Latent Factor Model for Momentum
Bayesian Version

1 Model

State-space representation:

Observation equation:

$$r_t = H'\xi_t + e_t = [0 \quad B \quad B]' \begin{pmatrix} \lambda_t \\ \lambda_{t-1} \\ f_t \end{pmatrix} + e_t$$

$$e_t \sim \mathcal{N}(0, \Sigma_e), \Sigma_e \text{ diagonal}$$

Transition equation:

$$\xi_t = \alpha + F\xi_{t-1} + \omega_t \Rightarrow \begin{pmatrix} \lambda_t \\ \lambda_{t-1} \\ f_t \end{pmatrix} = \begin{bmatrix} (I-\Phi)\mu \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \Phi & 0 & 0 \\ I_K & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \lambda_{t-1} \\ \lambda_{t-2} \\ f_{t-1} \end{pmatrix} + \begin{pmatrix} v_t \\ 0 \\ f_t \end{pmatrix}$$

$$\omega_t \sim \mathcal{N}(0, \mathbf{\Sigma}), \mathbf{\Sigma} = \begin{bmatrix} \Sigma_v & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Sigma_f \end{bmatrix}, \Sigma_v, \Sigma_f \text{ diagonal}$$

2 Procedure

First, initialize $\{\xi_t\}$ using principal components and data available for the entire sample. Based on these estimates of $\{\xi_t\}$, get initial values of F, H, Σ_e and $\mathbf{\Sigma}$ (specifically $\Phi, B, \Sigma_e, \Sigma_v, \Sigma_f$). Then each iteration of the Gibbs sampler is as follows:

1. Conditional on F, H, Σ_e and $\mathbf{\Sigma}$, draw $\{\xi_t\}$ using the Carter-Kohn (1994) procedure with the generalization that allows $\mathbf{\Sigma}$ to be singular.
2. Conditional on $\{\xi_t\}$, draw F, H, Σ_e and $\mathbf{\Sigma}$.
3. Data augmentation: Conditional on $\{\xi_t\}, H$ and Σ_e, sample $\{r_t\}$ for those with missing values.