MAT 101	LINEAR ALGEBRA AND CALCULUS	CATEGORY	L	T	P	CREDIT	Year of Introduction
		BSC	3	1	0	4	2019

Preamble: This course introduces students to some basic mathematical ideas and tools which are at the core of any engineering course. A brief course in Linear Algebra familiarises students with some basic techniques in matrix theory which are essential for analysing linear systems. The calculus of functions of one or more variables taught in this course are useful in modelling and analysing physical phenomena involving continuous change of variables or parameters and have applications across all branches of engineering.

Prerequisite: A basic course in one-variable calculus and matrix theory.

Course Outcomes: After the completion of the course the student will be able to

CO 1	solve systems of linear equations, diagonalize matrices and characterise quadratic forms
CO 2	compute the partial and total derivatives and maxima and minima of multivariable functions
CO 3	compute multiple integrals and apply them to find areas and volumes of geometrical shapes, mass and centre of gravity of plane laminas
CO 4	perform various tests to determine whether a given series is convergent, absolutely convergent or conditionally convergent
CO 5	determine the Taylor and Fourier series expansion of functions and learn their applications.

Mapping of course outcomes with program outcomes

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO 1	3	3	3	3	2	1			1	2		2
CO 2	3	3	3	3	2	1			1	2		2
CO 3	3	3	3	3	2	1			1	2		2
CO 4	3	2	3	2	1	1			1	2		2
CO 5	3	3	3	3	2	1			1	2		2

Assessment Pattern

Bloom's Category	Continuous Assessment Tests		End Semester Examination (Marks)
	Test 1 (Marks)	Test 2 (Marks)	
Remember	10	10	20
Understand	20	20	40
Apply	20	20	40
Analyse			
Evaluate			
Create			

Mark distribution

Total Marks	CIE marks	ESE marks	ESE Duration
150	50	100	3 hours

Continuous Internal Evaluation Pattern:

Attendance : 10 marks
Continuous Assessment Test (2 numbers) : 25 marks
Assignment/Quiz/Course project : 15 marks
Assignments: Assignment should include specific problems highlighting the applications of the methods introduced in this course in science and engineering.

End Semester Examination Pattern: There will be two parts; Part A and Part B. Part A contain 10 questions with 2 questions from each module, having 3 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which student should answer any one. Each question can have maximum 2 sub-divisions and carry 14 marks.

Course Level Assessment Questions

Course Outcome 1 (CO1): Solve systems of linear equations, diagonalize matrices and characterise quadratic forms

1. A is a real matrix of order $3 \times 3$and $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. What can you say about the solution of $AX = 0$if rank of A is 1? 2 ?3?

2. Given$A = \begin{bmatrix} 3 & 0 & 2 \\ 0 & 2 & 0 \\ -2 & 0 & 0 \end{bmatrix}$, find an orthogonal matrix Pthat diagonalizes A.

3. Find out what type of conic section the following quadratic form represents

$$17x^2 - 30x_1x_2 + 17x_2^2 = 128$$

4. The matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$has an eigen value5 with corresponding Eigen vector$X = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$. Find A^5X

Course Outcome 2 (CO2): compute the partial and total derivatives and maxima and minima of multivariable functions

1. Find the slope of the surface $z = x^2y + 5y^3$in the x-direction at the point (1,-2)

2. Given the function $w = xy + z$, use chain rule to find the instantaneous rate of change of $w$at each point along the curve $x = cost, y = sint, z = t$

3. Determine the dimension of rectangular box open at the top , having a volume 32 cubic ft and requiring the least amount of material for it's construction.

Course Outcome 3(CO3): compute multiple integrals and apply them to find areas and volumes of geometrical shapes, mass and centre of gravity of plane laminas.

1. Evaluate $\iint_D (x + 2y)\, DA$where D is the region bounded by the parabolas $y = 2x^2$and $y = 1 + x^2$

2. Explain how you would find the volume under the surface $z = f(x, y)$and over a specific region Din the xyplane using (i) double integral (ii) triple integral?

3. Find the mass and centre of gravity of a triangular lamina with vertices (0,0), (2,1), (0,3) if the density function is $f(x, y) = x + y$

4. Use spherical coordinates to evaluate $\iiint_B (x^2 + y^2 + z^2)^3\, dV$where B is the unit ball defined by $B = \{(x, y, z): x^2 + y^2 + z^2 \leq 1\}$

Course Outcome 4 (CO4): perform various tests to determine whether a given series is convergent, absolutely convergent or conditionally convergent.

1. What is the difference between a sequence and a series and when do you say that they are convergent? Divergent?

2. Determine whether the series $\sum_{n=1}^{n=\infty} \frac{5}{2n^2+4n+3}$converges or diverges.

3. Is the series $\sum_{n=1}^{n=\infty} \frac{(-1)^{n-1}}{n}$convergent? Absolutely convergent? Conditionally convergent?

Course Outcome 5 (CO5): determine the Taylor and Fourier series expansion of functions and learn their applications.

1. Assuming the possibility of expansion find the Maclaurin series expansion of

   $f(x) = (1 + x)^k$for$|x| < 1$where $k$is any real number. What happens if $k$is a positive

   integer?

2. Use Maclaurin series of $ln(1 + x), -1 < x \leq 1$to find an approximate value of$ln2$.

3. Find the Fourier series of the function$f(x) = x^2, -2 \leq x < 2, f(x + 4) = f(x)$. Hence using Parseval's identity prove that $1 + \frac{1}{2^4} + \frac{1}{3^4} + \ldots = \frac{\pi^4}{90}$

4. Expand the function $f(x) = x\ (0 < x < 1/2)$ into a (i) Fourier sine series (ii) Fourier cosine series.

Model Question paper

QP CODE: PAGES:3

Reg No:_______________

Name :_______________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY FIRST SEMESTER B.TECH DEGREE EXAMINATION, MONTH & YEAR

Course Code: **MAT 101**

Max. Marks: 100 **Duration: 3 Hours**

LINEAR ALGEBRA AND CALCULUs

(2019-Scheme)

(Common to all branches)

PART A

(Answer **all** questions, **each** question carries 3 marks)

1. Determine the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ -2 & -4 & 2 \\ 3 & 6 & -3 \end{bmatrix}$.
2. Write down the eigen values of $= \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$. What are the eigen values of $P^{-1}AP$ where $P = \begin{bmatrix} -4 & 2 \\ 3 & -1 \end{bmatrix}$?
3. Find $f_x(1,3)$ and $f_y(1,3)$ for the function $f(x,y) = 2x^3y^2 + 2y + 4x$.
4. Show that the function $u(x,t) = \sin(x - ct)$ is a solution of the equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$.
5. Use double integral to find the area of the region enclosed between the parabolas $y = \frac{1}{2}x^2$ and the line $y = 2x$.
6. Use polar coordinates to evaluate the area of the region bounded by $x^2 + y^2 = 4$, the line $y = x$ and the y axis in the first quadrant
7. Test the convergence of the series $\sum_{k=1}^{\infty} \frac{k}{k+1}$.
8. Test the convergence of the alternating series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k}$ using Leibnitz test.
9. Find the Taylor series expansion of $sin\pi x$ about $x = \frac{1}{2}$.
10. Find the values to which the Fourier series of

$f(x) = x$ for $-\pi < x < \pi$, with $f(x + 2\pi) = f(x)$ converges (10x3=30)

PART B

(Answer **one full** question from each module, each question carries **14** marks)

Module -I

11. (a) Solve the following system of equations

$$y + z - 2w = 0$$
$$2x - 3y - 3z + 6w = 2$$
$$4x + y + z - 2w = 4$$

(b) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

12. (a) Diagonalize the matrix $\begin{bmatrix} -1 & 2 & -2 \\ 2 & 4 & 1 \\ 2 & 1 & 4 \end{bmatrix}$

(b) What kind of conic section the quadratic form $3x_1^2 + 22x_1x_2 + 3x_2^2 = 0$ represents? Transform it to principal axes.

Module - II

13. (a) Find the local linear approximation to $f(x, y) = \sqrt{x^2 + y^2}$ at the point $(3, 4)$. Use it to approximate $f(3.04, 3.98)$

(b) Let $w = \sqrt{x^2 + y^2 + z^2}$, $x = cos\theta, y = sin\theta, z = tan\theta$. Use chain rule to find $\frac{dw}{d\theta}$ when $\theta = \frac{\pi}{4}$.

14. (a) Let $z = f(x, y)$ where $x = rcos\theta, y = rsin\theta$, prove that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$.

(b) Locate all relative maxima, relative minima and saddle points $f(x, y) = xy + \frac{a^3}{x} + \frac{b^3}{y} (a \neq 0, b \neq 0)$.

Module - III

15. (a) Evaluate $\iint_D (2x^2y + 9y^3)\, dxdy$ where D is the region bounded by $y = \frac{2}{3}x$ and $y = 2\sqrt{x}$

(b) Evaluate $\int_0^4 \int_{\sqrt{y}}^2 e^{x^3}\, dxdy$ changing the order of integration.

16. (a) Find the volume of the solid bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$..

(b) Evaluate $\iiint \sqrt{1 - x^2 - y^2 - z^2}\, dxdydz$, taken throughout the volume of the sphere $x^2 + y^2 + z^2 = 1$, by transforming to spherical polar coordinates

Module - IV

17. (a) Test the convergence of the series

(i) $\sum_{k=1}^{\infty} \frac{k^k}{k!}$ (ii) $\sum_{k=2}^{\infty} \left(\frac{4k-5}{2k+1}\right)^k$

(b) Determine the convergence or divergence of the series $\sum_{k=1}^{\infty} (-1)^k \frac{(2k-1)!}{3^k}$

18. (a) Check whether the series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{(2k)!}{(3k-2)!}$ is absolutely convergent, conditionally convergent or divergent.

(b) Test the convergence of the series $1+\frac{1.2}{1.3}+\frac{1.2.3}{1.3.5}+\frac{1.2.3.4}{1.3.5.7}+\cdots$

Module - V

19. (a) Obtain the Fourier series of for $f(x) = e^{-x}$, in the interval $0 < x < 2\pi$.with $f(x+2\pi) = f(x)$. Hence deduce the value of $\sum_{n=2}^{\infty} \frac{(-1)^n}{1+n^2}$.

(b) Find the half range sine series of $f(x) = \begin{cases} \frac{2kL}{x} & if\ 0 < x < \frac{L}{2} \\ \frac{2k(L-x)}{L} & if\ \frac{L}{2} < x < L \end{cases}$

20. (a)Expand $(1+x)^{-2}$.as a Taylor series about $x = 0$and state the region of convergence of the series.

(b) Find the Fourier series for $f(x) = x^2$ in the interval $-\pi < x < \pi$

with $f(x+2\pi) = f(x)$.Hence show that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\cdots = \frac{\pi^4}{90}$. (14X5=70)

Syllabus

Module 1 (Linear algebra)

(Text 2: Relevant topics from sections 7.3, 7.4, 7.5, 8.1,8.3,8.4)

Systems of linear equations, Solution by Gauss elimination, row echelon form and rank of a matrix, fundamental theorem for linear systems (homogeneous and non-homogeneous, without proof), Eigen values and eigen vectors. Diagonaliztion of matrices, orthogonal transformation, quadratic forms and their canonical forms.

Module 2 (multivariable calculus-Differentiation)

(Text 1: Relevant topics from sections 13.3, 13.4, 13.5, 13.8)

Concept of limit and continuity of functions of two variables, partial derivatives, Differentials, Local Linear approximations, chain rule, total derivative, Relative maxima and minima, Absolute maxima and minima on closed and bounded set.

Module 3(multivariable calculus-Integration)

(Text 1: Relevant topics from sections 14.1, 14.2, 14.3, 14.5, 14.6, 14.8)

Double integrals (Cartesian), reversing the order of integration, Change of coordinates (Cartesian to polar), finding areas and volume using double integrals, mass and centre of gravity of inhomogeneous laminas using double integral. Triple integrals, volume calculated as triple integral, triple integral in cylindrical and spherical coordinates (computations involving spheres, cylinders).

Module 4 (sequences and series)

(Text 1: Relevant topics from sections 9.1, 9.3, 9.4, 9.5, 9.6)

Convergence of sequences and series, convergence of geometric series and p-series(without proof), test of convergence (comparison, ratio and root tests without proof); Alternating series and Leibnitz test, absolute and conditional convergence.

Module 5 (Series representation of functions)

(Text 1: Relevant topics from sections 9.8, 9.9. Text 2: Relevant topics from sections 11.1, 11.2, 11.6)

Taylor series (without proof, assuming the possibility of power series expansion in appropriate domains), Binomial series and series representation of exponential, trigonometric, logarithmic functions (without proofs of convergence); Fourier series, Euler formulas, Convergence of Fourier series (without proof), half range sine and cosine series, Parseval's theorem (without proof).

Text Books

1. H. Anton, I. Biven,S.Davis, "Calculus", Wiley, 10th edition, 2015.
2. Erwin Kreyszig, Advanced Engineering Mathematics, 10thEdition, John Wiley & Sons, 2016.

Reference Books

1. J. Stewart, Essential Calculus, Cengage, 2nd edition, 2017
2. G.B. Thomas and R.L. Finney, Calculus and Analytic geometry, 9 th Edition, Pearson, Reprint, 2002.
3. Peter V. O'Neil, Advanced Engineering Mathematics , Cengage, 7th Edition, 2012
4. Veerarajan T., Engineering Mathematics for first year, Tata McGraw-Hill, New Delhi, 2008.
5. B.S. Grewal, Higher Engineering Mathematics, Khanna Publishers, 36 Edition, 2010.

Course Contents and Lecture Schedule

No	**Topic**	**No. of Lectures**
1	**Linear Algebra (10 hours)**	
1.1	Systems of linear equations, Solution by Gauss elimination	1
1.2	Row echelon form, finding rank from row echelon form, fundamental theorem for linear systems	3
1.3	Eigen values and eigen vectors	2
1.4	Diagonaliztion of matrices, orthogonal transformation, quadratic forms	4

	and their canonical forms.	
2	**Multivariable calculus-Differentiation (8 hours)**	
2.1	Concept of limit and continuity of functions of two variables, partial derivatives	2
2.2	Differentials, Local Linear approximations	2
2.3	Chain rule, total derivative	2
2.4	Maxima and minima	2
3	**Multivariable calculus-Integration (10 hours)**	
3.1	Double integrals (Cartesian)-evaluation	2
3.2	Change of order of integration in double integrals, change of coordinates (Cartesian to polar),	2
3.3	Finding areas and volumes, mass and centre of gravity of plane laminas	3
3.4	Triple integrals	3
4	**Sequences and series (8 hours)**	
4.1	Convergence of sequences and series, geometric and p-series	2
4.2	Test of convergence(comparison, ratio and root)	4
4.3	Alternating series and Leibnitz test, absolute and conditional convergence	2
5	**Series representation of functions (9 hours)**	
5.1	Taylor series, Binomial series and series representation of exponential, trigonometric, logarithmic functions;	3
5.2	Fourier series, Euler formulas, Convergence of Fourier series(Dirichlet's conditions)	3
5.3	Half range sine and cosine series, Parseval's theorem.	3