LINEAR MODELS

EDWARD O'CALLAGHAN

Contents

1. Prelude

TODO: Fix notation here...

2. Introduction

In this course we builds up the rudiments of some important notions of algebraic structures. That is, a algebraic structure of an arbitrary set, or carrier set, coupled with various finitary operations defined on it. ..

3. Semi-groups

Definition 3.1 (binary operation)**.** A **binary operation** on a set $\mathcal{G}$ is a map $\circ : \mathcal{G} \times \mathcal{G} \to \mathcal{G}$. **N.B.** that the binary operation is *closed*.

Definition 3.2 (magma)**.** A **magma** is a set $\mathcal{M}$ equipped with a binary operation $\circ$. We denote the magma as the tuple pair $(\mathcal{M}, \circ)$.

Definition 3.3 (semi-group)**.** A **semi-group** is a set $\mathcal{G}$ equipped with binary operation that is *associative*. Hence, a semi-group is a magma where the operation is *associative*; That is, given any $x, y, z \in \mathcal{G}$ then $x \circ (y \circ z) = (x \circ y) \circ z \in \mathcal{G}$. We denote the semi-group as the tuple pair $(\mathcal{G}, \circ)$, not to be confused with a magma from context.

Definition 3.4 (monoid)**.** A **semi-group with idenitity** or, **monoid** for short, is a semi-group $(\mathcal{G}, \circ)$ with a unquie elememt $e \in \mathcal{G}$ such that $x \circ e = x = e \circ x \, \forall x \in \mathcal{G}$

Example 3.5. Given $\mathcal{G} = \mathbb{Z}$ with the binary law of composition $\circ$ to be defined as arithmetic addition $+$. Then, $(\mathbb{Z}, +)$ forms a semi-group with idenitity 0. Verify the axioms.