CATEGORY THEORY

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1. Introduction

We begin by defining what we mean by a Functor.

Definition 1.1 (Functor)**.** Let $\mathcal{C}$ and $\mathcal{K}$ be categories. A *functor* F from $\mathcal{C}$ to $\mathcal{K}$ is a mapping that:

(1) associates to each object $X \in \mathcal{C}$ an object $F(X) \in \mathcal{K}$
(2) associates to each morphism $f : X \to Y \in \mathcal{C}$ a morphism $F(f) : F(X) \to F(Y) \in \mathcal{K}$ satisfying:
  (a) $F(id_X) = id_{F(X)}$ for every object $X \in \mathcal{C}$
  (b) $F(g \circ f) = F(g) \circ F(f)$ for all morphisms $f : X \to Y$ and $g : Y \to Z$

Remark. That is, functors must preserve identity morphisms and composition of morphisms.

Abstracting again, a category is itself a type of mathematical structure, so we can look for "processes" which preserve this structure in some sense; such a process is called a functor. A functor associates to every object of one category an object of another category, and to every morphism in the first category a morphism in the second.

In fact, what we have done is define a category of categories and functors the objects are categories, and the morphisms (between categories) are functors.

By studying categories and functors, we are not just studying a class of mathematical structures and the morphisms between them; we are studying the relationships between various classes of mathematical structures.