Ambiguity and Corporate Yield Spreads

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Abstract

We derive a model of bond pricing under ambiguity, showing that ambiguity interacts with risk to determine spreads. Since default is an inherently "unfavorable" outcome, ambiguity-averse bondholders overweigh its probability and demand higher yields for bonds with higher ambiguity. Empirically, the economic effect of ambiguity on credit spreads is of the same magnitude as that of risk. Furthermore, ambiguity and risk amplify each-other; spreads on higher risk bonds are more sensitive to ambiguity and vice versa. Incorporating ambiguity substantially improves the model's fit relative to observed spreads, providing a potential resolution to the credit spread puzzle.

Keywords and Phrases: Yield Spreads, Knightian Uncertainty, Ambiguity Aversion, Bond Ambiguity, Perceived Probabilities.

JEL Classification Numbers: C65, D81, D83, G12, G32

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1 Introduction

The theoretical and empirical literature on credit spreads has identified a number of factors that relate credit risk and spreads. Bond ratings are the primary empirical drivers of spreads; although ratings themselves are inextricably linked to leverage ratios, volatility, and other firm fundamentals.[1] Nevertheless, much of the level of, and variation in, credit spreads—after accounting for ratings—remains unexplained. Similarly, while factor models have been used extensively to explain expected returns estimated for portfolios of bonds (He, Kelly, and Manela, 2017), traditional measures of systematic risk can only partially explain spreads or expected returns at the individual bond or firm level (e.g., Berndt, 2015; Lewis, 2019).

Recent literature documents that ambiguity—the uncertainty over probabilities—represents an important dimension of uncertainty for financial decision-making (e.g., Izhakian, Yermack, and Zender, 2021; Malenko and Tsoy, 2020). Intuitively, in the presence of ambiguity, ambiguity-averse decision-makers tend to overweigh the likelihood of unfavorable events and underweight the likelihood of favorable events (e.g., Tversky and Kahneman, 1992; Izhakian, 2017), suggesting that ambiguity will have a meaningful impact on the pricing of risky assets. For the case of bonds, the possibility of default—an unfavorable event for a bondholder—is the primary differentiating factor between risky and risk-free assets. If bond investors are ambiguity averse, bond level ambiguity should be an important factor in understanding the difference in yield between risky and risk-free bonds; the credit or yield spread. The bond market is, therefore, a natural laboratory for examining the impact of ambiguity on asset pricing.

Developing a model for the pricing of risky and ambiguous debt with ambiguity- and risk-averse investors, we demonstrate that the ambiguity associated with a bond has a first-order impact on pricing. Moreover, the model establishes that ambiguity matters more for firms that are more likely to default and thus ambiguity amplifies the impact of default risk. We confirm these predictions empirically by showing that *(i)* bond level ambiguity has an impact on spreads that is of the same magnitude as that of standard measures of default risk; *(ii)* the economic magnitude of the effect of ambiguity more than doubles for firms with high default risk relative to firms less likely to default; and *(iii)* bond ambiguity helps explain the level and the variation in observed spreads.

The ambiguity associated with a firm/asset may not have a symmetric impact on the pricing of the firm's debt and its equity for two reasons. First, the nature of the ambiguity associated with the firm's cash flow may be such that it affects bondholders and equity holders differently. Ambiguity

[1] See Duffie and Singleton (2012) and the citations therein for a summary of this literature.

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reflects the uncertainty over the probabilities of the relevant possible events from the perspective of the stake holder. So, for example, the uncertainty regarding the probabilities of the asset's returns may be concentrated in one tail of the return distribution. A low-levered firm's debt may not be affected by ambiguity if the uncertainty regarding the probabilities of asset returns is concentrated in the probabilities associated with high asset returns. In such a case, the firm's equity should be strongly affected but not its debt. In contrast, if the uncertainty over probabilities is concentrated in the left tail of the return distribution, debt holders will be heavily impacted, while equity holders will not be affected.

Second, the way the standard financial instruments distribute ownership of the firm's cash flow across its claimants imposes a different partition of the state space for the different claimants. An ambiguity-averse decision-maker is only concerned with uncertainty over the probabilities of events that are distinguished by different payoffs. Therefore, the owner of a bond is concerned with the uncertainty associated with the probabilities of different default states but views all non-default states as a single event. In contrast, an ambiguity-averse owner of a firm's equity is exposed to the uncertainty associated with the probabilities of different non-default states but views all default states as a single event. These standard payout structures imply that, regardless of the nature of the ambiguity of the asset, the uncertainty over probabilities associated with the firm's debt may be very different from that associated with its equity.

Similar to the well-known results regarding risk, ambiguity-averse investors require a premium for exposure to ambiguity (e.g., Izhakian, 2020). Therefore, an increase in the ambiguity associated with a firm's risky debt should increase its credit spread just as the ambiguity of its equity affects the equity's expected return. Furthermore, because ambiguity-averse investors overweigh the likelihood of unfavorable outcomes, the impact of ambiguity on credit spreads should be stronger the higher is the bond's default risk. We confirm these intuitions in a static model of bond pricing with risk and ambiguity-averse investors. Firm's debt and equity holders both "price" the ambiguity associated with their respective securities. Importantly, because of the difference between the ambiguity a debt holder faces as compared to that an equity holder faces, there may appear to be differential pricing in these two markets when ambiguity is not examined. In other words, ambiguity may represent an important difference between the uncertainty facing debt holders and equity holders, creating the appearance of differential pricing across markets.

To empirically examine the theoretical predictions about the impact of bond ambiguity on credit spreads, we apply the technique derived in Izhakian (2020) to risky corporate bonds. Consistent

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with our model, we find that bond ambiguity has a direct and important relation with credit spreads of corporate bonds.[2] In a univariate setting, the importance of bond ambiguity for explaining credit spreads is of the same order of magnitude as that of standard measures of asset volatility or default risk. Moreover, this finding is robust to the inclusion of bond rating by time and maturity by time fixed effects and other traditional controls.

Our empirical findings also confirm the model's prediction that the relation between bond ambiguity and credit spreads is most powerful when risk is high. The evidence that ambiguity has its greatest impact when default risk is high is consistent with the intuition that bond ambiguity is important for credit spreads because it exaggerates ambiguity-averse investors' perceived probability of default. This finding is robust to the use of a variety of different measures of risk and default probability.

To further explore the effect of ambiguity on bond pricing, we provide evidence regarding the relation between bond index ambiguity and credit spreads. Contrary to the ambiguity of a single bond, the ambiguity of a large portfolio of bonds is unlikely to reflect uncertainty regarding the probability of default. Rather, we conjecture that the ambiguity of a bond index portfolio is a measure of uncertainty regarding the probabilities of future interest rate changes. If higher bond index ambiguity reflects greater uncertainty regarding the likelihood of future increases in interest rates, then, consistent with Longstaff and Schwartz (1995), who find that credit spreads are smaller as interest rates rise, higher bond index ambiguity should be negatively related to credit spreads. Using the ambiguity measured for portfolios of treasury bonds, we find that this conjecture holds true, even when controlling for individual bond ambiguity. Consistent with our conjecture, the same relationship is found using the ambiguity measured for portfolios of treasury bonds.

Starting with the theoretical modeling of credit spreads in Merton (1974), an extensive literature has focused on explaining the level of credit spreads for risky bonds. Duffie and Singleton (2012) provide a comprehensive review of the key reduced form determinants of credit spreads: bond rating, volatility, the leverage ratio, liquidity, and macroeconomic conditions are among the most important. Our paper contributes to this literature by identifying another important determinant in pricing the cross-section of bonds: the ambiguity of the firm's bonds. Calibration of our model indicates that bond ambiguity is an important input for explaining the level of credit spreads, conditioned on bond ratings and years to maturity.

The now canonical study, Huang and Huang (2012), examines the credit spread puzzle: essen-

[2] Our findings are consistent with those in Augustin and Izhakian (2020) regarding credit default swaps.

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tially, yield spreads are high relative to default rates. Several studies attempt to resolve this puzzle, primarily by developing models with a high covariance between the few defaults that do occur and investor's marginal utility (e.g., Ericsson, Reneby, and Wang, 2005; Chen, Collin-Dufresne, and Goldstein, 2009).[3] Structural models of credit spreads that utilize a standard asset value–default boundary framework are difficult to reconcile with the data because asset risk is not closely related to credit spreads, a puzzle highlighted in Berndt (2015) and Lewis (2019). Furthermore, enhancing the cost of default to increase the predicted spreads on high-quality bonds will tend to over-predict spreads on low-quality bonds. Ambiguity, however, differs in the cross-section of corporate bonds. As we show, the ambiguity associated with AAA bonds is much higher than the ambiguity associated with BBB bonds. Calibrating credit spreads using both ambiguity and risk as sources of uncertainty premia better reflects observed spreads, reducing the absolute percent deviations between calibrated and observed spreads summed across ratings classes for 10 year bonds by about 60%. Thus, our analysis helps to reconcile these puzzles by adding a price for bond ambiguity that is independent of risk.

The impact of ambiguity on the pricing of corporate bonds is a relatively unexplored issue. An exception is Zhao (2020), which uses differential ambiguity associated with long- and short-term bonds to jointly explain upward sloping (real and nominal) yield curves and the violation of the expectations hypothesis. Our focus is on credit spreads rather than the expectations hypothesis and we take as given the yield curve for risk-free bonds. Further, while Zhao (2020) uses the Blue Chip financial forecast dispersion as a proxy for ambiguity. This proxy for ambiguity is based on dispersions of predicted outcomes rather than dispersion of probabilities. In contrast, our measure of ambiguity is based on Izhakian (2020) and estimates the uncertainty associated with the distributions of the returns of traded corporate bonds.

2 The model

We develop a static model of the impact of ambiguity and ambiguity aversion on the pricing of risky debt. The model has two important, distinguishing features. First, investors' preferences for ambiguity are outcome independent and, therefore, independent of risk and attitude toward risk. Outcome-independent preferences for ambiguity are necessary to differentiate completely the effects of ambiguity and risk. Second, investors are averse to both ambiguity and risk.

Prices are determined by perceived expected utility (value), which is determined in part by the

[3] Feldhütter and Schaefer (2018) argue that data measurement issues can explain a good portion of the "credit spread puzzle".

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perceived expected payoffs. These perceived expected payoffs are affected by risk and ambiguity. To illustrate, suppose an investment of $400 whose payoff is determined by a flip of an unbalanced coin for which the investor does not know the odds. The payoff is $1,000 for heads and $0 for tails. Suppose information arrives indicating the payoff for heads is now $2,000. In this case, risk (measured by the variance of payoff) and the expected payoff increase, such that the investment is more attractive. However, ambiguity has not changed since there is no new information regarding likelihoods. Suppose instead the new information indicates a greater assessed degree of ambiguity about the coin. An ambiguity-averse investor lowers (raises) her perceived probability of the high (low) payoff and the investment opportunity becomes less attractive.[4] In this case, the investor has no reason to change her assessment of the payoffs in the different events, as the new information concerns only the likelihoods.

This example highlights a critical property: ambiguity is outcome independent, up to a state-space partition. That is, if the outcomes associated with events change, while the induced partition of the state space into events (the set of events) remains unchanged, then the degree of ambiguity remains unchanged since the probabilities remain unchanged. Furthermore, outcome dependence for ambiguity would imply risk dependence, confounding the two effects. To identify the effect of ambiguity independently of risk, the underlying preference for ambiguity must apply exclusively to the probabilities of events, independent of the outcomes associated with these events.

2.1 The decision-theoretic framework

To introduce ambiguity into asset pricing, we employ a version of Cumulative Prospect Theory (CPT, Tversky and Kahneman, 1992; Wakker and Tversky, 1993) using strictly concave utility (i.e., risk aversion for losses and gains), augmented with the theoretical framework of Expected Utility with Uncertain Probabilities (EUUP, Izhakian, 2017).[5] The EUUP model provides an axiomatic construct of the capacities (the nonadditive probabilities) employed in CPT, based upon

[4] The cumulative prospect theory (Tversky and Kahneman, 1992) explicitly formalize this weighting, and the max-min expected utility (Gilboa and Schmeidler, 1989) and the Choquet expected utility (Schmeidler, 1989) implicitly deliver it. Experiments show that this weighting holds for unfavorable and favorable outcomes (Abdellaoui, Attema, and Bleichrodt, 2010), as well as for unlikely and likely events (Crockett, Izhakian, and Jamison, 2019).

[5] Choquet Expected Utility (CEU, Schmeidler, 1989), can be viewed as a special case of CPT, in which all outcomes are considered favorable. Our theoretical results are unchanged if we utilize the CEU model rather than the CPT model with a reference point distinguishing favorable and unfavorable outcomes. Under some conditions, the max-min expected utility model (MEU, Gilboa and Schmeidler, 1989) can generate the same conclusions as CEU. However, MEU does not allow a separation of beliefs regarding ambiguity from attitudes toward ambiguity. Further, it is challenging to utilize the MEU or the smooth model of ambiguity aversion (Klibanoff, Marinacci, and Mukerji, 2005) in comparative statics of the effect of ambiguity, since preferences for ambiguity in these models are outcome-dependent and, therefore, risk-dependent. As a result, ambiguity and risk in these models are confounded, such that an impact of risk may mistakenly be attributed to ambiguity.

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the ambiguity in the environment (beliefs) and the decision-makers' aversion to (attitude toward) ambiguity. Under the EUUP model, preferences for ambiguity apply exclusively to the uncertain probabilities of future events (are outcome independent), such that aversion to ambiguity is defined as an aversion to mean-preserving spreads in probabilities. Thereby, ambiguity can be examined independently of risk, as the volatility of the uncertain probabilities of future events. This separation allows theoretical examination of the distinct impacts of ambiguity and risk. It also provides a model-derived, risk-independent measure of ambiguity that can be employed to test the theory (Izhakian, 2020).[6]

Formally, the investor, who values a risky and ambiguous payoff $X : \mathcal{S} \to \mathbb{R}$, possesses a set $\mathcal{P}$ of prior probability distributions P over events (subsets of states in the set of states $\mathcal{S}$). The set $\mathcal{P}$ of prior distributions is equipped with a second-order prior probability distribution ξ (a distribution over probability distributions).[7] Each probability distribution $\mathrm{P} \in \mathcal{P}$ is represented by a cumulative probability distribution $\mathrm{P}(\cdot)$ and a marginal probability function $\varphi(\cdot)$.[8] The investor evaluates the expected utility of a risky and ambiguous payoff by the CPT model (Wakker and Tversky, 1993)

$$\mathrm{V}(X) \;=\; \int_{\neg\mathcal{F}} \mathrm{U}(\cdot)\, d\left[\mathrm{Q}-1\right] + \int_{\mathcal{F}} \mathrm{U}(\cdot)\, d\mathrm{Q}, \tag{1}$$

where Q stands for the capacity, $\mathcal{F} \subseteq \mathcal{S}$ is the set of favorable states and $\neg\mathcal{F} \subseteq \mathcal{S}$ is the set of unfavorable states. Unfavorable states are associated with payoffs smaller than k, and favorable states are associated with payoffs greater than k. That is, k is the reference point.

In the EUUP model, a capacity is referred to as a perceived probability and is formed by the minimum (maximum) unique certain probability value that the investor is willing to accept in exchange for the uncertain probability of a given favorable (unfavorable) event, based upon her attitude toward the uncertainty of probabilities. Accordingly, the perceived probability is assessed by

$$\mathrm{Q}(x) = \Upsilon^{-1}\left(\int_{\mathcal{P}} \Upsilon\left(1-\mathrm{P}(x)\right) d\xi\right). \tag{2}$$

[6] The measurement of ambiguity independent from risk poses a challenge for other frameworks that do not separate ambiguity from attitudes toward ambiguity (e.g., Gilboa and Schmeidler, 1989; Schmeidler, 1989) or in which preferences for ambiguity are outcome-dependent (e.g., Klibanoff et al., 2005; Chew and Sagi, 2008).

[7] Specifically, there is a probability space $(\mathcal{S}, \mathcal{E}, \mathrm{P})$, where $\mathcal{S}$ is an infinite state space; $\mathcal{E}$ is a σ-algebra of subsets of the state space (a set of events); a λ-system $\mathcal{H} \subset \mathcal{E}$ contains the events with an unambiguous probability (i.e., events with a known, objective probability); and $\mathrm{P} \in \mathcal{P}$ is an additive probability measure. The set of all probability measures $\mathcal{P}$ is assumed to be endowed with an algebra $\Pi \subset 2^{\mathcal{P}}$ of subsets of $\mathcal{P}$ that satisfies the structure required by Kopylov (2010). Π is equipped with a unique countably-additive probability measure ξ that assigns each subset $A \in \Pi$ with a probability $\xi(A)$.

[8] To simplify notation, $\mathrm{P}(x)$ stands for $\mathrm{P}(\{s \in \mathcal{S} \mid X(s) \le x\})$ and $\varphi(x)$ for $\varphi(\{s \in \mathcal{S} \mid X(s) = x\})$.

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The marginal perceived probability for unfavorable outcomes can be usefully approximated as[9]

$$dQ = \mathrm{E}\left[\varphi\left(x\right)\right]\left(1-\frac{\Upsilon''\left(1-\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}{\Upsilon'\left(1-\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}\mathrm{Var}\left[\varphi\left(x\right)\right]\right) \tag{3}$$

and for favorable outcomes as

$$dQ = \mathrm{E}\left[\varphi\left(x\right)\right]\left(1+\frac{\Upsilon''\left(1-\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}{\Upsilon'\left(1-\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}\mathrm{Var}\left[\varphi\left(x\right)\right]\right), \tag{4}$$

where the function Υ, called the outlook function, captures the investor's attitude toward ambiguity;[10] the expected marginal and cumulative probabilities are computed using ξ, such that $\mathrm{E}\left[\varphi\left(\cdot\right)\right]=\int_{\mathcal{P}}\varphi\left(\cdot\right)d\xi$ and $\mathrm{E}\left[\mathrm{P}\left(\cdot\right)\right]=\int_{\mathcal{P}}\mathrm{P}\left(\cdot\right)d\xi$; and $\mathrm{Var}\left[\varphi\left(\cdot\right)\right]=\int_{\mathcal{P}}\left(\varphi\left(\cdot\right)-\mathrm{E}\left[\varphi\left(\cdot\right)\right]\right)^2 d\xi$ defines the variance of the marginal probability. In contrast to Tversky and Kahneman (1992), in our model, agents do not apply a probability weighting function to the perceived probabilities.

Using perceived probabilities, the investor assesses the expected utility of a risky and ambiguous payoff by

$$\begin{aligned}\mathrm{V}\left(X\right) = &\int_{x\le k}\mathrm{U}\left(x\right)\underbrace{\mathrm{E}\left[\varphi\left(x\right)\right]\left(1-\frac{\Upsilon''\left(1-\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}{\Upsilon'\left(1-\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}\mathrm{Var}\left[\varphi\left(x\right)\right]\right)}_{\text{Perceived Probability of Unfavorable Outcome } x}dx+ \\ &\int_{x>k}\mathrm{U}\left(x\right)\underbrace{\mathrm{E}\left[\varphi\left(x\right)\right]\left(1+\frac{\Upsilon''\left(1-\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}{\Upsilon'\left(1-\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}\mathrm{Var}\left[\varphi\left(x\right)\right]\right)}_{\text{Perceived Probability of Favorable Outcome } x}dx,\end{aligned} \tag{5}$$

where k is a reference point that distinguishes between unfavorable and favorable outcomes. In this representation of expected utility, risk aversion is distinctly captured by a strictly-increasing, concave, and twice-differentiable continuous utility function $\mathrm{U}:\mathbb{R}\to\mathbb{R}$ applied to the uncertain outcomes. Ambiguity aversion is captured by a strictly-increasing, concave, and twice-differentiable continuous outlook function $\Upsilon:[0,1]\to\mathbb{R}$ applied to the uncertain probabilities.

The ambiguity-averse investor compounds the set of priors $\mathcal{P}$ using the second-order prior ξ over $\mathcal{P}$ in a nonlinear way, as reflected in the perceived probabilities provided in Equation (5).[11] The (subadditive) perceived probabilities are a function of the extent of ambiguity (of each event),

[9] Izhakian (2020) shows that the residual of this approximation is $R_2\left(\mathrm{P}\left(x\right)\right)=o\left(\mathrm{E}\left[\left|\mathrm{P}\left(x\right)-\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right|^3\right]\right)$ as $\left|\mathrm{P}\left(x\right)-\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right|\to 0$, which is negligible. Therefore, to simplify notation, we use the equal sign instead of the approximation sign. The use of the approximate perceived probabilities rather than the perceived probabilities has no impact on our results.

[10] The outlook function is assumed to satisfy $\left|\frac{\Upsilon''\left(1-\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}{\Upsilon'\left(1-\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}\right|\le\frac{1}{\mathrm{Var}\left[\varphi\left(x\right)\right]}$, which bounds the concavity and convexity of Υ to assure that the approximated marginal perceived probabilities are always positive and satisfy set monotonicity.

[11] Ambiguity aversion is exhibited when the investor prefers the expectation of an uncertain probability over a probability drawn from the set of uncertain probabilities. This is analogous to the idea that risk aversion is exhibited when an investor prefers the expectation of the uncertain outcomes over an outcome drawn from the set of uncertain outcomes.

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measured by $\mathrm{Var}[\varphi(\cdot)]$, and the investor's aversion to ambiguity, captured by $-\frac{\Upsilon''}{\Upsilon'} > 0$ and evaluated at the expected cumulative probability. A higher aversion to ambiguity or a greater extent of ambiguity result in lower (more underweighted relative to the expected probabilities) perceived probabilities of favorable events and higher perceived probabilities of unfavorable events.[12]

To simplify the analysis, we assume constant absolute ambiguity aversion (CAAA). That is,

$$\Upsilon(\mathrm{P}) = -\frac{e^{-\eta \mathrm{P}}}{\eta}, \tag{6}$$

where η is the coefficient of absolute ambiguity aversion. The assumption of CAAA, which is sufficient but not necessary for our results, is consistent with behavioral evidence (e.g., Baillon and Placido, 2019).

Based on the functional form in Equation (5), the degree of ambiguity can be measured by the expected probability-weighted average volatility of probabilities across the relevant events (Izhakian, 2020). Formally, the measure of ambiguity is given by

$$\mho^2[X] = \int \mathrm{E}[\varphi(x)]\,\mathrm{Var}[\varphi(x)]\,dx. \tag{7}$$

This statistic can be estimated using trading data. Risk independence represents another major advantage of $\mho^2$; it reflects the uncertainty associated with the probabilities of the events but in contrast to risk measures, does not depend upon the magnitudes of the outcomes associated with them, only upon the partition the outcomes induce over the state space.

2.2 The asset pricing framework

We employ a standard asset pricing framework, where the only variation in our structure is the specification of probabilities. There are two points in time, 0 and 1. The state at time 0 is known and the states at time 1 are ordered by their associated levels of aggregate consumption from lowest to highest. Like much of the asset pricing literature, we assume that investors, endowed with initial wealth w_0 and time 1 wealth w_1, trade in a perfectly competitive capital market. We assume that assets in the market allow the construction of a complete set of state contingent claims. To illustrate our ideas as simply as possible, we assume two distinct types of investors, equity investors and debt investors, and the existence of a representative investor of each type.[13] It is assumed that

[12]When the investor is ambiguity neutral, Υ is linear, the perceived probabilities become the (additive) expected probabilities (the linear reduction of compound lotteries), and Equation (5) collapses to the standard expected utility. The same reduction of the model occurs in the absence of ambiguity, when $\mathrm{Var}[\varphi(\cdot)] = 0$).

[13]The restriction to debt and equity investors ensures that the marginal traders in debt securities do not also hold equity and vice versa. It can be motivated most simply by an institutional restriction.

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the capital market is perfect, participants are price takers, and no arbitrage opportunities exist (i.e., the law of one price holds).

The asset pricing framework is used only to extract the form of the state prices. Therefore, we assume a single product in the economy, with an uncertain payoff x at time 1. A risk- and ambiguity-averse investor's portfolio problem can then be written in the usual way

$$\max_{c_0,\theta} \quad \mathrm{V}(c_0) + \mathrm{V}(c_1), \tag{8}$$

subject to the budget constraints

$$c_0 = w_0 - \theta \int q(x) x dx \qquad \text{and} \qquad c_1 = w_1 + \theta x,$$

where $q(x)$ is the time 0 price of a pure state-contingent claim on state $s \in \mathcal{S}$ associated with outcome x; and θ identifies the investor's holding of the outcome x. Using the functional form of expected utility in Equation (5), the state prices can be extracted as follows.

Theorem 1. *Suppose a time-separable utility function. The state price of state x is then*[14]

$$q(x) \quad = \quad \pi(x) \frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}, \tag{9}$$

where

$$\pi(x) \quad = \quad \begin{cases} \mathrm{E}[\varphi(x)](1 + \eta \mathrm{Var}[\varphi(x)]), & x \le k \\ \mathrm{E}[\varphi(x)](1 - \eta \mathrm{Var}[\varphi(x)]), & x > k \end{cases};$$

and $q(x)$ is unique and positive.

The state price $q(x)$ is the price of a claim to one unit of the consumption good contingent on the occurrence of state x.[15] Theorem 1 illustrates the distinct impacts that risk, ambiguity, and the attitudes toward these aspects of uncertainty have on market pricing. Ambiguity and aversion to ambiguity impact state prices through the consolidation of the uncertain probabilities to the perceived probability, $\pi(x)$, of each state x. Risk and aversion to risk impact the state price via the utility function U and the magnitude of outcomes x relative to consumption at time 0.[16,17]

[14] We identify a state of nature $s \in \mathcal{S}$ with its associated outcome x. We denote the partial derivative of the utility function with respect to x as ∂_xu and its partial derivative with respect to c_0 as ∂_0u.

[15] Chapman and Polkovnichenko (2009) extract the state prices in a rank-dependent expected utility framework.

[16] Note that the separation of the impacts of risk and ambiguity, as illustrated by Theorem 1, is delivered by the separability inherent in the underlying preference representation. Such a separation is not feasible within other preference representations (e.g., Gilboa and Schmeidler, 1989; Klibanoff et al., 2005).

[17] In the absence of ambiguity (i.e., if probabilities are perfectly known) state prices $q(x)$ reduce to the conventional

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For a given investor, the state price for a specific state will depend upon the partition of the state space that is induced by that investor's optimal portfolio. This is true since the uncertainty regarding the probabilities associated with the relevant events as perceived by the investor is dependent upon the partition of the state space relevant for that investor. This partition of the state space is determined by the state contingent payoffs of the investor's optimal portfolio. In this sense, while there continue to be no economies of scale in the capital market, there may be economies of scope. As the investor forms a portfolio with different payoffs in different states of nature, the uncertainty over probabilities perceived by that investor may change. For example, if the market portfolio of risky securities (or a mix of the market and the risk-free asset) is efficient for all investors, then all investors will perceive the same partition of the state space.

In the view of the debt and equity holders, there are two additional events with important state (or "event") prices: solvency and bankruptcy. Suppose F is the face value of a specific bond. The event of solvency is defined by $S = \{s \in \mathcal{S} \,|\, X(s) \geq F\}$ and of bankruptcy by $B = \{s \in \mathcal{S} \,|\, X(s) < F\}$. The state price for solvency, as viewed by a holder of this bond, can be defined by

$$q(S) \quad = \quad \pi\,(S)\,\frac{\partial_F \mathrm{U}}{\partial_0 \mathrm{U}}, \tag{10}$$

where

$$\pi\,(S) \quad = \quad \mathrm{E}\,[1 - \mathrm{P}\,(F)]\,(1 - \eta \mathrm{Var}\,[1 - \mathrm{P}\,(F)])\,. \tag{11}$$

Similarly, the state price for bankruptcy, as viewed by an equity holder, can be defined by[18]

$$q(B) \quad = \quad \pi\,(B)\,\frac{\partial_B \mathrm{U}}{\partial_0 \mathrm{U}}, \tag{12}$$

where

$$\pi\,(B) \quad = \quad \mathrm{E}\,[\mathrm{P}\,(F)]\,(1 + \eta \mathrm{Var}\,[\mathrm{P}\,(F)])\,. \tag{13}$$

representation $q(x) = \varphi\,(x)\,\frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}$. Similarly, in the presence of ambiguity with ambiguity-neutral investors, consistent with the conventional representation, state prices are $q(x) = \mathrm{E}\,[\varphi\,(x)]\,\frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}$.

[18] This presumes the equity holder receives no payment in default. To distinguish consumption in the case of (e.g.) bankruptcy at time $t = 1$ from consumption at time $t = 0$, we abuse notation and denote the former by $\partial_B \mathrm{U}$ and the latter by $\partial_0 \mathrm{U}$.

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With these definitions in place, the value of the debt (D) for a bond investor is defined by

$$\mathrm{V}(D) \quad = \quad \int_{x<F} xq(x)dx + Fq(S), \tag{14}$$

and the value of the equity (E) for an equity investor is defined by

$$\mathrm{V}(E) \quad = \quad 0 \times q(B) + \int_{F \leq x} (x - F)\, q(x)dx. \tag{15}$$

By the methodology in Izhakian (2020), it can be shown that the ambiguity of the debt is[19]

$$\mho^2 [D] \quad = \quad \int_{x<F} \mathrm{E}[\varphi(x)] \operatorname{Var}[\varphi(x)]\, dx + \mathrm{E}[1 - \mathrm{P}(F)] \operatorname{Var}[\mathrm{P}(F)] \tag{16}$$

and the ambiguity of the equity is

$$\mho^2 [E] \quad = \quad \mathrm{E}[\mathrm{P}(F)] \operatorname{Var}[\mathrm{P}(F)] + \int_{F \leq x} \mathrm{E}[\varphi(x)] \operatorname{Var}[\varphi(x)]\, dx. \tag{17}$$

The ambiguity associated with the debt and the ambiguity associated with the equity are both determined by the uncertainty of the probabilities of the relevant states of nature. A change in the variance of the probability of some outcome x, $\operatorname{Var}[\varphi(x)]$, may affect the variance of the probability of some outcome y, $\operatorname{Var}[\varphi(y)]$, for two reasons. First, a change in $\operatorname{Var}[\varphi(x)]$ is caused by a change in at least one distribution P in the set of prior $\mathcal{P}$. Since every $\mathrm{P} \in \mathcal{P}$ is additive, this change may affect also $\operatorname{Var}[\varphi(y)]$. Second, the probabilities $\varphi(x)$ and $\varphi(y)$ may be correlated.

In this respect, note that the variance of the probabilities of the unified bankruptcy (solvency) state is determined by the variance of probabilities of all events associated with a payoff $x < F$ $(x \geq F)$ and their covariances. To see this, suppose two payoffs x and y associated with the events $A = \{s \in \mathcal{S} \,|\, X(s) = x\}$ and $B = \{s \in \mathcal{S} \,|\, X(s) = y\}$, respectively. The variance of the probability of $A \cup B$ is then $\operatorname{Var}[\mathrm{P}(A \cup B)] = \operatorname{Var}[\mathrm{P}(A)] + \operatorname{Var}[\mathrm{P}(B)] + 2\operatorname{Cov}[\mathrm{P}(A), \mathrm{P}(B)]$. In general, this variation will not be the same as the variation in probabilities for the solvency states as perceived by the equity holder.

To simplify the proofs of the propositions, we assume that payoffs are symmetrically distributed. In a framework with uncertain probabilities, symmetry is defined as follows (see, Izhakian, 2020).

Definition 1. *Returns are symmetrically distributed around a point of symmetry μ if and only if, for any $x, y \in X$ that satisfy $|x - \mu| = |y - \mu|$,*

$$\mathrm{E}[\varphi(x)] = \mathrm{E}[\varphi(y)] \qquad \textit{and} \qquad \mathrm{E}\left[\left(\varphi(x) - \mathrm{E}[\varphi(x)]\right)^n\right] = \mathrm{E}\left[\left(\varphi(y) - \mathrm{E}[\varphi(y)]\right)^n\right],$$

[19]Note that $\operatorname{Var}[1 - \mathrm{P}(F)] = \operatorname{Var}[\mathrm{P}(F)]$.

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for $n = 2, 3, 4, \ldots$.[20]

Consistent with the existing literature, the face value of debt, F, is assumed to be lower than the expected payoff of the firm, $F < \mathbb{E}[x]$, where $\mathbb{E}[x]$ is the expectation taken using the expected probabilities; otherwise, default is expected. By the symmetry of x, $\mathbb{E}[x] = \mu$ where μ denotes the point of symmetry. This assumption implies that the marginal default probability increases in F.

This framework allows us to identify several properties regarding the relation between the pricing of a firm's debt and ambiguity. The proofs of the propositions appear in Section A.1 in the Appendix.

Proposition 1. *The higher is the firm's debt ambiguity, the lower is its bond price and the higher is its credit spread.*

Proposition 2. *The higher is the firm's equity ambiguity, the (weakly) lower is its bond price and the (weakly) higher is its credit spread.*

We model risk by a mean-preserving spread in outcomes (Rothschild and Stiglitz, 1970), evaluated based upon the expected probabilities, rather than perceived probabilities.

Proposition 3. *The higher is the firm's debt risk, the lower is its bond price and the higher is its credit spread.*

Proposition 4. *The higher is the firm's equity risk, the (weakly) lower is its bond price and the (weakly) higher is its credit spread.*

Finally, the presence of ambiguity amplifies the effect of risk on bond pricing, as the next proposition indicates.

Proposition 5. *The higher is the firm's debt ambiguity, the stronger is the negative (positive) effect of risk of the firm's debt on the bond's price (spread).*

2.3 Calibrating Yield Spreads

An alternative representation of bond prices uses risk and ambiguity neutral probabilities rather than state prices. The representation is commonly used in the literature providing calibrations of credit spreads for variations of Merton's (1974) model (e.g., Chen et al., 2009; Huang and Huang, 2012). The following corollary is an immediate consequence of Theorem 1.

[20] Since the measure $\mho^2$ considers only the first two moments of the distribution of probabilities, $\mathrm{E}[\varphi(x)] = \mathrm{E}[\varphi(y)]$ and $\mathrm{Var}[\varphi(x)] = \mathrm{Var}[\varphi(y)]$ are sufficient.

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Corollary 1. *The risk and ambiguity neutral probability of state x for a given investor is*

$$\pi^*(x) \;=\; \frac{q(x)}{\displaystyle\int q(x)dx} \;=\; \frac{\pi\,(x)\,\frac{\partial_x \mathrm{u}}{\partial_0 \mathrm{u}}}{\displaystyle\int \pi\,(x)\,\frac{\partial_x \mathrm{u}}{\partial_0 \mathrm{u}}dx}, \tag{18}$$

and the associated discount rate is

$$r_f \;=\; \frac{1}{\displaystyle\int q(x)dx} - 1 \;=\; \frac{1}{\displaystyle\int \pi\,(x)\,\frac{\partial_x \mathrm{u}}{\partial_0 \mathrm{u}}dx} - 1. \tag{19}$$

Ambiguity and aversion to ambiguity transform the standard representation of risk-neutral probabilities via the term $\pi\,(x)$. Because the perceived probabilities may differ across investors, each investor may perceive different risk and ambiguity neutral probabilities. Note that, while the risk and ambiguity neutral probabilities depend upon the subadditive perceived probabilities, the risk and ambiguity neutral probabilities perceived by each investor are additive across all states by construction.

Also note that, as in the traditional framework, the absence of arbitrage implies that the "discount rate" applied to the expectation of cash flows under the risk neutral measure must be equal to the risk-free rate. As discussed above, if investors have different optimal portfolios they may perceive different partitions of the state space and so view the state prices differently. Regardless of the partition of the state space induced by an investor's optimal portfolio, if the investor purchases an Arrow-Debreu security for each possible state of nature (Corollary 1), there is a single relevant event for this portfolio whose probability must, therefore, unambiguously equal 1. It will, therefore, be risk and ambiguity free and its return will be equal to the risk-free rate (the absence of arbitrage opportunities).

For the structural empirical application, following Chen et al. (2009), we assume that in case of default, bondholders receive a constant payment $F - L > 0$, where $L > 0$. The price of a risky and ambiguous discount bond, $D\,(x)$, can then be written

$$\begin{aligned} D\,(x) \;&=\; \int_{x<F} \frac{(F-L)}{R_f}\pi^*\,(x)\,dx + \int_{x\geq F} \frac{F}{R_f}\pi^*\,(x)\,dx \\ &=\; \frac{1}{R_f}\left(F - \Pi^*\,(B)\,L\right), \end{aligned} \tag{20}$$

where $R_f = 1 + r_f$, and $\Pi^*\,(B)$ is the cumulative probability of default under the risk neutral probabilities.

From Merton (1974) and Chen et al. (2009), defining the bond's yield to maturity as $Y = \frac{F}{D(x)}$,

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the credit spread on the bond can be written[21]

$$Y - R_f = \frac{L}{D\left(x\right)} \times \Pi^*\left(B\right). \tag{21}$$

By Corollary 1, the probability of default under the risk neutral measure as a function of the probability of default under the physical measure is

$$\Pi^*\left(B\right) = R_f \times \left(\mathrm{E}\left[\varphi\left(B\right)\right]\left(1 + \eta \mathrm{Var}\left[\varphi\left(B\right)\right]\right)\frac{\partial_B \mathrm{U}}{\partial_0 \mathrm{U}}\right), \tag{22}$$

where $\mathrm{E}\left[\varphi\left(B\right)\right]$ is the expected physical probability of default and $\mathrm{Var}\left[\varphi\left(B\right)\right]$ is the variance of the physical probability of default. Together, Equations (21) and (22), allow us to incorporate ambiguity and ambiguity aversion into the analysis of the credit spread puzzle.

3 Testable Hypotheses

Our main hypotheses are delivered by the propositions developed above. These hypotheses demonstrate that under ambiguity-averse preferences, both aspects of uncertainty (risk and ambiguity) have distinct and measurable impacts on the credit spreads of risky bonds. To test these hypotheses, we examine the impacts bond ambiguity and risk have on credit spreads.

Proposition 1 establishes that bond ambiguity positively affects credit spreads. Intuitively, ambiguity-averse investors overweigh the probabilities associated with unfavorable outcomes (default) and underweight the probabilities of favorable (solvency) outcomes. Thus, as bond ambiguity rises, ambiguity-averse investors will perceive a higher probability of default and require a higher credit spread for holding the bonds.

Hypothesis 1. *Bond ambiguity positively affects credit spreads.*

For completeness, Proposition 3 establishes that, as in the standard model, credit spreads increase in the debt's risk. The model we employ allows for the separation of risk and ambiguity. Risk is measured using the expected probabilities—the probabilities that would be used by a ambiguity neutral Bayesian investor in the presence of ambiguity. The hypotheses with respect to risk are, therefore, not surprising.

Hypothesis 2. *The risk of the debt positively affects its credit spread.*

Proposition 2 and Proposition 4 establish similar relations between credit spreads and equity ambiguity and risk. These propositions deliver the following hypotheses.

[21] Equation (21) is analogous to Equation (A.11) in Chen et al. (2009).

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Hypothesis 3. *The issuing firm's equity ambiguity weakly positively affects the credit spread on its debt.*

Hypothesis 4. *The issuing firm's equity risk weakly positively affects the credit spread on its debt.*

Proposition 5 establishes that the impacts on credit spreads of the two aspects of uncertainty, risk and ambiguity, reinforce one another. Intuitively, ambiguity, which serves to exaggerate default risk, will have a greater impact on bond prices and credit spreads when the risk of default is high rather than low. We expect that risk, ambiguity, and their interaction will have a positive relation to bond credit spreads.

Hypothesis 5. *The interaction between a bond's ambiguity and its risk positively affects its credit spread.*

4 Data and Empirical design

4.1 Bond characteristics

Our primary sample of bond returns and spreads come from the Trade Reporting and Compliance Engine (TRACE) dataset and the Mergent Fixed Income Security Database (FISD) for the period 2002 - 2019. The data are accessed via the Wharton Research Data Services (WRDS) bond returns data, which restructures the TRACE data from trade by trade based frequency to a monthly frequency and adds additional information from the FISD Database. The WRDS bond returns data drop any canceled, duplicate, subsequently corrected, or commission trades and filter price sequences for outliers. WRDS provides both the price and yield of each bond, as well as monthly returns, rating information and some additional issuance information. We complement this with data from the FISD on pay-downs and debt retirements to obtain corrected measures of the amount outstanding.

To account for variation in the risk-free rate of return, we transform yields from WRDS into credit spreads as follows. First, we identify the swap rate from Bloomberg by interpolating yearly swap yields to match the time to maturity of each bond. We then subtract this swap rate from the WRDS yield to find the credit spread.

We utilize the TRACE–CRSP crosswalk file to link bond data to the equity of the corresponding issuer and link the resulting data with CRSP equity returns and Compustat annual firm fundamentals to collect firm specific information. Thereby, we can calculate asset level characteristics including leverage, Z-score, and distance to default for each bond.

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4.2 Estimating ambiguity

The use of the CPT model augmented with the EUUP framework delivers a risk-independent measure of ambiguity (Izhakian, 2020). Within this framework, the degree of ambiguity is measured by the volatility of the uncertain probabilities, just as the degree of risk is measured by the volatility of the uncertain outcomes. Formally, the measure of ambiguity, denoted $\mho^2$ and modeled in Equation (7), represents an expected probability-weighted average of the variances of the probabilities.

We follow the literature to estimate ambiguity and risk of equity from intraday Trade and Quote (TAQ) data (e.g., Izhakian and Yermack, 2017; Brenner and Izhakian, 2018; Augustin and Izhakian, 2020; Izhakian et al., 2021).[22] This will allow us to contrast the impact of debt and equity ambiguity of a given firm on the pricing of its bonds. For completeness, Section A.2 in the Appendix details the empirical methodology implementing the ambiguity measure modeled in Equation (7).

The empirical implementation of the ambiguity measure as discussed above for debt faces two challenges. The first is measuring the asset returns of individual firms. The second is identifying the default point based upon asset returns in order to identify the default and solvency events, and estimate debt and equity ambiguity. Data and calculation obstacles prevent us from deriving a satisfactory measure of bond ambiguity in this way. Therefore, rather than measuring the uncertainty regarding the probabilities of very rare default events directly from the ambiguity associated with asset returns, we presume that returns to holding corporate bonds are primarily influenced by perceptions of the uncertain probability of default. Therefore, we estimate this uncertainty from the observed distributions of returns on the bonds themselves.

To estimate the monthly degree of ambiguity for each bond, we apply the same methodology to intraday bond returns from the TRACE enhanced dataset that we apply to equity returns. To estimate ambiguity, we require that bonds trade multiple times in a given day and multiple days within a month to develop a set of distributions from which to estimate the variance of probabilities. Given the relatively illiquid trading of many bonds we have had to adapt the process used for equity returns. To measure the monthly ambiguity for any bond, we require the bond to have at least five trading days within the month with at least five trades in different five-minute intervals for each of these days. From these data, we calculate the variation across days in the month in the frequency with which given return realizations occur. Since even this approach leaves gaps (missing months)

[22] The measure of ambiguity, defined in Equation (7), is distinct from aversion to ambiguity. The former, which is a matter of beliefs (or information), is estimated from the data, while the latter is a matter of subjective attitudes.

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in the monthly ambiguity measure, we take the rolling six month average of this measure as our main measure of bond ambiguity. Robustness analysis assures that our empirical findings are not driven by specific choices within this adapted approach.

4.3 Estimating risk

Following earlier studies (e.g., Frank and Goyal, 2009), we use equity return volatility as one proxy for default risk. Equity return volatility has been used in the literature on pricing default risk for debt (e.g., Bharath and Shumway, 2008) and as an input for the "distance to default" measure derived from the Merton (1974) model. For consistency, we compute the variance of equity returns with the same five-minute returns used to estimate equity ambiguity. For each stock on each day, we compute the variance of intraday returns, applying the Scholes and Williams (1977) correction for non-synchronous trading and a correction for heteroscedasticity.[23] We estimate bond volatility using the two year rolling window from month $t-1$ to month $t-25$ from WRDS last trade in monthly returns data. These data compute returns between the last trade in the previously traded month to the last trade in the reference month and account for any interest accrual or distributions. Since some bonds do not trade every month, we exclude bonds with gaps in robustness tests.

4.4 Summary statistics

Table 1 presents summary statistics for the key variables used in the paper. Our sample includes monthly bond observations from the WRDS bond returns dataset for which we can measure bond ambiguity. Regressions with additional controls presented below reduce the sample, especially where we require information concerning the issuing firm's equity. The average (median) time to maturity for the bonds is 9 (6) years, with an average (median) yield of 5% (5%). The average (median) return of the bonds is 1% (0%) and the average (median) volatility is 3% (2%). The average (median) equity volatility is 9% (7%). The distance to default measure is skewed with a mean of 3.19 and a median of 2.39.

[Table 1]

To provide some perspective on our measure of bond ambiguity, Figure 1 plots the time series of average bond ambiguity for investment grade and junk bonds. Bond ambiguity is clearly higher for junk bonds than it is for investment grade bonds. Bond ambiguity also appears to have an important time series component that is strongly related to the aggregate economy. Figure 1 shows

[23] See, for example, French, Schwert, and Stambaugh (1987).

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that bond ambiguity rises in anticipation of, and through the recession, spiking just prior to the recovery.

[Figure 1]

Figure 2 provides an illustration of the "term structure" of bond ambiguity for investment grade and junk bonds. It plots the average bond ambiguity for bonds sorted based on the number of years to maturity. Figure 2 confirms a finding from Figure 1: for each level of years to maturity, the ambiguity of junk bonds is higher than that of investment grade bonds. Figure 2 also illustrates that the ambiguity of investment grade bonds seems to gradually rise as the number of years to maturity increases until about maturities of 18 - 20 years, at which bond ambiguity shows a distinct decline. The ambiguity of junk bonds has a distinct "V" shape for maturities between 1 and 20 years, followed by a sharp decline for longer term bonds. For the longer term bonds, the drop in ambiguity may be related to the liquidity of the bonds or to the probability of default for longer term bonds. The sharp drop in the ambiguity of junk bonds with 8-10 years to maturity is a puzzling feature of the data.

[Figure 2]

To examine the relation between bond ambiguity and different bond and issuing firm characteristics we first sort observations by bond ambiguity, grouping them into quintiles. Table 2 reports averages for bond and issuing firm level characteristics within these quintiles.

[Table 2]

Table 2 shows that years to maturity have a negative correlation with bond ambiguity, with bond ambiguity showing a sharp decrease between the second highest and highest quintile of bond ambiguity, mirroring Figure 2. The ambiguity of the issuing firm's equity, the offering amount, and the liquidity of the bond (inverse of the bid-ask spread) are also negatively related to bond ambiguity. Bond ambiguity shows a positive correlation with measures of return on the bonds (yield to maturity, yield spread, and coupon level), measures of the riskiness of the bonds and the issuing firm (bond rating, equity beta, bond volatility, equity volatility, asset volatility, Z-score, and distance to distance to default).

Table 2 reveals that average debt ambiguity is larger than the average equity ambiguity in total and across the quintiles. This is, in part, derived from two different characteristics of debt as compared to equity. The first is that theoretically, risk and ambiguity should have a small

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negative correlation. As the possible distributions of returns on the bonds become more diffuse the variation in the possible probabilities for any outcome declines. Secondly, as seen in Table 2, liquidity is negatively related to ambiguity in the cross section of bonds. Equity is much riskier and much more liquid than debt which contributes to this difference in magnitudes of estimated ambiguity. Finally, the very different frequencies of price transparencies across these different markets is also likely to contribute to this difference.[24]

A patterns similar to that in Table 2 is emerged when separating bonds into investment grade and junk bonds (not tabulated). A general difference is that the patterns for the relationship between bond ambiguity and measures of return and risk are muted in the investment grade sample relative to the whole sample and exaggerated in the junk bond sample. The most distinct difference from Table 2 is the relation between bond ambiguity and the number of years to maturity. For investment grade bonds, there is a "U" shape relation, in which bonds in the ambiguity based quintiles 1, 4, and 5 have the greatest average years to maturity. For junk bonds, there is a "hump" shaped relation, in which bonds in quintiles 2 and 3 have the greatest average years to maturity.

5 Empirical findings

5.1 Ambiguity and credit spreads

To evaluate the hypotheses presented in Section 3 we estimate the general regression specification

$$\begin{aligned} Spread_{i,t} &= \alpha + \beta_1 \times BondAmbiguity_{i,t-1} + \beta_2 \times EquityAmbiguity_{i,t-1} + \\ &\quad \gamma \times \mathbf{X}_{i,t-1} + y_{r,t-1} + z_{m,t-1} + e_{i,t}, \end{aligned} \tag{23}$$

where i denotes a bond, t a month, $\mathbf{X}$ is a set of control variables, $y_{r,t-1}$ is rating-time fixed effects, and $z_{m,t-1}$ is maturity-time fixed effects. To address concerns of reverse causality, we lag ambiguity, risk, and other bond and firm characteristics by one month. For robustness, we conduct the same analysis with all right hand side variables including fixed effects as contemporaneous variables and find that the key findings are unchanged. The findings of the contemporaneous regression are reported in Appendix A.

The set of controls includes: issuing firm's equity beta, bond's return volatility, return volatility of the issuing firm's equity, issuing firm's Z-score, issuing firm's distance to default measure, and the bond's bid-ask spread as a measure of liquidity. To aid in interpretability, especially for the interaction regressions, we standardize each independent variable by dividing by its own standard

[24] We thank Darrell Duffie for this comment.

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deviation and subtracting its mean. Thereby, the coefficients can be read as the basis point change in the credit spread associated with a one standard deviation change in the independent variable. Subtracting the mean of each variable eases the interpretation of coefficients on the interactions between explanatory variables.

Our first analysis focuses on the univariate comparison between the effect of bond ambiguity on credit spreads with the effect of other well-known determinants of credit spreads. Table 3 presents the findings of this analysis. Column (1) shows that a one standard deviation increase in Bond Ambiguity results in a 196.9 bps increase in credit spreads. This effect is economically large (the mean credit spread is 295 bps) and highly statistically significant. Economically, the effect of Bond Ambiguity on spreads is nearly identical to that of Equity Volatility. Moreover, the R^2 in the Bond Ambiguity univariate regression is 0.112, indicating that its statistical explanatory power is close to that of Equity Volatility (R^2 of 0.147) and larger than that of Equity Beta, Distance to Default, and Z-Score. Only the volatility of bond returns (Bond Volatility) has a more significant impact on spreads (R^2's of 0.208) than does Bond Ambiguity. Finally, consistent with Chen, Lesmond, and Wei (2007) less liquid bonds have higher credit spreads.

[Table 3]

The findings in Table 3 imply that bond level ambiguity, despite not appearing in the canonical Merton (1974) formula for the price of debt, has a large impact on bond prices. Just like volatility, Bond Ambiguity is correlated with the Rating of the bond and with the bond's Maturity, and also has a significant time series component. Therefore, our next analysis addresses the question whether ambiguity impacts spreads after controlling for the well known time series and cross-section of Rating and Maturity. Table 4 presents the findings of tests of Hypotheses 1 to 4. It represents an initial examination of the effect of uncertainty (risk and ambiguity) measures on yield spread. Column (1) presents the findings of yields spreads regressed on only Bond Ambiguity and a Time (year-month) fixed effect, as the explanatory variables. The R^2 of this regression test indicates that these minimal explanatory variables explain about 15.2% of the variation in yield spreads.

[Table 4]

Ambiguity appears to be negatively related to Ratings (safer rated bonds tend to have lower ambiguity) and to Years to Maturity. To the extent that bond ratings proxy for default risk, absorbing rating by month fixed effects allows us to peer beyond the impact of spreads associated with higher or lower ratings and focus on the impact of ambiguity, holding default risk relatively

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constant. Column (2) of Table 4 introduces Time × Rating and Time × Years to Maturity fixed effects into the regression test.[25] Not surprisingly, introducing these fixed effects results in a large reduction in the estimated coefficient on Bond Ambiguity. Nonetheless, Bond Ambiguity remains highly significant and the economic impact of a one standard deviation shift in this variable implies an increase in spreads of approximately 52.46bps.

Column (3) of Table 4 adds a bond level (CUSIP) fixed effect to the specification, controlling for the contribution of any bond specific characteristics that may be jointly correlated with ambiguity and spreads. For example, low liquidity bonds may have higher ambiguity and higher spreads (Chen et al., 2007). With the addition of this fixed effect, time series, or within bond variation in spreads and ambiguity are all that remain to drive our findings in Column (3). Nevertheless, we observe a positive and highly significant effect of Bond Ambiguity on credit spreads.

Column (4) of Table 4 presents the findings of a "kitchen sink" approach to controlling for default probabilities. The findings indicate that doing so does not eliminate the effect of bond ambiguity. This demonstrates that the impact of ambiguity is not directly related to the physical probability of default but instead amplifies the uncertainty of probabilities of the default states. Many of the controls for default risk in Column (4) are highly correlated with one another (e.g., distance to default is a direct function of both risk measures such as volatility and accounting variables similar to those included in Z-Score), therefore, we hesitate to directly interpret the coefficients presented. The specification in column (4) also includes the bond specific average bid-ask spread as a direct measure of bond liquidity. Intuitively, less liquid bonds tend to have higher spreads. Overall, including variables designed to reflect direct measures of default risk and liquidity does not eliminate the significant effect on ambiguity. These finding provide strong support for Hypothesis 1.

Column (5) of Table 4 substitutes Equity Ambiguity for Bond Ambiguity in the model presented in Column (4). It demonstrate that Equity Ambiguity has an insignificant effect on yield spreads. Hypothesis 3 indicates that equity ambiguity is expected to weakly positively affect credit spreads. Consistent with this hypothesis, bond investors appear to be pricing the ambiguity associated with the credit portion of the capital structure when setting the credit spread. Equity Ambiguity does not capture the relevant uncertainty facing bond investors and, thus, does not significantly affect spreads. Column (6) of Table 4 indicates that including both Bond Ambiguity and Equity Ambiguity as explanatory variables does not change these conclusions.

[25]Introducing Time × Rating fixed effects eliminates any significant relation between bond ambiguity and whether or not a bond ends in default.

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Finally, Column (4) and Column (5) of Table 4 show that Bond Volatility and Equity Volatility positively affect yield spread, supporting Hypothesis 2 and Hypothesis 4.

5.2 Ambiguity and the amplified effect of default risk

Tables 5 and 6 report the findings from regression tests of Hypothesis 5 that ambiguity amplifies the effect of default risk. Table 5 considers volatility measures, while Table 6 consider default probability measures developed in the literature. To cleanly assess the interaction of Bond Ambiguity with Equity Volatility and Bond Volatility, the regression test presented in Table 5 include the set of controls from Column (4) of Table 4, however, to mitigate the impact of multicollinearity, we leave out measures of volatility from this set of controls.

Column (1) of Table 5 indicates that Bond Ambiguity and the Equity Volatility of the issuing firm are both positively and highly significantly affect credit spreads. Testing Hypothesis 5, the estimated coefficient on the interaction term between these two uncertainty measures is also positive and highly significant. Moreover, the effect of Bond Ambiguity on yield spreads increases by about 160% (an increase from 15.38bps to 15.38bps+24.01bs = 39.39bps) when considering bonds that have a one standard deviation increase in Equity Volatility. Or, alternatively, the impact of Equity Volatility on yield spreads also increases by about 150% (an increase from 19.87bps to 19.87bps+24.01bs = 45.88bps) when looking at bonds with a one standard deviation increase in bond ambiguity.

[Table 5]

Similar findings are reported in Column (2) of Table 5 concerning Bond Volatility. Bond Ambiguity and Bond Volatility are both highly significant explanatory variables for credit spreads, and the estimated coefficient on their interaction is also positive and highly significant. Ambiguity amplifies the effect of Bond Volatility on credit spreads: the raw coefficient on Bond Volatility is 26.19bps for a deviation increase while it increases in about 130% for a bond with Bond Ambiguity that is one standard deviation above the mean.

Column (3) of Table 5 includes both Bond Volatility and Equity Volatility (alone and interacted with Bond Ambiguity) as explanatory variables. Bond Volatility and its interaction with Bond Ambiguity are more significant determinants of credit spreads than Equity Volatility and its interaction with Bond Ambiguity. The findings in Table 5 support the intuition that Bond Ambiguity is an important explanatory variable for credit spreads *because* ambiguity-averse investors exaggerate the default risk of bonds when ambiguity is high. Ambiguity-averse investors overweighing the

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perceived probabilities of unfavorable events implies that ambiguity matter most for bonds with a high probability of default.

Table 6 considers an indicator for investment grade (IG) bonds, Z-score, and distance to default as more direct (inverse) measures of the probability of default. As in Table 5, to focus on the effect of a single default variable, we limit the set of control variables to only include those that do not deal with volatility. In each column, Bond Ambiguity significantly positively affects credit spreads. In columns (1) and (3) the measure of default probability (the investment grade dummy and Z-Score, respectively) have the predicted sign and are statistically significant. The estimated coefficient on the interaction term between the measure of the default probability and Bond Ambiguity is also statistically significant and of the expected sign in both columns. In column (2), Distance to Default as a measure of default probability is insignificantly related to yield spreads. The significance of the interaction between Distance to Default and Bond Ambiguity suggests that Distance to Default significantly affect yield spreads only when Bond Ambiguity is high. Overall, findings presented in Tables 5 and 6 strongly support Hypothesis 5.

Column (4) of Table 6 examines a hypothesis that is outside of our model.[26] If liquidity is (in part) driven by issues of asymmetric information, then the interaction between liquidity and ambiguity should also be important for explaining credit spreads. Intuitively, when ambiguity is high, informed and uninformed parties will trade less aggressively on their information, lowering liquidity. Consistent with this hypothesis, the estimated coefficient on the interaction between Liquidity and Bond Ambiguity is positive and highly significant.

[Table 6]

5.3 Systematic risk and distance to default

Table 7 considers a robustness test related to Hypothesis 5. While volatility is closely associated to the probability of default, systematic volatility is closely associated with credit spreads. Firms that default during economic downturns should see a larger required return on their debt securities because they default in those states for which returns are most valuable. If this is important for credit spreads, we should see a stronger effect of Systematic Volatility and its interaction with Bond Ambiguity on yield spread than the effect of total volatility and its interaction with Bond Ambiguity. Using the Capital Asset Pricing Model (CAPM) betas of the bonds and of the issuing

[26] We thank Darrell Duffie for suggesting this regression.

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firm's equity, Table A.5 explores whether this relation is found in the data. While Bond Ambiguity remains significantly positive with the inclusion of the systematic risk measures (the direct effects in these regressions are captured by the CUSIP fixed effect), the interaction terms between Bond Ambiguity and the measures of Beta, while positive and highly significant, do not indicate that systematic risk more strongly affect yield spreads than total risk (see also Berndt, 2015; Lewis, 2019).

[Table 7]

5.4 Bond index ambiguity

Table 8 provides an empirical examination of the effect of ambiguity of bond portfolios (or indices) on individual bonds' credit spreads. For a large, diversified portfolio of bonds, we do not expect that portfolio ambiguity will be related to individual bonds' probability of default. However, from the perspective of owners of bond portfolio, an increase in the risk-free rate is an unfavorable event. Therefore, the ambiguity of a bond portfolio may reflect the uncertainty regarding the probability of a future interest rate increase. If this is true, then bond index ambiguity should negatively affect yield spreads on bonds (Longstaff and Schwartz, 1995). Table 8 examines this conjecture for several bond indices: investment grade and junk bond portfolios as well as short-, medium-, and long-term treasury securities. Columns (1) and (2) consider the effect of bond index ambiguity on yield spreads, while Columns (3) and (4) also include Bond Ambiguity measure as an explanatory variable.

[Table 8]

The findings in Table 8 are consistent with the intuition that greater bond index ambiguity is associated with an increase in the perceived probability of a future increase in the risk-free rate. They also reinforce the idea that individual bond ambiguity reflects the uncertainty of the probability of default. For the investment grade index, with or without the inclusion of bond ambiguity in the regression test, index ambiguity is found to significantly negatively affect bond credit spreads, while bond ambiguity continues to significantly positively affect spreads.

For the junk bond index, while the estimated coefficients on index ambiguity are negative they are insignificant. Robustness tests show that if contemporaneous junk bond index ambiguity is used in the regression the coefficient is significantly negative. However, it could also be the case

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that changes in the risk free rate have less of an impact on junk bonds than they do on investment grade bonds.

Table 8 also explores the effect of the ambiguity of treasury bond indexes, for which the probability of default is effectively zero. Index ambiguity for the Treasury indexes are negatively related to credit spreads, however, the estimated coefficient for the Long-Term Treasury index ambiguity is insignificant. This may be due to the fact that the Long-Term Treasury index is less closely associated with the relevant portion of the yield curve than are the Short- and Medium-Term indices for our sample of corporate bonds. It seems implausible that the ambiguity associated with treasury indexes reflect uncertainty regarding the probability of default and so these findings further support the notion that bond index ambiguity reflects uncertainty regarding the probability of future rate changes, while bond ambiguity reflects uncertainty regarding default probability.

5.5 Credit spread calibration

Finally, we utilize our theoretical framework to provide a potential resolution of the credit spread puzzle (Huang and Huang, 2012). Since our model is a two-date model, we cannot directly adapt the dynamic settings from recent work on credit spreads (e.g. Chen, 2010; Chen et al., 2009; McQuade, 2013). Instead, we follow a two-step calibration procedure. First, we calibrate the bond spread formula in Equations (21) and (22) in a world without ambiguity aversion (setting η to 0). Specifically, we select the level of risk aversion and the marginal rates of substitution in high and low consumption states that generate a risk-free rate that match existing estimate; as in Chen et al. (2009) and Huang and Huang (2012) without time-varying risk aversion. In the second step, we add ambiguity estimated from the data while keeping risk aversion, the risk-free rate, and the marginal rates of substitution fixed from the first step in order to reduce the degrees of freedom in this second step.

In the first calibration step, we set η to zero in Equations (21) and (22) (to eliminate any impact of ambiguity aversion) such that spreads solely reflect risk and risk preferences identified by the coefficient of absolute risk aversion, γ. Such a model is known to fail to match spreads. Indeed this type of model is unable to simultaneously match the moments of consumption growth, volatility, and the risk-free rate. Nevertheless, we are able to provide a model with a similar estimate of the risk-free rate, which delivers spreads very similar to those documented in Huang and Huang (2012). We consider Equation (21) with ambiguity neutral ($\eta = 0$) and CARA utility function with risk-aversion coefficient of 4, which is the upper bound of levels considered reasonable from

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experimental analysis)[27] Using observed probabilities of default, and consumption growth averaging 1.5% with -0.5% annualized consumption growth in the "down" state (Lewis, 2016, e.g.,), we then generate the spreads provided in the model labeled "Risk" in Table 9. In both, 4 year (Panel A) and 10 year (Panel B) bonds, the model based on risk and risk aversion alone delivers spreads that are too low across the board for investment grade bonds. Furthermore, the AAA-BBB rated bond spread is also low relative to the data. The resulting spreads are very close to the calibrated spreads reported in existing work (Huang and Huang, 2012; Chen et al., 2009).

[Table 9]

Next, we incorporate ambiguity and aversion to ambiguity into these spread estimates. We do so by taking the bond ambiguity measures from the data averaged across time and each rating category.[28] It is important to note that, across bonds, the relative estimated variance of probabilities is informative but the absolute scale of these estimates is not. The impact of the variance of probabilities can only be understood in conjunction with an estimate of absolute ambiguity aversion, η. Given that there is no guidance from experimental analysis regarding our underlying preference model, we pick an η of 0.0048 to match exactly the target AAA spread on 10 year bonds. To eliminate degrees of freedom in this version of the calibration, we take as fixed the estimates of the marginal rates of substitution and the risk-free rate from the model based on risk alone.

The "Risk and Ambiguity" column in Panels A and B of Table 9 presents the model implied spreads for 4 and 10 year corporate bonds incorporating ambiguity and ambiguity aversion. In both cases, incorporating ambiguity and ambiguity aversion alleviates the issue that credit spreads are way too low for higher rated bonds. For example, that observed spreads are 30 times higher for AAA rated 4 year bonds and four times higher for AAA rated 10 year bonds relative to the spreads generated by a model based on risk aversion alone. While AAA bonds are significantly less risky than other corporate bonds, they have higher levels of ambiguity, $\mathrm{Var}\left[\varphi\left(B_i\right)\right]$, and so higher variation in the probability of default than other corporate bonds, which leads ambiguity averse investors to demand more compensation for the perceived uncertainty.

Most resolutions of the credit spread puzzle face a tradeoff: to fit high spreads on safe debt, risk aversion must be high. High levels of risk aversion, unfortunately, means that the spreads on lower

[27]See for example, Chetty (2006), and Booij and van Praag (2009).

[28]The ambiguity we measure in the data is the expected probability of default weighted variance across possible probability distributions. To recover $\mathrm{Var}\left[\varphi\left(B_i\right)\right]$, we divide ambiguity by the estimated probably of default for each rating category. We then scale the resulting variance of probabilities of default for each rating class by η the measure of ambiguity aversion.

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rated (particularly Junk) bonds explode. But, because ambiguity is lower for riskier bonds, the increased spreads derived from aversion to variation in default probabilities (ambiguity aversion) does not explode the model predicted spread for speculative grade debt. Ambiguity-averse investors end up demanding a larger premium relative to a risk-only preferences on safe debt, but price risky debt in a similar manner to risk averse investors.

As shown in Table 9, incorporating ambiguity aversion does not fully solve the credit spread puzzle, especially at nearer maturities. It does however significantly decrease the sum of absolute deviations between calibrated and observed spreads. We find that the absolute percent deviations from the data for 4 year (10 year) bonds, summed across all rating classes, decreases from 395% (163%) for the model based solely on risk aversion to 220% (66%) for the model that incorporates both risk and ambiguity aversion. Overall, our empirical results suggest that ambiguity and aversion to ambiguity may have an important impact on the pricing of risky bonds. Bond ambiguity serves as a significant explanatory variable for the level of and the variation in credit spreads in our panel of corporate bonds.

6 Conclusion

Since the seminal work of Ellsberg (1961), it has been known that individuals are averse to making decisions when probabilities are uncertain; i.e., decision making under ambiguity. It is also clear that ambiguity characterizes every significant economic or financial decision. Nevertheless, the majority of theoretical and empirical models in finance concentrate on risk (an uncertain outcome with known probabilities). Models of ambiguity averse preferences (e.g., Tversky and Kahneman, 1992; Izhakian, 2017) suggest that, in the presence of ambiguity (an uncertain outcome with uncertain probabilities), ambiguity-averse individuals overweigh the probabilities of unfavorable outcomes and underweight the probabilities of favorable outcomes. This simple intuition suggests that if ambiguity has an important impact on the pricing of financial assets, this impact seems likely to be reflected in an examination of credit spreads on risky bonds.

We introduce a simple model of the pricing of risky bonds in a market with ambiguity-averse investors. The model highlights the idea that, the distribution of the firm's cash flow across the standard financial instruments implies that the ambiguity of the firm's debt and the ambiguity of its equity may be very different. The ambiguity related to a firm's bonds (bond ambiguity) is primarily focused on the uncertainty associated the probabilities of the default states, while the ambiguity of its equity concentrates on the uncertainty associated with the probabilities of

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its various non-default states. The model establishes that bond ambiguity and risk have distinct and complementary impacts on the credit spreads of risky bonds. Furthermore, because ambiguity causes bond holders to overweigh the likelihood of unfavorable outcomes, ambiguity will have an impact on bond pricing and credit spreads that is most pronounced when default risk is high.

Our empirical findings demonstrate that risk and ambiguity provide roughly equivalent explanatory power for bond credit spreads. Risk and ambiguity are shown to positively and significantly affect credit spreads, after controlling for bond ratings, maturity, volatility, and fixed effects that capture other factors known to impact credit spreads. Consistent with the theoretical model, we find that the interaction between risk and ambiguity also has a positive effect on bond credit spreads. Because ambiguity causes ambiguity-averse investors to overweigh the probabilities of unfavorable outcomes, ambiguity has the strongest impact on yield spreads when default risk is high.

We argue that the ambiguity associated with a bond index will not reflect the uncertainty associated with the probability of default of individual bonds. Rather, we conjecture that bond index ambiguity primarily reflects the uncertainty associated with the probabilities of future interest rate changes. For a holder of corporate bonds, future interest rate increases are unfavorable events. Therefore, consistent with Longstaff and Schwartz (1995), we expect bond index ambiguity to be negatively related to credit spreads. Empirically, we find that the ambiguity associated with investment grade and junk bond indices are both significantly negatively associated with yield spreads, even when controlling for individual bond level ambiguity. The same result is found for the ambiguity associated with a short- and medium-term treasury bond index.

Our theoretical model and empirical findings shed light on a new important determinant of credit spreads. They provide insight into the implications of ambiguity—uncertainty of probabilities—for credit spreads for risky bonds. The insights this paper provides have important theoretical and practical implications. It paves the way for further research into the way ambiguity and perceive probabilities affect pricing. Our findings also motivate further analysis of theoretical dynamic models of credit spreads under ambiguity.

Our analysis focuses on bond contracts. The results should provide, however, insights that are more broadly applicable to the pricing of other financial contracts and insurance claims. We leave a detailed theoretical and empirical analysis of such applications for future research.

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References

Abdellaoui, M., Attema, A., Bleichrodt, H., 2010. Intertemporal tradeoffs for gains and losses: An experimental measurement of discounted utility. Economic Journal 120, 845–866.

Andersen, T. G., Bollerslev, T., Diebold, F. X., Ebens, H., 2001. The distribution of realized stock return volatility. Journal of Financial Economics 61, 43–76.

Anderson, E. W., Ghysels, E., Juergens, J. L., 2009. The impact of risk and uncertainty on expected returns. Journal of Financial Economics 94, 233–263.

Augustin, P., Izhakian, Y., 2020. Ambiguity, volatility, and credit risk. The Review of Financial Studies 33, 1618–1672.

Baillon, A., Huang, Z., Selim, A., Wakker, P. P., 2018. Measuring ambiguity attitudes for all (natural) events. Econometrica 86, 1839–1858.

Baillon, A., Placido, L., 2019. Testing constant absolute and relative ambiguity aversion. Journal of Economic Theory 181, 309–332.

Bandi, F. M., Russell, J. R., 2006. Separating microstructure noise from volatility. Journal of Financial Economics 79, 655–692.

Bayes, T., Price, R., Canton, J., 1763. An essay towards solving a problem in the doctrine of chances. C. Davis, Printer to the Royal Society of London London, U. K.

Ben-Rephael, A., Izhakian, Y., 2020. Should I stay or should I go? trading behavior under ambiguity. SSRN eLibrary 3628757.

Berndt, A., 2015. A Credit Spread Puzzle for Reduced-Form Models. The Review of Asset Pricing Studies 5, 48–91.

Bernoulli, J., 1713. Ars Conjectandi (The Art of Conjecturing).

Bharath, S. T., Shumway, T., 2008. Forecasting default with the merton distance to default model. The Review of Financial Studies 21, 1339–1369.

Booij, A. S., van Praag, B. M., 2009. A simultaneous approach to the estimation of risk aversion and the subjective time discount rate. Journal of Economic Behavior and Organization 70, 374–388.

Brenner, M., Izhakian, Y., 2018. Asset prices and ambiguity: Empirical evidence. Journal of Financial Economics 130, 503–531.

Chapman, D. A., Polkovnichenko, V., 2009. First-order risk aversion, heterogeneity, and asset market outcomes. The Journal of Finance 64, 1863–1887.

Chen, H., 2010. Macroeconomic conditions and the puzzles of credit spreads and capital structure. The Journal of Finance 65, 2171–2212.

Electronic copy available at: https://ssrn.com/abstract=4066052

Chen, L., Collin-Dufresne, P., Goldstein, R. S., 2009. On the relation between the credit spread puzzle and the equity premium puzzle. The Review of Financial Studies 22, 3367–3409.

Chen, L., Lesmond, D. A., Wei, J., 2007. Corporate yield spreads and bond liquidity. The Journal of Finance 62, 119–149.

Chetty, R., 2006. A new method of estimating risk aversion. The American Economic Review 96, 1821–1834.

Chew, S. H., Sagi, J. S., 2008. Small worlds: modeling attitudes toward sources of uncertainty. Journal of Economic Theory 139, 1–24.

Crockett, S., Izhakian, Y., Jamison, J., 2019. Ellsberg's hidden paradox. SSRN eLibrary 3423534.

Duffie, D., Singleton, K. J., 2012. Credit risk: pricing, measurement, and management. Princeton university press.

Ellsberg, D., 1961. Risk, ambiguity, and the savage axioms. Quarterly Journal of Economics 75, 643–669.

Ericsson, J., Reneby, J., Wang, H., 2005. Can structural models price default risk? new evidence from bond and credit derivative markets. Quarterly Journal of Finance 05.

Faria, G., Correia-da Silva, J., 2014. A closed-form solution for options with ambiguity about stochastic volatility. Review of Derivatives Research 17, 125–159.

Feldhütter, P., Schaefer, S. M., 2018. The myth of the credit spread puzzle. The Review of Financial Studies 31, 2897–2942.

Frank, M. Z., Goyal, V. K., 2009. Capital structure decisions: which factors are reliably important? Financial management 38, 1–37.

Franzoni, L. A., 2017. Liability law under scientific uncertainty. American Law and Economics Review 19, 327–360.

French, K. R., Schwert, G. W., Stambaugh, R. F., 1987. Expected stock returns and volatility. Journal of Financial Economics 19, 3–29.

Gilboa, I., Schmeidler, D., 1989. Maxmin expected utility with non-unique prior. Journal of Mathematical Economics 18, 141–153.

He, Z., Kelly, B., Manela, A., 2017. Intermediary asset pricing: New evidence from many asset classes. Journal of Financial Economics 126, 1–35.

Herron, R., Izhakian, Y., 2019. Mergers and ambiguity: The role of ambiguity. SSRN eLibrary 3100611.

Huang, J.-Z., Huang, M., 2012. How Much of the Corporate-Treasury Yield Spread Is Due to Credit Risk? The Review of Asset Pricing Studies 2, 153–202.

Izhakian, Y., 2017. Expected utility with uncertain probabilities theory. Journal of Mathematical Economics 69, 91–103.

Electronic copy available at: https://ssrn.com/abstract=4066052

Izhakian, Y., 2020. A theoretical foundation of ambiguity measurement. Journal of Economic Theory Forthcoming.

Izhakian, Y., Yermack, D., 2017. Risk, ambiguity, and the exercise of employee stock options. Journal of Financial Economics 124, 65–85.

Izhakian, Y., Yermack, D., Zender, J., 2021. Ambiguity and the trade off theory of capital structure. Management Science Forthcoming.

Jaynes, E. T., 1957. Information theory and statistical mechanics. Physical review 106, 620.

Kendall, M., Stuart, A., 2010. The Advanced Theory of Statistics. vol. 1: Distribution Theory. London: Griffin, 2010, 6th ed. 1.

Klibanoff, P., Marinacci, M., Mukerji, S., 2005. A smooth model of decision making under ambiguity. Econometrica 73, 1849–1892.

Kopylov, I., 2010. Simple axioms for countably additive subjective probability. Journal of Mathematical Economics 46, 867–876.

Laplace, P. S., 1814. Théorie analytique des probabilitiés.

Lengwiler, Y., 2009. Microfoundations of financial economics: an introduction to general equilibrium asset pricing. Princeton University Press.

LeRoy, S. F., Werner, J., 2014. Principles of financial economics. Cambridge University Press.

Lewis, R., 2016. Corporate debt markets and recovery rates with vulture investors. Available at SSRN: https://ssrn.com/abstract=2539585 .

Lewis, R., 2019. Same firm, different betas. Different Betas (May 7, 2019) .

Liu, L. Y., Patton, A. J., Sheppard, K., 2015. Does anything beat 5-minute rv? a comparison of realized measures across multiple asset classes. Journal of Econometrics 187, 293–311.

Longstaff, F. A., Schwartz, E. S., 1995. A simple approach to valuing risky fixed and floating rate debt. The Journal of Finance 50, 789–819.

Malenko, A., Tsoy, A., 2020. Asymmetric information and security design under knightian uncertainty. SSRN library 3100285.

McQuade, T. J., 2013. Stochastic volatility and asset pricing puzzles. Available at SSRN 3222902 .

Merton, R. C., 1974. On the pricing of corporate debt: The risk structure of interest rates. The Journal of Finance 29, 449–470.

Rothschild, M., Stiglitz, J. E., 1970. Increasing risk: I. A definition. Journal of Economic Theory 2, 225–243.

Schmeidler, D., 1989. Subjective probability and expected utility without additivity. Econometrica 57, 571–587.

Electronic copy available at: https://ssrn.com/abstract=4066052

Scholes, M., Williams, J., 1977. Estimating betas from nonsynchronous data. Journal of Financial Economics 5, 309–327.

Tversky, A., Kahneman, D., 1992. Advances in prospect theory: cumulative representation of uncertainty. Journal of Risk and Uncertainty 5, 297–323.

Wakker, P. P., Tversky, A., 1993. An axiomatization of cumulative prospect theory. Journal of Risk and Uncertainty 7, 147–175.

Zhao, G., 2020. Ambiguity, nominal bond yields, and real bond yields. American Economic Review: Insights 2, 177–92.

Electronic copy available at: https://ssrn.com/abstract=4066052

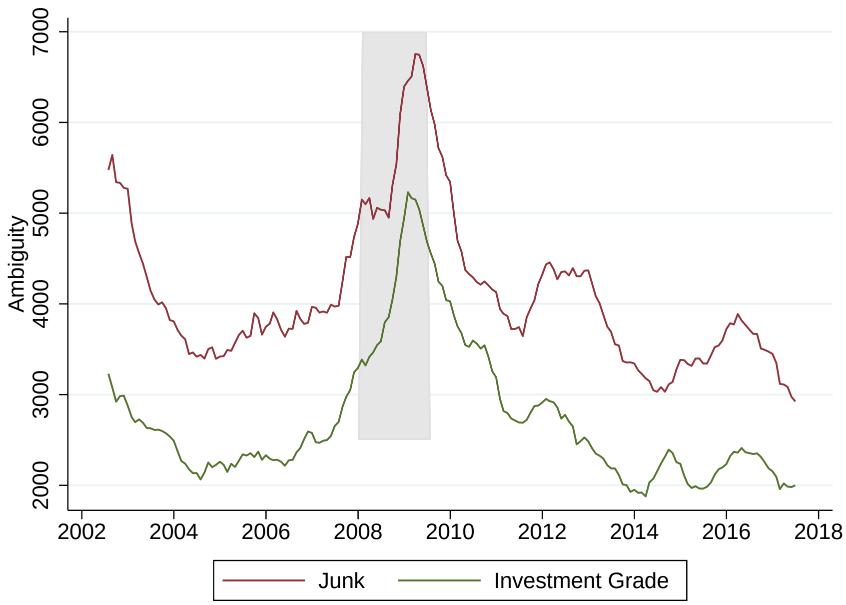

Figure 1: **Average Bond Ambiguity**

This figure presents a time series of the average bond ambiguity over the sample period for investment grade (IG) bonds and for junk bonds. The shaded region identifies the NBER recession that occurred during the sample period.

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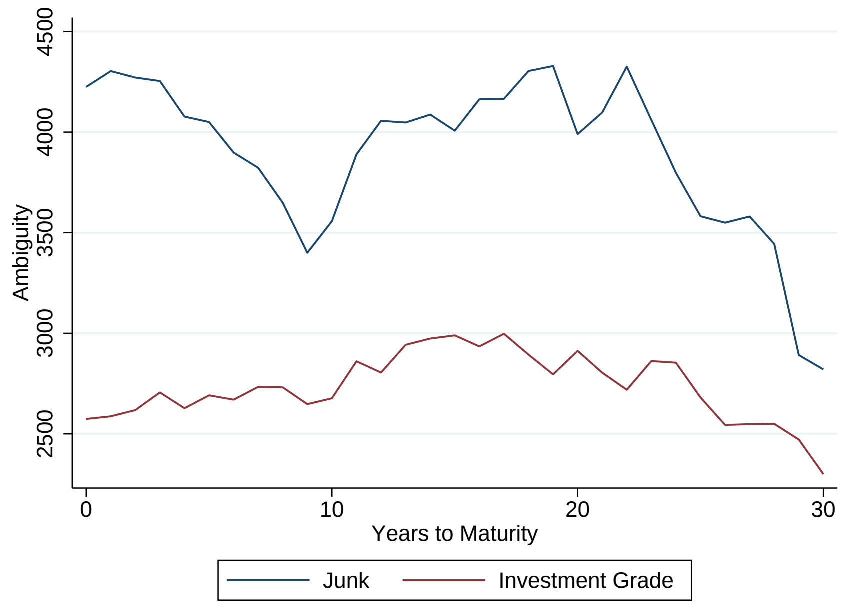

Figure 2: **"Term Structure" of Bond Ambiguity**

This figure presents the average measure of bond ambiguity for bonds with different times to maturity over the sample period for investment grade (IG) bonds and for junk bonds.

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Table 1: **Summary Statistics**

The sample consists of bond month observations from the WRDS bond returns dataset for the years 2002-2019 for which we can measure bond ambiguity and credit spreads. Additional controls may further reduce the sample, especially where we require the issuing firm to have publicly traded equity. Years to maturity measures the remaining life of the bond in years. Yield is the promised yield on the bond. Spread measures the promised yield on the bond less the contemporaneous risk-free swap rate. FISD Coupon Rate measures the coupon return based on the offering price of the bond. Offering Amount measures the size of the offering in which the bond was sold, expressed in millions of dollars. Numerical Rating represents the average bond rating translated to integer values. IG is an indicator variable for the bond being Investment Grade. Bond Return is the return from holding the bond over the prior year. Flag for Default is an indicator for whether the bond is or has been in default. Equity Beta is an estimate of the systematic risk of the issuing firm's equity as measured by its CAPM Beta. Bond Volatility measures the standard deviation of the bond return. Equity Volatility measures the standard deviation of the issuing firm's equity return. Asset Volatility is an estimate of the issuing firm's standard deviation of unlevered equity return. Z-Score is Altman's estimate of the credit worthiness of the issuing firm. Distance to Default is the estimate of the number of standard deviations of equity return the issuing firm is from defaulting on the bond. Average Bid/Ask Spread for a bond is taken from the WRDS bond return data and reflects the monthly average of the differences between buy and sell transaction prices. Bond Ambiguity is an estimate of the uncertainty of probabilities relevant for investors in the bond, estimated as detailed in the text. Equity Ambiguity is an estimate of the uncertainty of probabilities relevant for investors in the issuing firm's equity, estimated as detailed in the text.

	Mean	25th Pctl	Median	75th Pctl	StDev	N
Bond Characteristics						
Years to Maturity	9.04	3.22	6.09	9.85	9.02	552,871
Yield	0.05	0.03	0.05	0.06	0.06	548,967
Spread	294.96	67.67	149.11	322.99	597.55	431,196
Coupon	5.88	4.70	6.00	7.12	1.99	554,576
Offering Amount ($ mm)	741.71	349.64	500.00	1000.00	638.61	554,576
Numerical Rating (Avg)	8.81	6.00	8.00	10.00	3.70	552,116
Bond Return	0.01	-0.00	0.00	0.02	0.04	543,813
Equity Beta	1.06	0.71	1.01	1.37	0.49	451,883
Bond Volatility	0.03	0.01	0.02	0.03	0.03	523,681
Equity Volatility	0.09	0.05	0.07	0.10	0.06	478,870
Asset Volatility	0.05	0.03	0.04	0.06	0.03	478,095
Z-Score	1.87	0.82	1.48	2.68	2.48	476,999
Distance to Default	3.19	1.37	2.39	3.85	9.43	451,929
Avg Bid/Ask Spread (pct)	0.74	0.26	0.49	0.90	1.06	536,203
Ambiguity						
Bond Ambiguity	3129.25	1727.48	2608.73	3929.96	2162.18	554,576
Equity Ambiguity	132.33	46.24	101.16	180.41	175.47	476,752

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Table 2: **Bond Ambiguity Quintile Sorts**

This table presents average bond characteristics for bond-month observations sorted into quintiles of bond ambiguity. The sample consists of bond month observations from the WRDS bond returns dataset for the years 2002-2019 for which we can measure bond ambiguity and credit spreads. Additional controls may further reduce the sample, especially where we require the issuing firm to have publicly traded equity. Years to maturity measures the remaining life of the bond in years. Yield is the promised yield on the bond. Spread measures the promised yield on the bond less the contemporaneous risk-free swap rate. FISD Coupon Rate measures the coupon return based on the offering price of the bond. Offering Amount measures the size of the offering in which the bond was sold, expressed in millions of dollars. Numerical Rating represents the average bond rating translated to integer values. Equity Beta is an estimate of the systematic risk of the issuing firm's equity as measured by its CAPM Beta. Bond Volatility measures the standard deviation of the bond return. Equity Volatility measures the standard deviation of the issuing firm's equity return. Asset Volatility is an estimate of the issuing firm's standard deviation of unlevered equity return. Z-Score is Altman's estimate of the credit worthiness of the issuing firm. Distance to Default is the estimate of the number of standard deviations of equity return the issuing firm is from defaulting on the bond. Average Bid/Ask Spread for a bond is taken from the WRDS bond return data and reflects the monthly average of the differences between buy and sell transaction prices. Bond Ambiguity is an estimate of the uncertainty of probabilities relevant for investors in the bond, estimated as detailed in the text. Equity Ambiguity is an estimate of the uncertainty of probabilities relevant for investors in the issuing firm's equity, estimated as detailed in the text.

	(1) Mean	(2) Mean	(3) Mean	(4) Mean	(5) Mean
Bond Characteristics					
Years to Maturity	9.42	9.21	9.17	9.18	8.22
Yield	0.04	0.04	0.05	0.06	0.08
Spread	139.34	158.66	215.28	319.96	616.88
Coupon	5.30	5.42	5.80	6.20	6.67
Offering Amount ($ mm)	932.65	917.47	767.83	613.22	471.91
Numerical Rating (Avg)	7.37	7.29	8.30	9.76	11.39
Equity Beta	0.99	1.05	1.05	1.08	1.15
Bond Volatility	0.02	0.02	0.03	0.03	0.04
Equity Volatility	0.07	0.08	0.08	0.10	0.11
Asset Volatility	0.04	0.04	0.04	0.05	0.05
Z-Score	2.31	1.90	1.71	1.68	1.74
Distance to Default	3.77	3.81	3.41	2.54	2.31
Avg Bid/Ask Spread (pct)	0.50	0.59	0.70	0.86	1.05
Ambiguity					
Bond Ambiguity	1237.85	2049.64	2687.42	3586.54	6135.13
Equity Ambiguity	155.77	154.39	137.53	112.49	97.14

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Table 3: **Spreads Horse Race**

OLS regression tests of the effect of the determinants of credit spreads. All independent variables are one-month lagged. The sample consists of bond month observations from the WRDS bond returns dataset for the years 2002-2019 for which we can measure bond ambiguity and credit spreads. Additional controls may further reduce the sample, especially where we require the issuing firm to have publicly traded equity. Bond Ambiguity is an estimate of the uncertainty of probabilities relevant for investors in the bond, estimated as detailed in the text. Equity Volatility measures the standard deviation of the issuing firm's equity return. Bond Volatility measures the standard deviation of the bond return. Equity Beta is an estimate of the systematic risk of the issuing firm's equity as measured by its CAPM Beta. Distance to Default is the estimate of the number of standard deviations of equity return the issuing firm is from defaulting on the bond. Z-Score is Altman's estimate of the credit worthiness of the issuing firm. Average Bid/Ask Spread for a bond is taken from the WRDS bond return data and reflects the monthly average of the differences between buy and sell transaction prices. Equity Ambiguity is an estimate of the uncertainty of probabilities relevant for investors in the issuing firm's equity, estimated as detailed in the text. (z) appearing after a variable name indicates the variable has been normalized by it own mean and standard deviation. (w,z) appearing after a variable name indicates the variable has been normalized by it own mean and standard deviation and Winsorized at the 1^{st} and 99^{th} percentiles. In parentheses are standard errors clustered by year-month and CUSIP, *p<0.1; **p<0.05; ***p<0.01.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Bond Ambiguity (z)	196.91***							
	(11.57)							
Equity Volatility (z)		186.63***						
		(15.28)						
Bond Volatility (z)			273.09***					
			(20.34)					
Equity Beta (z)				95.20***				
				(5.98)				
Distance to Default (w,z)					-129.57***			
					(8.00)			
Z-Score (w,z)						-91.21***		
						(6.28)		
Avg Bid/Ask Spread (w,z)							260.38***	
							(14.24)	
Equity Ambiguity (z)								-88.80***
								(10.80)
Constant	288.18***	245.20***	301.86***	249.91***	249.96***	249.38***	301.64***	246.83***
	(9.10)	(10.41)	(12.79)	(9.70)	(9.56)	(9.56)	(8.12)	(8.53)
R^2	0.112	0.147	0.208	0.038	0.069	0.035	0.165	0.032
Observations	423,541	365,980	397,831	349,028	349,060	364,673	411,988	364,534

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Table 4: **Bond Ambiguity and Credit Spreads**

OLS regression tests of the effect of risk and ambiguity on credit spreads. All independent variables are one-month lagged. The sample consists of bond month observations from the WRDS bond returns dataset for the years 2002-2019 for which we can measure bond ambiguity and credit spreads. Additional controls may further reduce the sample, especially where we require the issuing firm to have publicly traded equity. Bond Ambiguity is an estimate of the uncertainty of probabilities relevant for investors in the bond, estimated as detailed in the text. Equity Ambiguity is an estimate of the uncertainty of probabilities relevant for investors in the issuing firm's equity, estimated as detailed in the text. Equity Volatility measures the standard deviation of the issuing firm's equity return. Bond Volatility measures the standard deviation of the bond return. Equity Beta is an estimate of the systematic risk of the issuing firm's equity as measured by its CAPM Beta. Distance to Default is the estimate of the number of standard deviations of equity return the issuing firm is from defaulting on the bond. Average Bid/Ask Spread for a bond is taken from the WRDS bond return data and reflects the monthly average of the differences between buy and sell transaction prices. Z-Score is Altman's estimate of the credit worthiness of the issuing firm. Time is a year by month fixed effect. Rating is a bond rating fixed effect. CUSIP is a bond fixed effect. Years to Mat is a years to maturity fixed effect. (z) appearing after a variable name indicates the variable has been normalized by it own mean and standard deviation. (w,z) appearing after a variable name indicates the variable has been normalized by it own mean and standard deviation and Winsorized at the 1^{st} and 99^{th} percentiles. In parentheses are standard errors clustered by year-month and CUSIP, *p<0.1; **p<0.05; ***p<0.01.

	(1)	(2)	(3)	(4)	(5)	(6)
Bond Ambiguity (z)	185.40***	52.46***	30.11***	17.78***		17.51***
	(10.40)	(4.52)	(4.02)	(3.22)		(3.22)
Equity Ambiguity (z)					-2.07	-2.24
					(1.44)	(1.41)
Equity Beta (z)				-16.12	-16.96	-16.24
				(28.25)	(28.17)	(28.25)
Bond Volatility (z)				46.26***	46.90***	45.82***
				(10.07)	(10.08)	(10.03)
Equity Volatility (z)				28.57**	30.53**	29.65**
				(9.81)	(9.88)	(9.79)
Z-Score (w,z)				-48.38***	-48.26***	-48.40***
				(10.18)	(10.24)	(10.25)
Distance to Default (w,z)				11.18*	11.69*	11.82*
				(5.35)	(5.32)	(5.31)
Avg Bid/Ask Spread (w,z)				92.32***	92.55***	91.85***
				(10.32)	(10.41)	(10.36)
Time	Y					
Time x Rating		Y	Y	Y	Y	Y
Time x Years to Mat		Y	Y	Y	Y	Y
CUSIP			Y	Y	Y	Y
R^2	0.152	0.666	0.785	0.799	0.797	0.798
Observations	423,541	355,838	355,749	262,967	262,072	262,072

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Table 5: **Bond Ambiguity Interactions with Risk**

OLS regression tests of the interaction effect of risk and ambiguity on credit spreads. All independent variables are one-month lagged. The sample consists of bond month observations from the WRDS bond returns dataset for the years 2002-2019 for which we can measure bond ambiguity and credit spreads. Additional controls may further reduce the sample, especially where we require the issuing firm to have publicly traded equity. Bond Ambiguity is an estimate of the uncertainty of probabilities relevant for investors in the bond, estimated as detailed in the text. Equity Volatility measures the standard deviation of the issuing firm's equity return. Bond Volatility measures the standard deviation of the bond return. Controls include the issuing firm's equity beta, the issuing firm's Z-score, the issuing firm's distance to default measure, and the bond's average bid-ask spread as a measure of liquidity. Time is a year by month fixed effect. Rating is a bond rating fixed effect. CUSIP is a bond fixed effect. Years to Mat is a years to maturity fixed effect. (z) appearing after a variable name indicates the variable has been normalized by it own mean and standard deviation. In parentheses are standard errors clustered by year-month and CUSIP, $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$.

	(1)	(2)	(3)
Bond Ambiguity (z)	15.38***	19.86***	17.91***
	(2.92)	(3.48)	(3.36)
Equity Volatility (z)	19.87*		18.32*
	(8.99)		(9.26)
Bond Ambiguity (z) x Equity Volatility (z)	24.01***		10.31
	(4.82)		(5.37)
Bond Volatility (z)		26.19*	23.01*
		(10.43)	(9.93)
Bond Ambiguity (z) x Bond Volatility (z)		32.61***	26.03***
		(5.90)	(6.35)
Default Controls	Y	Y	Y
Time x Rating	Y	Y	Y
Time x Years to Mat	Y	Y	Y
CUSIP	Y	Y	Y
R^2	0.798	0.800	0.801
Observations	280,470	262,996	262,996

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Table 6: **Bond Ambiguity Interactions with Default Measures**

OLS regression tests of the interaction effect of default risk and ambiguity on credit spreads. All independent variables are one-month lagged. The sample consists of bond month observations from the WRDS bond returns dataset for the years 2002-2019 for which we can measure bond ambiguity and credit spreads. Additional controls may further reduce the sample, especially where we require the issuing firm to have publicly traded equity. Bond Ambiguity is an estimate of the uncertainty of probabilities relevant for investors in the bond, estimated as detailed in the text. IG is an indicator variable for the bond being Investment Grade. Distance to Default is the estimate of the number of standard deviations of equity return the issuing firm is from defaulting on the bond. Z-Score is Altman's estimate of the credit worthiness of the issuing firm. Average Bid/Ask Spread for a bond is taken from the WRDS bond return data and reflects the monthly average of the differences between buy and sell transaction prices. Controls include the issuing firm's equity beta, the bond's return volatility, the volatility of the return on the issuing firm's equity, and the bond's average bid-ask spread as a measure of liquidity. Time is a year by month fixed effect. Rating is a bond rating fixed effect. CUSIP is a bond fixed effect. Years to Mat is a years to maturity fixed effect. (z) appearing after a variable name indicates the variable has been normalized by it own mean and standard deviation. (w,z) appearing after a variable name indicates the variable has been normalized by it own mean and standard deviation and Winsorized at the 1^{st} and 99^{th} percentiles. In parentheses are standard errors clustered by year-month and CUSIP, $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$.

	(1)	(2)	(3)	(4)
Bond Ambiguity (z)	33.01***	16.03***	19.70***	15.99***
	(6.95)	(2.99)	(3.71)	(3.15)
Bond Ambiguity (z) x IG	-31.66***			
	(7.09)			
Distance to Default (w,z)		2.99		
		(5.00)		
Bond Ambiguity (z) x Distance to Default (w,z)		-17.20***		
		(3.37)		
Z-Score (w,z)			-39.89***	
			(9.30)	
Bond Ambiguity (z) x Z-Score (w,z)			-24.25***	
			(4.30)	
Bond Ambiguity (z) x Avg Bid/Ask Spread (w,z)				48.89***
				(6.15)
Volatility Controls	Y	Y	Y	Y
Time x Rating	Y	Y	Y	Y
Time x Years to Mat	Y	Y	Y	Y
CUSIP	Y	Y	Y	Y
R^2	0.799	0.799	0.797	0.803
Observations	276,739	263,687	275,616	276,739

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Table 7: **Bond Ambiguity Interactions with Systematic Risk Measures**

OLS regression tests of the interaction effect of systematic risk and ambiguity on credit spreads. All independent variables are one-month lagged. The sample consists of bond month observations from the WRDS bond returns dataset for the years 2002-2019 for which we can measure bond ambiguity and credit spreads. Additional controls may further reduce the sample, especially where we require the issuing firm to have publicly traded equity. Bond Ambiguity is an estimate of the uncertainty of probabilities relevant for investors in the bond, estimated as detailed in the text. Equity Beta is an estimate of the systematic risk of the issuing firm's equity as measured by its CAPM Beta. Bond Beta is an estimate of the systematic risk of the bond as measured by the CAPM Beta of the bond. Controls include the bond's return volatility, the volatility of the return on the issuing firm's equity, the issuing firm's Z-score, the issuing firm's distance to default measure, and the bond's average bid-ask spread as a measure of liquidity. Time is a year by month fixed effect. Rating is a bond rating fixed effect. CUSIP is a bond fixed effect. Years to Mat is a years to maturity fixed effect. (z) appearing after a variable name indicates the variable has been normalized by it own mean and standard deviation. In parentheses are standard errors clustered by year-month and CUSIP, *p<0.1; **p<0.05; ***p<0.01.

	(1)	(2)
Bond Ambiguity (z)	16.70***	18.52***
	(3.20)	(3.32)
Bond Ambiguity (z) x Bond Beta (z)	31.29**	
	(10.69)	
Bond Ambiguity (z) x Equity Beta (z)		14.36***
		(3.51)
Controls	Y	Y
Time x Rating	Y	Y
Time x Years to Mat	Y	Y
CUSIP	Y	Y
R^2	0.796	0.799
Observations	259,361	262,967

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Table 8: **Bond Index Ambiguity and Credit Spreads**

OLS regression tests of the effect of the ambiguity of bond portfolios on credit spreads. All independent variables are one-month lagged. The sample consists of bond month observations from the WRDS bond returns dataset for the years 2002-2019 for which we can measure bond ambiguity and credit spreads. Additional controls may further reduce the sample, especially where we require the issuing firm to have publicly traded equity. Bond Ambiguity is an estimate of the uncertainty of probabilities relevant for investors in the bond, estimated as detailed in the text. Short Treasury Ambiguity is an estimate of the ambiguity associated with a portfolio of short-term treasury bills. Medium Treasury Ambiguity is an estimate of the ambiguity associated with a portfolio of medium-term treasury notes. Long Treasury Ambiguity is an estimate of the ambiguity associated with a portfolio of long-term treasury bonds. IG Index is an estimate of the ambiguity associated with an index portfolio of investment grade bonds. Junk Index is an estimate of the ambiguity associated with an index portfolio of junk bonds. Rating is a bond rating fixed effect. CUSIP is a bond fixed effect. Years to Mat is a years to maturity fixed effect. Controls include issuing firm's equity beta, the bond's return volatility, the volatility of the return on the issuing firm's equity, the issuing firm's Z-score, the issuing firm's distance to default measure, and the bond's average bid-ask spread as a measure of liquidity. In parentheses are standard errors clustered by year-month and CUSIP, $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$.

	(1)	(2)	(3)	(4)
Bond Ambiguity (z)			65.49***	38.34***
			(8.25)	(6.08)
Short Treasuries (z)	-17.52*		-18.36*	
	(7.98)		(7.54)	
Medium Treasuries (z)	-38.27***		-30.16***	
	(9.21)		(7.67)	
Long Treasuries (z)	-11.61		-13.01	
	(8.66)		(7.56)	
IG Index (z)		-21.60**		-20.16**
		(6.56)		(6.38)
Junk Index (z)		-7.98		-6.24
		(5.93)		(5.83)
Controls	Y	Y	Y	Y
Rating	Y	Y	Y	Y
Years to Mat	Y	Y	Y	Y
CUSIP	Y	Y	Y	Y
R^2	0.652	0.647	0.658	0.649
Observations	168,365	237,129	168,365	237,129

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Table 9: **Calibration of Credit Spreads with Ambiguity**

This table reports model based credit spreads under risk and ambiguity aversion (Column 1), risk aversion alone (Column 2) and the observed spreads (Column 3). The risk aversion coefficient, consumption and state probabilities are chosen to approximate the baseline model results from Chen et al. (2009) and McQuade (2013). Columns (4)-(5) present the relative decimal distance to the observed spreads for the risk and ambiguity aversion and the risk aversion models, respectively.

Panel (A) : Credit Spreads on 4-Year Maturity Debt

	Annual Credit Spread			% Difference From Observed	
	Risk and Ambiguity	Risk	Observed	Risk and Ambiguity	Risk
Aaa	0.44%	0.01%	0.46%	0.042	0.974
Aa	0.35%	0.04%	0.56%	0.383	0.922
A	0.53%	0.13%	0.87%	0.395	0.848
Baa	0.88%	0.34%	1.49%	0.412	0.774
Ba	2.77%	1.89%	3.10%	0.105	0.391
B	6.10%	4.77%	4.70%	0.299	0.015

Panel (B) : Credit Spreads on 10-Year Maturity Debt

	Annual Credit Spread			% Difference From Observed	
	Risk and Ambiguity	Risk	Observed	Risk and Ambiguity	Risk
Aaa	0.48%	0.12%	0.47%	0.030	0.741
Aa	0.51%	0.24%	0.69%	0.258	0.652
A	0.93%	0.62%	0.96%	0.036	0.352
Baa	1.41%	1.01%	1.50%	0.062	0.324
Ba	3.54%	3.16%	3.10%	0.142	0.020
B	5.30%	4.93%	4.70%	0.127	0.049

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A Appendix

A.1 Proofs

Proof of Theorem 1. Substituting the budget constraints into the objective function in Equation (8) and solving the maximization problem using point-wise optimization (i.e., differentiation with respect to θ conditional on a given event associated with x), as is standard in the literature (e.g., Lengwiler, 2009; LeRoy and Werner, 2014), provides

$$q(x)\partial_0 \mathrm{u} \quad = \quad \mathrm{E}\left[\varphi\left(x\right)\right]\left(1 \pm \eta \mathrm{Var}\left[\varphi\left(x\right)\right]\right)\partial_x \mathrm{u}.$$

The absence of arbitrage implies that the payoff pricing functional assigns a positive price to each state contingent claim. Organizing terms completes the proof. □

Proof of Proposition 1. A higher debt ambiguity implies that the variance of probabilities $\mathrm{Var}\left[\varphi\left(x\right)\right]$ of at least one unfavorable payoff $x < F$ increases. By symmetry, the variance of the probabilities $\mathrm{Var}\left[\varphi\left(y\right)\right]$ of a favorable payoff y, where $|x-\mu| = |y-\mu|$, also increases. Let Δ denote the change in the variable of interest. The overall change in the bond price is then

$$\begin{aligned}\Delta p \quad &= \quad \Delta q(x)x + \Delta q(y)y \\ &= \quad \mathrm{E}\left[\varphi\left(x\right)\right]\eta\Delta\mathrm{Var}\left[\varphi\left(x\right)\right]\frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}x - \mathrm{E}\left[\varphi\left(y\right)\right]\eta\Delta\mathrm{Var}\left[\varphi\left(y\right)\right]\frac{\partial_y \mathrm{U}}{\partial_0 \mathrm{U}}y.\end{aligned}$$

By symmetry, $\mathrm{E}\left[\varphi\left(x\right)\right] = \mathrm{E}\left[\varphi\left(y\right)\right]$ and $\mathrm{Var}\left[\varphi\left(x\right)\right] = \mathrm{Var}\left[\varphi\left(y\right)\right]$. Therefore,

$$\Delta p \quad = \quad \mathrm{E}\left[\varphi\left(x\right)\right]\eta\Delta\mathrm{Var}\left[\varphi\left(x\right)\right]\left(\frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}x - \frac{\partial_y \mathrm{U}}{\partial_0 \mathrm{U}}y\right).$$

Since $\frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}} \geq \frac{\partial_y \mathrm{U}}{\partial_0 \mathrm{U}}$,

$$\Delta p \quad \leq \quad \mathrm{E}\left[\varphi\left(x\right)\right]\eta\Delta\mathrm{Var}\left[\varphi\left(x\right)\right]\frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}\left(x-y\right).$$

Since $y > x$, $\Delta p < 0$. □

Proof of Proposition 2. A higher equity ambiguity implies that the variance of probabilities $\mathrm{Var}\left[\varphi\left(y\right)\right]$ of at least one payoff $y \geq F$ increases. By symmetry, the variance of the probabilities $\mathrm{Var}\left[\varphi\left(x\right)\right]$ of a payoff x such that $|\mu - x| = |\mu - y|$ also increases. If $x \geq F$, then the bond price remains the same. If $x < F$, the rest of the proof is then similar to the proof of Proposition 1. □

Proof of Proposition 3. An increase in debt's risk implies the addition of a mean-preserving spread in outcomes to at least one state whose payoff $x < F$. In particular, the payoff x becomes

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$x-\delta$ or $x+\delta$, each with probability $\frac{1}{2}$. As a result, the bond price would change as follows

$$\Delta p \quad = \quad q(x-\delta)\,(x-\delta)+q(x+\delta)\,(x+\delta)-q(x)x.$$

Since the partition of the state space changes, state prices change such that

$$\begin{aligned}\Delta p \quad = \quad & \mathrm{E}\left[\varphi\left(x\right)\right]\left(1+\eta \mathrm{Var}\left[\varphi\left(x\right)\right]\right)\frac{1}{2}\frac{\partial_{x-\delta}\mathrm{U}}{\partial_0\mathrm{U}}\left(x-\delta\right)+\\ & \mathrm{E}\left[\varphi\left(x\right)\right]\left(1+\eta \mathrm{Var}\left[\varphi\left(x\right)\right]\right)\frac{1}{2}\frac{\partial_{x+\delta}\mathrm{U}}{\partial_0\mathrm{U}}\left(x+\delta\right)-\\ & \mathrm{E}\left[\varphi\left(x\right)\right]\left(1+\eta \mathrm{Var}\left[\varphi\left(x\right)\right]\right)\frac{\partial_{x}\mathrm{U}}{\partial_0\mathrm{U}}x.\\ = \quad & \mathrm{E}\left[\varphi\left(x\right)\right]\left(1+\eta \mathrm{Var}\left[\varphi\left(x\right)\right]\right)\left(\frac{1}{2}\frac{\partial_{x-\delta}\mathrm{U}}{\partial_0\mathrm{U}}+\frac{1}{2}\frac{\partial_{x+\delta}\mathrm{U}}{\partial_0\mathrm{U}}-\frac{\partial_{x}\mathrm{U}}{\partial_0\mathrm{U}}\right)x+\\ & \mathrm{E}\left[\varphi\left(x\right)\right]\left(1+\eta \mathrm{Var}\left[\varphi\left(x\right)\right]\right)\frac{1}{2}\left(\frac{\partial_{x+\delta}\mathrm{U}}{\partial_0\mathrm{U}}-\frac{\partial_{x-\delta}\mathrm{U}}{\partial_0\mathrm{U}}\right)\delta.\end{aligned}$$

By the convexity of $\partial_x\mathrm{U}$, and since $\frac{\partial_{x-\delta}\mathrm{U}}{\partial_0\mathrm{U}} \geq \frac{\partial_{x+\delta}\mathrm{U}}{\partial_0\mathrm{U}}$,

$$\Delta p \quad \leq \quad \mathrm{E}\left[\varphi\left(x\right)\right]\left(1+\eta \mathrm{Var}\left[\varphi\left(x\right)\right]\right)\frac{1}{2}\left(\frac{\partial_{x+\delta}\mathrm{U}}{\partial_0\mathrm{U}}-\frac{\partial_{x-\delta}\mathrm{U}}{\partial_0\mathrm{U}}\right)\delta \;\leq\; 0. \tag{24}$$

□

Proof of Proposition 4. An increase in equity's risk implies the addition of a mean-preserving spread in outcomes to at least one state whose payoff is $y \geq F$. In particular, the payoff y becomes $y-\delta$ or $y+\delta$, each with probability $\frac{1}{2}$. By symmetry, an mean preserving spread in outcomes is applied to some x that satisfies $|\mu - x| = |\mu - y|$. If $x-\delta > F$, then the bond price remains the same. If $x-\delta \leq F$, the rest of the proof is then similar to the proof of Proposition 3. □

Proof of Proposition 5. An increase in debt's risk implies an addition of mean-preserving spread of at leat for one $x \leq F$. The effect of this increase in risk is summarized in Equation (24):

$$\Delta p \quad \leq \quad \mathrm{E}\left[\varphi\left(x\right)\right]\left(1+\eta \mathrm{Var}\left[\varphi\left(x\right)\right]\right)\frac{1}{2}\left(\frac{\partial_{x+\delta}\mathrm{U}}{\partial_0\mathrm{U}}-\frac{\partial_{x-\delta}\mathrm{U}}{\partial_0\mathrm{U}}\right)\delta \;\leq\; 0.$$

A higher debt's ambiguity implies a greater $\mathrm{Var}\left[\varphi\left(x\right)\right]$, implying a stronger negative effect of the increase in risk. □

A.2 Estimating equity ambiguity

The use of the CPT model augmented with the EUUP framework, Equation (5) naturally delivers a risk-independent measure of ambiguity (Izhakian, 2020). Within this framework, the degree of ambiguity can be measured by the volatility of the uncertain probabilities, just as the degree of risk

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can be measured by the volatility of the uncertain outcomes. Formally, the measure of ambiguity, denoted by $\mho^2$ and defined by Equation (7), represents an expected probability-weighted average of the variances of probabilities. We follow the recent literature (e.g., Izhakian and Yermack, 2017; Brenner and Izhakian, 2018; Augustin and Izhakian, 2020) and estimate the monthly degree of ambiguity for each firm's equity using intraday stock return data from TAQ.[29]

To estimate ambiguity as implemented in Equation (25) below, the expectation of and the variation in return probabilities across the set of possible prior probability distributions must be measured. We assume that the intraday equity return distribution for each day in a given month represents a single distribution, P, in the set of possible distributions, $\mathcal{P}$, and the number of priors in the set is assumed to depend on the number of trading days in the month. The set of priors for each month, $\mathcal{P}$, therefore, consists of the 18-22 realized daily distributions in that month. For practical implementation, we discretize return distributions into n bins of equal size, and represent each distribution as a histogram, as illustrated in Figure A.1 in the Appendix. Each bin $B_j = (r_{j-1}, r_j]$ represents an interval of return outcomes. The bin height is computed as the fraction of daily intraday returns observed in that bin, and thus represents the probability of the returns in that bin. Equipped with these 18-22 daily return histograms, we compute the expected probability of each bin across the daily return distributions for each month, $\mathrm{E}\left[\mathrm{P}\left(B_j\right)\right]$, as well as the variance of these probabilities, $\mathrm{Var}\left[\mathrm{P}\left(B_j\right)\right]$, by assigning an equal likelihood (ξ) to each histogram.[30] Using these values, the monthly degree of ambiguity for firm i is computed as

$$\mho^2\left[r_i\right] = \frac{1}{w^2\left(1-w\right)^2}\sum_{j=1}^{n}\mathrm{E}\left[\mathrm{P}_i\left(B_j\right)\right]\mathrm{Var}\left[\mathrm{P}_i\left(B_j\right)\right]. \tag{25}$$

To minimize the impact of the selected bin size on the value of ambiguity, we apply a variation of Sheppard's correction and scale the weighted-average volatilities of probabilities to the bin size by $\frac{1}{w^2(1-w)^2}$, where $w = r_{i,j} - r_{i,j-1}$.

[29]The measure of ambiguity, defined in Equation (7), is distinct from aversion to ambiguity. The former, which is a matter of beliefs (or information), is estimated from the data, while the latter is a matter of subjective attitudes. Baillon, Huang, Selim, and Wakker (2018) suggest an elicitation of "matching probabilities," which are similar to the perceived probabilities in the EUUP model. Based upon matching probabilities, they suggest an ambiguity insensitivity index. By construction, this index supports only three possible events, and it is attitude dependent, making it less useful for this application. The measure employed here supports an unlimited number of events and is attitude independent.

[30]Equal likelihoods is consistent with: the principle of insufficient reason, which states that given n possibilities that are indistinguishable except for their names, each possibility should be assigned a probability equal to $\frac{1}{n}$ (Bernoulli, 1713; Laplace, 1814); the simplest non-informative prior in Bayesian probability (Bayes, Price, and Canton, 1763), which assigns equal probabilities to all possibilities; and the principle of maximum entropy (Jaynes, 1957), which states that the probability distribution which best describes the current state of knowledge is the one with the largest entropy.

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In our implementation, we sample five-minute stock returns from 9:30 to 16:00, as this frequency has been shown to eliminate microstructure effects (Andersen, Bollerslev, Diebold, and Ebens, 2001; Bandi and Russell, 2006; Liu, Patton, and Sheppard, 2015). Thus, we obtain daily histograms of up to 78 intraday returns, each of which represents a single $\mathrm{P} \in \mathcal{P}$. If we observe no trade in a specific time point for a given stock, we compute returns based on the volume-weighted average of the nearest trading prices within 150 seconds distance from that time point. If there is no price change within this distance, we drop this five minute observation of the given stock. We ignore returns between closing and next-day opening to eliminate the impact of overnight price changes and dividend distributions. We drop all days with less than 15 five-minute returns and months with less than 15 intraday return distributions. In addition, we drop extreme returns ($\pm 5\%$ log returns over five minutes), as many such returns occur due to improper orders subsequently canceled by the stock exchange. We normalize the intraday five-minute returns to daily returns.[31]

For the bin formation, we divide the range of daily returns into 1002 intervals. We form a grid of 1000 bins, from -100% to 100%, each of width 0.2%, in addition to the left and right tails, defined as $(-\infty, -100\%]$ and $[+100\%, +\infty)$, respectively. We compute the mean and the variance of probabilities for each interval by assigning equal likelihood to each histogram.[32] Some bins may not be populated with return realizations, which makes it difficult to compute their probability. Therefore, we assume a normal return distribution and use its moments to extrapolate the missing return probabilities. That is, $\mathrm{P}_i(B_j) = \Phi(r_j; \mu_i, \sigma_i) - \Phi(r_{j-1}; \mu_i, \sigma_i)$, where $\Phi(\cdot)$ denotes the cumulative normal probability distribution, characterized by its mean μ_i and the variance σ_i^2 of the returns.[33]

The EUUP measure of ambiguity is outcome-independent (up to a state-space partition), which allows for a risk-independent examination of the impact of ambiguity on financial decisions. Specifically, the measure of ambiguity described above captures the variation in the frequencies (probabilities) of outcomes without incorporating the magnitudes of outcomes. In contrast, the measure of risk captures the variation in the magnitudes of outcomes without incorporating the variation in the frequencies of outcomes. Thus, the measure of ambiguity is risk-independent just as standard

[31]Our results are robust to the inclusion of extreme price changes, as well as for a cutoff of $\pm 1\%$ in terms of log returns over five minutes. Note that a $\pm 5\%$ five-minute return implies a $\pm 390\%$ daily return.

[32]Equal likelihoods are equivalent to daily ratios $\frac{\mu_i}{\sigma_i}$ that are Student-t distributed. When $\frac{\mu}{\sigma}$ is Student-t distributed, cumulative probabilities are uniformly distributed (e.g., Kendall and Stuart, 2010, page 21, Proposition 1.27).

[33]Scholes and Williams (1977) suggest adjusting the volatility of returns for non-synchronous trading as $\sigma_t^2 = \frac{1}{N_t} \sum_{\ell=1}^{N_t} (r_{t,\ell} - \mathrm{E}[r_{t,\ell}])^2 + 2\frac{1}{N_t - 1} \sum_{\ell=2}^{N_t} (r_{t,\ell} - \mathrm{E}[r_{t,\ell}])(r_{t,\ell-1} - \mathrm{E}[r_{t,\ell-1}])$. We also perform all estimations using the Scholes-Williams correction for non-synchronous trading. The findings are essentially the same.

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measures of risk are ambiguity-independent, implying that these two measures capture distinct aspects of uncertainty. Empirically, the monthly measure of ambiguity and the monthly measure of risk are weakly negatively correlated with a correlation coefficient of -0.06.

Other proxies for ambiguity in the literature include the volatility of mean return (Franzoni, 2017), the volatility of return volatility (Faria and Correia-da Silva, 2014), or the dispersion in (disagreement of) analyst forecasts (Anderson, Ghysels, and Juergens, 2009). As these measures are sensitive to changes in the set of outcomes (i.e., are outcome-dependent), they are therefore risk-dependent and less useful for this study.[34] Similarly, skewness, kurtosis, and other higher moments of the return distribution are outcome-dependent and so different from $\mho^2$. Jumps, time-varying mean and time-varying volatility are also outcome-dependent.

Brenner and Izhakian (2018) and Augustin and Izhakian (2020) show that $\mho^2$ does not simply reflect other well-known "uncertainty" factors including skewness, kurtosis, variance of mean, variance of variance, downside risk, mixed data sampling measure of forecasted volatility (MIDAS), jumps, or investors' sentiment, among many others.

A.3 Estimating risk

Following many earlier studies of leverage (e.g., Frank and Goyal, 2009), we use equity return volatility (or unlevered return volatility) as a proxy for default risk. Return volatility has also been used in the literature on pricing default risk for debt (e.g., Bharath and Shumway, 2008) and as an input for the "distance to default" measure derived from the Merton (1974) model.

For consistency, we compute the variance of equity returns with the same intraday returns used to estimate ambiguity. For each stock on each day, we compute the variance of intraday returns.[35] We also estimate risk as the average monthly variance of daily returns and find no meaningful change in results.

[34] For example, Ben-Rephael and Izhakian (2020) report a correlation between daily volatility of mean return and volatility of return (risk) of 0.81 and a correlation between daily volatility of return volatility and volatility of return (risk) equal to 0.72, implying that these proposed proxies for ambiguity are very closely related to risk. Herron and Izhakian (2019) find even higher values (0.86 and 0.91, respectively) for the monthly correlations.

[35] For robustness we have computed variance applying the Scholes and Williams (1977) correction for non-synchronous trading and for heteroscedasticity and found the same results. See, for example, French et al. (1987).

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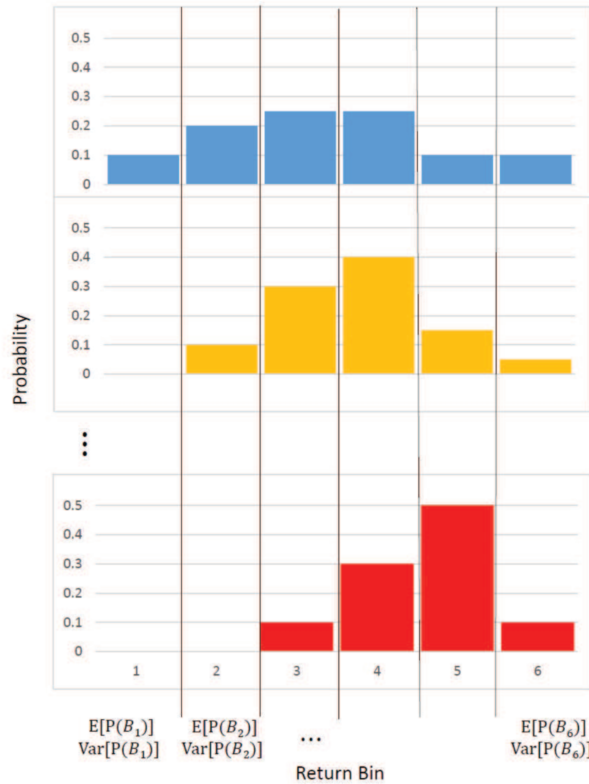

Figure A.1: **Ambiguity measurement**

This figure illustrates the computation of the ambiguity measure, which is derived for each firm-month based on intraday stock-returns sampled at five-minute frequencies from 9:30 to 16:00. Thus, we obtain up to 22 daily histograms of up to 78 intraday returns in each month. We discretize the daily return distributions into n bins of equal size $B_j = (r_j, r_{j-1}]$ across histograms. The height of the histogram for a particular bin is computed as the fraction of daily intraday returns observed in that bin, and thus represents the probability of that particular bin outcome. We compute the expected probability of a particular bin across the daily return distributions, $\mathrm{E}[\mathrm{P}_i(B_j)]$, as well as the variance of these probabilities, $\mathrm{Var}[\mathrm{P}_i(B_j)]$. Ambiguity is then computed as $\mho^2[r_i] \equiv 1/(w(1-w))^2 \sum_{j=1}^{n} \mathrm{E}[\mathrm{P}_i(B_j)] \mathrm{Var}[\mathrm{P}_i(B_j)]$, where we scale the weighted-average volatilities of probabilities to the bins' size $w = r_{i,j} - r_{i,j-1}$.

A.4 Robustness tests

This section replicates the main regression tests for contemporaneous independent variables. In particular, Table A.1 mimics Table 3, Table A.2 mimics Table 4, Table A.4 mimics Table 6, Table A.5 mimics Table 7, Table A.5 mimics Table 7, and Table A.6 mimics Table 8.

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Table A.1: **Spreads Horse Race**

OLS regression tests of the effect of the determinants of credit spreads. All independent variables are contemporaneous. The sample consists of bond month observations from the WRDS bond returns dataset for the years 2002-2019 for which we can measure bond ambiguity and credit spreads. Additional controls may further reduce the sample, especially where we require the issuing firm to have publicly traded equity. Bond Ambiguity is an estimate of the uncertainty of probabilities relevant for investors in the bond, estimated as detailed in the text. Equity Volatility measures the standard deviation of the issuing firm's equity return. Bond Volatility measures the standard deviation of the bond return. Equity Beta is an estimate of the systematic risk of the issuing firm's equity as measured by its CAPM Beta. Distance to Default is the estimate of the number of standard deviations of equity return the issuing firm is from defaulting on the bond. Z-Score is Altman's estimate of the credit worthiness of the issuing firm. (z) appearing after a variable name indicates the variable has been normalized by it own mean and standard deviation. (w,z) appearing after a variable name indicates the variable has been normalized by it own mean and standard deviation and Winsorized at the 1^{st} and 99^{th} percentiles. In parentheses are standard errors clustered by year-month and CUSIP, *p<0.1; **p<0.05; ***p<0.01.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Bond Ambiguity (z)	203.57***							
	(12.44)							
Equity Volatility (z)		193.86***						
		(15.40)						
Bond Volatility (z)			292.28***					
			(21.06)					
Equity Beta (z)				94.26***				
				(5.76)				
Distance to Default (w,z)					-131.94***			
					(7.98)			
Z-Score (w,z)						-91.45***		
						(6.30)		
Avg Bid/Ask Spread (w,z)							274.49***	
							(14.55)	
Equity Ambiguity (z)								-83.77***
								(10.83)
Constant	289.20***	244.85***	304.10***	249.40***	249.33***	248.23***	305.03***	246.82***
	(8.70)	(9.99)	(12.31)	(9.62)	(9.15)	(9.33)	(7.75)	(8.35)
R^2	0.121	0.164	0.240	0.040	0.075	0.038	0.183	0.032
Observations	431,196	372,057	406,900	351,458	351,489	370,615	419,437	370,525

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Table A.2: **Bond Ambiguity and Credit Spreads**

OLS regression tests of the effect of risk and ambiguity on credit spreads. All independent variables are contemporaneous. The sample consists of bond month observations from the WRDS bond returns dataset for the years 2002-2019 for which we can measure bond ambiguity and credit spreads. Additional controls may further reduce the sample, especially where we require the issuing firm to have publicly traded equity. Bond Ambiguity is an estimate of the uncertainty of probabilities relevant for investors in the bond, estimated as detailed in the text. Equity Ambiguity is an estimate of the uncertainty of probabilities relevant for investors in the issuing firm's equity, estimated as detailed in the text. Equity Volatility measures the standard deviation of the issuing firm's equity return. Bond Volatility measures the standard deviation of the bond return. Equity Beta is an estimate of the systematic risk of the issuing firm's equity as measured by its CAPM Beta. Distance to Default is the estimate of the number of standard deviations of equity return the issuing firm is from defaulting on the bond. Average Bid/Ask Spread for a bond is taken from the WRDS bond return data and reflects the monthly average of the differences between buy and sell transaction prices. Z-Score is Altman's estimate of the credit worthiness of the issuing firm. Time is a year by month fixed effect. Rating is a bond rating fixed effect. CUSIP is a bond fixed effect. Years to Mat is a years to maturity fixed effect. (z) appearing after a variable name indicates the variable has been normalized by it own mean and standard deviation. (w,z) appearing after a variable name indicates the variable has been normalized by it own mean and standard deviation and Winsorized at the 1^{st} and 99^{th} percentiles. In parentheses are standard errors clustered by year-month and CUSIP, *p<0.1; **p<0.05; ***p<0.01.

	(1)	(2)	(3)	(4)	(5)	(6)
Bond Ambiguity (z)	190.29***	53.56***	33.33***	20.45***		20.16***
	(11.17)	(4.68)	(4.13)	(3.10)		(3.08)
Equity Ambiguity (z)					-1.69	-1.88
					(1.37)	(1.33)
Equity Beta (z)				-10.00	-11.13	-10.16
				(27.27)	(27.17)	(27.28)
Bond Volatility (z)				55.71***	56.33***	55.11***
				(10.25)	(10.26)	(10.18)
Equity Volatility (z)				26.06**	28.58**	27.53**
				(8.90)	(9.00)	(8.89)
Z-Score (w,z)				-47.13***	-47.20***	-47.36***
				(9.83)	(9.90)	(9.91)
Distance to Default (w,z)				13.46**	13.99**	14.13**
				(5.15)	(5.14)	(5.13)
Avg Bid/Ask Spread (w,z)				95.06***	95.16***	94.33***
				(9.47)	(9.58)	(9.50)
Time	Y					
Time x Rating		Y	Y	Y	Y	Y
Time x Years to Mat		Y	Y	Y	Y	Y
CUSIP			Y	Y	Y	Y
R^2	0.157	0.695	0.803	0.822	0.821	0.822
Observations	431,196	362,383	362,290	266,145	265,233	265,233

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Table A.3: **Bond Ambiguity Interactions with Risk**

OLS regression tests of the interaction effect of risk and ambiguity on credit spreads. All independent variables are contemporaneous. The sample consists of bond month observations from the WRDS bond returns dataset for the years 2002-2019 for which we can measure bond ambiguity and credit spreads. Additional controls may further reduce the sample, especially where we require the issuing firm to have publicly traded equity. Bond Ambiguity is an estimate of the uncertainty of probabilities relevant for investors in the bond, estimated as detailed in the text. Equity Volatility measures the standard deviation of the issuing firm's equity return. Bond Volatility measures the standard deviation of the bond return. Controls include the issuing firm's equity beta, the issuing firm's Z-score, the issuing firm's distance to default measure, and the bond's average bid-ask spread as a measure of liquidity. Time is a year by month fixed effect. Rating is a bond rating fixed effect. CUSIP is a bond fixed effect. Years to Mat is a years to maturity fixed effect. (z) appearing after a variable name indicates the variable has been normalized by it own mean and standard deviation. In parentheses are standard errors clustered by year-month and CUSIP, $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$.

	(1)	(2)	(3)
Bond Ambiguity (z)	17.40***	23.03***	20.87***
	(2.81)	(3.31)	(3.22)
Equity Volatility (z)	15.42		13.91
	(7.83)		(7.87)
Bond Ambiguity (z) x Equity Volatility (z)	29.55***		11.70*
	(4.79)		(5.11)
Bond Volatility (z)		26.58**	24.86**
		(9.85)	(9.50)
Bond Ambiguity (z) x Bond Volatility (z)		42.21***	34.81***
		(5.92)	(6.35)
Default Controls	Y	Y	Y
Time x Rating	Y	Y	Y
Time x Years to Mat	Y	Y	Y
CUSIP	Y	Y	Y
R^2	0.822	0.825	0.826
Observations	282,532	266,174	266,174

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Table A.4: **Bond Ambiguity Interactions with Default Measures**

OLS regression tests of the interaction effect of default risk and ambiguity on credit spreads. All independent variables are contemporaneous. The sample consists of bond month observations from the WRDS bond returns dataset for the years 2002-2019 for which we can measure bond ambiguity and credit spreads. Additional controls may further reduce the sample, especially where we require the issuing firm to have publicly traded equity. Bond Ambiguity is an estimate of the uncertainty of probabilities relevant for investors in the bond, estimated as detailed in the text. IG is an indicator variable for the bond being Investment Grade. Distance to Default is the estimate of the number of standard deviations of equity return the issuing firm is from defaulting on the bond. Z-Score is Altman's estimate of the credit worthiness of the issuing firm. Controls include the issuing firm's equity beta, the bond's return volatility, the volatility of the return on the issuing firm's equity, and the bond's average bid-ask spread as a measure of liquidity. Time is a year by month fixed effect. Rating is a bond rating fixed effect. CUSIP is a bond fixed effect. Years to Mat is a years to maturity fixed effect. (z) appearing after a variable name indicates the variable has been normalized by it own mean and standard deviation. (w,z) appearing after a variable name indicates the variable has been normalized by it own mean and standard deviation and Winsorized at the 1^{st} and 99^{th} percentiles. In parentheses are standard errors clustered by year-month and CUSIP, *p<0.1; **p<0.05; ***p<0.01.

	(1)	(2)	(3)	(4)
Bond Ambiguity (z)	37.49***	18.54***	22.02***	18.87***
	(6.63)	(2.86)	(3.58)	(2.87)
Bond Ambiguity (z) x IG	-35.29***			
	(6.74)			
Distance to Default (w,z)		4.76		
		(4.86)		
Bond Ambiguity (z) x Distance to Default (w,z)		-19.86***		
		(3.29)		
Z-Score (w,z)			-37.24***	
			(9.03)	
Bond Ambiguity (z) x Z-Score (w,z)			-25.62***	
			(4.22)	
Bond Ambiguity (z) x Avg Bid/Ask Spread (w,z)				54.60***
				(6.25)
Volatility Controls	Y	Y	Y	Y
Time x Rating	Y	Y	Y	Y
Time x Years to Mat	Y	Y	Y	Y
CUSIP	Y	Y	Y	Y
R^2	0.820	0.823	0.820	0.826
Observations	282,819	266,872	281,599	282,819

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Table A.5: **Bond Ambiguity Interactions with Systematic Risk Measures**

OLS regression tests of the interaction effect of systematic risk and ambiguity on credit spreads. All independent variables are contemporaneous. The sample consists of bond month observations from the WRDS bond returns dataset for the years 2002-2019 for which we can measure bond ambiguity and credit spreads. Additional controls may further reduce the sample, especially where we require the issuing firm to have publicly traded equity. Bond Ambiguity is an estimate of the uncertainty of probabilities relevant for investors in the bond, estimated as detailed in the text. Equity Beta is an estimate of the systematic risk of the issuing firm's equity as measured by its CAPM Beta. Bond Beta is an estimate of the systematic risk of the bond as measured by the CAPM Beta of the bond. Controls include the bond's return volatility, the volatility of the return on the issuing firm's equity, the issuing firm's Z-score, the issuing firm's distance to default measure, and the bond's average bid-ask spread as a measure of liquidity. Time is a year by month fixed effect. Rating is a bond rating fixed effect. CUSIP is a bond fixed effect. Years to Mat is a years to maturity fixed effect. (z) appearing after a variable name indicates the variable has been normalized by it own mean and standard deviation. In parentheses are standard errors clustered by year-month and CUSIP, $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$.

	(1)	(2)
Bond Ambiguity (z)	19.22***	21.31***
	(3.05)	(3.22)
Bond Ambiguity (z) x Bond Beta (z)	34.06**	
	(11.00)	
Bond Ambiguity (z) x Equity Beta (z)		17.15***
		(3.39)
Controls	Y	Y
Time x Rating	Y	Y
Time x Years to Mat	Y	Y
CUSIP	Y	Y
R^2	0.821	0.823
Observations	262,316	266,145

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Table A.6: **Bond Index Ambiguity and Credit Spreads**

OLS regression tests of the effect of the ambiguity of bond portfolios on credit spreads. All independent variables are contemporaneous. The sample consists of bond month observations from the WRDS bond returns dataset for the years 2002-2019 for which we can measure bond ambiguity and credit spreads. Additional controls may further reduce the sample, especially where we require the issuing firm to have publicly traded equity. Bond Ambiguity is an estimate of the uncertainty of probabilities relevant for investors in the bond, estimated as detailed in the text. Short Treasury Ambiguity is an estimate of the ambiguity associated with a portfolio of short-term treasury bills. Medium Treasury Ambiguity is an estimate of the ambiguity associated with a portfolio of medium-term treasury notes. Long Treasury Ambiguity is an estimate of the ambiguity associated with a portfolio of long-term treasury bonds. IG Index is an estimate of the ambiguity associated with an index portfolio of investment grade bonds. Junk Index is an estimate of the ambiguity associated with an index portfolio of junk bonds. Rating is a bond rating fixed effect. CUSIP is a bond fixed effect. Years to Mat is a years to maturity fixed effect. Controls include issuing firm's equity beta, the bond's return volatility, the volatility of the return on the issuing firm's equity, the issuing firm's Z-score, the issuing firm's distance to default measure, and the bond's average bid-ask spread as a measure of liquidity. In parentheses are standard errors clustered by year-month and CUSIP, $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$.

	(1)	(2)	(3)	(4)
Bond Ambiguity (z)			69.55***	43.04***
			(8.98)	(6.45)
Short Treasuries (z)	-17.17*		-18.07*	
	(8.04)		(7.54)	
Medium Treasuries (z)	-43.91***		-35.30***	
	(9.41)		(7.92)	
Long Treasuries (z)	-2.95		-4.44	
	(7.40)		(6.50)	
IG Index (z)		-17.88**		-16.24**
		(5.31)		(5.06)
Junk Index (z)		-16.15**		-14.22**
		(4.90)		(4.62)
Controls	Y	Y	Y	Y
Rating	Y	Y	Y	Y
Years to Mat	Y	Y	Y	Y
CUSIP	Y	Y	Y	Y
R^2	0.681	0.678	0.688	0.681
Observations	170,193	239,905	170,193	239,905

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