Electricity and Magnetism

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Contents:

Kirchhoff's law

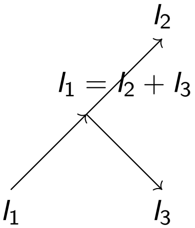
Kirchhoff's laws are fundamental in circuit analysis, consisting of two rules: the Kirchhoff's Current Law (KCL) and the Kirchhoff's Voltage Law (KVL).

1. Kirchhoff's Current Law (KCL)

Statement: The total current entering a junction is equal to the total current leaving the junction. Mathematically,

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

Explanation: At any junction in an electrical circuit, charge is conserved, so the sum of incoming currents equals the sum of outgoing currents.

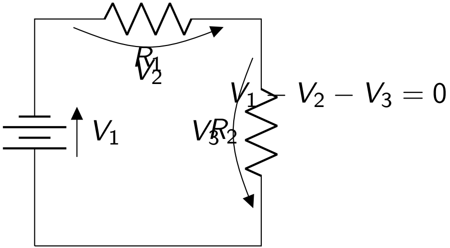


2. Kirchhoff's Voltage Law (KVL)

Statement: The sum of all voltages around a closed loop in a circuit is zero. Mathematically,

$$\sum V = 0$$

Explanation: In any closed loop, the energy gained by charges from sources (like batteries) is equal to the energy lost through resistors or other components.

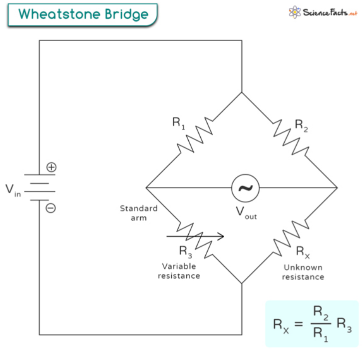


Wheatstone bridge Circuit

Wheatstone Bridge

The Wheatstone Bridge is an electrical circuit used to measure an unknown resistance by balancing two legs of a bridge circuit. It operates based on the principle of null deflection, where no current flows through the galvanometer when the bridge is balanced.

Circuit Diagram:



Balance Condition:

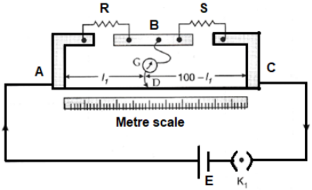
$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

When the bridge is balanced, the ratio of resistances in one leg equals the ratio in the other leg, and no current flows through the galvanometer.

Meter Bridge

The Meter Bridge is a practical form of the Wheatstone Bridge used to measure unknown resistances. It consists of a one-meter-long wire of uniform cross-sectional area stretched over a meter scale.

Circuit Diagram:



Working Principle: The balance condition is achieved by adjusting the jockey on the wire until the galvanometer shows zero deflection. The unknown resistance R is then calculated using:

$$\frac{R}{S} = \frac{l}{(100 - l)}$$

where l is the balancing length.

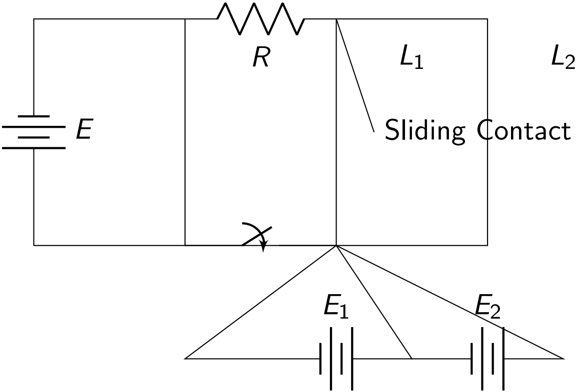
Potentiometer

The potentiometer is an instrument used to measure the electromotive force (emf) of a cell, compare the emfs of two cells, and determine the internal resistance of a cell. It works on the principle that the potential difference across a uniform wire is directly proportional to its length when a constant current flows through it.

1. Comparison of EMFs

The potentiometer can be used to compare the emfs of two cells.

Circuit Diagram:



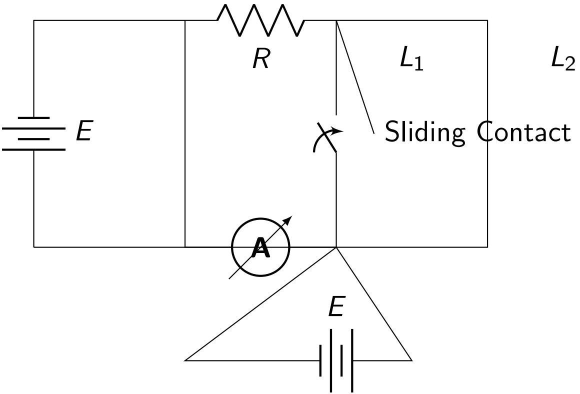
Mathematical Formulation: Let L_1 and L_2 be the balancing lengths for the two cells E_1 and E_2, respectively. Since the potentiometer works on the principle of a null deflection, we have:

$$\frac{E_1}{E_2} = \frac{L_1}{L_2}$$

2. Measurement of Internal Resistance of a Cell

The potentiometer can also be used to measure the internal resistance r of a cell.

Circuit Diagram:



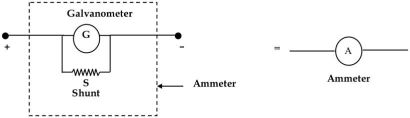
1. Let L_1 be the balancing length when the cell is in open circuit. 2. Let L_2 be the balancing length when the cell is connected to a known resistance R.

The internal resistance r is given by:

$$r = R\left(\frac{L_1}{L_2} - 1\right)$$

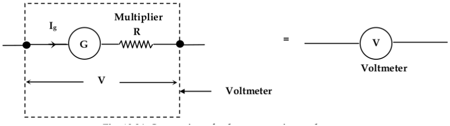
Explanation: The balancing length L_1 corresponds to the emf of the cell, while L_2 corresponds to the potential difference across the cell when it delivers current through R. The difference allows the calculation of r.

Conversion of Galvanometer to ammeter



- **Galvanometer:** A galvanometer is a sensitive device used to detect small currents.
- **Add Shunt Resistor:** Connect a low-resistance shunt resistor in parallel with the galvanometer.
- **Low Resistance:** The shunt should have a very low resistance to allow most current to bypass the galvanometer.
- **Current Range:** The total resistance determines the ammeter's current range.
- **Calibration:** Calibrate the scale to show current instead of voltage.
- **Adjust Sensitivity:** Ensure the galvanometer can measure small currents accurately without damage.
- Calculate the shunt resistance using the formula: $R_s = \frac{I_g G}{I - I_g}$

Conversion of Galvanometer to voltmeter



- **Add Shunt Resistor:** Connect a large resistor in series.
- **High Resistance:** Ensure the shunt has high resistance.
- **Calibrate:** Adjust for accurate voltage readings.
- **Voltage Range:** Set the total resistance for desired range.
- **Adjust Scale:** Modify the scale to read voltage.

  The voltage across the galvanometer at full-scale deflection is:

$$V_g = I_g \cdot G$$

To measure the maximum voltage V_{max}, the total resistance of the voltmeter should be:

$$R_{\text{total}} = \frac{V_{\text{max}}}{I_g}$$

Now, the multiplier resistance R_m is the difference between the total resistance and the resistance of the galvanometer:

$$R_m = R_{\text{total}} - G = \frac{V_{\text{max}}}{I_g} - G$$

Thus, the multiplier resistance required to convert the galvanometer into a voltmeter is:

$$R_m = \frac{V_{\text{max}}}{I_g} - G$$

Joule's Law of Heating

According to this law, the amount of heat (H) developed in an ohmic conductor by the passage of current I .

1. Directly proprtional to the square of the current $i.e. H \propto I^2$
2. Directly proportional to the resistance of the conductor $H \propto R$
3. Directly proportiona to the time flow of current $H \propto t$

On Combining above reltion we get,

$$H \propto I^2 Rt$$
$$H = kI^2 Rt$$

Where, k is the proportionality constant. The value of k depends on the system of unit of heat.

$k = \frac{1}{j}$ is the unit conversion factor. and Its value is 4.2 J/calorie.

Verification of Joule's Law

To verify Joule's Law experimentally, we focus on the following three points:

1. **Heat is directly proportional to the square of the current (I^2):**
   By varying the current and keeping R and t constant, measure the heat produced. A plot of H vs I^2 should yield a straight line.
2. **Heat is directly proportional to the resistance (R):**
   By varying the resistance and keeping I and t constant, measure the heat produced. A plot of H vs R should yield a straight line.
3. **Heat is directly proportional to the time (t):**
   By varying the time and keeping I and R constant, measure the heat produced. A plot of H vs t should yield a straight line.

Thermoelectric Effects

Thermoelectric effects describe the direct conversion of temperature differences into electric voltage and vice versa. The two primary effects are:

- **Seebeck Effect**
- **Peltier Effect**

Mathematical Explanation

Seebeck Effect:

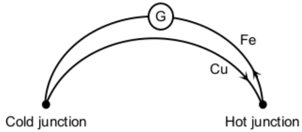
The Seebeck effect generates a voltage when a temperature gradient is applied across a conductor. The voltage V is given by:

$$V = S \cdot \Delta T$$

where:

- S is the Seebeck coefficient (material property),
- ΔT is the temperature difference.

Thermocouple:



A couple of wires of dissimilar metals forming a loop and producing thermoelectricity is called a thermocouple.

Thermoelectric series:

Antimony, Iron, Zinc, Silver, Gold, Tin, Lead, Copper, Platinum, Nickel, Bismuth.

- This series has two main advantages: (a) to know the direction of current flow in the couple, (b) to find the thermo-emf in the thermo-couple.
- In this series,Antimony-Bismuth (Sb–Bi) couple produces the maximum thermo-emf among any possible couple in the given series.
- Therefore, thermocouple is usually made up of Antimony and Bismuth.

Causes of Seebeck Effect

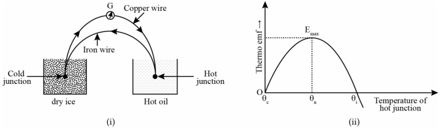
The Seebeck Effect occurs due to the generation of a voltage when a temperature gradient is applied across a conductor or semiconductor. The primary causes are:

Causes of the Seebeck Effect

1. **Temperature Gradient:**
   A temperature difference (ΔT) across a material causes charge carriers (electrons or holes) to diffuse from the hot end to the cold end, creating a voltage.
2. **Charge Carrier Diffusion:**
   Electrons in the hotter region have higher kinetic energy and diffuse toward the colder region, leading to an accumulation of charge and an electric potential.
3. **Material Properties:**
   The Seebeck coefficient (S) of a material determines how effectively it converts a temperature gradient into a voltage. Different materials have different S values.

Verifaction of Thermo-emf(E) with temperature (θ):

Thermo-emf increases until it attains maximum value and the temperature at which Thermo-emf becomes maximum is called neutral temperature. Therefore, the temperature of hot junction at which the Thermo-emf becomes maximum is called neutral temperature (θ_n).



- In such condition, the relation between Thermo-emf and temperature is given by,

  $$\text{i.e. } E = \alpha\theta + \frac{1}{2}\beta\theta^2$$

  where , α and β are constants which are called thermoelectric coefficients.
- **At neutral temperature:** The first derivative of emf w.r.t. temperature must be zero.

  $$\text{i.e. } \frac{dE}{d\theta_n} = 0$$

$$\frac{d(\alpha\theta_n+\frac{1}{2}\beta\theta_n^2)}{d\theta}=0$$

or, $$\alpha+\beta\theta_n=0$$

$\Longrightarrow$ $$\theta_n=-\frac{\alpha}{\beta}$$

- **At temperature of inversion:** The thermo emf is zero at the temperature of inversion (θ_i) Beyond this temperature the thermo emf changes sign and direction current reverses. and E=0.

  or, $$\alpha\theta_i+\frac{1}{2}\beta\theta_i^2=0$$

  or, $$\alpha+\frac{\beta\theta_i}{2}=0$$

  $\Longrightarrow$ $$\theta_i=-\frac{2\alpha}{\beta}$$

- Therefore, the neutral temperature can be determined by taking average of temperature of inversion (θ_i) and temperature of cold junction (θ_c).

  or. i.e. $\theta_n=\frac{\theta_i+\theta_c}{2}$

- In Cu-Fe thermocouple, neutral temperature is about 270^0C when cold junction is maintained at 0^0C.

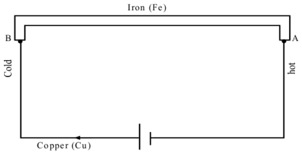
Peltier Effect:

The Peltier effect describes heat absorption or release at the junction of two materials when an electric current flows through it. The heat Q is given by:

$$Q = \Pi \cdot I$$

where:

- Π is the Peltier coefficient,
- I is the electric current.



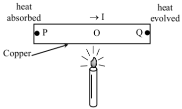
Cause of Peltier effect

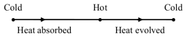
- Due to dissimilar metals have different electron densities.
- Electrons tend to diffuse from higher potential metal to lower potential metal.

Thomson's Effect

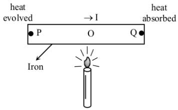
- When two ends of a metal conductor are maintained at different temperatures and current is passed through it, heat is evolved from one end and heat is absorbed at another end.

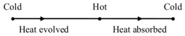
1 Positive thomson effect: The evolution of heat in the part of conductor along which current flows in the direction of temperature fall.





2 Negative thomson effect: When a current is sent in the iron rod in the direction from P to Q, the point P becomes hotter than point Q i.e., heat energy is transferred from Q to P.

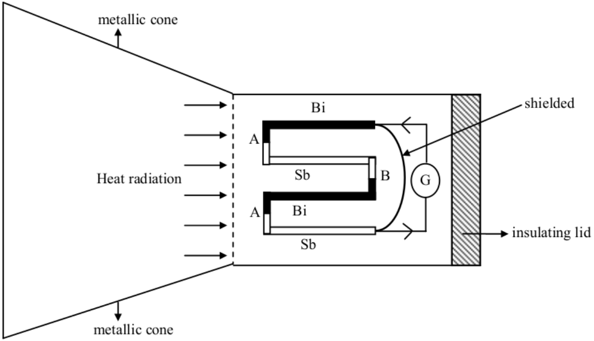




- If the direction of current in either of the above cases is reversed, the Thomson effect is also reversed. In lead, the Thomson effect is zero.

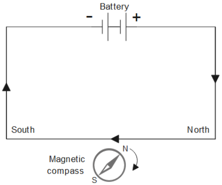
Thermopile:

- Thermopile is an electrical device that uses Seebeck effect to detect and measure the intensity of thermal radiation.
- It works on the principle of thermoelectric effect.
- t is constructed with the series combination of thermocouple made up of Antimony (Sb) and Bishmuth (Bi).



Magnetic effect of current

Oersted's experiment



- Oersted's experiment demonstrated that an electric current creates a magnetic field, establishing a fundamental connection between electricity and magnetism.
    - **Electric Current and Magnetism:** A compass needle deflected when placed near a wire carrying electric current, showing that electricity produces magnetism.
    - **Direction of Magnetic Field:** The magnetic field's direction depends on the current's direction, forming circular lines around the wire.
    - **Foundation for Electromagnetism:** This discovery laid the groundwork for further studies in electromagnetism.

Rules of finding direction of magnetic field

1. maxwell's cork screw rule:
    - **Direction of Magnetic Field:** If a right-handed cork screw is rotated in the direction of the current, the direction in which the screw moves represents the direction of the magnetic field around the current-carrying conductor.
    - **Circular Field Lines:** The magnetic field forms concentric circular loops around the wire, perpendicular to the direction of the current.
2. Right hand thumb rule:
    - **Direction of Magnetic Field:** If you grasp a current-carrying wire with your right hand.
    - The thumb pointing in the direction of the current, the curled fingers show the direction of the magnetic field around the wire.
3. Fleming left hand rule:
    - **Force Direction:** Stretch the thumb, forefinger, and middle finger of your left hand mutually perpendicular to each other;
    - The forefinger points in the direction of the magnetic field, the middle finger in the direction of the current, and the thumb indicates the direction of the force on the conductor.

Lorentz force:

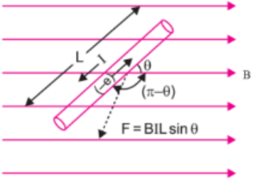
The force experienced by a charged particle moving in an electromagnetic field

- Electric force experienced by charge q is $\vec{F_e} = q\vec{E}$
- Magnetic force experienced by charge q is $\vec{F} = q(\vec{v} \times \vec{B})$
- Net force experienced by charge q is given by $\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]$
  This is called the lorentz force at Electic force zero is.

$$or, \vec{F} = Bqv \sin\theta \tag{1}$$

- At the charge q is at rest v=0 and so $\vec{F} = 0$
- At $\theta = 0$ or $\theta = 180^0$ the charge q is in parallel or antiparallel to the field so Minimum magnetic force is $\vec{F} = 0$
- At $\theta = 90^0$ The charge q is in perpendicular to the field then it expericence the maximum magnetic force is $\vec{F} = Bqv$

Magnetic force on a current carrying conductor



- Let us consider a straight conductor of length L and cross-sectional area A carrying a steady current I from bottom to top placed in uniform magnetic field $\vec{B}$
- Let θ be the angle of inclination ,
- If v_d be the drift velocity of charges then force experienced by charge is .
- $\vec{F} = q(\vec{v_d} \times \vec{B})$ $F = Bqv_d \sin\theta$
- Total number of charge in conductor is $N = nV = nAL$ (where n= charge per unit volume and V is colume of conductor)
- Net magnetic force $F = NBqv_d \sin\theta$

$$F = (nAL)Bqv_d \sin\theta$$

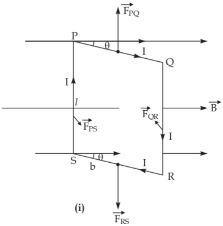
$$\therefore F = (nqAv_d)(BL\sin\theta)$$

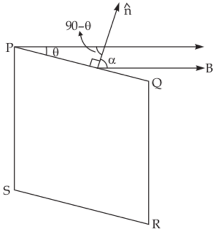
We know drift velocity $v_d = \frac{I}{nAq}$ By replacing terms by I (current) then we get

$$F = IBL\sin\theta \implies \vec{F} = I(\vec{L} \times \vec{B})$$

This is the total force exeperienced by the conductor placed in uniform

Torque on Rectangular Current Loop





- Consider a rectangular current loop PQRS of a conducting wire with linear dimension L and b carrying current I through it, and is placed in a uniform magnetic field $\vec{B}$.
- The force acting on each side of the loop is described below.

$$F_{PQ} = IBb\sin\theta, \text{ (upward)}$$
$$F_{RS} = IBb\sin\theta, \text{ (downward) both are equal and}$$

opposite and cancel each other.

Here, PQ=RS=b (into the plane of paper)

$\therefore F_{QR} = IBL\sin 90^0 = IBL$

or, $F_{PS} = IBL\sin 90^0 = IBL$

- The Torque (τ) = Magnitude of either force $\times$ perpendicular distanace between them.

$$\tau = IBL \times b\cos\theta \implies \tau = IBA\cos\theta \text{ (Lb=A)}$$

- Let N be the number of turns of rectangular coils then total torque is given by

$$\tau = BINA\cos\theta$$

- Let α be the angle betwen $\vec{B}$ and normal $\hat{n}$ to the plane of loop, then $\alpha = 90^0 - \theta$

$$\tau = BINA\cos(90^0 - \alpha)$$

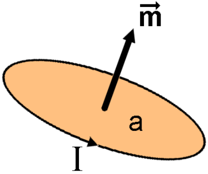
$$\therefore \tau = BINA\sin\alpha \tag{2}$$

  This is the required expression if torque on a rectangular current loop.

- **Loop perpendicular to B:** The net force is zero as forces on opposite sides cancel out, and no torque is produced.
- **Loop at an angle to B:** Unequal force alignment creates a couple, generating torque (τ) that rotates the loop.

Magnetic Moment:

- When current flows in a closed loop, it acts as a tiny magnet in which the magnetic field is directed axially outward.
- The current loop acts as a dipole. This produces the magnetic moment $\mu = IA$

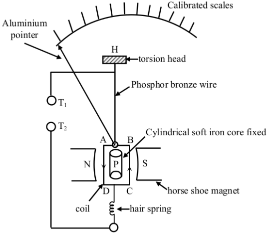


- Magnetic moment of a coil containing N turns is $\mu = NIA$
- As we know that , $\tau = \mu B \sin\theta$ ($\because ab\sin\theta = \vec{a} \times \vec{b}$)
  or, $\vec{\tau} = \vec{\mu} \times \vec{B} \implies \vec{\tau} = NI\vec{A} \times \vec{B}$

Moving Coil Galvanometer

- **Principle:** It works on the principle that a current-carrying coil placed in a magnetic field experiences a torque, causing it to rotate.
- **Construction:** It consists of a rectangular coil wound on a metallic frame, suspended between the poles of a strong magnet. A soft iron core enhances the magnetic field, and a suspension wire provides restoring torque.
- **Theory:** The torque acting on the coil is given by:

- Torque $\tau = BINA \sin\theta$
- At Equilibrium $BINA = k\theta \implies I \propto \theta$
- This shows, the deflection of the coil is proportional to the current flowing through it.
- This means corrent and voltage can me measured by angular deflection of galvanometer.

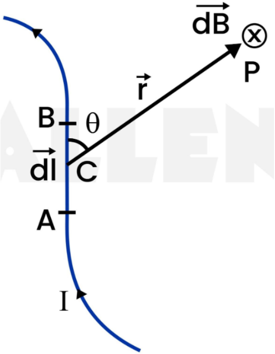


1. Current sensitivity
   It is the deflection per unit current. $S_I = \frac{\theta}{I} = \frac{NBA}{K}$
   where N is the number of turns, A is the coil area, B is the magnetic field, and k is the restoring torque per unit deflection.

2. Voltage sensitivity
   It is the deflection per unit voltage. $S_v = \frac{\theta}{v} = \frac{NBA}{kR}$
   The sensitivity can be increased by increasing N, A and B and decreasing k and R.

Biot-Savart Law

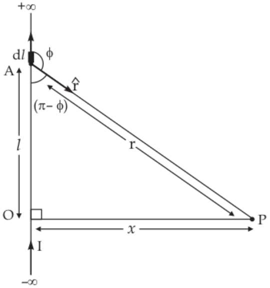
- This law is used to find net magnetic field due to any distribution of currents by first writing differential magnetic field $d\vec{B}$
- The magnitude of differential magnetic field (dB) at point P at distance r due to current length element $Id\vec{L}$ is,
  - Directly proportional to the current element IdL, $dB \propto IdL$
  - Inversely proportional to the square of radial distance (r), $dB \propto \frac{1}{r^2}$
  - Directly proportional to the sine angle between $d\vec{L}$ and $\vec{r}$, $dB \propto \sin\phi$



- Combining above conditions $dB = \frac{\mu_0}{4\pi}\frac{IdL\sin\phi}{r^2}$ in terms of vectors $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{L}\times\hat{r}}{r^2}$ where μ_0 is absolute permeability and its value is $4\pi \times 10^{-7}Hm^{-1}$
- This is the required expression of Biot and Savart's law for magnetic field.

1. Magnetic field due to an infinitely long straight conductor carrying current

- Let us consider an infinitely long straight conductor carrying steady current I.
- Let P be a point, at a perpendicular distance x from the conductor where the magnetic field $\vec{B}$ is to be determined.
- consider an element of conductor of length dL = dy at point A that is r distance away from point P.
- The small magneitc field at p due to length element dl is



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\vec{dl}\times\hat{r}}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{IdL\sin\phi}{r^2} \tag{3}$$

- In right angled traingle AOP $r^2 = x^2 + L^2$ and $\sin(\pi-\phi) = \frac{x}{r} \implies \sin\phi = \frac{x}{\sqrt{(x^2+L^2)}}$

- Then small magnetic field $dB = \frac{\mu_0 I}{4\pi}\frac{xdL}{(x^2+L^2)^{\frac{3}{2}}}$
- Total magnetic fiels is given by integrating from $-\infty$ to ∞ we get

$$B = \int_{-\infty}^{\infty} dB = \int_{-\infty}^{\infty} \frac{\mu_0}{4\pi}\frac{xdL}{(x^2+L^2)^{\frac{3}{2}}} \tag{4}$$

- Further, $cot(180-\phi) = \frac{L}{x} \implies -\cot\phi = \frac{L}{x}$
  Differentiating , $L = -\cot\phi$ and $dL = xcosec^2\phi d\phi$
- Here when $I \to \infty, \phi \to \pi$ and when $I \to -\infty, \phi \to 0$
- By changing the limiting values, we can write,

$$B = \frac{\mu_0 I}{4\pi}\int_0^\pi \frac{x.xcosec^2\phi d\phi}{(xcosec^2\phi)^{\frac{3}{2}}} = \frac{\mu_I}{4\pi}\int_0^\pi \frac{\sin\phi d\phi}{x}$$

or, $\quad B = \frac{\mu_0 I}{4\pi x}[-\cos\phi]_0^\pi = \frac{\mu_0 I}{4\pi x}[\cos\pi + \cos 0]$

or, $\quad B = \frac{\mu_0 I}{4\pi x}[-(-1)+1]$

$$\therefore B = \frac{\mu_0 I}{2\pi x} \tag{5}$$

2 (i). Magnetic field due to a circular coil at its centre

- Let us consider a coil carrying a current I .Let O be the centre and a be the radius.
- Let dL be the length element at point p then small magnetic field dB is given by.

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{L} \times \hat{a}}{a^2} \text{ where } \hat{a} \text{ is unit vector along PO.}$$

- In terms of magnitude magnitic field is
$$dB = \frac{\mu_0 I dL \sin\phi}{4\pi a^2} = \frac{\mu_0 I dL \sin 90^0}{4\pi a^2} \implies dB = \frac{\mu_0}{4\pi}\frac{IdL}{a^2}$$
- So, the total magnetic field is the sum of magnetic fields due to individual length elements is

$$B = \int_0^{2\pi a} \frac{\mu_0}{4\pi}\frac{Idl}{a^2}$$

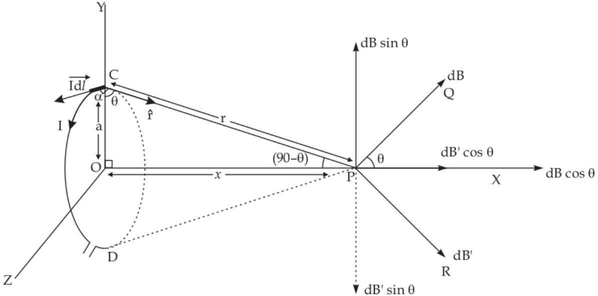
$$B = \frac{\mu_0 I}{4\pi a^2}\int_0^{2\pi a} dl = \frac{\mu_0 I}{4\pi a^2}(2\pi a) = \frac{\mu_0 I}{2a}$$

- If N be the number of turns Then ,

$$B = \frac{\mu_0 NI}{2a} \tag{6}$$

2(ii) Magnetic field due to a circular coil at any point on the axis

Let us consider a circular coil of mean radius 'a' carrying a steady current I in anti-clockwise direction.Let 'P' be a point at a distance x on the x-axis from centre O, where the magnetic field is to be determined.



- Let us consider an elemental length ='dL' on the upper half.
- current element I dl at C is along Z-axis out of the plane of the paper. and perpendicular to it.
- Let, r be the distance between elemental length and the point P.

- From Biot-Savart's law

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{L} \times \hat{r}}{r^2}$$

- The magnitude of magnetic field at P is $dB = \frac{\mu_0}{4\pi}\frac{IdL\sin\alpha}{r^2}$,
  Here but angle between $d\vec{L}$ and $\hat{r}$ is $\alpha = 90^0$ so

$$dB = \frac{\mu_0 IdL}{4\pi r^2}$$

  The angle θ made by dB with x-axis
  Then Components of dB are

$$dB_x = dB\cos\theta = \frac{\mu_0}{4\pi}\frac{IdL\cos\theta}{r^2}$$
$$dB_y = dB\sin\theta = \frac{\mu_0}{4\pi}\frac{IdL\sin\theta}{r^2}$$

The Net field at P, due to the whole coil is given by

$$B = \int_0^{2\pi a} dB_x = \frac{\mu_0}{4\pi}\int_0^{2\pi a}\frac{IdL\cos\theta}{r^2} = \frac{\mu_0 I}{4\pi r^2}\cos\theta\int_0^{2\pi a} dL$$
$$B = \frac{\mu_0 I}{4\pi r^2}\cos\theta.[2\pi a - 0] = \frac{\mu_0 I}{4\pi r^2}\cos\theta.2\pi a$$

$$\therefore B = \frac{\mu_0 Ia}{2\pi r^2}\cos\theta \qquad (7)$$

From ΔCOP, $\cos\theta = \frac{a}{r}$ and $r = \sqrt{(a^2 + x^2)}$
The (7) becomes,

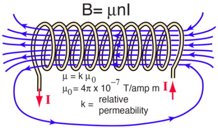
$$B = \frac{\mu_0 I a^2}{2(a^2+x^2)^{\frac{3}{2}}} \qquad \text{(using } a^2 = \frac{A}{\pi}\text{) we get,}$$

$$\therefore B = \frac{\mu_N I A}{2\pi(a^2 + x^2)^{\frac{3}{2}}} \tag{8}$$

Special cases:

- When point P lies at center of coil, x=0 $\implies B = \frac{\mu_0 N I}{2a}$
- when point p lies on axis at x=a $\implies B = \frac{\mu_0 N I}{\sqrt{2^5} a}$
- when point p lies on axis at $x >> a, \therefore a^2 + x^2 = x^2 \implies B = \frac{\mu_0 N I a^2}{2x^3}$

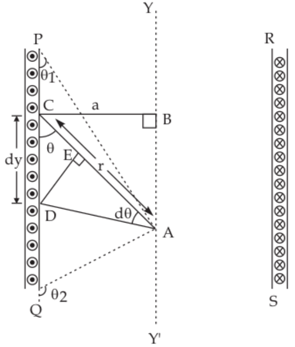
3. Magnetic field due to Solenoid:



Consider a solenoid of radius a with n turns per unit length carrying a current I.
We know magnetic field for identical circulal coil of radius a is

$$dB = \frac{\mu_0 I a^2}{2r^3} \tag{9}$$

To find the magnetic field along its axis, we consider a small element of length dy of n truns is to be determined.



Step 1: Magnetic Field Due to a Small Element a small current element IdL is given by:

$$dB = \frac{\mu_0 I a^2}{2r^3} ndy \tag{10}$$

where AC=AD=r is the radius vector and angle $\angle DCA= \theta$,
From Geometry $\angle CDA = \angle CAB = \theta$ and $\sin\theta = \frac{DE}{CD} = \frac{DE}{dy}$

$$dy = \frac{DE}{\sin\theta} \implies DE = dy\sin\theta$$

Again, In $\Delta DAE, \sin\theta = \theta = \frac{DE}{DA} = \frac{DE}{r} \implies DE = rd\theta$
using in above $dy = \frac{rd\theta}{\sin\theta}$ and from $\angle CAB$, $a = r\sin\theta$,
Finally (10) becomes , $dB = \frac{\mu_0 I}{2r^3}(r\sin\theta)^2.\frac{(nrd\theta)}{\sin\theta}$

$$\therefore dB = \frac{1}{2}\mu_0.nI\sin\theta.d\theta \tag{11}$$

Step 2: Summing Contributions from All Elements
To find the total magnetic field is taken from θ_1 to θ_2:

$B = \int_{\theta_1}^{\theta_2} dB = \int_{\theta_1}^{\theta_2} \frac{1}{2}\mu_0 nI \sin\theta . d\theta = \frac{\mu_0 nI}{2}[-\cos\theta]_{\theta_1}^{\theta_2}$

$B = \frac{\mu_0 nI}{2}[-\cos\theta_2 - (-\cos\theta_1)]$

$B = \frac{\mu_0 nI}{2}(\cos\theta_1 - \cos\theta_2)$

Step 3: Approximation for Long Solenoid

If the solenoid is infite ly long, then θ varies from 0^0 to 180^0 as below.

$$B = \frac{1}{2}\mu_0 nI(\cos 0^0 - \cos 180^0) = \frac{1}{2}\mu_0 nI[1 - (-1)]$$

For a long solenoid where $L \to \infty$, we approximate:

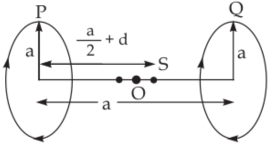
$$B = \mu_0 nI \tag{12}$$

This shows that the field is uniform inside a long solenoid.

If N be the total number of turns, then , $n = \frac{N}{L} \implies B = \frac{\mu_0 NI}{L}$

4. Helmholtz Coil:

A helmholtz coil is a parallel pair of identical circular coils spaced one radius apart and arranged co-axially such that current flow in same direction.



- Let us consider two identical coils each of radius a and carrying steady current I which are arranged co-axially separated by a distance a from each other.
- Let S be any point on the axis d distance away from mid- point O where magnetic field is to be determined.
- If N be the number of turns in each coil, then magnetic field at S due to coil P is.

$$B_p = \frac{\mu_0 N I a^2}{2[(\frac{a}{2}+d)^2+a^2]^{\frac{3}{2}}}$$

Similarly, field at S due to Q is ,

$B_Q = \frac{\mu_0 NIa^2}{2[(\frac{a}{2}-d)^2+a^2]^{\frac{3}{2}}}$ The resultant field is given by

$$B = B_P + B_Q = \frac{\mu_0 NIa^2}{2}\left[\frac{1}{\left\{\left(\frac{a}{2}+d\right)^2+a^2\right\}^{\frac{3}{2}}}+\frac{1}{\left\{\left(\frac{a}{2}-d\right)^2+a^2\right\}^{\frac{3}{2}}}\right]$$

$$= \frac{\mu_0 NIa^2}{2}\left[\frac{1}{\left\{\frac{a^2}{4}+ad+d^2+a^2\right\}^{\frac{3}{2}}}+\frac{1}{\left\{\frac{a^2}{4}-ad+d^2+a^2\right\}^{\frac{3}{2}}}\right]$$

$$= \frac{\mu_0 NIa^2}{2}\left[\frac{1}{\left\{\frac{a^2}{4}\left(1+\frac{4d}{a}+\frac{4d^2}{a^2}\right)+a^2\right\}^{\frac{3}{2}}}+\frac{1}{\left\{\frac{a^2}{4}\left(1-\frac{4d}{a}+\frac{4d^2}{a^2}\right)+a^2\right\}^{\frac{3}{2}}}\right]$$

Here, $d \ll a$, so neglecting higher powers of $\frac{2d}{a}$, we get,

$$B = \frac{\mu_0 N I a^2}{2}\left[\frac{1}{\left\{\frac{a^2}{4}\left(1+\frac{4d}{a}\right)+a^2\right\}^{\frac{3}{2}}}+\frac{1}{\left\{\frac{a^2}{4}\left(1-\frac{4d}{a}\right)+a^2\right\}^{\frac{3}{2}}}\right]$$

$$= \frac{\mu_0 N I a^2}{2}\left[\frac{1}{\left(\frac{5a^2}{4}+ad\right)^{\frac{3}{2}}}+\frac{1}{\left(\frac{5a^2}{4}-ad\right)^{\frac{3}{2}}}\right]$$

$$= \frac{\mu_0 N I a^2}{2}\left[\frac{1}{\left(\frac{5a^2}{4}\right)^{\frac{3}{2}}\left(1+\frac{4d}{5a}\right)^{\frac{3}{2}}}+\frac{1}{\left(\frac{5a^2}{4}\right)^{\frac{3}{2}}\left(1-\frac{4d}{5a}\right)^{\frac{3}{2}}}\right]$$

$$= \frac{\mu_0 N I a^2}{2}\left(\frac{4}{5a^2}\right)^{\frac{3}{2}}\left[\left(1+\frac{4d}{5a}\right)^{-\frac{3}{2}}+\left(1-\frac{4d}{5a}\right)^{-\frac{3}{2}}\right]$$

$$= \frac{\mu_0 NIa^2}{2}\left(\frac{4}{5}\right)^{\frac{3}{2}} \cdot \frac{1}{(a^2)^{\frac{3}{2}}}\left[1 - \frac{3}{2}\left(\frac{4d}{5a}\right) + \cdots + 1 + \frac{3}{2}\left(\frac{4d}{5a}\right) - \dots\right]$$

($\because$ Using binomial expansion and neglecting higher powers of $\frac{4d}{5a}$)

$$= \frac{\mu_0 NI}{2a}\left(\frac{4}{5}\right)^{\frac{3}{2}}[1+1]$$

$$= \frac{\mu_0 NI}{2a}\left(\frac{4}{5}\right)^{\frac{3}{2}} = 0.72\frac{\mu_0 NI}{a} \quad \text{(approx.)}$$

Now, at mid-point 0,

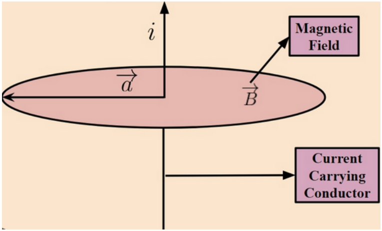
$$B = 2 \times \text{magnetic field due to each} = 2 \cdot \frac{\mu_0 NIa^2}{2\left(\frac{a^2}{4} + a^2\right)^{\frac{3}{2}}}$$

$$B = \frac{\mu_0 N I a^2}{\left(\frac{5a^2}{4}\right)^{\frac{3}{2}}} = 0.72 \frac{\mu_0 N I}{a}$$

This is the magnetic field due to helmholtz coil.

Ampere's Circital Law

The line integral around a closed path of the component of the magnetc field tangent to the direction of the path equals μ_0 times the current intercepted by the area within the path.



- **Mathematical Form:** It is expressed as: $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}$ where B is the magnetic field, dl is a small element of the closed path,μ_0 is the permeability of free space, and I_{enc} is the total current enclosed by the loop.
- **Application in Symmetric Cases:** Ampère's Law is useful for determining magnetic fields in highly symmetric cases such as long straight conductors, solenoids, and toroids.

Magnetic Field due to a long straight conductor carrying current

Consider a long, straight conductor carrying a steady current I. The magnetic field at a perpendicular distance r from the wire is determined. Since the magnetic field B is tangential to the circular path and has the same magnitude at all points on the loop, the line integral simplifies to:
$\oint \mathbf{B} \cdot d\mathbf{l} = B \oint d\mathbf{l} = B(2\pi r)$
From Ampère's Law, $B(2\pi r) = \mu_0 I$
Solving for B: $B = \frac{\mu_0 I}{2\pi r}$
Thus, the magnetic field due to a long straight conductor carrying current is:

$$B = \frac{\mu_0 I}{2\pi r}$$

This result shows that the magnetic field is directly proportional to the current I and inversely proportional to the radial distance r.

Magnetic field due to solenoid:

For a solenoid of length L, number of turns N, and current I per turn, we consider an Amperian loop inside the solenoid along its axis. The total enclosed current is:

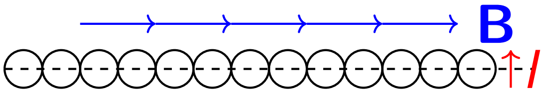
$$I_{\text{enc}} = NI \tag{13}$$

The loop is chosen such that the integral simplifies to:

$$BL = \mu_0 NI \tag{14}$$

Since the number of turns per unit length is $n = \frac{N}{L}$, we get:

$$B = \mu_0 nI \tag{15}$$



Magnetic field due to torid:

A toroid is a circular coil with N turns carrying a current I. We consider an Amperian loop of radius r inside the toroid along a circular path.
Since the toroid has N turns, the total enclosed current is:

$$I_{\text{enc}} = NI \tag{16}$$

Since the magnetic field inside the toroid is tangential and constant along the loop, the integral simplifies to:
$B(2\pi r) = \mu_0 NI$ On Solving for B:

$$B = \frac{\mu_0 NI}{2\pi r}$$

Thus, the magnetic field inside a toroid is:

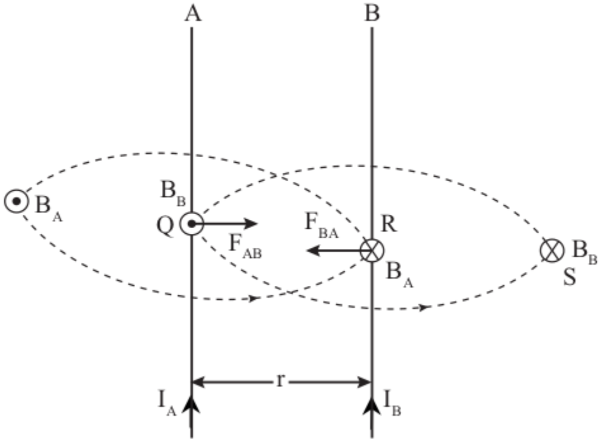
$$B = \frac{\mu_0 NI}{2\pi r}$$

where r is the radial distance from the center of the toroid. The field is strongest inside the toroid and zero outside.

Force between two conductors carrying current:

When currents are in the same direction:

Let us conductors A and B in the form of wires placed parallel, r distance away from each other. Let I_A and I_B be the steady currents flowing on wires A and B respectively from bottom to top (i.e.,along same direction).



$B_A = \frac{\mu_0 I_A}{2\pi r}$ due to I_A is directed to inward,
$B_B = \frac{\mu_0 I_B}{2\pi r}$ due to I_B is directed to outward .
Force F_{BA} due to magnetic field B_A is ,
$F_{BA} = B_A I_B L \sin\theta$
where L is segment of wire B for $\theta = 90^0$

$$F_{BA} = B_A I_B L$$

$$\therefore F_{BA} = \frac{\mu_0 I_A I_B L}{2\pi r} \tag{17}$$

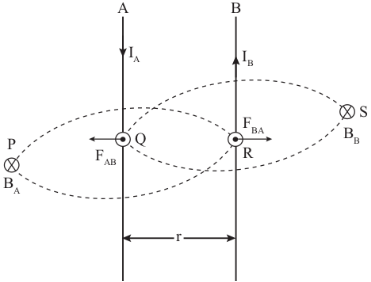
Similarly, Force F_{AB} due to magnetic field B_B is

$$F_{AB} = \frac{\mu_0 I_A I_B L}{2\pi r} \tag{18}$$

Thus, $F_{AB} = F_{BA}$ The force exerted by the conductors on one another is equal in magnitude and act towards another and force is in attractive nature.

When currents are in opposite direction:

If the direction of the current say I_A is reversed then we see this looks below.



The field B_A and $B_B$due to current I_A, I_B is

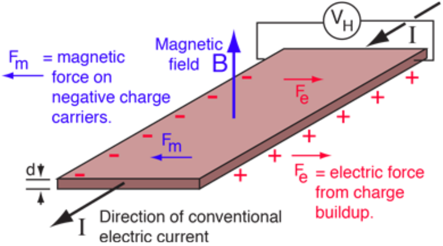
$$B_A = \frac{\mu_0 I_A}{2\pi r} \text{ and } B_B = \frac{\mu_0 I_B}{2\pi r}$$

The force experience by each force are same as eariler case but in opposite direction like ,

$F_{AB} = F_{BA}$ This means force are eqaul in magnitude , act away from one another, repulsive in nature.

Hall effect:

Definition: The Hall effect is the production of a voltage difference across an electrical conductor when it is placed in a magnetic field perpendicular to the current flow.



- **Voltage Generation** – A voltage is induced perpendicular to both current and magnetic field.
- **Used in Sensors** – Commonly used in measuring magnetic fields and detecting position or speed.
- **Discovered in 1879** – Found by Edwin Hall, demonstrating charge carrier movement in conductors.

This electric force on the electrons opposes the magnetic force on them and an equilibrium is reached soon at which these forces balance each other.

$$i.e. F_E = F_B$$

Let V_H is the potential difference associated with electric field also known as hall voltage ,

$$E = \frac{V_H}{d}$$

The Electric force F_E experiences by each electron of charge e is ,

$$F_E = eE$$

Then magnetic force experiences by each electron is

$$BeV_d = eE$$

$$V_d = \frac{eV_H}{Bed} \tag{19}$$

If n be the number of conduction electrons per unit volume of conductor whose crossectional area A then,

$V_d = \frac{I}{nAe}$

Finally , $\frac{I}{nAe} = \frac{eV_H}{Bed}$

or, $V_H = \frac{BId}{nAe}$ (but $A = d \times t$)

$$V_H = \frac{BI}{net}$$

we see that hall voltage is inversely proportional to charge density.

using $H_c = \frac{1}{ne}$ is hall's coefficiecnt then,

$V_H = \frac{BIH_c}{t}$

If R_H is the hall resistance $V_H = IR_H \implies R_H = \frac{BH_c}{t}$

If J be the current density $J = \frac{I}{A} \implies I = JA$ we get, or, $V_H = \frac{BJAH_c}{t} = BJH_c d$ and $H_c = (\frac{V_H}{d}) \times \frac{1}{BJ} = \frac{E_H}{BJ}$

These are the required expression of Hall voltage and hall coeficient by hall effect.

Significance of Hall Effect:

1. Measuring the drift velocity of charge carries:
2. Detecting nature of charge carriers:
3. Hall effect permits the direct measurement of the concentration of the charge carriers (n) in the material.

Magnetic Properties of Materials

Relative permeability:

Relative permeability is the ratio of a fluid's effective permeability to the absolute permeability of the porous medium, indicating how easily the fluid flows in the presence of other fluids.
Its denoted by (μ_r) and given by

$$\mu_r = \frac{\mu}{\mu_0}$$

And we have this one relation ,

$$\mu_r = 1 + \chi$$

where χ is magnetic susptibility and is defined by

Magnetic suptibility:-

Magnetic susceptibility is a measure of how much a material becomes magnetized when exposed to an external magnetic field. Its denoted by χ
And the Formula is given by,

$$\chi = \frac{M}{H}$$

Where,
M = magnetization(magnetic moment per unit per unit volume)
H = Applied magnetic field strength.

Intensity of Magnetization (M):

It is the magnetic moment per unit volume of a material.

$$M = \frac{m}{v}$$

where m is the magnetic moment and V is the volume

Magneitc Intensity(H):

It is the external magnetic field applied to a material, which induces magnetization.

$$H = \frac{B}{\mu} - M$$

where B is the total magnetic field and μ is permeability.

Total magnetic field(B):

It is the total magnetic field inside a material, including both external and induced fields.

$$B = \mu_0(H + M)$$

where, μ_0 is the permeability in free space.

Relation between Relative permeability and magnetic susptibility:

As we know definition of permeability for a material medium is givne by

$$\mu = \frac{B}{H} \tag{20}$$

Substituting B from Equation TBM;

$$\mu = \frac{\mu_0(H + M)}{H}$$

Dividing by μ_0:

$$\mu_r = \frac{H + M}{H} \tag{21}$$

Using Equation From definition of Susptibility;

$$M = \chi H$$

Substituting into Equation (21);

$$\mu_r = \frac{H + \chi H}{H}$$

Factoring out H:

$$\mu_r = \frac{H(1+\chi)}{H} \tag{22}$$

Canceling H:

$$\mu_r = 1 + \chi \tag{23}$$

This is the required expression of magnetic susptibility and relative permeability.

Curie Law:

The Curie Law (named after Pierre Curie) describes the magnetic behavior of paramagnetic materials.

Statemant: It states that the magnetic susceptibility (χ) of a paramagnetic material is inversely proportional to its absolute temperature (T),

$$\chi = \frac{C}{T}$$

where,

χ Magnetic Susceptibility,

C Curie constant

T Absolute temperature in (K).

- Temperature of a paramagnetic material increases, its magnetic susceptibility decreases.
- At higher temperatures, the random motion of the atoms dominates over the alignment of magnetic moments.
- weakening the material's response to an external magnetic field.

Understanding Diamagnetic, Paramagnetic, and Ferromagnetic Materials

1. Diamagnetic Materials

Diamagnetic materials are substances that create an opposing magnetic field when exposed to an external magnetic field. They exhibit a weak negative magnetization.

Characteristics:

- Weakly repelled by a magnetic field.
- Magnetic susceptibility (χ) is **negative** ($\chi < 0$).
- Relative permeability (μ_r) is **slightly less than 1** ($\mu_r < 1$).
- No permanent dipoles; induced dipoles oppose the external field.
- Independent of temperature.

Examples: Copper (Cu), Gold (Au), Silver (Ag), Lead (Pb), Water, Graphite.

2. Paramagnetic Materials

Paramagnetic materials are substances that develop a weak magnetization in the direction of an applied magnetic field. Their magnetic effect disappears when the external field is removed.

Characteristics:

- Weakly attracted to a magnetic field.
- Magnetic susceptibility (χ) is **positive but small** ($\chi > 0$).
- Relative permeability (μ_r) is **slightly greater than 1** ($\mu_r > 1$).
- Presence of unpaired electrons causing small net magnetic moments.
- Magnetization follows **Curie's Law** ($\chi \propto \frac{1}{T}$), meaning it decreases with increasing temperature.

Examples: Aluminum (Al), Platinum (Pt), Magnesium (Mg), Oxygen (O_2).

3. Ferromagnetic Materials

Ferromagnetic materials are substances that exhibit strong magnetization in the direction of an applied magnetic field and retain magnetization even after the field is removed (permanent magnetism).

Characteristics:

- Strongly attracted to a magnetic field.
- Magnetic susceptibility (χ) is **very large and positive** ($\chi \gg 1$).
- Relative permeability (μ_r) is **much greater than 1** ($\mu_r \gg 1$).
- Presence of **magnetic domains** where atomic dipoles align

- Retains magnetization even after removing the field (hysteresis effect).
- Has a **Curie temperature** (T_C) above which ferromagnetism disappears and behaves as paramagnetic.

Examples: Iron (Fe), Cobalt (Co), Nickel (Ni), Steel.

Comparison Table

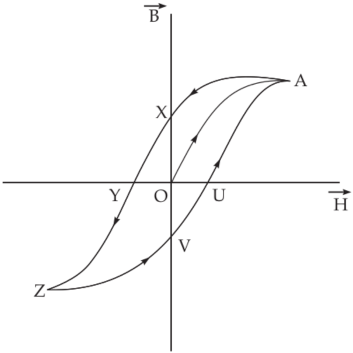
Property	Diamagnetic	Paramagnetic	Ferromagnetic
Response to Field	Weak repulsion	Weak attraction	Strong attraction
Magnetic Susceptibility (χ)	$\chi < 0$ (Negative)	$\chi > 0$ (Small Positive)	$\chi \gg 1$ (Large Positive)
Relative Permeability (μ_r)	$\mu_r < 1$	$\mu_r > 1$ (Slightly)	$\mu_r \gg 1$
Permanent Magnetism	No	No	Yes
Temperature Effect	No effect	Decreases with temperature	Disappears above Curie temperature

Hysteresis:

The lag between the applied magnetic field and the material's magnetization, where the material "remembers" past states.

Hysteresis loop:

TA graph showing the relationship between the applied magnetic field (H) and the resulting magnetization (B), demonstrating the material's history-dependent response.



- Coercivity (H_c): The field strength required to reduce magnetization to zero after full magnetization. High coercivity indicates hard magnets.
- Remanence (B_r): The remaining magnetization when the field is zero. High remanence indicates permanent magnet properties.
- Saturation Magnetization (Bs): The maximum magnetization a material can achieve, beyond which the magnetization does not increase even with a higher field.
- Area of the Loop: Represents energy loss in one cycle of magnetization. Larger area means more energy loss, typical of hard magnetic materials.

Use for Material Types:

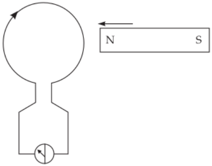
- **Hard materials:** High coercivity and remanence.
- **Soft materials:** Low coercivity and remanence, low energy loss.

Electromagnetic Induction:

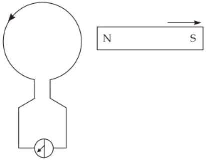
Electromagnetic Induction is the process by which a changing magnetic field generates an electric current in a conductor. This phenomenon, discovered by Michael Faraday, forms the basis of many electrical devices, such as generators and transformers. It occurs when a conductor (like a wire) is exposed to a varying magnetic field, causing electrons to move and produce an electric current.

Key: characterostics:

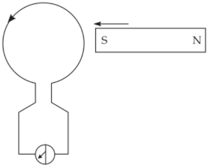
- Changing Magnetic Field: Induction occurs when the magnetic field around a conductor changes.
- Induced Current: A changing magnetic field creates an electric current in the conductor.
- Faraday's Law: The induced current is proportional to the rate of change of the magnetic field.
- Lenz's Law: The induced current opposes the change in the magnetic field.
- Applications: Used in generators, transformers, and induction motors.



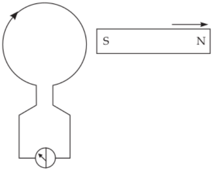
a. When the magnet is moved towards coil with its north pole facing the coil, the current is clockwise (say) in the coil.



b. When the magnet is moved away from the coil and its north pole still faces the coil, the current is anticlockwise in the coil.

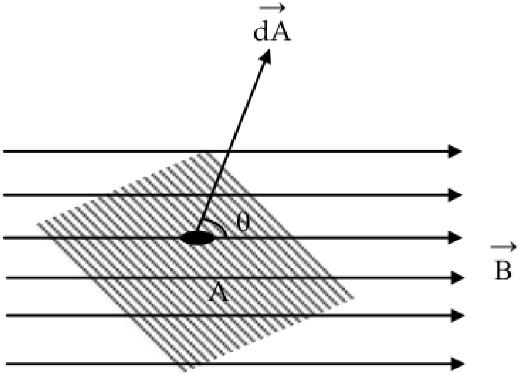


c. When the magnet is moved towards coil with its south pole facing the coil, current is again anti-clockwise in the coil.



d. When the magnet with its south pole facing the coil is moved away from the coil, current is in clockwise direction.

Magnetic Field: The region at where the magnetic source modifies the space around it in some manner so that any other magnetic material experience force due to it



Magnetic flux:

Magnetic flux is the total magnetic field passing through a given surface. It is measured in Weber (Wb) and given by $\phi = BA\cos\theta$ where B is the magnetic field, A is the area, and θ is the angle between them.

Faraday's law of electromagnetic induction:

Faraday's law states that the induced electromotive force (EMF) in a circuit is directly proportional to the rate of change of magnetic flux through the circuit.

- Formula:is magnetic flux. $emf = -\frac{d\phi}{dt}$ where, ϕ is magnetic flux.
- Negative Sign: Indicates Lenz's Law, meaning the induced EMF opposes the change in flux.
- Faster Change, More EMF: A rapid change in magnetic flux induces a stronger EMF.
- Practical Applications: Used in transformers, generators, and electric induction devices.

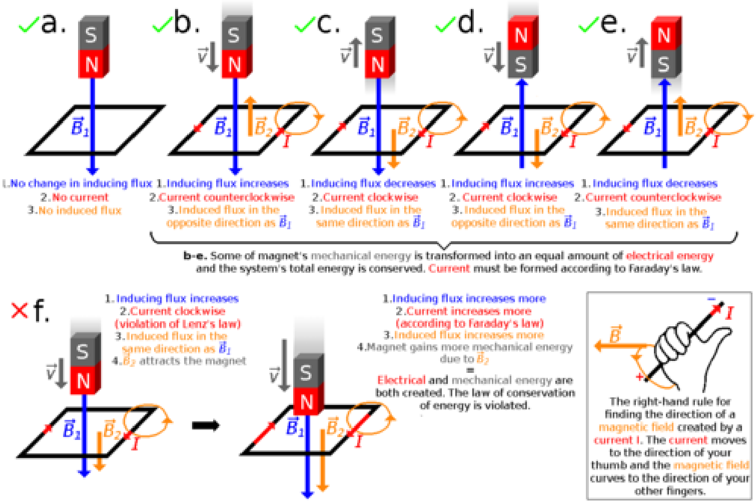
Lenz Law:

Lenz's law states that the direction of the induced current always opposes the change in magnetic flux that caused it.

The induced current has a direction such as to oppose the cause producing it

Lenz law is the direct consequence of conservation of energy.

Lenz law and direction of induced emf



Construction and working of A.C. generators

An A.C. generator (alternator) is a device that converts mechanical energy into electrical energy in the form of alternating current (A.C.) based on the principle of electromagnetic induction.

Construction:

- Stator: Stationary part with magnetic field.
- Rotor: Rotating armature coil.
- Field Magnets: Produce a magnetic field.
- Slip Rings Brushes: Transfer A.C. to external circuit.

Working:

- The armature coil rotates inside a magnetic field.
- Due to electromagnetic induction, an A.C. voltage is generated.
- The slip rings and brushes transfer the alternating current to an external circuit.

Opposition to Change: Prevents self-sustaining energy generation.

Energy Input Required: Work must be done to change flux, converting mechanical to electrical energy.

No Perpetual Motion: Stops infinite energy creation, ensuring physics laws hold

Fleming Right Hand Rule

Fleming's Right-Hand Rule:

- Thumb $\rightarrow$ Motion
- Index $\rightarrow$ Field
- Middle $\rightarrow$ Current

Eddy Current:

Eddy currents are looping currents induced in conductors when exposed to a changing magnetic field, following Faraday's Law of electromagnetic induction.

Self-Inductance (L):

Property of a coil that opposes changes in current by inducing EMF in itself.

$$V_L = -L\frac{dL}{dt}$$

Uses:

- Inductors in circuits (filters, oscillators).
- Chokes (block AC, allow DC).
- Energy storage in magnetic fields.

Mutual Inductance(M):

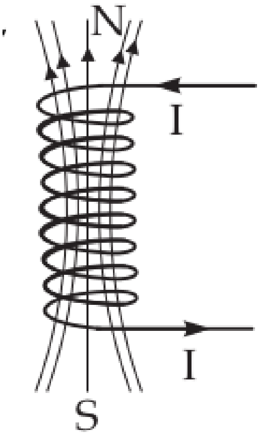
Property where a changing current in one coil induces voltage in another coil.

$$V_M = -M\frac{dI}{dt}$$

Uses:

- Transformers (power transmission).

- Wireless charging.
- Coupled circuits (radio, TV tuners).



Energy Stored in an inductor:

The energy required in the inductor in the form of work done till the current attains a maximum steady value.
As we know the emf related to the self inductance

$$E = L\frac{dI}{dt}$$

If P be the rate at which energy is delivered (power) $P = I.E$

$$P = L\frac{dI}{dt}.I$$

Rearranging, $Pdt = LIdI$
Integrating from 0 to t and 0 to I_0 we get

$$\int_0^t Pdt = \int_0^{I_0} LIdI$$

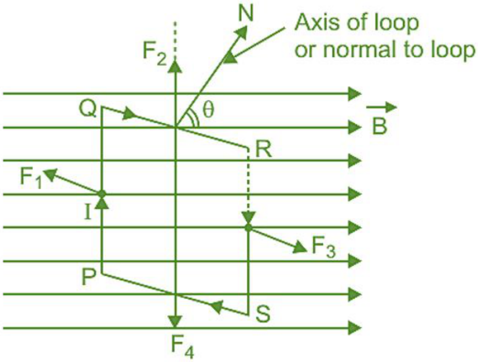
$$W = L\int_0^{I_0} IdI \implies W = L[\tfrac{I}{2}]_0^{I_0} \implies W = \tfrac{1}{2}LI_0^2 = U$$

This is the required expression of energy stored in an inductor.

Emf induced in a rotating coil in uniform magnetic field

Consider a rectangular coil of surface area A and having N number of turns. The coil is rotated in the uniform magnetic field of . Then magnetic flux linked with coil is ,

$$\phi = NBA\cos\theta$$



Let ω be the angular velocity of coil then $\theta = \omega t$

$$\phi = BNA \cos \omega t$$

Differentiating both side we get,

$$\frac{d\phi}{dt} = -NBA\omega \sin \omega t$$

Also we know $E = -\frac{d\phi}{dt}$ we get,

$$E = -(-NBA\omega \sin \omega t) \implies E = NBA\omega \sin \omega t$$

The value of induced emf is $E = E_0$ at $\sin \omega t = 1$ we get,

$$\therefore E = E_0 \sin \omega t$$

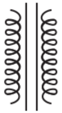
This is the alternating emf (voltage) induced in an inductor. Also E=IR then

$$I = I_0 \sin \omega t$$

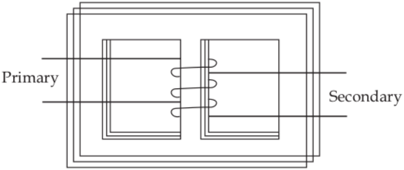
This is the alternating current induced in an inductor.

Transformer:

An electrical device which transforms (changes) an alternating voltage from one value to another of greater or smaller value by using the principle of mutual induction is called transformer.



(i)



(ii)

If $\frac{d\phi}{dt}$ be the rate of flux linked with secondary coil, Then by Faraday law emf (E_s) in secondary coil is.

$$E_s = -N_s \frac{d\phi}{dt}$$

Here N_s is the turns in the secondary coil. similarly E_P for primary coil is ,

$$E_p = -N_p \frac{d\phi}{dt}$$

where N_P is the number of turns in primary coil. Form above this two relations we get,

$$\frac{E_s}{E_p} = \frac{N_s}{N_p}$$

This is the known as the transformer equation.
If the transformer is 100% efficient , then from energy considerations ,

$$V_p I_p = V_s I_s \implies \frac{V_s}{V_p} = \frac{I_p}{I_s}$$

comparing this with above relation we get,

$$\frac{N_s}{N_p} = \frac{I_p}{I_s}$$

Efficiency of transformer:

The efficiency of transformer is defined as the ratio of output power to input power. It is ususally expressed in percentage, So,

$$\eta = \frac{I_s V_s}{I_p V_p} \times 100\%$$

Alternating Currents

Introduction:

Alternating Current (AC) is an electric current that periodically reverses direction, commonly used in homes and industries. Its ability to be easily transformed makes it ideal for efficient power transmission.

Instantenous value of Ac:

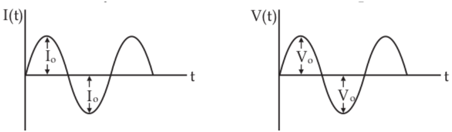
Instantaneous AC refers to the value of alternating current or voltage at any specific moment in time. It varies continuously with the AC waveform. instantaneous alternating current is given by,

$$I(t) = I_0 \sin \omega t$$

where, I_0 is the peak value of current.

Peak value of current:

The peak value of alternating quantity is the value of voltage or current at the positive or negative maximum with respect to zero. This is also called as amplitudes of alternating quantity I_0.



Average or mean value of A.C

The average (mean) value of AC over a full cycle is zero because the positive and negative halves cancel out. However, over a half-cycle, it is given by:

$$i.e \qquad I_{av} = \frac{2}{\pi} I_0 \qquad or, \qquad V_{av} = \frac{2}{\pi} V_0$$

Where, I_0 and V_0 are the maximum value of current and voltage.

Proof:

The average value of an alternating current (AC) or voltage is calculated over a half-cycle, as the full-cycle average is zero due to symmetry.
For a sinusoidal AC voltage or current:

$$V = V_0 \sin\theta$$

where V_0 is the peak value and θ is the phase angle (in radians).
Step 1: Formula for Average Value The average value over a half-cycle (0 to π) is given by:

$$V_{\text{avg}} = \frac{1}{\pi}\int_0^{\pi} V_0 \sin\theta \, d\theta$$

Step 2: Evaluating the Integral

$$V_{\text{avg}} = \frac{V_0}{\pi}\int_0^{\pi} \sin\theta \, d\theta$$

We know that: $\int \sin\theta \, d\theta = -\cos\theta$
Applying limits from 0 to π:

$$[-\cos\theta]_0^{\pi} = -\cos\pi + \cos 0 = -(-1) + 1 = 1 + 1 = 2$$

Step 3: Final Expression

$$V_{\text{avg}} = \frac{V_0}{\pi} \times 2 = \frac{2V_0}{\pi}$$

Thus, the **average or mean value of AC** over a half-cycle is:

$$V_{\text{avg}} = \frac{2V_0}{\pi} \qquad or \qquad I_{\text{avg}} = \frac{2I_0}{\pi}$$

RMS value of AC

The RMS value of an alternating current (AC) or voltage is the effective value that produces the same heating effect as a direct current (DC) of the same magnitude. It is mathematically defined as:

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2 dt}$$

For Sinusoidal AC voltage or current:

$$V = V_0 \sin\theta$$

The RMS value over one full cycle is defined as:

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

Similarly, For current:

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

The RMS value is widely used in electrical calculations, as it represents the practical or effective value of AC in power systems.

Proof For I_{rms} value of AC

The RMS value of AC current over one full cycle is given by:

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T I^2 dt}$$

For a sinusoidal AC current: $I = I_0 \sin\theta$

where I_0 is the peak current , And the RMS formula becomes:

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} I_0^2 \sin^2\theta \, d\theta}$$

Using the standard integral identity: $\int_0^{2\pi} \sin^2\theta \, d\theta = \frac{2\pi}{2} = \pi$

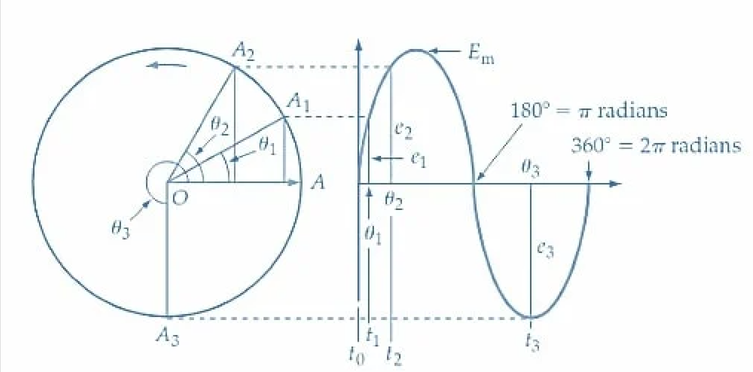
$$I_{\text{rms}} = \sqrt{\frac{I_m^2}{2\pi} \times \pi}$$

$I_{\text{rms}} = \sqrt{\frac{I_0^2}{2}} \implies I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$

RMS value of a sinusoidal AC waveform is $\frac{1}{\sqrt{2}}$ times its peak value.

Phasors:

Phasors are complex numbers representing sinusoidal signals, showing both magnitude and phase. Phasor representation is used to study the phase relationship between two sinusoidally varying quantities having same frequency.



Phase shift between two wave forms:

The phase shift between two waveforms is the difference in their phase angles,

$$\Delta\theta = \theta_2 - \theta_1$$

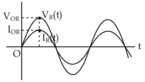
Interpretation:

- Positive $\Delta\theta \rightarrow$ Waveform 2 leads Waveform 1.
- Negative $\Delta\theta \rightarrow$ Waveform 2 lags Waveform 1.
- 180^0 phase shift $\rightarrow$ The waveforms are completely out of phase.
- 0^0 or 360^0 phase shift $\rightarrow$ The waveforms are in phase (aligned).

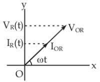
If the waveforms have the same frequency f, the phase shift can also be expressed in time t as:

$$\Delta t = \frac{\Delta\theta}{360^0} \times T$$

A.C. Through Resistor:



(i) Plot of $V_R(t)$ vs $I_R(t)$



(ii) Phasor diagram

Apply Kirchhoff's Voltage Law (KVL)
For a purely resistive circuit with an applied AC voltage:

$$V(t) = V_0 \sin(\omega t)$$

where: V_0 = peak voltage $\omega = 2\pi f$ = angular frequency and t = time
Using **Ohm's Law**: $I(t) = \frac{V(t)}{R}$

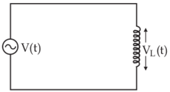
$$I(t) = \frac{V_0 \sin(\omega t)}{R}$$

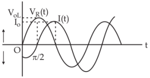
Define peak current as: $I_0 = \frac{V_0}{R}$ Thus, the equation simplifies to:

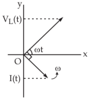
$$I(t) = I_0 \sin(\omega t)$$

Thus: The current $I(t)$ is a sinusoidal function with the same frequency ω as the voltage, proving that the current through a resistor is sinusoidal.

A.C. Through Inductor:







Apply Kirchhoff's Voltage Law (KVL)

For an inductor L in an AC circuit, the applied voltage is:

$$V(t) = V_0 \sin(\omega t)$$

Using the inductor voltage-current relationship:

$$V(t) = L\frac{dI(t)}{dt}$$

Step 2: Solve for $I(t)$

$$L\frac{dI(t)}{dt} = V_0 \sin(\omega t)$$

Dividing both sides by L:

$$\frac{dI(t)}{dt} = \frac{V_0}{L}\sin(\omega t)$$

Integrating both sides:

$$I(t) = \int \frac{V_0}{L}\sin(\omega t)dt$$

$$I(t) = \frac{V_0}{L}\int \sin(\omega t)dt$$

Using the integral:

$$\int \sin(\omega t)dt = -\frac{\cos(\omega t)}{\omega}$$

$$I(t) = -\frac{V_0}{L\omega}\cos(\omega t)$$

Since $\cos(\omega t) = \sin(\omega t + 90^\circ)$, we rewrite:

$$I(t) = I_0 \sin(\omega t - 90^\circ)$$

where:

$$I_0 = \frac{V_0}{\omega L}$$

Step 3: Phase Relationship

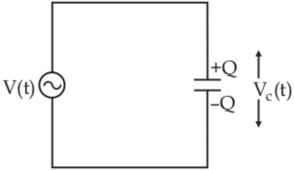
- The current **lags** the voltage by **90°** (or $\frac{\pi}{2}$ radians).
- The inductor **opposes** changes in current, causing a **phase shift**.

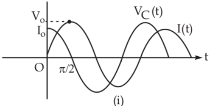
Conclusion

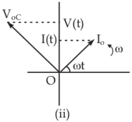
For AC through an inductor: $I(t) = I_0 \sin(\omega t - 90^\circ)$

- The current **lags** voltage by **90°**.
- The inductive **reactance** is given by $X_L = \omega L$, where $\omega = 2\pi f$.
- The peak current is $I_0 = \frac{V_0}{X_L}$.

A.C. Through Capacitor:







Apply Kirchhoff's Voltage Law (KVL)

For a capacitor C in an AC circuit, the applied voltage is:

$$V(t) = V_0 \sin(\omega t)$$

Using the capacitor voltage-current relationship:

$$I(t) = C\frac{dV(t)}{dt}$$

Solve for $I(t)$ $\quad I(t) = C\frac{d}{dt}\left(V_0 \sin(\omega t)\right)$

Taking the derivative:

$$I(t) = CV_0\omega\cos(\omega t)$$

Since $\cos(\omega t) = \sin(\omega t + 90^\circ)$, we rewrite:

$$I(t) = I_0 \sin(\omega t + 90^\circ)$$

where:

$$I_0 = V_0\omega C$$

Phase Relationship

- The current **leads** the voltage by **90°** (or $\frac{\pi}{2}$ radians).
- The capacitor **opposes** changes in voltage, causing a **phase shift**.

Capacitive Reactance

The capacitive reactance X_C is given by:

$$X_C = \frac{1}{\omega C}$$

Thus, the peak current can be written as:

$$I_0 = \frac{V_0}{X_C}$$

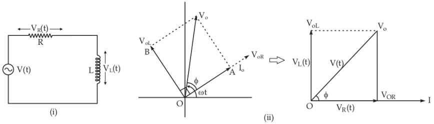
Conclusion

For AC through a capacitor:

$$I(t) = I_0 \sin(\omega t + 90^\circ)$$

- The current **leads** voltage by **90°**.
- The capacitive **reactance** is given by $X_C = \frac{1}{\omega C}$.
- The peak current is $I_0 = \frac{V_0}{X_C}$.

A.C. Through R–L Series Circuit:



Apply Kirchhoff's Voltage Law (KVL)

Consider an R-L series circuit with an applied voltage $V(t) = V_0 \sin(\omega t)$. The total voltage across the resistor and inductor is:

$$V(t) = V_R(t) + V_L(t)$$

Using Ohm's Law for the resistor:

$$V_R(t) = I(t)R$$

And for the inductor:

$$V_L(t) = L\frac{dI(t)}{dt}$$

Thus, applying Kirchhoff's Voltage Law (KVL):

$$V_0 \sin(\omega t) = I(t)R + L\frac{dI(t)}{dt}$$

Solve the Differential Equation

This is a first-order linear differential equation:

$$L\frac{dI(t)}{dt} + I(t)R = V_0 \sin(\omega t)$$

Solve the homogeneous equation first:

$$L\frac{dI_h(t)}{dt} + I_h(t)R = 0$$

The solution to the homogeneous equation is:

$$I_h(t) = Ae^{-\frac{R}{L}t}$$

Next, solve the non-homogeneous equation using the method of undetermined coefficients. Assume a solution of the form:

$$I(t) = I_0 \sin(\omega t + \phi)$$

Substitute this into the equation $L\frac{dI(t)}{dt} + I(t)R = V_0 \sin(\omega t)$:

$$L \cdot I_0 \omega \cos(\omega t + \phi) + I_0 R \sin(\omega t + \phi) = V_0 \sin(\omega t)$$

Using trigonometric identities, we get:

$$I_0(L\omega \cos(\omega t + \phi) + R \sin(\omega t + \phi)) = V_0 \sin(\omega t)$$

Equating the coefficients of $\sin(\omega t)$ and $\cos(\omega t)$, we get two equations:

$$I_0 R \cos(\phi) = V_0 \quad (\text{coefficient of } \sin(\omega t))$$

$$I_0 L\omega \sin(\phi) = 0 \quad (\text{coefficient of } \cos(\omega t))$$

Solve for I_0 and ϕ

From the first equation:

$$I_0 = \frac{V_0}{\sqrt{R^2 + (L\omega)^2}}$$

From the second equation, solving for the phase angle ϕ:

$$\tan(\phi) = \frac{L\omega}{R}$$

Thus, the phase angle ϕ is:

$$\phi = \tan^{-1}\left(\frac{L\omega}{R}\right)$$

Current Expression

The current in the circuit is:

$$I(t) = I_0 \sin(\omega t + \phi)$$

where:

$$I_0 = \frac{V_0}{\sqrt{R^2 + (L\omega)^2}}, \quad \phi = \tan^{-1}\left(\frac{L\omega}{R}\right)$$

Conclusion

For an R-L series circuit, the current $I(t)$ is sinusoidal with:

- The peak current $I_0 = \frac{V_0}{\sqrt{R^2+(L\omega)^2}}$.
- The phase shift $\phi = \tan^{-1}\left(\frac{L\omega}{R}\right)$, indicating that the current **lags** the voltage by a phase angle ϕ.

Impedance of R-L Circuit

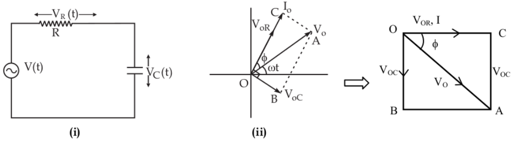
The total impedance Z of the R-L circuit is:

$$Z = \sqrt{R^2 + (L\omega)^2}$$

Thus, the peak current can be expressed as:

$$I_0 = \frac{V_0}{Z}$$

A.C. Through R-C Circuit:



Apply Kirchhoff's Voltage Law (KVL)

Consider an R-C series circuit with an applied AC voltage $V(t) = V_0 \sin(\omega t)$. The total voltage across the resistor and capacitor is:

$$V(t) = V_R(t) + V_C(t)$$

Using Ohm's Law for the resistor:

$$V_R(t) = I(t)R$$

For the capacitor, the voltage is related to the current by:

$$V_C(t) = \frac{1}{C}\int I(t)dt$$

Thus, applying Kirchhoff's Voltage Law:

$$V_0 \sin(\omega t) = I(t)R + \frac{1}{C}\int I(t)dt$$

Solve the Differential Equation

Taking the derivative of both sides of the equation with respect to time:

$$\frac{d}{dt}\left(V_0 \sin(\omega t)\right) = R\frac{dI(t)}{dt} + \frac{I(t)}{C}$$

Simplifying:

$$V_0\omega \cos(\omega t) = R\frac{dI(t)}{dt} + \frac{I(t)}{C}$$

This is a first-order linear differential equation. To solve it, we use the method of undetermined coefficients, assuming a solution of the form:

$$I(t) = I_0 \sin(\omega t + \phi)$$

Substitute the Assumed Solution

Substitute $I(t) = I_0 \sin(\omega t + \phi)$ into the differential equation. First, calculate the derivative:

$$\frac{dI(t)}{dt} = I_0 \omega \cos(\omega t + \phi)$$

Substitute into the equation:

$$V_0 \omega \cos(\omega t) = R I_0 \omega \cos(\omega t + \phi) + \frac{I_0 \sin(\omega t + \phi)}{C}$$

Using trigonometric identities, we get:

$$I_0 \left(R\omega \cos(\omega t + \phi) + \frac{\sin(\omega t + \phi)}{C} \right) = V_0 \omega \cos(\omega t)$$

Solve for the Amplitude and Phase Shift

By equating the coefficients of $\sin(\omega t)$ and $\cos(\omega t)$, we get:

$$I_0 \left(R\omega \cos(\phi) + \frac{1}{C} \sin(\phi) \right) = V_0 \omega$$

From this, the magnitude of the current is:

$$I_0 = \frac{V_0}{\sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2}}$$

The phase angle ϕ is given by:

$$\tan(\phi) = \frac{\frac{1}{C\omega}}{R}$$

Thus, the phase shift ϕ is:

$$\phi = \tan^{-1}\left(\frac{1}{RC\omega}\right)$$

Current Expression

Finally, the current in the circuit is:

$$I(t) = I_0 \sin(\omega t + \phi)$$

where:

$$I_0 = \frac{V_0}{\sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2}}, \quad \phi = \tan^{-1}\left(\frac{1}{RC\omega}\right)$$

Conclusion

For an R-C series circuit, the current $I(t)$ is sinusoidal with:

- The peak current $I_0 = \frac{V_0}{\sqrt{R^2+\left(\frac{1}{C\omega}\right)^2}}$.
- The phase shift $\phi = \tan^{-1}\left(\frac{1}{RC\omega}\right)$, indicating that the current **lags** the voltage.

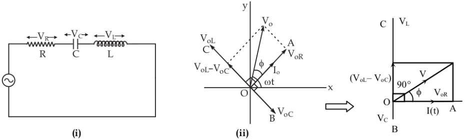
The **impedance** Z of the circuit is:

$$Z = \sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2}$$

The peak current can also be written as:

$$I_0 = \frac{V_0}{Z}$$

L-C-R Series Circuit in A.C:



Apply Kirchhoff's Voltage Law (KVL), Consider an L-C-R series circuit with an applied AC voltage $V(t) = V_0 \sin(\omega t)$. The total voltage across the resistor, inductor, and capacitor is:

$$V(t) = V_R(t) + V_L(t) + V_C(t)$$

Using Ohm's Law for the resistor:
$V_R(t) = I(t)R$
For the inductor:

$$V_L(t) = L\frac{dI(t)}{dt}$$

For the capacitor:

$$V_C(t) = \frac{1}{C} \int I(t)dt$$

Thus, applying Kirchhoff's Voltage Law:

$$V_0 \sin(\omega t) = I(t)R + L\frac{dI(t)}{dt} + \frac{1}{C} \int I(t)dt$$

Take the derivative of both sides with respect to time:

$$\frac{d}{dt}\left(V_0 \sin(\omega t)\right) = R\frac{dI(t)}{dt} + L\frac{d^2I(t)}{dt^2} + \frac{I(t)}{C}$$

Simplifying:

$$V_0\omega \cos(\omega t) = R\frac{dI(t)}{dt} + L\frac{d^2I(t)}{dt^2} + \frac{I(t)}{C}$$

This is a second-order linear differential equation. To solve it, we assume a solution of the form:

$$I(t) = I_0 \sin(\omega t + \phi)$$

Substitute $I(t) = I_0 \sin(\omega t + \phi)$ into the differential equation. First, compute the derivatives:

$$\frac{dI(t)}{dt} = I_0 \omega \cos(\omega t + \phi)$$

$$\frac{d^2 I(t)}{dt^2} = -I_0 \omega^2 \sin(\omega t + \phi)$$

Substitute these into the equation:

$$V_0 \omega \cos(\omega t) = R I_0 \omega \cos(\omega t + \phi) + L(-I_0 \omega^2 \sin(\omega t + \phi)) + \frac{I_0 \sin(\omega t + \phi)}{C}$$

Using trigonometric identities, we get:

$$I_0 \left(R\omega \cos(\omega t + \phi) + \frac{\sin(\omega t + \phi)}{C} - L\omega^2 \sin(\omega t + \phi) \right) = V_0 \omega \cos(\omega t)$$

Amplitude and Phase Shift

By equating the coefficients of $\sin(\omega t)$ and $\cos(\omega t)$, we obtain:

- Coefficients of $\cos(\omega t)$:

$$I_0\omega\left(R\cos(\phi) - L\omega\sin(\phi)\right) = V_0\omega$$

- Coefficients of $\sin(\omega t)$:

$$I_0\omega\left(R\sin(\phi) + L\omega\cos(\phi)\right) = 0$$

Solve these two equations for the amplitude I_0 and phase ϕ.
The amplitude of the current is:

$$I_0 = \frac{V_0}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

The phase shift ϕ is given by:

$$\tan(\phi) = \frac{L\omega - \frac{1}{C\omega}}{R}$$

Thus, the phase shift ϕ is:

$$\phi = \tan^{-1}\left(\frac{L\omega - \frac{1}{C\omega}}{R}\right)$$

Finally, the current in the circuit is:

$$I(t) = I_0 \sin(\omega t + \phi)$$

where:

$$I_0 = \frac{V_0}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}, \quad \phi = \tan^{-1}\left(\frac{L\omega - \frac{1}{C\omega}}{R}\right)$$

The total impedance Z of the L-C-R circuit is:

$$Z = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$

Thus, the peak current can also be written as:

$$I_0 = \frac{V_0}{Z}$$

Conclusion

For an L-C-R series circuit, the current $I(t)$ is sinusoidal with:

- The peak current $I_0 = \frac{V_0}{\sqrt{R^2+\left(L\omega-\frac{1}{C\omega}\right)^2}}$.
- The phase shift $\phi = \tan^{-1}\left(\frac{L\omega-\frac{1}{C\omega}}{R}\right)$, indicating that the current may **lead or lag** the voltage depending on the relative values of L, C, and R.

The impedance of the circuit is:

$$Z = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$

Where:

- R is the resistance,
- L is the inductance,
- C is the capacitance,
- ω is the angular frequency of the AC source.

Condition for Resonance

Resonance occurs when the reactive component (inductive reactance $X_L = L\omega$ and capacitive reactance $X_C = \frac{1}{C\omega}$) cancel each other out. This happens when:

$$X_L = X_C \quad \Rightarrow \quad L\omega = \frac{1}{C\omega}$$

for Angular Frequency

From the condition $L\omega = \frac{1}{C\omega}$, we solve for ω:

$$L\omega^2 = \frac{1}{C} \quad \Rightarrow \quad \omega^2 = \frac{1}{LC}$$

Thus, the angular frequency at resonance is:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Resonance Frequency

The resonance frequency f_0 in Hertz is related to the angular frequency ω_0 by:

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

Three special Cases:

1. Inductive Case (When the circuit is inductive)

Condition: $X_L > X_C$ (Inductive reactance greater than capacitive reactance)

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Where $X_L = L\omega$ and $X_C = \frac{1}{C\omega}$.

Phase: The current *lags* the voltage by an angle ϕ where:

$$\tan(\phi) = \frac{X_L - X_C}{R}$$

2. Capacitive Case (When the circuit is capacitive)
Condition: $X_L < X_C$ (Capacitive reactance greater than inductive reactance)

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

Where $X_L = L\omega$ and $X_C = \frac{1}{C\omega}$.
Phase: The current *leads* the voltage by an angle ϕ where:

$$\tan(\phi) = \frac{X_C - X_L}{R}$$

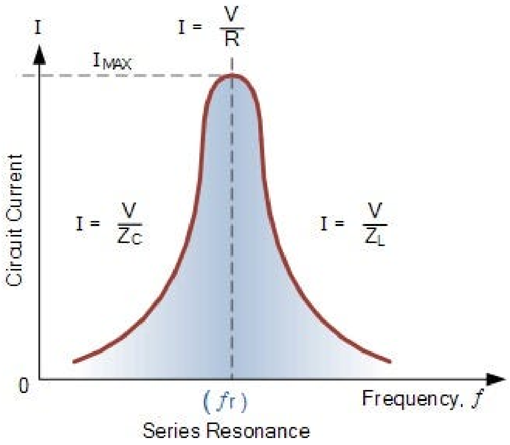
3. Resonance Case (When the circuit is at resonance)
Condition: $X_L = X_C$ (Inductive reactance equals capacitive reactance)

$$L\omega = \frac{1}{C\omega}$$

Impedance:

$$Z = R$$

The impedance is minimized, and the current is maximized.
Phase: The current and voltage are *in phase* (i.e., $\phi = 0$).



Power in LCR Circuit:

Voltage and Current in an LCR Circuit

For an AC voltage source:

$$V = V_0 \cos(\omega t)$$

Let the current through the circuit be:

$$I = I_0 \cos(\omega t - \phi)$$

where:

- V_0 and I_0 are the peak voltage and peak current,
- ω is the angular frequency,
- ϕ is the phase angle between voltage and current.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X_L = \omega L \quad \text{(Inductive Reactance)}$$
$$X_C = \frac{1}{\omega C} \quad \text{(Capacitive Reactance)}$$

The phase angle is given by:

$$\tan\phi = \frac{X_L - X_C}{R}$$

Instantaneous Power $P(t) = V(t) \cdot I(t)$ Substituting the values of V and I:

$$P(t) = V_0 \cos(\omega t) \cdot I_0 \cos(\omega t - \phi)$$

Using the trigonometric identity:

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

we get:

$$P(t) = \frac{V_0 I_0}{2}[\cos\phi + \cos(2\omega t - \phi)]$$

Average Power (Real Power) The average power over a complete cycle is given by:

$$P_{\text{avg}} = \frac{1}{T}\int_0^T P(t)dt$$

Since the integral of $\cos(2\omega t - \phi)$ over one cycle is zero, we obtain:

$$P_{\text{avg}} = \frac{V_0 I_0}{2} \cos\phi$$

Using RMS values ($V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$ and $I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$), we rewrite the expression as:

$$P_{\text{avg}} = V_{\text{rms}} I_{\text{rms}} \cos\phi$$

$P_{\text{avg}} = V_{\text{rms}} I_{\text{rms}} \cos\phi$, which determines the actual energy consumed in the circuit.

Q Factor in LCR Circuit

The Q Factor (Quality Factor) of an LCR circuit is a dimensionless parameter that measures the sharpness of resonance in the circuit.
It is defined as:

$$Q = \frac{Resonant\ \ Frequency(f_0)}{Bnadwidth(\Delta f)}$$

Equivalently,

$$Q = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Where: $f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$ is resonant frequency, $\Delta f = \frac{R}{2\pi L}$ is Bandwidth
$\omega_0 = 2\pi f_0$ is angular resonant frequency, R,L,C have their usual meaning.

Thank you!

Reference

1 **R. Pd.Koirala ,PRINCIPLES OF PHYSI CS - II, Asmita Books Publishers and Distributors (P) Ltd.**

2 **https://www.deepseek.com and https://openai.com/index/chatgpt/**