CATEGORY THEORY

EDWARD O'CATLAGHAN

Contents

1. Introduction

We begin by defining what we mean by a Functor.

Definition 1.1 (Functor)**.** Let $\mathcal{C}$ and $\mathcal{K}$ be categories. A *functor* $\mathcal{K}$ from $\mathcal{C}$ to $\mathcal{K}$ is a mapping that:

(1) associates to each object $X \in \mathcal{C}$ an object $\mathcal{K}(X) \in \mathbb{K}$
(2) associates to each morphism $f : X \to Y \in \mathcal{C}$ a morphism $\mathcal{K}(f)i : \mathcal{K}(X) \to \mathcal{K}(Y) \in \mathbb{K}$ satisfying:
   (a) $\mathcal{K}(id_X) = id_{\mathcal{K}(X)}$ for every object $X \in \mathcal{C}$
   (b) $\mathcal{K}(g \circ f) = \mathcal{K}(g) \circ \mathcal{K}(f)$ for all morphisms $f : X \to Y$ and $g : Y \to Z$

Remark. That is, functors must preserve identity morphisms and composition of morphisms.

Abstracting again, a category is itself a type of mathematical structure, so we can look for ”processes” which preserve this structure in some sense; such a process is called a functor. A functor associates to every object of one category an object of another category, and to every morphism in the first category a morphism in the second.

In fact, what we have done is define a category of categories and functors the objects are categories, and the morphisms (between categories) are functors.

By studying categories and functors, we are not just studying a class of mathematical structures and the morphisms between them; we are studying the relationships between various classes of mathematical structures.