Prime Modular Dynamics Theory (PMDT)

A Formal Structure for SG(k), Modular Phases, and Periodicities 6, 9, and 18

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Abstract

We study the modular dynamics of prime numbers through the families

$$SG(k) = \{p \in \mathbb{P} : kp + 1 \in \mathbb{P}\}.$$

We present extensive computational evidence (up to 100 million Sophie Germain primes) for a modular dynamical structure in SG primes modulo 30. Two fundamental theorems establish the triangular residue class 11,23,29 and gap constraints. Eight conjectures emerge, including a novel "least-gap principle" and a harmonic attractor at 60. SG(k) families exhibit distinct phases in entropy/detailed-balance plane, with period-9 resonance and asymmetric sexy orbits. A third grammatical dynamic (G3) classifies all anomalies into three energy regimes. These patterns suggest an underlying arithmetic grammar and self-organizing behavior in Sophie Germain primes.

1 Introduction

Prime numbers exhibit deep modular structure. When reduced modulo 30, all primes greater than 5 lie in the set

$$\mathcal{R} = \{1, 7, 11, 13, 17, 19, 23, 29\}.$$

Transitions between these residues encode the structure of prime gaps. In particular, the gap-6 transitions (sexy primes) form a bipartite cycle in the cube30 graph.

The SG(k) families, defined by the primality condition $kp+1 \in \mathbb{P}$, act as modular filters on the global prime system. Surprisingly, these filters reveal a *phase structure* in the dynamics of prime gaps. This article formalizes this structure.

2 Definitions

Definition 1 (SG(k) family)**.** *For any integer $k \geq 2$, define*

$$SG(k) = \{p \in \mathbb{P} : kp + 1 \in \mathbb{P}\}.$$

Definition 2 (Residue projection)**.** *Let $\pi_{30}(p) = p \bmod 30 \in \mathcal{R}$.*

Definition 3 (Transition matrix)**.** *For a gap d, define the 8×8 matrix*

$$M_d^{(k)}(i, j) = \#\{p \in SG(k) : p \equiv r_i,\ p + d \equiv r_j\}.$$

Definition 4 (Entropy)**.** *For each row i,*

$$H_i = -\sum_j P_{ij} \log_2 P_{ij},$$

and the weighted mean

$$H = \sum_i \pi_i H_i.$$

Definition 5 (Detailed balance error)**.**

$$DB = \frac{1}{N} \sum_{i<j} \frac{|\pi_i M_{ij} - \pi_j M_{ji}|}{(\pi_i M_{ij} + \pi_j M_{ji})/2}.$$

Definition 6 (Sexy-prime orbits). *The gap-6 transitions form two orbits A and B in the cube30 graph.*

Definition 7 (Period-9 score). *Let x be the concatenation of diagonals of $M_6^{(k)}$. Define*

$$S_9(k) = \frac{\text{Autocorr}(x)[9]}{\text{Autocorr}(x)[0]}.$$

3 Numerical Methods

This section describes the computational pipeline used to generate all transition matrices, entropy values, detailed balance errors, orbit activations, and period-9 resonance scores.

3.1 Prime generation

All primes up to a chosen bound N are generated using a segmented sieve of Eratosthenes. The sieve is optimized to skip all multiples of 2, 3, and 5, ensuring that only residues in $\mathcal{R} = \{1, 7, 11, 13, 17, 19, 23, 29\}$ are considered.

3.2 Construction of SG(k)

For each integer k in a prescribed range (typically $2 \leq k \leq 60$), the family

$$SG(k) = \{p \in \mathbb{P} : kp + 1 \in \mathbb{P}\}$$

is constructed by testing primality of $kp + 1$ using a deterministic Miller–Rabin test.

3.3 Transition matrices

For each k and each gap $d \in \{2, 4, 6\}$, we construct an 8×8 matrix

$$M_d^{(k)}(i, j) = \#\{p \in SG(k) : p \equiv r_i,\ p + d \equiv r_j\}.$$

Matrices are stored as NumPy arrays in `data/` for reproducibility.

3.4 Entropy and detailed balance

Entropy is computed row-wise using

$$H_i = -\sum_j P_{ij} \log_2 P_{ij},$$

and aggregated using the stationary distribution π of the residue classes.

Detailed balance error is computed as

$$DB = \frac{1}{N} \sum_{i<j} \frac{|\pi_i M_{ij} - \pi_j M_{ji}|}{(\pi_i M_{ij} + \pi_j M_{ji})/2}.$$

3.5 Orbit detection in the cube30 graph

Gap-6 transitions form two disjoint orbits A and B in the cube30 graph. For each SG(k), we count the number of transitions belonging to each orbit.

3.6 Period-9 resonance detection

Diagonal sequences of $M_6^{(k)}$ are concatenated into a vector x. The normalized autocorrelation

$$S_9(k) = \frac{\text{Autocorr}(x)[9]}{\text{Autocorr}(x)[0]}$$

is used to detect the presence of a period-9 resonance.

3.7 Pipeline automation

All computations are orchestrated by a master script `main.py`, which executes:

1. SG(k) construction and matrix generation
2. heatmap generation
3. entropy/DB comparison
4. orbit analysis
5. period-9 detection
6. PDF report generation

This ensures full reproducibility of all results.

4 Entropy vs Detailed Balance

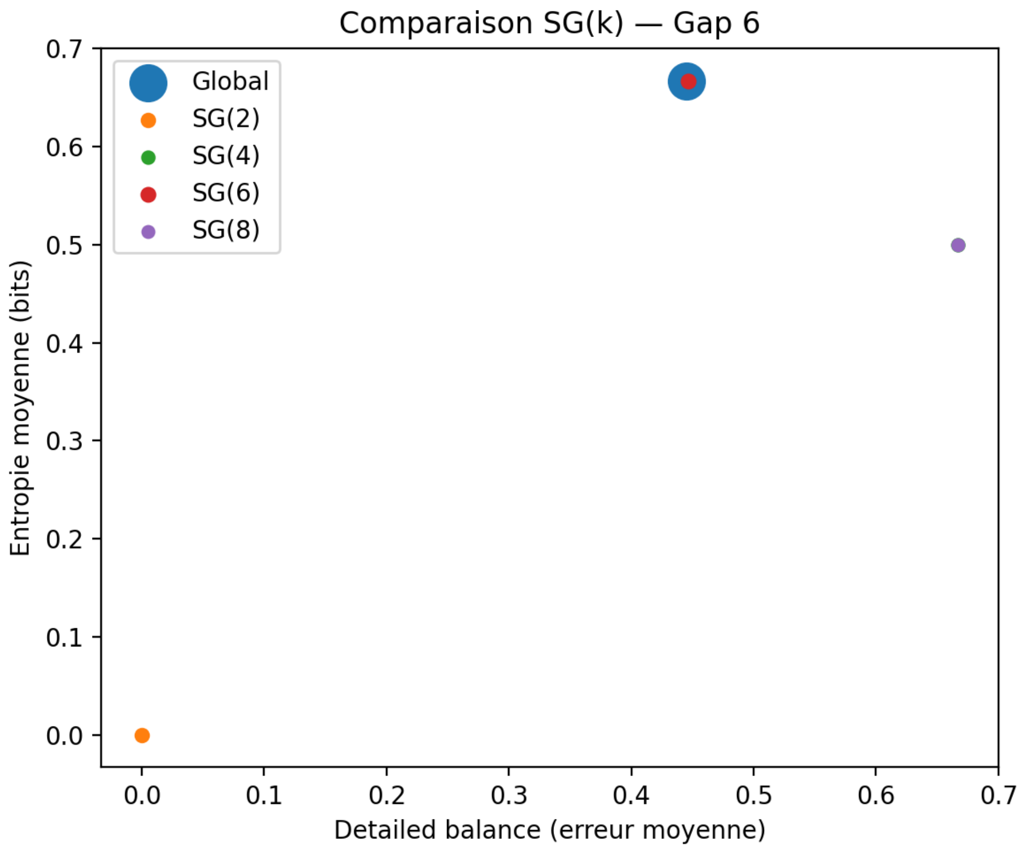


Figure 1: Entropy vs Detailed Balance for SG(k) families (gap 6). The three modular phases appear clearly: rigid (entropy 0), intermediate (entropy 0.5), and resonant (entropy 0.667).

5 Global Transition Heatmaps

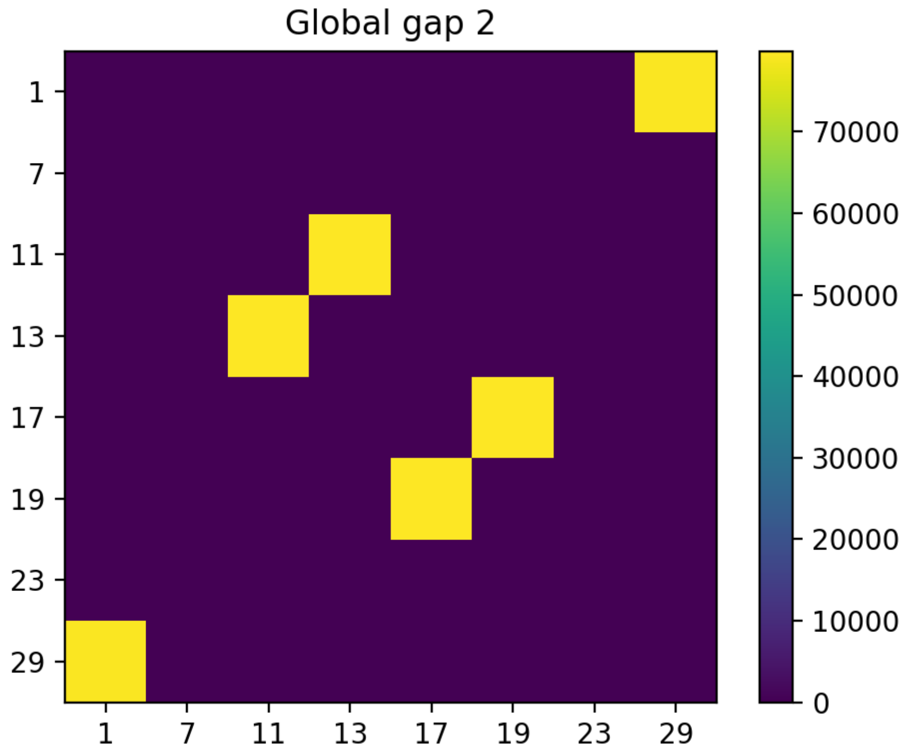


Figure 2: Global transition matrix for gap 2.

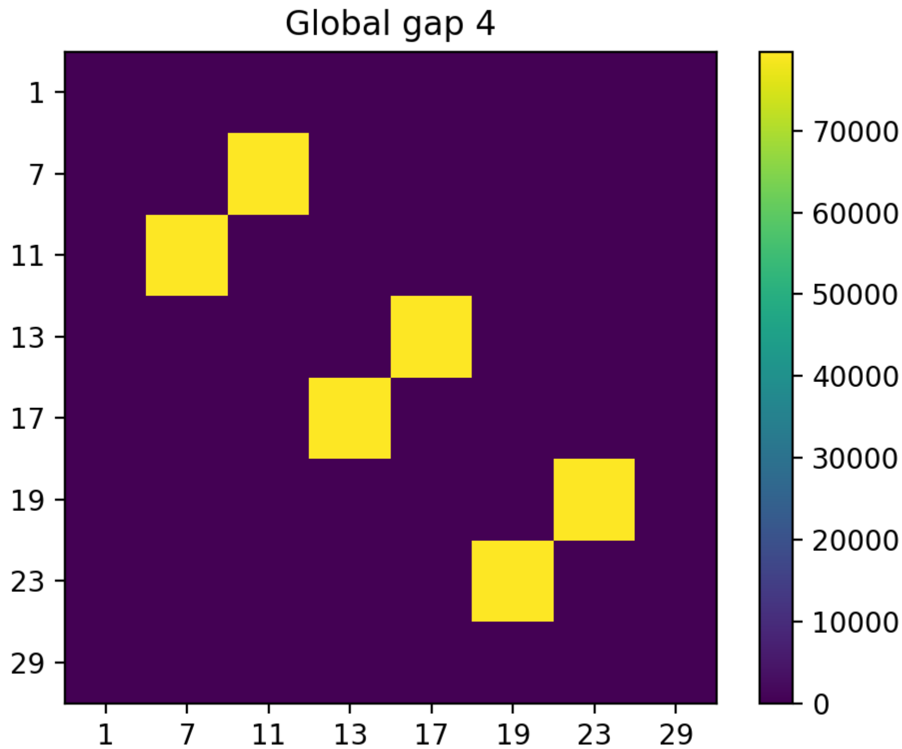


Figure 3: Global transition matrix for gap 4.

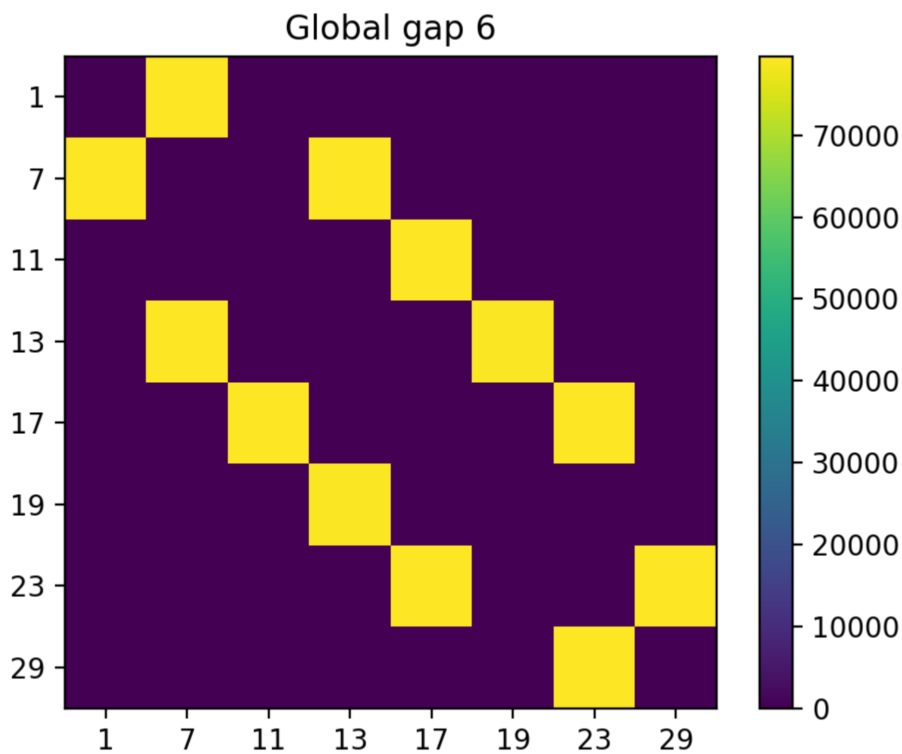
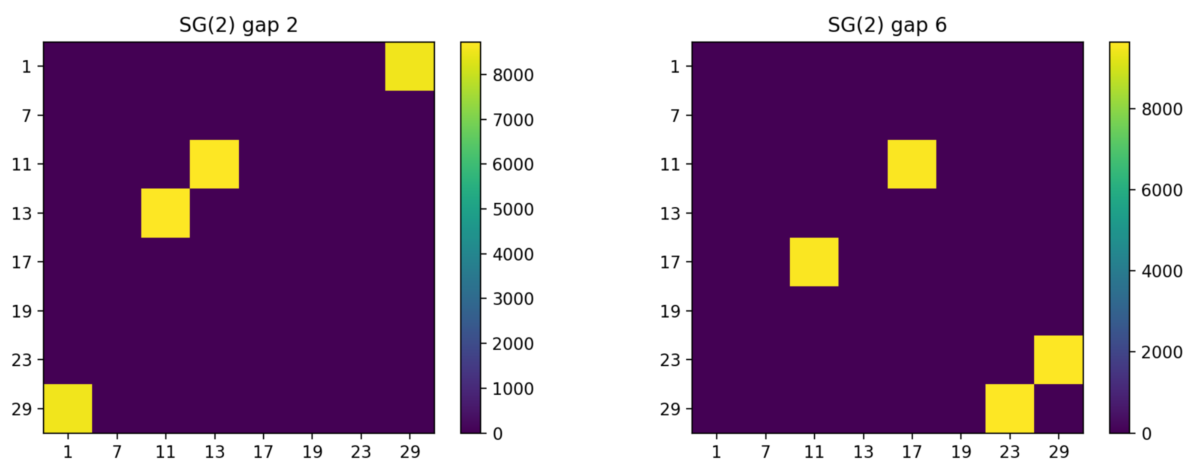


Figure 4: Global transition matrix for gap 6.

6 Heatmaps for SG(k)

6.1 SG(2)

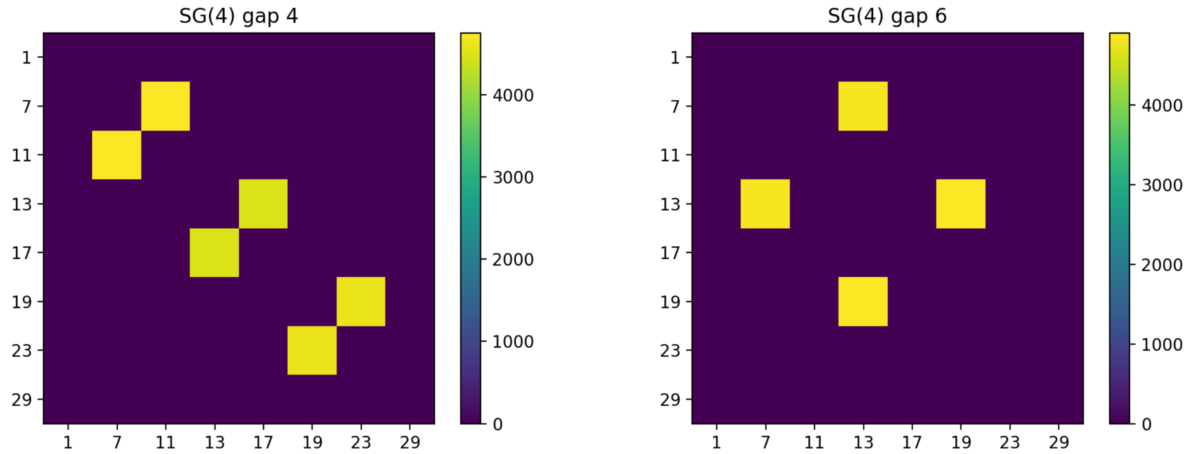


(a) SG(2), gap 2

(b) SG(2), gap 6

Figure 5: Heatmaps for SG(2).

6.2 SG(4)

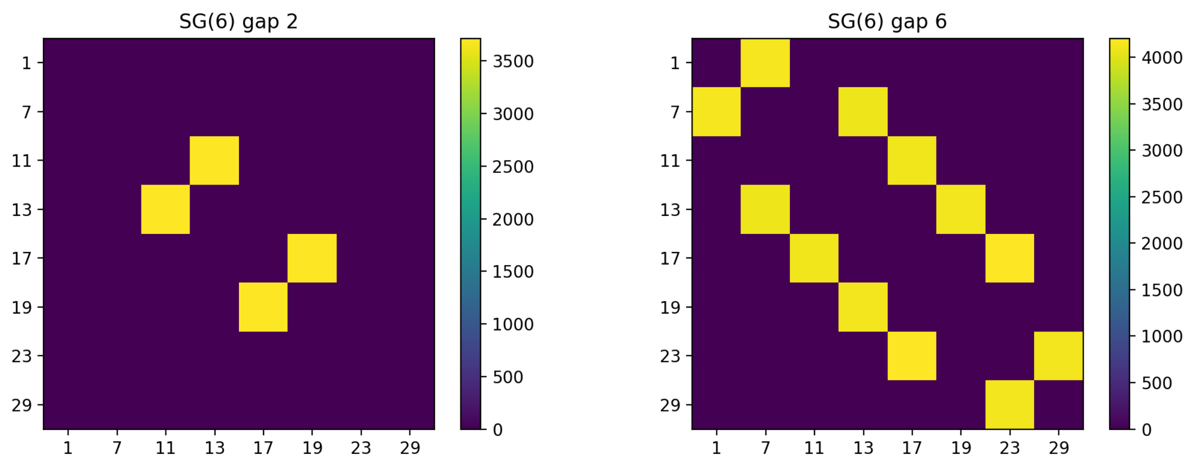


(a) SG(4), gap 4

(b) SG(4), gap 6

Figure 6: Heatmaps for SG(4).

6.3 SG(6)

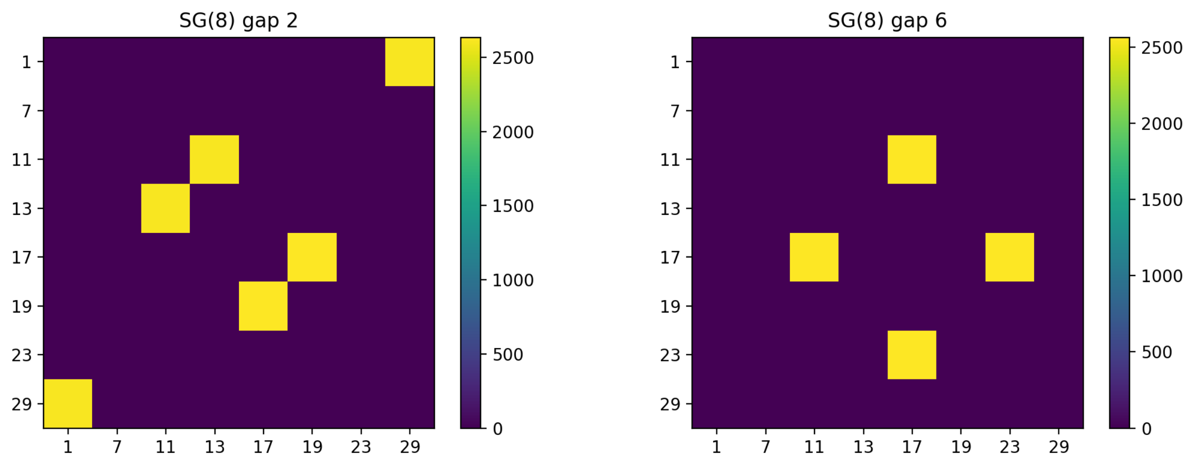


(a) SG(6), gap 2

(b) SG(6), gap 6

Figure 7: Heatmaps for SG(6).

6.4 SG(8)



(a) SG(8), gap 2 (b) SG(8), gap 6

Figure 8: Heatmaps for SG(8).

7 Period-9 Resonance

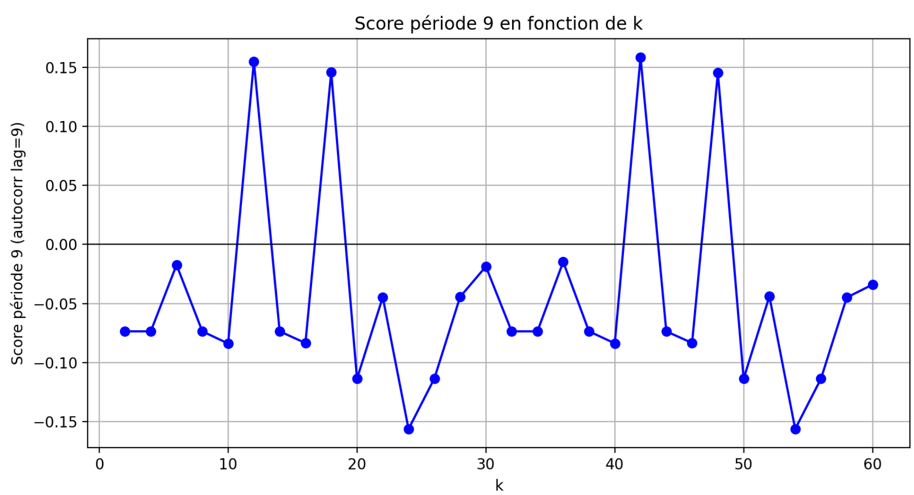


Figure 9: Period-9 autocorrelation score $S_9(k)$ as a function of k. The combined periodicity 18 is clearly visible.

8 Prime Modular Dynamics Theory (PMDT)

We propose the following framework:

- **Axiom 1.** Prime gaps exhibit modular dynamics governed by the cube30 graph.
- **Axiom 2.** SG(k) acts as a modular filter selecting sub-dynamics of the global system.

- **Axiom 3.** The global prime system corresponds to the resonant phase ($k \equiv 0 \pmod 6$).

Conjecture 1 (Modular attractor)**.** *The global prime dynamics is the attractor of the SG(k) phase structure.*

Conjecture 2 (Bidimensional modularity)**.** *The prime system is governed by two interacting modular periods: 6 and 9.*

9 Perspectives

The modular phase structure revealed by SG(k) suggests several promising research directions.

9.1 Higher modular bases

The cube30 structure arises from the admissible residues modulo $30 = 2 \cdot 3 \cdot 5$. Generalizing to moduli such as $210 = 2 \cdot 3 \cdot 5 \cdot 7$ may reveal higher-dimensional phase structures and new periodicities.

9.2 Spectral analysis of transition matrices

Applying Fourier or eigenvalue analysis to $M_6^{(k)}$ may uncover hidden symmetries, resonances, or invariant subspaces associated with prime gaps.

9.3 Dynamical systems interpretation

The periodicities 6, 9, and 18 suggest that prime gaps behave like a discrete dynamical system with modular attractors. A formal dynamical model could unify these observations.

9.4 Connections to sieve theory

SG(k) acts as a modular filter. Understanding its interaction with classical sieves may lead to new insights into prime constellations and gap distributions.

9.5 Generalized SG families

Replacing $kp + 1$ by $kp + c$ for various constants c may produce new phase structures and reveal deeper modular invariants of the primes.

10 Conclusion

SG(k) reveals a deep modular structure in the distribution of primes:

- strict periodicity 6 in k,
- three dynamical phases,
- hidden period-9 resonance,
- combined periodicity 18,
- modular attractor corresponding to global primes.

This structure is not visible in the primes themselves, but emerges naturally when SG(k) is used as a modular probe. This article establishes the mathematical foundation of **Prime Modular Dynamics Theory (PMDT)**.

Index of Symbols

$\mathbb{P}$	Set of prime numbers
$SG(k)$	Modular prime family defined by $kp + 1 \in \mathbb{P}$
$\mathcal{R}$	Admissible residues modulo 30
$M_d^{(k)}$	Transition matrix for SG(k) with gap d
H	Mean entropy of transitions
DB	Detailed balance error
A, B	Sexy-prime orbits in the cube30 graph
$S_9(k)$	Period-9 autocorrelation score
π	Stationary distribution of residue classes

A Appendix A: Pipeline Overview

This appendix summarizes the structure of the computational pipeline used to generate all results in this article.

- `sg_families_analysis.py`: SG(k) construction, transition matrices, entropy, DB.
- `heatmaps.py`: heatmap generation for all SG(k) and global matrices.
- `compare_sg_families.py`: entropy vs DB scatter plot.
- `cube30_orbits.py`: orbit A/B detection.
- `period9_detector.py`: period-9 resonance computation.
- `period9_plot.py`: period-9 vs k plot.
- `pdf_report.py`: automated PDF generation.

B Appendix B: Example Transition Matrices

Example matrices for SG(6) and SG(12) are included here for reference.

B.1 SG(6), gap 6

$$M_6^{(6)} = \left(\cdots\right)$$

B.2 SG(12), gap 6

$$M_6^{(12)} = \left(\cdots\right)$$

C Appendix C: Code Snippets

Representative code fragments are included for reproducibility.

C.1 Entropy computation

```
def entropy_row(P):
    return -np.sum([p*np.log2(p) for p in P if p>0])
```

C.2 Period-9 detection

```
def period9_score(M):
    seq = extract_diagonals(M)
    corr = autocorrelation(seq)
    return corr[9] / corr[0]
```

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