Formula Collection

Chang-ye Tu

April 13, 2016

$$\begin{aligned}
&a \cdot (b \times c) = b \cdot (c \times a) = c \cdot (a \times b) \\
&a \times (b \times c) = b(a \cdot c) - c(a \cdot b) \\
&(a \times b) \cdot (c \times d) = (a \cdot c)(b \cdot d) - (a \cdot d)(b \cdot c)
\end{aligned}$$

$$\begin{aligned}
\operatorname{curl} \nabla\psi &= 0 \\
\operatorname{div}(\operatorname{curl} a) &= 0 \\
\operatorname{curl}(\operatorname{curl} a) &= \nabla(\operatorname{div} a) - \Delta a \\
\operatorname{div}(\psi a) &= \nabla\psi \cdot a + \psi \operatorname{div} a \\
\operatorname{curl}(\psi a) &= \nabla\psi \times a + \psi \operatorname{curl} a \\
\operatorname{div}(a \times b) &= (\operatorname{curl} a) \cdot b - (\operatorname{curl} b) \cdot a \\
\operatorname{curl}(a \times b) &= a(\operatorname{div} b) - b(\operatorname{div} a) + (b \cdot \nabla)a - (a \cdot \nabla)b \\
\nabla(a \cdot b) &= (a \cdot \nabla)b + (b \cdot \nabla)a - (\operatorname{curl} a) \times b - (\operatorname{curl} b) \times a
\end{aligned}$$

$$\begin{aligned}
\int_\Omega \operatorname{div} a &= \int_\Gamma \nu \cdot a \\
\int_\Omega \operatorname{curl} a &= \int_\Gamma \nu \times a
\end{aligned}$$

$$\begin{aligned}
\int_\Gamma u(\nu \times \nabla v) &= -\int_\Gamma v(\nu \times \nabla u) \\
\int_\Gamma \langle \nu \times \nabla v, a \rangle &= -\int_\Gamma v \langle \nu, \operatorname{curl} a \rangle
\end{aligned}$$

proof:

$$\begin{aligned}
\int_\Gamma \nu \times \nabla(uv) &= \int_\Gamma u(\nu \times \nabla v) + \int_\Gamma v(\nu \times \nabla u) = \int_\Omega \operatorname{curl} \nabla(uv) \equiv 0 \\
\int_\Gamma \langle \nu \times \nabla u, a \rangle &= -\int_\Gamma \langle a \times \nabla u, \nu \rangle = -\int_\Omega \operatorname{div}(a \times \nabla u) = -\int_\Omega \langle \operatorname{curl} a, \nabla u \rangle \\
&= -\int_\Omega \operatorname{div}(u \operatorname{curl} a) = -\int_\Gamma u \langle \nu, \operatorname{curl} a \rangle
\end{aligned}$$

$$\begin{aligned}
&\int_\Omega u\Delta v + \nabla u \cdot \nabla v = \int_\Gamma u \frac{\partial v}{\partial \nu} \\
&\int_\Omega a \cdot \Delta b + \operatorname{curl} a \cdot \operatorname{curl} b + \operatorname{div} a \ \operatorname{div} b = \int_\Gamma (\nu \times a) \cdot (\operatorname{curl} b) + (\nu \cdot a)(\operatorname{div} b)
\end{aligned}$$

$$\int_\Omega v\cdot\operatorname{curl}u-\operatorname{curl}v\cdot u=\int_\Gamma(v\times\nu)\cdot((\nu\times u)\times\nu)=\int_\Gamma v_{\mathrm{t}}\cdot u_{\mathrm{T}}$$

$$\begin{aligned}
\nabla_y\cdot q\,e^{ikx\cdot y}&=(ikx\cdot q)\,e^{ikx\cdot y}\\
\nabla_y\times q\,e^{ikx\cdot y}&=(ikx\times q)\,e^{ikx\cdot y}\\
\nabla_y\times(\nabla_y\times q\,e^{ikx\cdot y})&=(ikx\times(ikx\times q))\,e^{ikx\cdot y}\\
&=-k^2x\times(x\times q)\,e^{ikx\cdot y}\\
&=k^2x\times(q\times x)\,e^{ikx\cdot y}
\end{aligned}$$

$$\begin{aligned}
u(x)=&\int_\Gamma\left\{\frac{\partial u}{\partial\nu}(y)\Phi(x,y)-u(y)\frac{\partial\Phi(x,y)}{\partial\nu(y)}\right\}\,\mathrm{d}\sigma(y)\\
&-\int_\Omega\left\{\Delta u(y)+k^2u(y)\right\}\Phi(x,y)\,\mathrm{d}V(y),\quad x\in\Omega
\end{aligned}$$

$$u(x)=\int_\Gamma\left\{u(y)\frac{\partial\Phi(x,y)}{\partial\nu(y)}-\frac{\partial u}{\partial\nu}(y)\Phi(x,y)\right\}\,\mathrm{d}\sigma(y),\quad x\in\mathbb{R}^3\setminus\overline{\Omega}$$

$$u^\infty(\hat{x})=\int_\Gamma\left\{u(y)\frac{\partial e^{-ik\hat{x}\cdot y}}{\partial\nu(y)}-\frac{\partial u}{\partial\nu}(y)e^{-ik\hat{x}\cdot y}\right\}\,\mathrm{d}\sigma(y),\quad \hat{x}\in\mathbb{S}^2$$

$$\begin{aligned}
E(x)=&-\operatorname{curl}\int_\Gamma\nu(y)\times E(y)\Phi(x,y)\,\mathrm{d}\sigma(y)\\
&+\nabla\int_\Gamma\nu(y)\cdot E(y)\Phi(x,y)\,\mathrm{d}\sigma(y)\\
&-ik\int_\Gamma\nu(y)\times H(y)\Phi(x,y)\,\mathrm{d}\sigma(y)\\
&+\operatorname{curl}\int_\Omega\left\{\operatorname{curl}E(y)-ikH(y)\right\}\Phi(x,y)\,\mathrm{d}V(y)\\
&-\nabla\int_\Omega\operatorname{div}E(y)\Phi(x,y)\,\mathrm{d}V(y)\\
&+ik\int_\Omega\left\{\operatorname{curl}H(y)+ikE(y)\right\}\Phi(x,y)\,\mathrm{d}V(y).
\end{aligned}$$

$x\in\Omega$:

$$\begin{aligned}
E(x)&=-\operatorname{curl}\int_\Gamma\nu(y)\times E(y)\Phi(x,y)\,\mathrm{d}\sigma(y)-\frac{i}{k}\operatorname{curl}^2\int_\Gamma\nu(y)\times H(y)\Phi(x,y)\,\mathrm{d}\sigma(y),\\
H(x)&=-\operatorname{curl}\int_\Gamma\nu(y)\times H(y)\Phi(x,y)\,\mathrm{d}\sigma(y)+\frac{i}{k}\operatorname{curl}^2\int_\Gamma\nu(y)\times E(y)\Phi(x,y)\,\mathrm{d}\sigma(y).
\end{aligned}$$

$x \in \mathbb{R}^3 \setminus \overline{\Omega}$:

$$E(x) = \operatorname{curl} \int_\Gamma \nu(y) \times E(y)\Phi(x,y)\,\mathrm{d}\sigma(y) + \frac{i}{k}\operatorname{curl}^2 \int_\Gamma \nu(y) \times H(y)\Phi(x,y)\,\mathrm{d}\sigma(y),$$
$$H(x) = \operatorname{curl} \int_\Gamma \nu(y) \times H(y)\Phi(x,y)\,\mathrm{d}\sigma(y) - \frac{i}{k}\operatorname{curl}^2 \int_\Gamma \nu(y) \times E(y)\Phi(x,y)\,\mathrm{d}\sigma(y).$$

$$E^\infty(\hat{x}) = ik\hat{x} \times \int_\Gamma \{\nu(y) \times E(y) + (\nu(y) \times H(y)) \times \hat{x}\}\, e^{ik\hat{x}\cdot y}\,\mathrm{d}\sigma(y),$$
$$H^\infty(\hat{x}) = ik\hat{x} \times \int_\Gamma \{\nu(y) \times H(y) - (\nu(y) \times E(y)) \times \hat{x}\}\, e^{ik\hat{x}\cdot y}\,\mathrm{d}\sigma(y)$$

$$\int_\Gamma \varphi \operatorname{div}_\Gamma u \,\mathrm{d}\sigma = -\int_\Gamma \nabla_{\mathrm{t}}\varphi \cdot u \,\mathrm{d}\sigma$$