Economic modelling 1:
Tutorial Session 1
Full differential and implicit differentiation
to be prepared for October 9, 2020

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Exercise I - Full differential

Write the full differential of the following functions:

1. $f(x) = ln(g(x))$
2. $f(x, y) = x^2 e^{-3y}$
3. $f(x, y, z) = xy^3 + xln(z) - 5$
4. $f(x, y) = ln\left(\frac{g(x)}{h(y)^2}\right)$
5. $f(x, y, z) = \frac{g(x,y)}{h(z)h(x)}$

Exercise II - Economic effect of a sale tax

We study the effect of a sale tax in a partial equilibrium model. We denote CP the reciprocal demand function of the consumers ($CP(q)$ is the maximum price consumers are ready to pay to buy a quantity q of good. We denote FP the reciprocal supply function of the firms ($FP(q)$ is the minimum price required by firms to sell the quantity q. The state earns a sell tax t on each unit bought by the consumers. Thus the equilibrium quantity on this market is given by the following one equation model.

$$CP(q) - FP(q) = t$$

Questions:

1. Taking the equilibrium level on this market q^* as an implicit function of the sell tax t, with $t \in [0, CP(0) - FP(0)]$, give the derivative of the function q^*
2. We denote T the total amount of sell axe collected on this market. Express T as a function of $t \in [0, CP(0) - FP(0)]$. Does this function admit a maximum ?

Exercise III - Two period inter-temporal choice model

A consumer living two periods (denoted 0 and 1) has to plan her consumption in both period (c_0 and c_1) under her Life cycle budgetary constraint. The intertemporal preferences of the agent can be represented by an additively separable utility function. The decision can thus be modelled as the solution of the following program:

$$\left\{ \begin{array}{c} \max\limits_{c_0, c_1} u(c_0) + \frac{u(c_1)}{1+\theta} \\ s.t.\ c_0 + \frac{c_1}{1+r} = w_0 + \frac{w_1}{1+r} \end{array} \right.$$

with u assumed to be increasing and concave, θ, the subjective discount rate, w_i, the income in period i, and r, the rate of interest. The necessary conditions of optimality for the consumer's program are summarized in the following two equation system:

$$(S) \quad \left\{ \begin{array}{lr} \frac{u'(c_0)}{u'(c_1)} = \frac{1+r}{1+\theta} & \text{(Euler Equation)} \\ c_0 + \frac{c_1}{1+r} = w_0 + \frac{w_1}{1+r} & \text{(Lice-Cycle Budgetary Constraint)} \end{array} \right.$$

Questions: The system (S) define c_0 and c_1 as implicit functions of w_0, w_1 and r. Using the full differentiat method, characterize the partial derivative of these implicit functions? Comment. What is particularly remarkable concerning $\partial c_0 / \partial w_0$?

Exercise IV - The IS-LM model

We study the following program:

$$\left\{ \begin{array}{lr} Y = C(Y-T) + I(i) + G & (IS) \\ \frac{M}{p} = L(Y-T, i) & (LM) \end{array} \right.$$

with Y, the national income, C, the final consumption function of the disposable income, T, the Income Tax, i, the nominal rate of interest, M, the money supply, L, the real demand for money, function of the disposable income and the nominal interest rate, p the price level. We assume that p is constant, that $C' > 0$, $L'_Y > 0$, $L'_i < 0$ and that : M, T and G are exogenous policy variables.

Questions:

1. What are the endogenous variable of this model ?
2. Write $(\mathrm{d}IS)$ the full differential of the (IS) identity. When T and G are assumed constant, Express $\frac{\mathrm{d}i}{\mathrm{d}Y}\big|_{IS}$. What is the interpretation of this last expression ?
3. Same questions for $(\mathrm{d}LM)$
4. Considered the endogenous variable as implicit functions of the policy variables. After solving the $(\mathrm{d}IS)$ $(\mathrm{d}LM)$ system, express the partial derivatives of these implicit functions. Interpret them.
5. Let us consider the case of an increase of the public spending financed by a rise of a same amount of the income tax. To what is equal $\frac{\mathrm{d}Y}{\mathrm{d}G}$ in this case ? Comment.