most important factor in determining the power output of a wind turbine because power increases exponentially with the swept area.

not really

After rotor diameter, wind speed is crucial to wind energy production. Power is proportional to the cube of wind speed, which means that if you double the velocity, you increase the power eight-fold (i.e., $2^3 = 8$, and $4^3 = 64$).

v^3

There are a few steps to estimating the output of a wind turbine. While there are several approaches, the *swept area method* is most commonly used due to the significant impact of swept area on the power output of a wind turbine.

The properties of air affect the energy and power available in the wind. In the swept area method, we first find the *annual power density*, the watts per area of the wind stream intersected by the rotor. In order to find power density, we must know the *kinetic energy* of the wind. The wind's kinetic energy is based on the *mass (m)* of air and its *velocity (v)*. Its mass is based on its *density* and *volume*. Since air is constantly moving, its volume is its velocity times the *area (A)* it passes in a given period of time. Thus:

$$\textbf{Kinetic Energy (KE) of Wind} = \frac{1}{2}mv^2$$

m = mass
v = velocity

Where:

$$m = \rho Avt$$

ρ = air density
A = Area
v = velocity
t = time

Substituting the formula for mass into the KE of wind, we get:

$$\textbf{KE of Wind} = \frac{1}{2}\rho Atv^3$$

* Cubic in velocity
* Quadratic in diameter

Since power is energy divided by time, time cancels out and power is thus:

$$\textbf{Power} = \frac{\text{Energy}}{\text{Time}}$$

$$= \frac{1}{2}\rho Av^3$$

Annual Power Density is derived by dividing area from the formula above to get the watts per unit area.

$$\textbf{Annual Power Density}\left(\frac{\mathbf{W}}{\mathbf{m}^2}\right) = \frac{\text{Power}}{\text{Area}}$$

Thus:

$$\textbf{Annual Power Density} = \frac{1}{2}\rho v^3$$

IN SUMMARY

As you can see, in order to get annual power density, we need air density and velocity. *Air density* varies with pressure, temperature, and humidity. It also increases with decreasing temperature—air is denser in winter than summer, and denser at sea level than at higher elevations.

In order make some estimates of power output, we assume the turbine is at sea-level. This is an acceptable assumption in most locations, unless, of course, you are at very high altitudes and/or in extreme climates. The temperature is 15 degrees Celsius or 288.15 Kelvin (59 degrees F), and the air pressure is 1 standard atmosphere or 101,352.9 Newtons per square meter (N/m^2), or 14.7 pounds per square inch.

Air density (ρ) is found by the dividing the *air pressure (P)* in N/m^2 or Pascal, by the *gas constant (R)*, 287.04 joules per kilogram (J/kg), times *temperature (T)* in *Kelvin (K)*.

$$\textbf{Air Density} = \frac{\text{Air Pressure}}{(\text{Gas Constant} * \text{Temperature})}$$

$$\rho = \frac{P}{(R * T)}$$

Therefore, at sea level, the air density will be:

$$= \frac{101{,}352.9 \ \mathrm{N/m^2}}{(287.04\,\mathrm{J/kg} * 288.15\ \mathrm{K})}$$

$$= 1.225\ \mathrm{kg/m^3}\ (0.07651\mathrm{lbs/ft^3})$$

In order to find annual power density at sea level we plug air density into our formula for *annual power density*. Again, annual power density is the rate at which energy passes through a unit of area and is given in watts per square meter (W/m²).

$$\textbf{Annual Power Density} = \frac{1}{2}\rho v^3$$

$$\textbf{Annual Power Density at Sea Level} = \frac{1}{2} * 1.225 \frac{\mathrm{kg}}{\mathrm{m^3}} * v^3$$

$$= 0.6125\ \mathrm{kg/m^3} * v^3$$

?

Next, in order to find the rated power output in kW of our wind turbine, we multiply the annual power density times the area swept by the rotor.

RATED

$$\textbf{Power (kW)} = \frac{\text{Annual Power Density}\left(\frac{\mathrm{W}}{\mathrm{m^2}}\right) * \text{Swept Area}\left(\mathrm{m^2}\right)}{1{,}000\ \text{watts / kW}}$$

so up to here we derived the formula for RATED POWER from two convenient quantities

The swept area is simply the rotor sweeping the area in a circle. Thus, it is the area of a circle ($A = \pi r^2$, with r being the radius). If the rotor diameter is 50 meters (164 feet), which is an average utility scale turbine rotor diameter, then:

$$\textbf{Swept Area (A)} = \pi * (50/2)^2$$

$$= 1{,}963\ \mathrm{m^2}$$

In other words, the rotor intercepts 1,963 m² of the wind stream.

We now know the annual power density and the swept area. However, there is one more complication. We can never know for certain the wind speed in the future so the annual power density is based on predictions of wind speed that come from probability distributions. The one we use for calculating probabilities of wind speed is called the *Weibull Distribution*. It is comprised of two parameters and informed by average wind speed

k (shape parameter, dimensionless) and *C (scale parameter, provides meters/second units).*

In particular, we use the *Rayleigh Distribution*, a type of Weibull with a k of 2, which is sufficient for most locations in the world, but not areas with very high or low average wind speeds. The Rayleigh lets us use a mean wind speed, but this is not always accurate, as demonstrated by the graphs in Figure 2.26, which show the Rayleigh distribution overestimating actual measured data in Ethiopia.

An *Energy Pattern Factor (EPF)* is used to connect the average speed with the number of hours the wind blows at that speed, which, for a Rayleigh Distribution, is 1.91. We multiply this EPF term by the rest of our terms to calculate annual power density.

$$\textbf{Annual Power Density} = 0.6125\ \mathrm{kg/m^3} * v^3 * \mathrm{EPF}$$

v = meters/second (m/s)

EPF = Energy Pattern Factor = 1.91

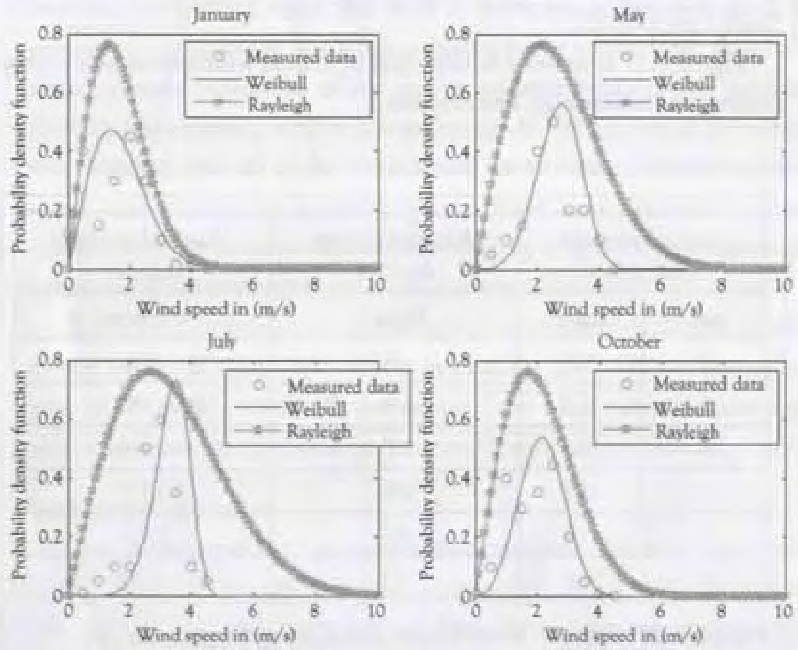

Figure 2.26 How Rayleigh distribution overestimates wind speed.

Source: Nage, American Journal of Modern Energy.[29]

Now, let's say the wind speed is 10 mph at our site, which is 4.47 meters per second. Then,

Annual Power Density = $0.6125\ kg/m^3 * 4.47\ m/s^3 * 1.91$
$= \mathbf{104\ W/m^2}$

The annual *wind energy density, energy per area (E/A in Wh/m²)* is the amount of energy available to a wind turbine at your site. It is simply the Annual Power Density multiplied by time.

Energy per Area (E/A) = Power Density * 8,760 hours per year

Taking our example above,

E/A = $104\ w/m^2 * 8{,}760$ hours per year / 1,000 watts per kW
$= \mathbf{911\ kWh/m^2}$

We have found that based on an average wind speed of 10 mph the annual amount of energy available to a wind turbine is 911 kWh/m^2. That's a lot of wind!

Figure 2.27 contains a handy table of annual wind power and energy density for the Rayleigh Distribution.

Annual average wind speed		Annual power density	Annual energy density
m/s	mph	W/m^2	kWh/m^2
4	9.0	75	656
5	11.2	146	1,281
6	13.4	253	2,214
7	15.7	401	3,515
8	17.9	599	5,247
9	20.2	853	7,471

Figure 2.27** **Annual Wind Power and Energy Density for the Rayleigh distribution.

Courtesy Paul Gipe, *Wind Energy for the Rest of Us: A Comprehensive Guide to Wind Power and How to Use It*. Bakersfield, California: wind-works.org, 2016. Page 238.

To find the power capacity in kW of our wind turbine, we multiply the Annual Power Density by the Swept Area:

FINALLY

Power (kW) = Annual Power Density (W/m²) × Swept Area (m²)

Recall from our example, for a rotor diameter of 50 meters:

Swept Area (A) = $\pi * (50/2)^2$
$= 1{,}963\ m^2$

Thus, for a wind speed of 4.47 meters per second (10 mph), and a rotor diameter of 50 meters, an average utility-scale turbine, we see that the wind turbine power output is:

Power (kW) = $104\ W/m^2 * 1{,}963\ m^2$
= 204,152 watts/1,000 watts/kW
= **204 kW**

Convert that capacity to annual energy (204 kW * 8,760 hours/year) and you get 1,787 MWh per year.

However, the **Annual Energy Output** must be derated because of the *Betz Limit*, which states that wind turbines can deliver, at most, 59.3 percent of the power in the wind available to the rotor. A wind turbine simply cannot capture all of the power available because they work by slowing down passing wind to extract energy. If 100 percent of the energy was extracted then all of the wind would have to stop, preventing additional air from entering, and halting the turbine. In reality, most turbines achieve 20 to 30 percent efficiency due to kinetic to electrical conversion losses and changes in wind speed and direction.*

Therefore, after derating, and assuming a conservative 20 percent efficiency, we could expect our turbine to produce 357 MWh of energy per year (1,787 MWh * 0.2). To determine if the wind turbine makes economic sense we can calculate payback and return on investment.

Let's take an example:

You are a developer with a turbine that produces 357 MWh/year and have a PPA with the utility for your energy of $110 per MWh. How long will it take to pay off the machine? What is the return on investment? Assume the total installed cost was $750,000.

* Downtime for maintenance is also included in this conversion efficiency loss.

To calculate annual revenue, we multiply 357 MWh times $110 per MWh to get $39,270.

Payback (years) = Total Cost / Annual Revenue

$750,000/$39,270

= 19 years

Return on Investment (ROI) = Annual Revenue / Total Cost

$39,270/$750,000 = 0.05 * 100

= 5 percent

The revenue from your investment will be paid back in 19 years and you will receive a 5 percent return. Note this is a simple payback period as it does not account for interest and inflation. Accounting for the *time value of money*, or changes to money over time, will be discussed in Chapter 3.

Summary

Calculating the output of a wind energy turbine is complex. Unlike fossil fuel generators, the generator size has little to do with the power output of a wind turbine. Estimating energy output of a wind turbine involves determining the power in the wind, and then the amount of that power captured by the wind turbine. With that information, we can calculate the economics of an investment in a wind turbine.

To recap, to estimate the energy output of our utility-scale wind turbine with a 50 meter diameter rotor and an average annual wind speed of 4.47 meters/second:

Step 1: Find the *power* density of the wind at a particular site. We assume sea level conditions and a Rayleigh Distribution.

Power (kW) = Annual Power Density (W/m^2) * Swept Area (m^2)

= 104 W/m^2 * 1,963 m^2

= 204,152 watts/1,000 watts/ kW

= 204 kW

Step 2: Find the *energy* passing through the rotor in one year.

Energy (kWh) = Power (kW) * Time (hours)

= 204 kW * 8760 hours (*1 year = 8,760 hours*)

= 1,787,040/1,000 kW/MW

= 1,787 MWh/year

Step 3: De-rate for Betz Limit, conversion losses, and efficiency.

Annual Estimated Output (AEO) = kWh * Overall Conversion Efficiency

= 1787 MWh/year * 0.20

= 357 MWh/year