laplace Transform of  $(s) = \frac{3s-137}{s^2+2s+4}$ ?

2. Find Laplace Transform of Unit step function.

3. Solve the differential equation of  $y'' + 9y = \delta\left(t - \frac{\pi}{2}\right)$ ? Given  $y(0) = 2$ ,  $y'(0) = 0$

**Course Outcome 5 (CO5):** Determine the Fourier transforms of functions and apply them to solve problems arising in engineering

1. Find the Fourier integral representation of function defined by

$$f(x) = e^{-x} \text{ for } x > 0 \text{ and } f(x) = 0 \text{ for } x < 0.$$

2. What are the conditions for the existence of Fourier Transform of a function  $f(x)$ ?

3. Find the Fourier transform of  $f(x) = 1$  for  $|x| < 1$  and  $f(x) = 0$  otherwise.

### Model Question paper

**QP CODE:**

**PAGES:3**

Reg No: \_\_\_\_\_

Name : \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY FIRST SEMESTER B.TECH DEGREE EXAMINATION,  
MONTH & YEAR**

**Course Code: MAT 102**

**Max. Marks: 100**

**Duration: 3 Hours**

**VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND TRANSFORMS**

**(2019-Scheme)**

**(Common to all branches)**

## PART A

(Answer all questions. Each question carries 3 marks)

1. Is the vector  $\mathbf{r}$  where  $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$  conservative. Justify your answer.
2. State Greens theorem including all the required hypotheses
3. What is the outward flux of  $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$  across any unit cube.
4. What is the relationship between Green's theorem and Stokes theorem?
5. Solve  $y'' + 4y' + 2.5y = 0$
6. Does the function  $y = C_1 \cos x + C_2 \sin x$  form a solution of  $y'' + y = 0$ ? Is it the general solution? Justify your answer.
7. Find the Laplace transform of  $e^{-t} \sinh 4t$
8. Find the Laplace inverse transform of  $\frac{1}{s(s^2 + \omega^2)}$ .
9. Given the Fourier transform  $\frac{1}{\sqrt{2}} e^{-\frac{\omega^2}{4}}$  of  $f(x) = e^{-x^2}$ , find the Fourier transform of  $x e^{-x^2}$
10. State the convolution theorem for Fourier transform

## PART B

(Answer one full question from each module. Each full question carries 14 marks)

### MODULE 1

- 11a) Prove that the force field  $\mathbf{F} = e^y \mathbf{i} + x e^y \mathbf{j}$  is conservative in the entire xy-plane
- b) Use Greens theorem to find the area enclosed by the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- 12 a) Find the divergence of the vector field  $\mathbf{F} = \frac{c}{(x^2 + y^2 + z^2)^{3/2}} (x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$
- b) Find the work done by the force field  $\mathbf{F}(x, y, z) = xy\mathbf{i} + yz\mathbf{j} + xz\mathbf{k}$  along C where C is the curve  $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$

### MODULE II

- 13 a) Use divergence theorem to find the outward flux of the vector field  $\mathbf{F} = 2x\mathbf{i} + 3y\mathbf{j} + z^3\mathbf{k}$  across the unit cube bounded by  $x = 0, y = 0, z = 0, x = 1, y = 1, z = 1$
- b) Find the circulation of  $\mathbf{F} = (x - z)\mathbf{i} + (y - x)\mathbf{j} + (z - xy)\mathbf{k}$  using Stokes theorem around the triangle with vertices  $A(1,0,0), B(0,2,0)$  and  $C(0,0,1)$
- 14 a) Use divergence theorem to find the volume of the cylindrical solid bounded by  $x^2 + 4x + y^2 = 7, z = -1, z = 4$ , given the vector field  $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$  across surface of the cylinder
- b) Use Stokes theorem to evaluate  $\int_C \mathbf{F} \cdot d\mathbf{r}$  where  $\mathbf{F} = x^2\mathbf{i} + 3x\mathbf{j} - y^3\mathbf{k}$  where C is

the circle  $x^2 + y^2 = 1$  in the  $xy$ - plane with counterclockwise orientation looking down the positive  $z$ -axis

### MODULE III

- 15 a) Solve  $y'' + 4y' + 4y = x^2 + e^{-x} \cos x$   
b) Solve  $y''' - 3y'' + 3y' - y = e^x - x - 1$   
16 a) Solve  $y''' + 3y'' + 3y' + y = 30e^{-x}$  given  $y(0) = 3, y'(0) = -3, y''(0) = -47$   
b) Using method of variation of parameters, solve  $y'' + y = \sec x$

### MODULE IV

- 17 a) Find the inverse Laplace transform of  $F(s) = \frac{2(e^{-s} - e^{-3s})}{s^2 - 4}$   
b) Solve the differential equation  $y'' + 16y = 4\delta(t - 3\pi); y(0) = 2, y'(0) = 0$  using Laplace transform  
18 a) Solve  $y'' + 3y' + 2y = f(t)$  where  $f(t) = 1$  for  $0 < t < 1$  and  $f(t) = 1$  for  $t > 1$  using Laplace transform  
b) Apply convolution theorem to find the Laplace inverse transform of  $\frac{1}{s^2(s^2 + \omega^2)}$

### MODULE V

- 19 a) Find the Fourier cosine integral representation for  $f(x) = e^{-kx}$  for  $x > 0$  and  $k > 0$  and hence evaluate  $\int_0^\infty \frac{\cos wx}{k^2 + w^2}$  the function  
b) Does the Fourier sine transform  $f(x) = x^{-1} \sin x$  for  $0 < x < \infty$  exist? Justify your answer  
20 a) Find the Fourier transform of  $f(x) = |x|$  for  $|x| < 1$  and  $f(x) = 0$  otherwise  
b) Find the Fourier cosine transform of  $f(x) = e^{-ax}$  for  $a > 0$

## Syllabus

### Module 1 (Calculus of vector functions)

(Text 1: Relevant topics from sections 12.1, 12.2, 12.6, 13.6, 15.1, 15.2, 15.3)

Vector valued function of single variable, derivative of vector function and geometrical interpretation, motion along a curve-velocity, speed and acceleration. Concept of scalar and vector fields, Gradient and its properties, directional derivative, divergence and curl, Line integrals of vector fields, work as line integral, Conservative vector fields, independence of path and potential function (results without proof).

### Module 2 ( Vector integral theorems)

(Text 1: Relevant topics from sections 15.4, 15.5, 15.6, 15.7, 15.8)

Green's theorem (for simply connected domains, without proof) and applications to evaluating line integrals and finding areas. Surface integrals over surfaces of the form  $z = g(x, y)$ ,  $y = g(x, z)$  or  $x = g(y, z)$ , Flux integrals over surfaces of the form  $z = g(x, y)$ ,  $y = g(x, z)$  or  $x = g(y, z)$ , divergence theorem (without proof) and its applications to finding flux integrals, Stokes' theorem (without proof) and its applications to finding line integrals of vector fields and work done.

### Module- 3 ( Ordinary differential equations)

(Text 2: Relevant topics from sections 2.1, 2.2, 2.5, 2.6, 2.7, 2.10, 3.1, 3.2, 3.3)

Homogenous linear differential equation of second order, superposition principle, general solution, homogenous linear ODEs with constant coefficients-general solution. Solution of Euler-Cauchy equations (second order only). Existence and uniqueness (without proof). Non homogenous linear ODEs-general solution, solution by the method of undetermined coefficients (for the right hand side of the form  $x^n, e^{kx}, \sin ax, \cos ax, e^{kx} \sin ax, e^{kx} \cos ax$  and their linear combinations), methods of variation of parameters. Solution of higher order equations-homogeneous and non-homogeneous with constant coefficient using method of undetermined coefficient.

### Module- 4 (Laplace transforms)

(Text 2: Relevant topics from sections 6.1, 6.2, 6.3, 6.4, 6.5)

Laplace Transform and its inverse, Existence theorem (without proof), linearity, Laplace transform of basic functions, first shifting theorem, Laplace transform of derivatives and integrals, solution of differential equations using Laplace transform, Unit step function, Second shifting theorems. Dirac delta function and its Laplace transform, Solution of ordinary differential equation involving unit step function and Dirac delta functions. Convolution theorem (without proof) and its application to finding inverse Laplace transform of products of functions.

## Module-5 (Fourier Transforms)

### (Text 2: Relevant topics from sections 11.7,11.8, 11.9)

Fourier integral representation, Fourier sine and cosine integrals. Fourier sine and cosine transforms, inverse sine and cosine transform. Fourier transform and inverse Fourier transform, basic properties. The Fourier transform of derivatives. Convolution theorem (without proof)

#### Text Books

1. H. Anton, I. Biven S.Davis, "Calculus", Wiley, 10<sup>th</sup> edition, 2015.
2. Erwin Kreyszig, "Advanced Engineering Mathematics", Wiley, 10<sup>th</sup> edition, 2015.

#### Reference Books

1. J. Stewart, Essential Calculus, Cengage, 2<sup>nd</sup> edition, 2017
2. G.B. Thomas and R.L. Finney, Calculus and Analytic geometry, 9<sup>th</sup> Edition, Pearson, Reprint, 2002.
3. Peter O Neil, Advanced Engineering Mathematics, 7th Edition, Thomson, 2007.
4. Louis C Barret, C Ray Wylie, "Advanced Engineering Mathematics", Tata McGraw Hill, 6<sup>th</sup> edition, 2003.
5. VeerarajanT."Engineering Mathematics for first year", Tata McGraw - Hill, 2008.
6. B.S. Grewal, Higher Engineering Mathematics, Khanna Publishers, 36<sup>th</sup> edition, 2010.
7. Srimanta Pal, Subodh C. Bhunia, "Engineering Mathematics", Oxford University Press, 2015.
8. Ronald N. Bracewell, "The Fourier Transform and its Applications", McGraw – Hill International Editions, 2000.

#### Course Contents and Lecture Schedule

No	Topic	No. of Lectures
<b>1</b>	<b>Calculus of vector functions (9 hours)</b>	
1.1	Vector valued function of a scalar variable - derivative of vector valued function of scalar variable t-geometrical meaning	2
1.2	Motion along a curve-speed, velocity, acceleration	1
1.3	Gradient and its properties, directional derivative, divergent and curl	3
1.4	Line integrals with respect to arc length, line integrals of vector fields. Work done as line integral	2
1.5	Conservative vector field, independence of path, potential function	1

<b>2</b>	<b>Vector integral theorems( 9 hours)</b>	
2.1	Green's theorem and it's applications	2
2.2	Surface integrals , flux integral and their evaluation	3
2.3	Divergence theorem and applications	2
2.4	Stokes theorem and applications	2
<b>3</b>	<b>Ordinary Differential Equations (9 hours)</b>	
3.1	Homogenous linear equation of second order, Superposition principle, general solution	1
3.2	Homogenous linear ODEs of second order with constant coefficients	2
3.3	Second order Euler-Cauchy equation	1
3.4	Non homogenous linear differential equations of second order with constant coefficient-solution by undetermined coefficients, variation of parameters.	3
3.5	Higher order equations with constant coefficients	2
<b>4</b>	<b>Laplace Transform (10 hours)</b>	
4.1	Laplace Transform , inverse Transform, Linearity, First shifting theorem, transform of basic functions	2
4.2	Transform of derivatives and integrals	1
4.3	Solution of Differential equations, Initial value problems by Laplace transform method.	2
4.4	Unit step function --- Second shifting theorem	2
4.5	Dirac Delta function and solution of ODE involving Dirac delta function	2
4.6	Convolution and related problems.	1
<b>5</b>	<b>Fourier Transform (8 hours)</b>	
5.1	Fourier integral representation	1
5.2	Fourier Cosine and Sine integrals and transforms	2
5.3	Complex Fourier integral representation, Fourier transform and its inverse transforms, basic properties	3
5.4	Fourier transform of derivatives, Convolution theorem	2