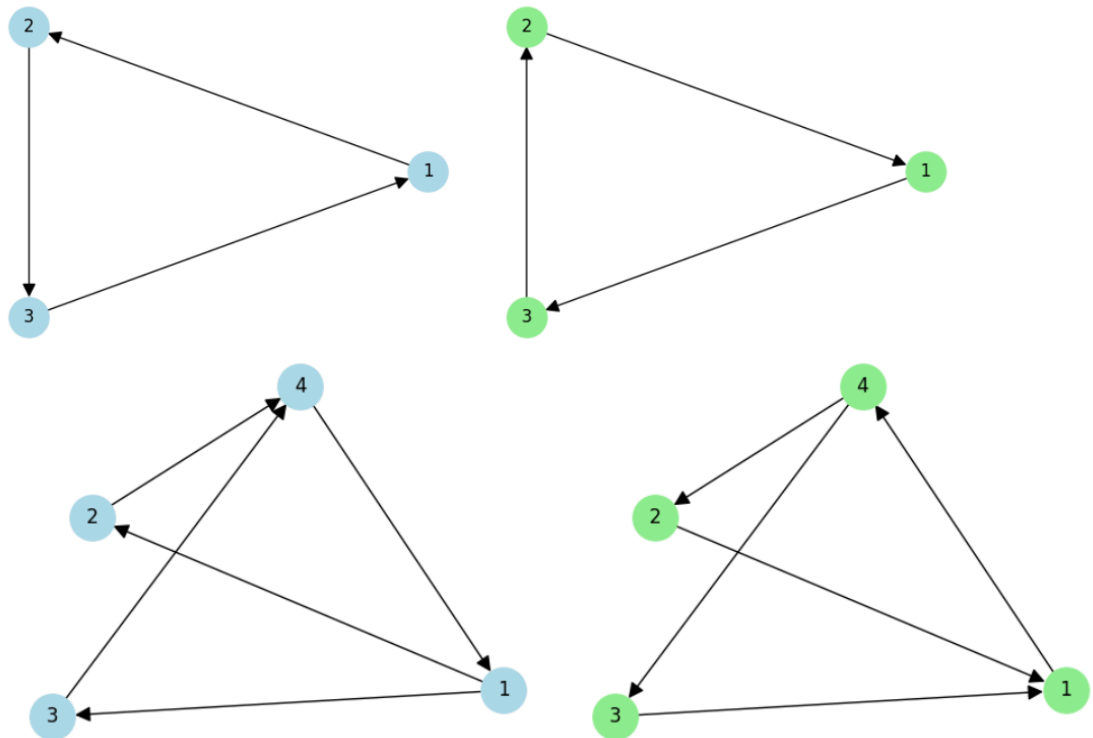


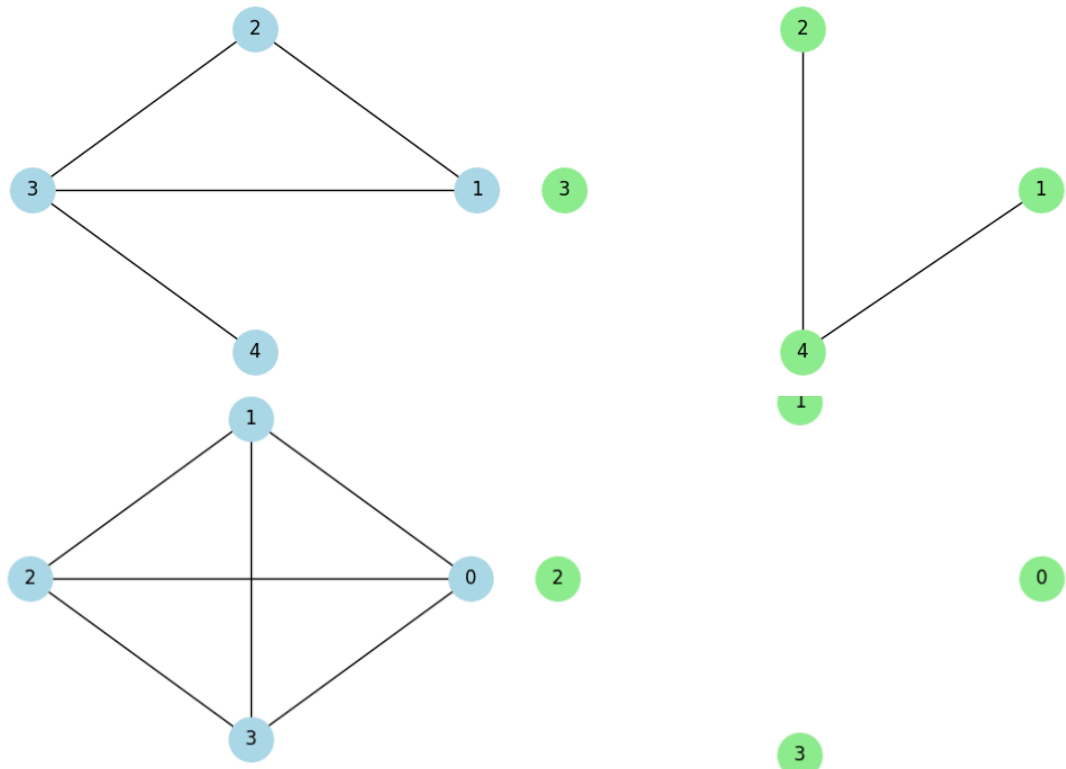
Problem 1:

1. The transposed graph (G^T) of a directed graph G is obtained by reversing the direction of every edge.

blue - directed original, green - transposed

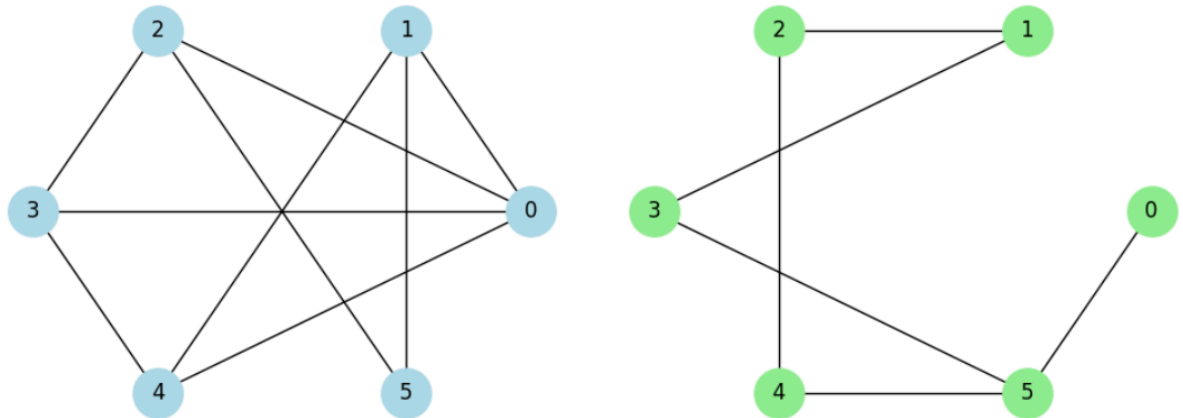


2. The complement graph (G^c or G') of an undirected graph G has the same vertices. Two vertices are connected by an edge in G^c if and only if they are NOT connected in G . blue - directed original, green - inverse

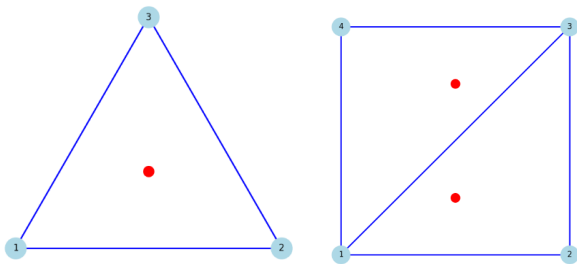


3. A dense graph is a graph that already contains most of the possible edges. The inverse (complement) graph keeps the same set of vertices but includes only the edges that are missing in the original graph.

Therefore: if the original graph is dense, its inverse graph will be sparse, because very few edges are missing, and only those missing edges appear in the inverse graph.



4. A dual graph is a concept used for planar graphs — graphs that can be drawn on a plane with no edges crossing.



5. The dual graph is intrinsically linked to the combinatorial structure of faces in a planar drawing. Since a non-planar graph like this cannot be embedded in the plane without edge crossings, it lacks a well-defined set of faces. Consequently, a geometric dual graph cannot be constructed for it. In abstract terms, a graph has a dual if and only if it is planar (as per Whitney's theorem).

