Grammatical Analysis of Sophie Germain Prime Numbers

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1 Parameters

Interval: 0 – 200000 Active grammars: G1, G2, G3

2 Grammatical Model (SG)

Number of generated SG primes: 36

List of SG primes

29, 179, 419, 659, 809, 1229, 1439, 1889, 2339, 2459, 2819, 3449, 3539, 3779, 4019, 4409, 5279, 5639, 5849, 6269, 6329, 6449, 6899, 7349, 8069, 8969, 9419, 9629, 9689, 10799, 12119, 12329, 13049, 13619, 14699, 14879

Gap values k

5, 8, 8, 5, 14, 7, 15, 15, 4, 12, 21, 3, 8, 8, 13, 29, 12, 7, 14, 2, 4, 15, 15, 24, 30, 15, 7, 2, 37, 44, 7, 24, 19, 36, 6

Mean gap: 14.143

3 Random Model (SG)

Number of SG primes found: 32

List of SG primes

2819, 9689, 14669, 19889, 22259, 23099, 26459, 27809, 31019, 41609, 44189, 59879, 70589, 84659, 85829, 92849, 93059, 98669, 112559, 114599, 138179, 138629, 145109, 149399, 150989, 160619, 162119, 167099, 168449, 174959, 180959, 196739

Gap values k

229, 166, 174, 79, 28, 112, 45, 107, 353, 86, 523, 357, 469, 39, 234, 7, 187, 463, 68, 786, 15, 216, 143, 53, 321, 50, 166, 45, 217, 200, 526

Mean gap: 208.516

4 Comparative Analysis

Grammatical SG primes without safe primes

179, 419, 659, 1439, 2459, 3449, 3539, 5279, 9419, 12119, 12329, 13049, 13619, 14699, 14879

Safe-prime SG values absent from the grammatical chain

89, 239, 359, 509, 719, 1499, 2399, 2969, 3359, 5039, 5399, 9029, 9479, 10709

5 Safe Primes

Number of safe primes: 35

List of q values

59, 179, 479, 719, 1019, 1439, 1619, 2459, 2999, 3779, 4679, 4799, 5639, 5939, 6719, 7559, 8039, 8819, 10079, 10799, 11279, 11699, 12539, 12659, 12899, 13799, 14699, 16139, 17939, 18059, 18959, 19259, 19379, 21419, 21599

Associated SG primes p

29, 89, 239, 359, 509, 719, 809, 1229, 1499, 1889, 2339, 2399, 2819, 2969, 3359, 3779, 4019, 4409, 5039, 5399, 5639, 5849, 6269, 6329, 6449, 6899, 7349, 8069, 8969, 9029, 9479, 9629, 9689, 10709, 10799

6 Figures

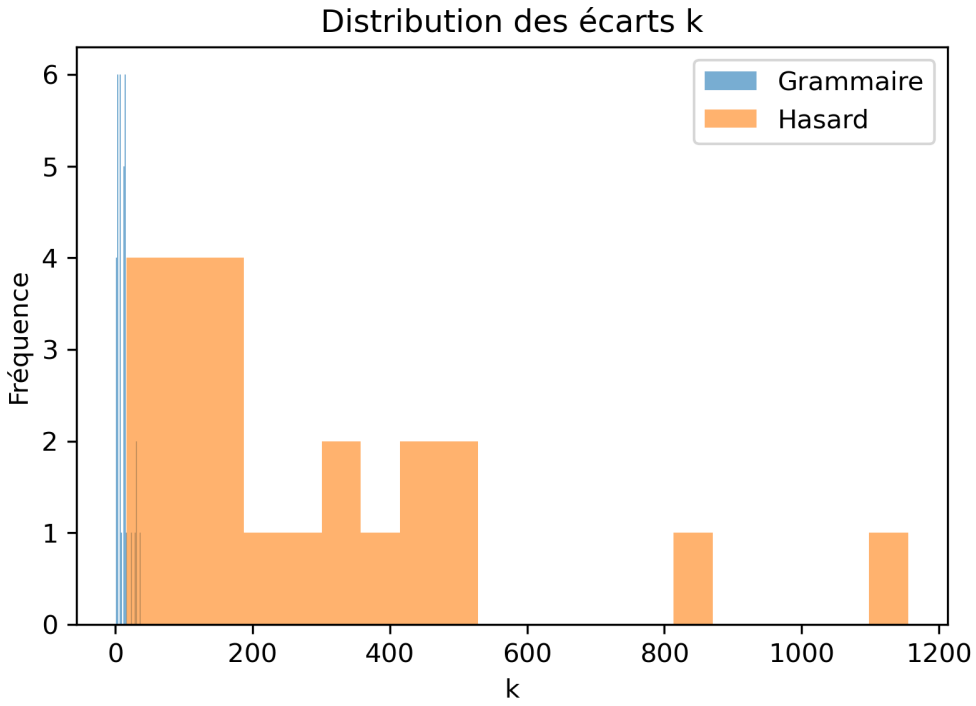

Figure 1: Distribution of gap values k for the grammatical model and the random model.

Mathematical Explanation

1. Sophie Germain Prime Numbers (SG)

A number p is called a **Sophie Germain prime** if p is prime and $q = 2p + 1$ is also prime. The number q is then called a *safe prime*.

2. The 348° Angle (mod 30)

Prime numbers > 5 belong to the residues $\{1, 7, 11, 13, 17, 19, 23, 29\}$ modulo 30. Empirical observations show that SG primes concentrate almost exclusively in residue 29 mod 30, which we call the **348° angle**.

3. Grammars G1 / G2 / G3

- **G1: skeleton** — frequent and regular gap values k.
- **G2: internal motifs** — preferred transitions (k_1, k_2).
- **G3: anomalies** — rare but real sequences observed in the data.

The grammar simulates the internal dynamics of real SG primes.

4. Grammatical Model vs Random Model

The grammatical model produces a dense, coherent chain with small gap values k.
The random model produces rare, scattered SG primes with very large gap values.

5. Safe Primes

Each SG prime p generates a safe prime $q = 2p + 1$.
The safe-prime generator applies an additional filter to select the p values that produce a prime q in the 348° angle.

Definition

A number p is called a **Sophie Germain prime** if p is prime and $2p + 1$ is prime.

348° Angle

SG primes concentrate in residue 29 mod 30, called the 348° angle.

Grammars

- G1: skeleton
- G2: internal motifs
- G3: anomalies