In [1]:

```
import random
import numpy as np
import matplotlib.pyplot as pt

from math import ceil
from typing import List, Tuple, Callable
```

In [2]:

```
def get_polynomial(x):
    return 5*x + 20*x**2 + 1*x**3
```

In [3]:

```
def get_dataset(number_of_samples, noise_scale) -> Tuple[np.ndarray]:
    x_rand = np.sort(25*(np.random.rand(number_of_samples, 1) - 0.8), -1).reshape((number_of_samples,))
    noise = (noise_scale*np.random.randn(number_of_samples, 1)).reshape((number_of_samples,))

    x = np.sort(x_rand)
    y = np.add(get_polynomial(x), noise)

    x_plot = np.linspace(x[0], x[-1], 100)
    y_plot = get_polynomial(x_plot)

    return (x, y, x_plot, y_plot, noise)
```

In [4]:

```
def plot_arbitrary_polynomial(x, coef, title: str = f"Polynomial"):
    x_plot = np.linspace(x[0], x[-1], 100)
    y_seq = np.zeros(len(x_plot))
    for i in range(len(coef)):
        y_seq = np.add(y_seq, coef[i]*x_plot**i)
    pt.plot(x_plot, y_seq)
    pt.title(title)
    pt.show()
```

In [5]:

```
def polyfit(x: np.array, y: np.array, order: int, title: str = None) -> Tuple[np.ndarray]:
    poly = np.polyfit(x, y, order, full=True)
    coefficients = poly[0]
    residuals = poly[1]
    pt.plot(x, y, 'ro')
    if title is not None:
        plot_arbitrary_polynomial(x, coefficients[::-1], title=title)
    else:
        plot_arbitrary_polynomial(x, coefficients[::-1], titlet=title)
    return (coefficients, residuals)
```

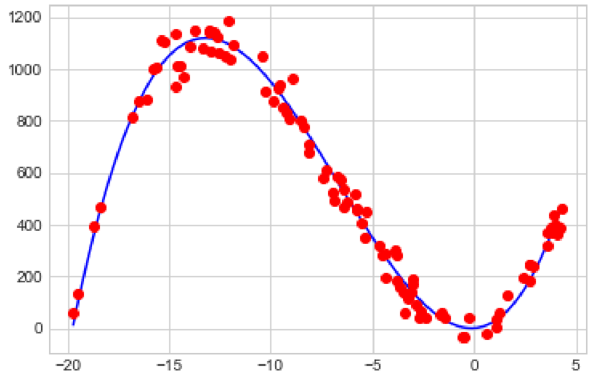
In [6]:

```
pt.style.use('seaborn-whitegrid')
print("True polynomial: 1*x**3 + 20*x**2 + 5*x + 0")
(x, y, x_plot, y_plot, noise) = get_dataset(100, 50)
pt.plot(x_plot, y_plot, 'b')
pt.plot(x, y, 'ro')
```

```
True polynomial: 1*x**3 + 20*x**2 + 5*x + 0
```

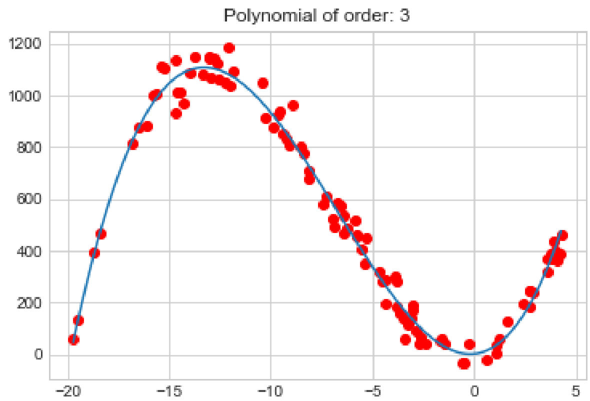
Out[6]:

```
[<matplotlib.lines.Line2D at 0x16a0b4164c0>]
```



In [7]:

```python
mse: List[float] = []
order: List[int] = [3]
hw1_coefs = None
for m in order:
    (coef, res) = polyfit(x, y, m, title=f"Polynomial of order: {m}")
    hw1_coefs = np.flip(coef)
    function_string = ""
    for i in range(len(coef)):
        function_string += f" {coef[i]}*x^{len(coef)-i-1} "
        if i != len(coef)-1:
            function_string += '+'
    print(f"Function: {function_string}\n")
    print(f"Coefficients: {coef}")
    print(f"Residuals: {res}")
    mse += res.tolist()
```




```
Function:  0.9842611395580602*x^3 + 19.891605879030973*x^2 + 7.31096351096582*x^1 +
1.2189682780935982*x^0

Coefficients: [ 0.98426114 19.89160588  7.31096351  1.21896828]
Residuals: [244640.57682334]
```

In [8]:

```python
class GeneticAlgorithm():
    def __init__(self, max_degree, population, max_init, mutation_rate, mutation_wiggle, crossover_rate):
        self.pop = np.random.random((population, max_degree+1)) * max_init
        self.crossover_rate = crossover_rate
        self.mutation_wiggle = mutation_wiggle
        self.mutation_rate = mutation_rate

    def fit(self, x: np.ndarray, y: np.ndarray, loss: Callable, percent_kept_per_generation: float, thres
        m, n = self.pop.shape
        self.best_coefs = np.zeros(n)
        self.best_loss = None
        # for each generation
        for generation in range(max_iter):
            print(f'generation: {generation}')
            # compute fitness of each baby and save it
            loss_scores = np.zeros(m)
            for i in range(m):
                # get the coef for that child
                sample_coefs = self.pop[i, :]
```

```
            # calculate that childs fitness
            loss_scores[i] = loss(sample_coefs, x, y)
            #stop condition
            if loss_scores[i] <= threshold:
                print(f'stopping condition: threshold on loss score {i}: {loss_scores[i]}')
                self.best_coefs = sample_coefs
                return

        best_coefs_index = np.argpartition(loss_scores, 1)[:1]
        self.best_coefs = self.pop[best_coefs_index, :]
        self.best_loss = loss_scores[best_coefs_index]

        # check how many children you're keeping per generation
        num_kept = int(m * percent_kept_per_generation)

        # find the num_kept with the minimum loss
        kept_ind = np.argpartition(loss_scores, num_kept)[:num_kept]

        # first, create the children as perfect copies of their parents
        new_generation = np.zeros_like(self.pop)
        new_gen_ind = np.array_split(np.arange(m), len(kept_ind))
        for i, ind_batch in enumerate(new_gen_ind):
            new_generation[ind_batch, :] = self.pop[kept_ind[i], :]

#         if self.mutation_rate > 0:
        # randomly sample a percentage of them to mutate
        inds_to_mutate = np.random.randint(low=0, high=m, size=(ceil(m * self.mutation_rate),))
        # generate the coefficients we will 'wiggle' each coef by
        wiggle = np.random.uniform(low=self.mutation_wiggle[0], high=self.mutation_wiggle[1], size=(l
        # generate the coefficient indices to mutate
        coef_inds_to_mutate = np.random.randint(low=0, high=n, size=(len(inds_to_mutate),))
        for i, ind_to_mutate in enumerate(inds_to_mutate):
            # randomly choose to wiggle a coefficient up or down
            if bool(random.getrandbits(1)):
                new_generation[ind_to_mutate, coef_inds_to_mutate[i]] += new_generation[ind_to_mutate
            else:
                new_generation[ind_to_mutate, coef_inds_to_mutate[i]] -= new_generation[ind_to_mutate

#         if self.crossover_rate > 0:
        # randomly sample a percentage of them to crossover
        num_to_crossover = ceil(m * self.crossover_rate) if ceil(m * self.crossover_rate) % 2 == 0 els
        inds_to_crossover = np.random.randint(low=0, high=m, size=(num_to_crossover,))
        crossover_ind_pairs = np.array_split(inds_to_crossover, num_to_crossover / 2)

        stop_index = int(n/2)
        for pair in crossover_ind_pairs:
            # grab the values
            sample1 = new_generation[pair[0], :]
            sample2 = new_generation[pair[1], :]

            # perform the swap
            temp = None
            temp = sample1[:stop_index].copy()
            sample1[:stop_index] = sample2[:stop_index]
            sample2[:stop_index] = temp

            # reset the values
            new_generation[pair[0], :] = sample1
            new_generation[pair[1], :] = sample2

        self.pop = new_generation
    print('stopping condition: max_iter')
```

In [9]:

```
ga = GeneticAlgorithm(
    max_degree = 3,
    population = 1000,
    max_init = 50,
    mutation_rate = 0.01,
    mutation_wiggle = (0.0, 2.0),
    crossover_rate = 0.5
)
```

In [10]:

```
def loss(coef, x, y):
    def calc_poly_value(degree, x, coef):
        output = 0
        for i in range(degree):
            output += coef[i]*x**i
        return output

    values = np.zeros(len(x))
    for i in range(len(x)):
        values[i] = calc_poly_value(len(coef), x[i], coef)
    return abs(sum(y - values))

ga.fit(
    x,
    y,
    loss=loss,
    percent_kept_per_generation=0.1,
    threshold=800,
    max_iter=500
)

best_coefs = ga.best_coefs
print(f"best coefs: {best_coefs}")
```

```
generation: 0
generation: 1
generation: 2
stopping condition: threshold on loss score 132: 222.86026757009319
best coefs: [36.85696503 14.41196728 19.01139074  0.9062793 ]
```

In [11]:

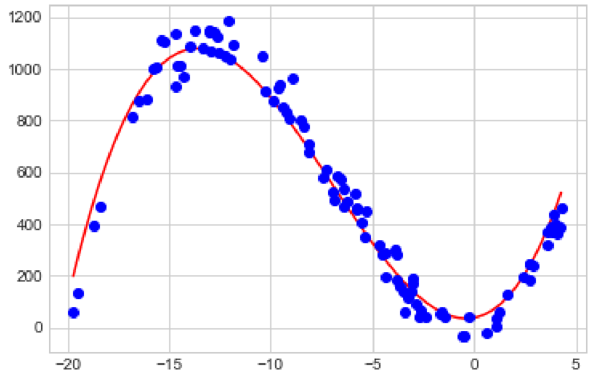
```
best_coefs
```

Out[11]:

```
array([36.85696503, 14.41196728, 19.01139074,  0.9062793 ])
```

In [12]:

```
x_plot = np.linspace(min(x), max(x), 100)
y_seq = np.zeros(len(x_plot))
for i in range(len(best_coefs)):
    y_seq = np.add(y_seq, best_coefs[i]*x_plot**i)
pt.plot(x_plot, y_seq, 'r')
pt.plot(x, y, 'bo')
pt.show()
```



Final plot with noisy input data, Genetic Algorithm Output, and HW1 Polyfit Output

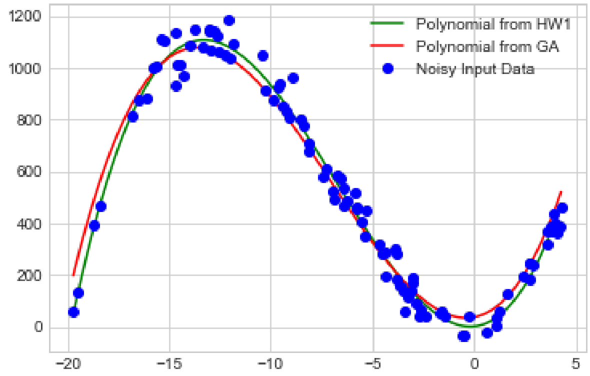
In [15]:

```
# x_plot = np.linspace(x[0], x[-1], 100)
y_seq_hw1 = np.zeros(len(x_plot))
for i in range(len(hw1_coefs)):
    y_seq_hw1 = np.add(y_seq_hw1, hw1_coefs[i]*x_plot**i)
```

In [17]:

```
pt.plot(x_plot, y_seq_hw1, 'g')
pt.plot(x_plot, y_seq, 'r')
pt.plot(x, y, 'bo')
pt.legend(["Polynomial from HW1", "Polynomial from GA", "Noisy Input Data"])
```

```
pt.show()
```



In [20]:

```
print(f"Coefficients from HW1: {hw1_coefs}")
print(f"Coefficients from GA: {best_coefs}")
```

```
Coefficients from HW1: [ 1.21896828  7.31096351 19.89160588  0.98426114]
Coefficients from GA: [36.85696503 14.41196728 19.01139074  0.9062793 ]
```