Mechanics

Birendra Bdr. Dhanuk

Namuna Secondary School Ladavir

March 23, 2025

Contents:

Rotational dynamics

Rotational dynamics is the branch of physics that studies the motion of objects rotating about a fixed axis, focusing on the relationships between torque, angular acceleration, moment of inertia, and angular momentum.

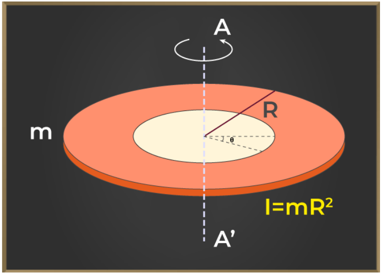
Comparison of Linear and Angular Motion:

Aspect	**Linear Motion**	**Angular Motion**
Displacement	s (m)	θ (rad)
Velocity	$v = \frac{ds}{dt}$ (m/s)	$\omega = \frac{d\theta}{dt}$ (rad/s)
Acceleration	$a = \frac{dv}{dt}$ (m/s^2)	$\alpha = \frac{d\omega}{dt}$ (rad/s^2)
Equations	1. $v = u + at$ 2. $s = ut + \frac{1}{2}at^2$ 3. $v^2 = u^2 + 2as$	$\omega = \omega_0 + \alpha t$ $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ $\omega^2 = \omega_0^2 + 2\alpha\theta$
Inertia	Mass (m) (kg)	Moment of inertia (I) (kg·m^2)
Force/Torque	$F = ma$ (N)	$\tau = I\alpha$ (N·m)
Energy	$KE = \frac{1}{2}mv^2$	$KE = \frac{1}{2}I\omega^2$
Momentum	$p = mv$ (kg·m/s)	$L = I\omega$ (kg·m^2/s)
Circular Motion	$v = r\omega$	$a = r\alpha$

Moment of inertia; Radius of gyration

1. **Moment of inertia(I):**

   Moment of inertia is a measure of an object's resistance to changes in its rotational motion. It depends on the mass distribution relative to the axis of rotation.

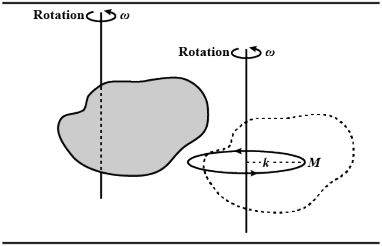


Concept:- It quantifies how mass is distributed around the axis of rotation. The farther the mass is from the axis, the greater the moment of inertia.

$$i.e. I = Mr^2 = \sum_{1}^{n}(m_i r_i^2) \text{ or } I = Mr^2 = \int_{1}^{n} r^2 dm$$

2 Radius of gyration(k):

The radius of gyration is the distance from the axis of rotation at which the total mass of the body could be concentrated without changing its moment of inertia.



Concept:- It simplifies the representation of the mass distribution of a rigid body.

$$K = \sqrt{\frac{I}{M}}$$

Where , I = MI and M is the total mass of the body.

Theorem of Moment of Inertia:

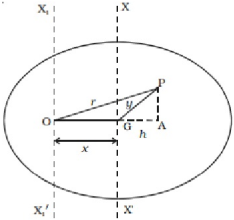


Fig . Parallel axes theorem

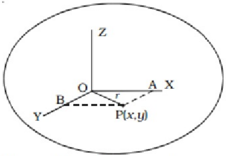


Fig Perpendicular axes theorem

1 Theorem of parallel:

Statement:- The moment of inertia I of a rigid body about any axis is equal to the sum of: (A) Its moment of inertia I_{cm} about a parallel axis passing through its center of mass, and (B) The product of its total mass M and the square of the perpendicular distance d between the two axes.

$$I = I_{cm} + Md^2$$

2 Theorem of Parpendicular:

Statement: For a planar (2D) object, the moment of inertia about an axis perpendicular to the plane (I_z) is the sum of the moments of inertia about two perpendicular axes in the plane (I_x) and (I_y)

$$I_z = I_x + I_y$$

Where, I_z is perpendicular axis, and (I_x, I_y) are axes in the plane.

Rigid Body:

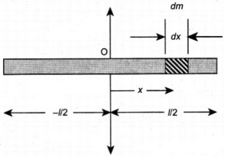
A rigid body is an idealized object in which the distance between any two points remains constant over time, meaning it does not deform under any forces.

Rigid Body	M.I.(I)	R.G.(k)
Thin Rod (tr-Center)	$I = \frac{1}{12}ML^2$	$k = \frac{L}{\sqrt{12}}$
Thin Rod (tr-End)	$I = \frac{1}{3}ML^2$	$k = \frac{L}{\sqrt{3}}$
Solid Sphere (tr-Center)	$I = \frac{2}{5}MR^2$	$k = \sqrt{\frac{2}{5}}R$
Hollow Sphere (tr-Center)	$I = \frac{2}{3}MR^2$	$k = \sqrt{\frac{2}{3}}R$

Moment of Inertia of a Thin Uniform Rod

1. About its Center:

- Consider the rod lying along the x-axis from $-\frac{L}{2}$ to $\frac{L}{2}$.
- The mass per unit length is $\lambda = \frac{M}{L}$.
- The moment of inertia is:



Moment of inertia of uniform rod

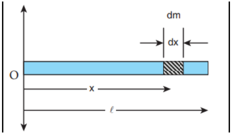
$$I_{\text{center}} = \int_{-L/2}^{L/2} x^2\, dm = \lambda \int_{-L/2}^{L/2} x^2\, dx$$

Solving the integral:

$$I_{\text{center}} = \frac{1}{12} ML^2$$

2. About its End:

- Use the **parallel axis theorem**: $I_{\text{end}} = I_{\text{center}} + Md^2$
- Here, $d = \frac{L}{2}$, so: $I_{\text{end}} = \frac{1}{12} ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{3} ML^2$



Expression of rotational Kinetic Energy

Linear Kinetic Energy: $KE = \frac{1}{2}mv^2$

As we use the relation of rotaional analog into velocity $v = r\omega$ we get,

$$KE = \frac{1}{2}mr^2\omega^2$$

Sum Over All Particles:

For i^{th} particles, Then

$$KE_{\text{rot}} = \frac{1}{2}\left(\sum m_i r_i^2\right)\omega^2$$

By Moment of Inertia:$I = \sum m_i r_i^2$ we get

The final Expression is;

$$KE_{\text{rot}} = \frac{1}{2}I\omega^2$$

Newton's Second Law for Rotation ($\tau = I\alpha$) From the definition of torque,

$$\tau = \mathbf{r} \times \mathbf{F} \implies \tau = rF\sin\theta$$

From the definition of angular acceleration,

$$\alpha = \frac{d\omega}{dt}$$

Since;

$$\mathbf{F} = m\mathbf{a} \implies \tau = \mathbf{r} \times (m\mathbf{a})$$

$$\mathbf{a} = r\alpha \implies \tau = \mathbf{r} \times (mr\alpha) \implies \tau = mr^2\alpha$$

Also we know,

$$I = \sum (m_i r_i^2)$$

Therefore;

$$\tau_{\text{net}} = \sum (m_i r_i^2)\alpha$$

Final expression is given by ,

$$\tau_{\text{net}} = I\alpha$$

Work and Power in Rotational Motion

In rotational motion, **work** and **power** describe how forces cause rotational motion and how energy is transferred. Here's a breakdown with mathematical expressions:

1. Work in Rotational Motion

Work is done when a torque (τ) causes an angular displacement (θ). The work done (W) is given by:

$$W = \int_{\theta_1}^{\theta_2} \tau \, d\theta$$

If the torque is constant, the work simplifies to:

$$W = \tau \Delta\theta$$

where:

- τ = torque (in N·m),
- $\Delta\theta$ = angular displacement (in radians).

Relation to Kinetic Energy: The work done by a torque changes the rotational kinetic energy of the object:

$$W = \Delta K = \frac{1}{2} I\omega_f^2 - \frac{1}{2} I\omega_i^2$$

- I = moment of inertia (in kg·m²),(ω_f) = final angular velocity (in rad/s) and (ω_i) = initial angular velocity (in rad/s).

2. Power in Rotational Motion

Power (P) is the rate at which work is done in rotational motion. It is given by:

$$P = \frac{dW}{dt}$$

Substituting $W = \tau\Delta\theta$, we get:

$$P = \tau\frac{d\theta}{dt}$$

Since $\frac{d\theta}{dt} = \omega$ (angular velocity), the power becomes:

$$P = \tau\omega$$

where: τ = torque (in N·m), and ω = angular velocity (in rad/s).

Angular Momentum:

Angular momentum (L) is a measure of the rotational motion of an object and is defined as:

$$\vec{L} = \vec{r} \times \vec{P} = I\omega$$

where, r= position vector p= linear momentum (P=mv)
I= moment of inertia ω=angular velocity

Principle of Conservation of Angular Momentum:

If no external torque (τ) acts on a system, the total angular momentum of the system remains constant:

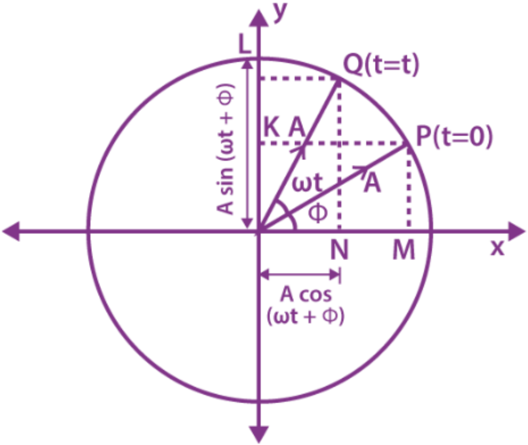
$$\tau_{ext} = \frac{dL}{dt}$$

If $\tau_{ext} = 0$, for isolated equilibrium system of body then:

$$\frac{dL}{dt} = 0 \implies L = constant$$

Thus, angular momentum is conserved in the absence of external torque.

Simple Harmonic Motion (SHM):
Simple Harmonic Motion is a type of periodic motion where the restoring force is directly proportional to the displacement and acts in the opposite direction. It is characterized by smooth oscillations.



Equation of SHM:

The displacement (x) of an object in SHM as a function of time (t) is given by:

$$x(t) = A\sin(\omega t + \phi)$$

where, A= amplitude , ϕ= phase constant
ω=angular velocity

Differential Equation of SHM:

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \implies \frac{d^2x}{dt^2} = -\omega^2 x$$

Where, $\frac{d^2x}{dt^2}$ is acceleration for angular motion.
Hence,

$$F \propto -x \implies a \propto -x$$

This means the restoring force or acceleration acting on the particle should always be proportional to the displacement of the particle and directed towards the equilibrium position.

Velocity in SHM

The velocity v(t) is the time derivative of the displacement x(t):

$$x(t) = A\sin(\omega t + \phi) \ \ is \ \ v(t)\frac{dx(t)}{dt} = \frac{d}{dt}[A\sin(\omega t + \phi)]$$

Since ω and ϕ are constants:

$$v(t) = A\omega\cos(\omega t + \phi) \implies v(t) = A\omega\sqrt{1 - \sin^2(\omega t + \phi)}$$

Therefore,

$$v(t) = \omega\sqrt{A^2 - A^2\sin^2(\omega t + \phi)} \implies v(t) = \omega(\sqrt{A^2 - x^2})$$

Case:

The velocity at extreme position $(\frac{\pi}{2}, ..\frac{n\pi}{2})$ is zero v(t)=0
The velocity at mean position $(0, 2\pi, ..n\pi)$ is maximum i.e $v(t) = A\omega$
This means the velocity of the particle excuting SHM is directly proportional to the amplitude and angular velocity of oscillating particle.

Energy in SHM

The kinetic energy $K.E.$ of the particle is given by:

$$K.E. = \frac{1}{2}mv^2$$

Substitute $v(t) = A\omega\cos(\omega t + \phi)$:

$$K.E. = \frac{1}{2}m\left(A\omega\cos(\omega t + \phi)\right)^2$$

$$K.E. = \frac{1}{2}mA^2\omega^2\cos^2(\omega t + \phi)$$

Maximum kinetic energy is

$$K.E = \frac{1}{2}mA^2\omega^2 = \frac{1}{2}kA^2$$

The potential energy $P.E.$ stored in the spring is given by:

$$P.E. = \frac{1}{2}kx^2$$

Substitute $x(t) = A\sin(\omega t + \phi)$:

$$P.E. = \frac{1}{2}k\left(A\sin(\omega t + \phi)\right)^2$$

$$P.E. = \frac{1}{2}kA^2\sin^2(\omega t + \phi)$$

Maximum potential energy is

$$P.E. = \frac{1}{2}m\omega^2x^2$$

The total energy E of the system is the sum of the kinetic and potential energies:

$$E = K.E. + P.E.$$

Substitute the expressions for $K.E$ and $P.E.$:

$$E = \frac{1}{2}kA^2\cos^2(\omega t + \phi) + \frac{1}{2}kA^2\sin^2(\omega t + \phi)$$

$$E = \frac{1}{2}kA^2\left(\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)\right)$$

$$E = \frac{1}{2}kA^2$$

Derivation of the Period for Vertical Oscillation of a Mass-Spring System

Consider a mass m suspended from a coiled spring. When the mass is displaced vertically from its equilibrium position, the spring exerts a restoring force F proportional to the displacement y, as described by **Hooke's Law**:

$$F = -ky$$

where:

- k is the **spring constant** (a measure of the stiffness of the spring),
- y is the displacement from the equilibrium position,
- The negative sign indicates that the force is restorative (opposite to the direction of displacement).

From Newton's second law, the force acting on the mass is also equal to the mass m times its acceleration a:

$$F = ma$$

Since ($a = \frac{dv}{dt}$), we can write:

$$F = m\frac{dv}{dt}$$

From Hooke's Law and Newton's second law, we equate the two expressions for F:

$$m\frac{dv}{dt} = -ky$$

Rearranging this equation, we obtain:

$$\frac{dv}{dt} + \frac{k}{m}y = 0$$

This is the **differential equation for simple harmonic motion**.

The general solution to the differential equation:

$$\frac{d^2y}{dt^2} + \omega^2 y = 0$$

is:

$$y(t) = A\cos(\omega t + \phi)$$

where:

- A is the amplitude of oscillation,
- ω is the angular frequency,
- ϕ is the phase constant.

Comparing this with our earlier equation, we see that:

$$\omega^2 = \frac{k}{m}$$

Thus, the angular frequency ω is:

$$\omega = \sqrt{\frac{k}{m}}$$

The period T is the time taken for one complete oscillation. It is related to the angular frequency ω by:

$$T = \frac{2\pi}{\omega}$$

Substituting $\omega = \sqrt{\frac{k}{m}}$, we get:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

The period T of vertical oscillation of a mass m suspended from a coiled spring is:

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

Key Points:

- The period T depends on the mass m and the spring constant k.
- A larger mass m increases the period, while a stiffer spring (larger k) decreases the period.
- This derivation assumes no damping (energy loss) and small oscillations, ensuring the motion is simple harmonic.

Simple Pendulum

A simple pendulum is an idealized model consisting of a point mass (bob) suspended from a massless, inextensible string or rod. When displaced from its equilibrium position and released, it oscillates under the influence of gravity.

Derivation for time period of simple pendulum:

To derive the expression for the period, we analyze the forces acting on the pendulum and use small-angle approximations.

Consider a simple pendulum of length L with a bob of mass m. When the pendulum is displaced by an angle θ from the vertical, the forces acting on the bob are:

- Gravitational force (mg) i.e. $mg\cos\theta$ acting vertically downward.
- Tension (T) i.e. $mg\sin\theta$ in the string acting along the string.

fig. simple pendulum

Equation of motion

The restoring force acting on the pendulum bob is:

$$F = -mg\sin\theta$$

Using Newton's second law, the tangential acceleration is:

$$a = -g\sin\theta$$

For small angles ($\theta \ll 1$), $\sin\theta \approx \theta$, so:

$$a = -g\theta$$

The tangential acceleration can also be expressed in terms of angular displacement:

$$a = \frac{d^2s}{dt^2} = L\frac{d^2\theta}{dt^2}$$

Substituting into the equation of motion:

$$L\frac{d^2\theta}{dt^2} = -g\theta$$

Rearranging:

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0$$

This is the differential equation for simple harmonic motion.

$$\omega^2 = \frac{g}{L}$$

The period T is related to the angular frequency by:

$$T = \frac{2\pi}{\omega}$$

Substituting $\omega = \sqrt{\frac{g}{L}}$:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Final Expression for the Period

$$T = 2\pi\sqrt{\frac{L}{g}}$$

1. Damped Oscillation: Oscillations that decrease in amplitude over time due to energy loss (e.g., friction or resistance).
2. Free Oscillation: Oscillations that occur naturally at a system's resonant frequency without external forces.
3. Forced Oscillation: Oscillations driven by an external periodic force, potentially at a frequency different from the system's natural frequency.

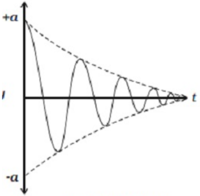


Fig. Damped oscillations

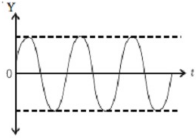


Fig. Maintained oscillations

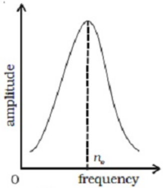


Fig. Resonance

Archimedes principle and Pascal's law

Archimedes' Principle:

Statement:- Archimedes' principle states that when a body is partially or fully submerged in a fluid (liquid or gas), it experiences an upward buoyant force equal to the weight of the fluid displaced by the body.

The buoyant force F_b is given by ,

$$F_b = \rho V g$$

Where:

ρ= density of fluid ,

V= volume of displaced fluid.

g= acceleration due to gravity

Applications:

- Determining the buoyancy of ships and submarines.
- Measuring the density of irregular objects.
- Understanding why objects float or sink in fluids.

Pascal's Law:

Statement:- Pascal's law states that when pressure is applied to a confined fluid, the pressure change is transmitted equally and undiminished to all parts of the fluid and to the walls of the container.

Mathematically, Pascal's law can be expressed as:

$$\Delta P = \frac{F}{A}$$

Where:

- ΔP is the change in pressure,
- F is the applied force,
- A is the area over which the force is applied.

Applications:

- Hydraulic systems, such as hydraulic lifts and brakes, which use Pascal's law to amplify force.
- Understanding fluid behavior in closed systems, such as pipes and tanks.
- Designing devices that rely on pressure transmission, like hydraulic jacks and presses.

Some Terminolgy:

1. Up-thrust:
   The upward force exerted by a fluid on an object submerged in it, equal to the weight of the fluid displaced by the object.
2. Pressure in Fluid:
   The force exerted per unit area within a fluid, given by $P = F/A$ where F is the force and A is the area.
3. Buoyancy:
   The ability of a fluid to exert an upward force on an object placed in it, causing the object to float or appear lighter.
4. Center of Buoyancy:
   The geometric center of the displaced fluid volume, where the buoyant force acts on the submerged object.
5. Meta Center:
   The point where the line of action of the buoyant force intersects the object's centerline when the object is tilted slightly; it determines the stability of floating objects.

Law of Floatation:

Statement:- A body floats in a fluid when the weight of the body is equal to the weight of the fluid displaced by it.
Mathematically:
Weight of body=Weight of displaced fluid.

- Explains why objects like ships and boats float.
- Used to design vessels to ensure they displace enough water to balance their weight.

Surface tension:

Definition: Surface tension is the property of a liquid's surface that allows it to resist external forces, acting like a stretched elastic membrane.
Cause: It arises due to cohesive forces between liquid molecules, which are stronger at the surface because surface molecules are not equally attracted in all directions.

Principle:

- Surface tension minimizes the surface area of a liquid, leading to phenomena like water droplets forming spherical shapes and insects walking on water.
- It is measured as force per unit length (N/m).

Surface Energy:

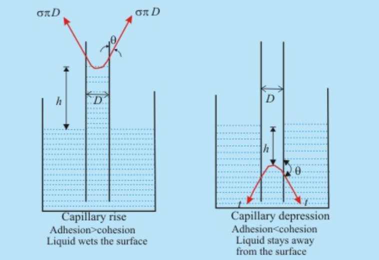
The work required to increase the surface area of a liquid by a unit amount. It is numerically equal to surface tension and is measured in J/m^2.

Relation Between Surface Tension and Surface Energy

- Consider a liquid film with a movable side of length L. Apply force F to stretch the side by Δx.
  Work done:
  $$W = F \cdot \Delta x.$$
  Increase in surface area:
  $$\Delta A = 2L \cdot \Delta x.$$
- Surface tension $T = \frac{F}{2L}$ (force per unit length, 2 surfaces).
  So, $F = T \cdot 2L$.
  $$\therefore W = T \cdot 2L \cdot \Delta x$$
- Work done is also: $W =$ Surface Energy (E) $\cdot \Delta A$.
- Equate work:
  $$T \cdot 2L \cdot \Delta x = E \cdot 2L \cdot \Delta x \implies T = E$$

Surface Tension (T) = Surface Energy (E): $\implies T = E$.

Angle of Contact and Capillarity with examples



Angle of Contact:

The angle (θ) formed between the tangent to the liquid surface and the solid surface at the point of contact, measured inside the liquid.
Example: Water on glass has a small angle of contact ($\theta < 90^0$), while mercury on glass has a large angle ($\theta > 90^0$).

Capillarity:

The rise or fall of a liquid in a narrow tube (capillary tube) due to surface tension and adhesive/cohesive forces.
Example: Water rises in a glass capillary tube, while mercury falls in the same tube.

Newton's Formula for viscosity of a liquid and Coefficient of viscosity

Newton's Formula for Viscosity

$$F = \eta A \frac{du}{dz}$$

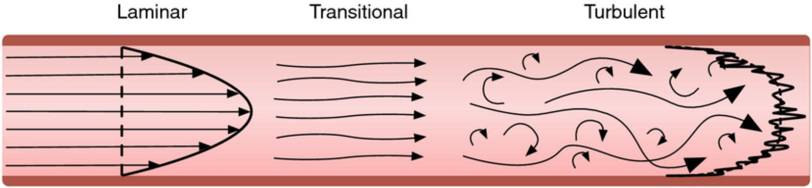
where:

- F: Viscous force,
- η: Coefficient of viscosity,
- A: Area of the liquid layer,
- $\frac{du}{dz}$: Velocity gradient perpendicular to the flow.

Coefficient of Viscosity (η)

- It is the measure of a fluid's resistance to flow.
- Defined as the tangential force per unit area required to maintain a unit velocity gradient between two parallel layers of fluid.
- Unit: Pa·s (Pascal-second).

Laminar and Turbulent flow



Laminar Flow: Smooth, orderly flow where fluid moves in parallel layers with no disruption between them.
Example: Flow of honey or slow-moving water.
Turbulent flow: Chaotic, irregular flow with random eddies and mixing of fluid layers.
Example: Flow in rivers or fast-moving air.

Reynolds Number (Re)

- A dimensionless number used to predict whether a flow will be laminar or turbulent.
- Formula:

$$Re = \frac{\rho v L}{\eta}$$

where:

  - ρ: Density of the fluid,
  - v: Velocity of the flow,
  - L: Characteristic length (e.g., diameter of a pipe),
  - η: Coefficient of viscosity.

Interpretation of Reynolds Number

- $Re < 2000$: Laminar flow.
- $2000 < Re < 4000$: Transitional flow.
- $Re > 4000$: Turbulent flow.

For water flowing in a pipe:

- If $Re = 1500$, the flow is laminar.
- If $Re = 5000$, the flow is turbulent.

Poiseuille's formula

Poiseuille's formula describes the volumetric flow rate (Q) of a viscous fluid through a cylindrical pipe under laminar flow conditions. It is given by:

$$Q = \frac{\pi \Delta P r^4}{8\eta L}$$

where:

- Q: Volumetric flow rate (m^3/s),
- ΔP: Pressure difference between the ends of the pipe (Pa),
- r: Radius of the pipe (m),
- η: Coefficient of viscosity of the fluid (Pa·s),
- L: Length of the pipe (m).

Derivation of Poiseuille's Formula

Assumptions

- The flow is laminar and steady.
- The fluid is incompressible and Newtonian.
- The pipe is cylindrical with a constant radius.

Take a cylindrical fluid element of radius y and thickness dy within the pipe. The viscous drag force (F_{viscous}) acting on the element is given by

$$F_{\text{viscous}} = \eta \cdot A \cdot \frac{dv}{dy}$$

where $A = 2\pi yL$ is the surface area of the element, and $\frac{dv}{dy}$ is the velocity gradient.

Force Balance The pressure difference (ΔP) across the pipe creates a driving force:

$$F_{\text{pressure}} = \Delta P \cdot \pi y^2$$

At steady state, the viscous drag force balances the pressure force:

$$\Delta P \cdot \pi y^2 = \eta \cdot 2\pi yL \cdot \frac{dv}{dy}$$

Simplifying:

$$\frac{dv}{dy} = -\frac{\Delta P}{2\eta L} \cdot y$$

Integrate the velocity gradient:

$$v(y) = -\frac{\Delta P}{4\eta L} \cdot y^2 + C$$

Apply the no-slip boundary condition ($v = 0$ at $y = r$):

$$0 = -\frac{\Delta P}{4\eta L} \cdot r^2 + C \implies C = \frac{\Delta P}{4\eta L} \cdot r^2$$

Thus, the velocity profile is:

$$v(y) = \frac{\Delta P}{4\eta L}(r^2 - y^2)$$

Calculate Volumetric Flow Rate (Q) Integrate the velocity profile over the cross-sectional area of the pipe:

$$Q = \int_0^r v(y) \cdot 2\pi y \, dy$$

Substitute $v(y)$:

$$Q = \int_0^r \frac{\Delta P}{4\eta L}(r^2 - y^2) \cdot 2\pi y \, dy$$

Simplify and integrate:

$$Q = \frac{\pi \Delta P}{2\eta L} \int_0^r (r^2 y - y^3)\, dy$$

$$Q = \frac{\pi \Delta P}{2\eta L} \left[\frac{r^2 y^2}{2} - \frac{y^4}{4} \right]_0^r$$

Evaluate the integral:

$$Q = \frac{\pi \Delta P}{2\eta L} \left(\frac{r^4}{2} - \frac{r^4}{4} \right)$$

$$Q = \frac{\pi \Delta P}{2\eta L} \cdot \frac{r^4}{4}$$

$$Q = \frac{\pi \Delta P r^4}{8\eta L}$$

This is **Poiseuille's Formula**, which describes the flow rate of a viscous fluid through a cylindrical pipe.

State Stoke's law and coefficient of viscosity of liquid

Stoke's law: Stoke's law describes the viscous drag force (F) acting on a small spherical object moving through a viscous fluid. It is given by:

$$F = 6\pi\eta rv$$

- F: Viscous drag force,
- η: Coefficient of viscosity,
- r: Radius of the object,
- v: Terminal velocity.

Determine the Coefficient of Viscosity (η):

Experimental Setup:

- A small spherical object (e.g., a ball bearing) is dropped into a viscous liquid (e.g., oil or glycerin) contained in a tall cylindrical tube.
- The object reaches terminal velocity (v) when the viscous drag force (F) balances the net downward force (weight - buoyant force).

Measure Terminal Velocity (v)

- Allow the object to fall through the liquid.
- Measure the time (t) it takes to travel a known distance (d).

Terminal velocity is calculated as: $v = \frac{d}{t}$

Balance Forces at Terminal Velocity

At terminal velocity, the net force on the object is zero. The forces acting are:

- Weight (W) : $W = mg = \rho_{object} V_g$
- Buoyant Force (F_b) : $F_b = \rho_{fluid} V_g$
- Viscous Drag Force (F) : $F = 6\pi\eta r v$

where:

- F: Viscous drag force, η: Coefficient of viscosity,
- r: Radius of the object, and, v: Terminal velocity.

Determine η

1. Measure terminal velocity (v) of a spherical object in the fluid.
2. Balance forces at terminal velocity:

$$(\rho_{\text{object}} - \rho_{\text{fluid}}) \cdot \frac{4}{3}\pi r^3 g = 6\pi\eta r v$$

3. Solve for η:

$$\eta = \frac{2(\rho_{\text{object}} - \rho_{\text{fluid}}) r^2 g}{9v}$$

Equation of continuity and its application

It states that the mass of fluid entering a system must equal the mass leaving the system, assuming no mass is created or destroyed within the system.

For an incompressible fluid (density ρ is constant), the equation of continuity is:

$$A_1 v_1 = A_2 v_2$$

where:

- A_1 and A_2 are the cross-sectional areas at two points in the flow,
- v_1 and v_2 are the velocities of the fluid at those points.

For compressible flow (general form)

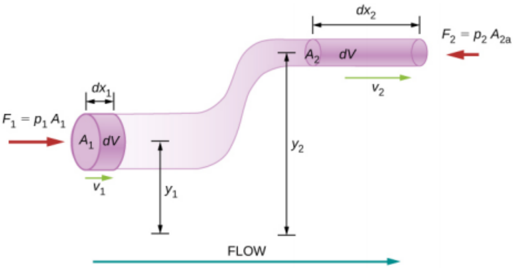
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

For steady flow (compressible)

$$\nabla \cdot (\rho \mathbf{v}) = 0$$

Bernoulli's equation and explain its uses

Bernoulli's equation is a fundamental principle in fluid dynamics that describes the conservation of energy in a flowing fluid. It relates pressure, velocity, and elevation in a fluid flow and is derived from the principles of conservation of energy.



Statement: for an incompressible, non-viscous, and steady flow, the total mechanical energy of the fluid remains constant along a streamline. Mathematically, it is expressed as:

$$P + \frac{1}{2}\rho v^2 + \rho gh = constant$$

where;

- $\rho =$ Density of fluid
- h=y=height of fluid from reference point.
- g= acceleration due to gravity
- v= velocity of fluid.

Derivation of Bernoulli's Equation

Let us assumptions

- The fluid is **incompressible** (density ρ is constant).
- The flow is **steady** (no change in flow properties with time).
- The flow is **non-viscous** (no energy loss due to friction).
- The flow is along a **streamline**.

Consider a fluid flowing through a pipe with varying elements.

At point 1:

- Cross-sectional area $= A_1$
- Pressure $= P_1$
- Velocity $= v_1$
- Height $= y_1$

At point 2:

- Cross-sectional area $= A_2$
- Pressure $= P_2$
- Velocity $= v_2$
- Height $= y_2$

The fluid element is pushed by pressure forces at both ends of the pipe. The work done by these forces is:

- **Work done at point 1 (W_1)**:

$$W_1 = P_1 A_1 \Delta x_1$$

- **Work done at point 2 (W_2)**:

$$W_2 = -P_2 A_2 \Delta x_2$$

The **net work done (W)** by the pressure forces is:

$$W = W_1 + W_2 = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2$$

Change in the fluid's kinetic and potential energy.

- **Change in Kinetic Energy (ΔKE)**:

$$\Delta KE = \frac{1}{2} m (v_2^2 - v_1^2)$$

- **Change in Potential Energy (ΔPE)**:

$$\Delta PE = mg(y_2 - y_1)$$

From the **work-energy principle**, the net work done equals the change in total energy:

$$W = \Delta KE + \Delta PE$$

Substitute the expressions for W, ΔKE, and ΔPE:

$$P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 = \frac{1}{2} m(v_2^2 - v_1^2) + mg(y_2 - y_1)$$

From the **continuity equation**, the mass flow rate is constant:

$$A_1 v_1 = A_2 v_2$$

The volume of fluid moved at each point is:

$$A_1 \Delta x_1 = A_2 \Delta x_2 = V$$

The mass of the fluid element is:$m = \rho V$
Substitute $m = \rho V$ and $A_1 \Delta x_1 = A_2 \Delta x_2 = V$ into the work-energy equation:

$$P_1 V - P_2 V = \frac{1}{2} \rho V(v_2^2 - v_1^2) + \rho V g(y_2 - y_1)$$

Divide through by V:

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1)$$

Rearrange the equation to group terms at points 1 and 2:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

By (replacing y by h)

Bernoulli's Equation

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

Which states that the total energy per unit volume remains constant along a streamline:

Thank you!

Reference

1 **R. Pd.Koirala ,PRINCIPLES OF PHYSI CS - II, Asmita Books Publishers and Distributors (P) Ltd.**

2 **https://www.deepseek.com and https://openai.com/index/chatgpt/**