CATEGORY THEORY

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1. Introduction

In category theory, a branch of mathematics, a natural transformation provides a way of transforming one functor into another while respecting the internal structure (i.e. the composition of morphisms) of the categories involved. Hence, a natural transformation can be considered to be a "morphism of functors". Indeed this intuition can be formalized to define so-called functor categories. Natural transformations are, after categories and functors, one of the most basic notions of category theory and consequently appear in the majority of its applications.

Definition 1.1 (Natural Transformation)**.** Let F and G be functors between the categories $\mathcal{C}$ and $\mathcal{K}$, then a *natural transformation* $\eta : F \to G$ associates to every object $X \in \mathcal{C}$ a morphism $\eta_X : F(X) \to G(X)$ between objects of $\mathcal{K}$, called the *component* of η at X, such that for every morphism $f : X \to Y \in \mathcal{C}$ we have:
$\eta_Y \circ F(f) = G(g)\eta_X$

Abstracting yet again, constructions are often "naturally related" a vague notion, at first sight. This leads to the clarifying concept of natural transformation, a way to "map" one functor to another. Many important constructions in mathematics can be studied in this context. "Naturality" is a principle, like general covariance in physics, that cuts deeper than is initially apparent.