Statistical Inference Project : Central Limit Theorem

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Introduction

The project intends to investigate the distribution of averages of 40 exponentials and compare it with the Central Limit Theorem.

In the simulations, two thousand (nsim = 2000) avearages of 40 (n = 40) exponential random variables (r.v.) are generated in R. The rate of the exponential distribution is $\lambda = 0.2$. Thus, the mean and standard deviation of the exponential distribution are both $\frac{1}{\lambda} = 5$.

The Central Limit Theorem states that the distribution of averages of iid random variables with well-defined mean μ and variance σ^2 will be approximately normally distributed when the sample size is sufficiently large. The distribution of the averages will have a mean center around the population mean and its standard deviation can be approximated with the standard error given by $se = \frac{\sigma}{\sqrt{(n)}}$. In the following, we will try to examine the CLT with simulations.

R code

The R code that generates the exponentials, calculates the mean, se, sd, etc of the distribution and produces the plots is as follows:

```
##
## Exponential distribution of rate 1/lambda : mean = 1/lambda, std = 1/lambda
##

rate <- 0.2     # rate parameter of exponential distribution (lambda)
n <- 40         # number of exponential variables for averaging
nsim <- 2000    # number of simulations
mu <- 1/rate    # mean of the exponential variable
sd <- 1/rate    # standard deviation of the exponential variable

x1 <- matrix(rexp(n*nsim, rate), nrow = n)  # matrix of 40x1000 of exponentials
y <- colMeans(x1)   # vector containing 'nsim' average of 'n' exponentials
```

```
## Plot the histogram of the sample mean
## Overlay a normal curve with mean =5 and std. deviation = se for comparison
##
hist(y, xlab=bquote("Mean of"~.(n)~"exponentials,"~bar(X)),
     main=bquote("Histogram of mean of"~.(n)~ "exponentials,"~bar(X)),
     ylim=c(0, 0.5), xlim=c(2,8),
     probability = T, cex.main=1)
curve(dnorm(x, mu, sd/sqrt(n)), add=T, col="red", lwd=2)
lines(5-rep(se,3), c(0, 0.1, 0.26), col="blue", lwd=1, lty=2)
lines(5+rep(se,3), c(0, 0.1, 0.26), col="blue", lwd=1, lty=2)
lines(rep(mean(y), 2), c(0,0.5), col="blue", lwd=2)
text(5,-0.01, ~mu, cex=0.8)
```

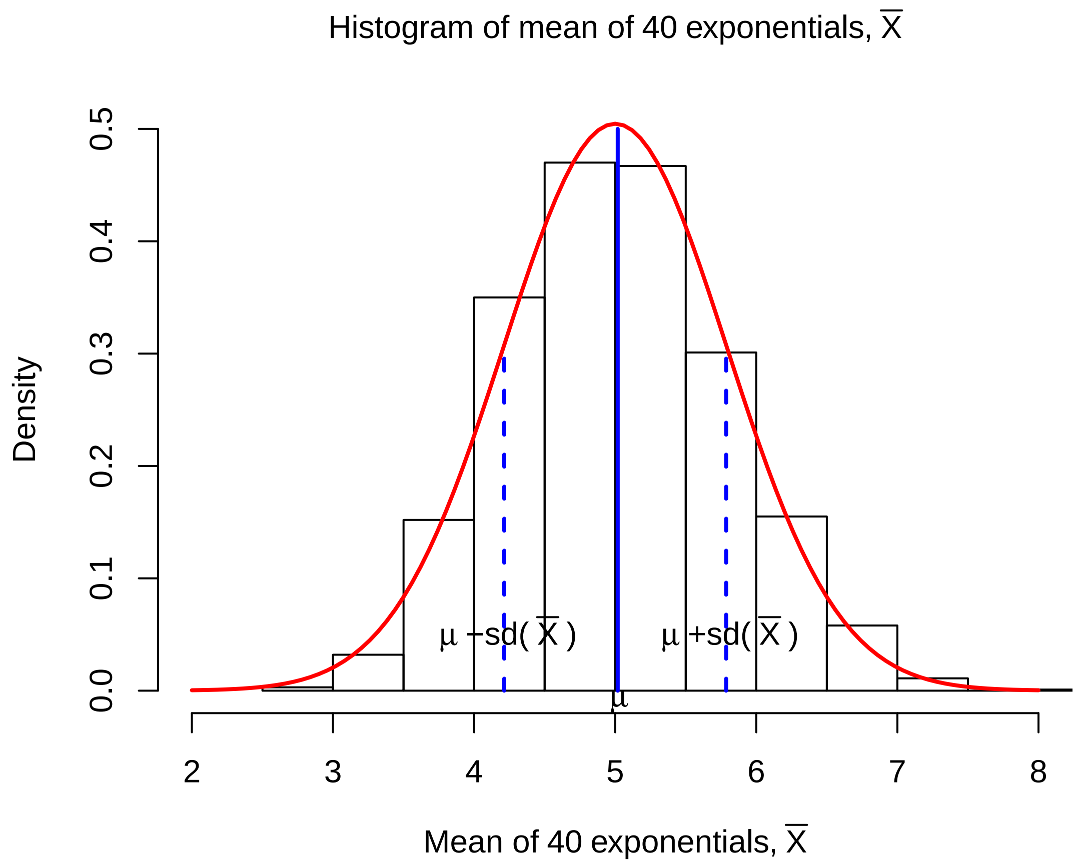
```r
text(3.5, 0.05, "5-sd("~bar(X)~")", cex = 0.6)
text(6.5, 0.05, "5+sd("~bar(X)~")", cex=0.6)

## Compare the standard deviation of the sample mean
## with the theoretical sample mean, i.e., sd/sqrt(n)
##
se <- sd(y)
se_t <- sd/sqrt(n)
```

```r
## Plot histogram of sample mean minus mean and divided by std error should give a std normal r.v
## Overlay a standard normal curve for comparison
##
y1 <- (y - mu)/se
hist(y, xlab=bquote("Mean of"~.(n)~"exponentials,"~bar(X)),
     main=bquote("Histogram of mean of"~.(n)~ "exponentials,"~bar(X)),
     ylim=c(0, 0.5), xlim=c(2,8),
     probability = T, cex.main=1)
curve(dnorm(x, mu, sd/sqrt(n)), add=T, col="red", lwd=2)
lines(c(5-se, 5-se, 5-se), c(0, 0.1, 0.3), col="blue", lwd=2, lty=2)
lines(c(5+se, 5+se, 5+se), c(0, 0.1, 0.3), col="blue", lwd=2, lty=2)
lines(rep(mean(y), 2), c(0,0.5), col="blue", lwd=2)
text(5,-0.01, ~mu, cex=1)
text(5-se, 0.05, ~mu~"-sd("~bar(X)~")", cex = 1)
text(5+se, 0.05, ~mu~"+sd("~bar(X)~")", cex=1)
```

Histogram of sample mean of 40 exponentials $\bar{X}$ approximates a normal distribution with mean $= \mu$ and standard dviation $= \frac{\sigma}{\sqrt{n}}$

The histogram of averages of 40 exponentials is shown in the plot below. For comparison, the normal curve with parameters mean$=\frac{1}{\lambda}$ and sd$=\frac{1}{\lambda\sqrt{n}}$ is also plotted for comparisson. It can be seen that the histogram approximates a normal distribution with mean around 5.



The mean of the distibution is indicated by the solid blue line, at 5.017983, which fall approximately around $\mu = 5$.

The dotted blue lines indicate one a shift of one standard deviation $\bar{X}$ from the mean. The standard deviation of sample mean $\bar{X}$ shows the variation of the sample mean from the population mean. From the simulation, the standard deviation of distribution of $\bar{X}$ is $\sigma_{\bar{X}} = 0.804$, which is close to the standard error which can be approximated with the population standard deviation as $\frac{\sigma}{\sqrt{n}} = 0.791$.