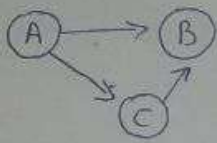


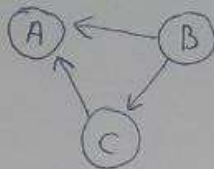
EXO 7

Problem 1: Draw a few undirected graphs and a few directed graphs

1. Directed graph \rightarrow transpose



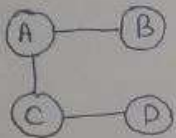
original



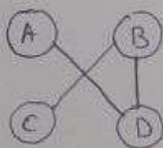
Transpose
reversed all arrow

Transposed graph = reverse all arrow directions. If original $A \rightarrow B$, transpose: $B \rightarrow A$
Transpose helps us find paths going backwards. If you want to know "who points to A", look at a transpose to see which vertices A points to.

2. Undirected graph \rightarrow inverse



original



inverse

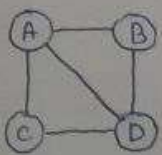
Inverse = complement, remove existing edges
add missing edges between all vertices

original $A-B \Rightarrow$ NOT $A-B$

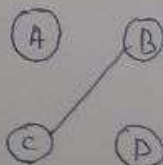
original missing $B-D \Rightarrow$ WILL $B-D$

Sometimes we want to find vertices that are NOT connected. Inverse shows us their clear

3. Dense graph \rightarrow inverse



original (4 vertices, 5 edges)



inverse

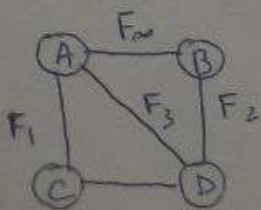
If original is dense (many edges), the
inverse graph become sparse few edges.

Dense \rightarrow Sparse

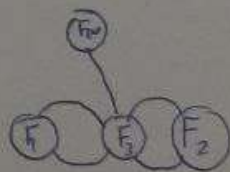
if original will have 5 vertices \rightarrow completed

Dense and sparse are opposite. If one is
big other is small, they balance each other

4. Dual graph



original



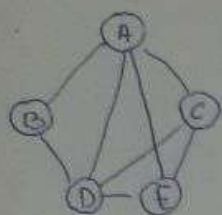
Dual (each face \rightarrow vertex)

Dual graph: each face (region) become a vertex.
If two faces share an edge, connect their vertex.
Face is region (area) in graph surrounded by edge
like room with walls

Dual helps us to solve problems about
region by turning them into vertex problems

5. Why dual need plane graph?

non planer graph

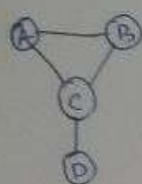


non planer cannot be drawn without edges crossing, so face are not well defined

dual graph only work for planar graphs where face are well-defined

when edges cross we cannot tell which region is which
only planer graphs can have dual graph. Non planer not.

Problem 2: Consider the following undirected graph G with vertices $V = \{A, B, C, D\}$ and edges $E = \{AB, AC, BC, CD\}$



graph = $\{ A: [B, C], B: [A, C], C: [A, B, D], D: [C] \}$

1. initial call to the algorithm

- R (the current clique being built): $[\]$ - empty set.
- P - the set of potential candidates to extend the clique: $[A, B, C, D]$
- X - the set of excluded vertices: $[\]$

2. First recursive call:

we need to try each vertex from P and make recursive call to extend R .

we choose A from P

$R = [A]$

$P = [B, C]$

$X = [\]$

recursive 1 with A :

$R = [A]$

$P = [B, C] \Rightarrow$

$X = [\]$

B :

$R = [A, B]$

$P = [C]$

$X = [\]$

recursive 2 with A and B :

$R = [A, B]$

$P = [C]$

$X = [\]$

C :

$\Rightarrow R = [A, B, C]$

$P = [\]$ no candidates

$X = [\]$

\Rightarrow The maximal clique: $[A, B, C]$

3. Second recursive call from the maximal clique.

After we return from the first recursive branch, we move to other vertices from P .
After many operation all branch, we find maximum clique.

Maximal clique of G :

$[A, B, C]$ - has 3 vertices.

$[C, D]$.

This is simple explanation Bron-Kerbosch algorithm works