EXO_5

Problem 1 (Graph and Tree Definitions, 5/10 pts). Prove that the following definitions are all equivalent:

1. A tree is a connected acyclic graph.
2. A tree is one component of a forest. (A forest is an acyclic graph.)
3. A tree is a connected graph with at most V - 1 edges.
4. A tree is a minimally connected graph; removing any edge disconnects the graph.
5. A tree is an acyclic graph with at least V - 1 edges.
6. A tree is a maximally acyclic graph; adding an edge between any two vertices creates a cycle.
7. A tree is a graph that contains a unique path between each pair of vertices.

Understanding Tree Definitions

A **tree** is a special type of graph. All the definitions of a tree are equivalent they just describe the same idea in different ways.

Step-by-Step Explanation

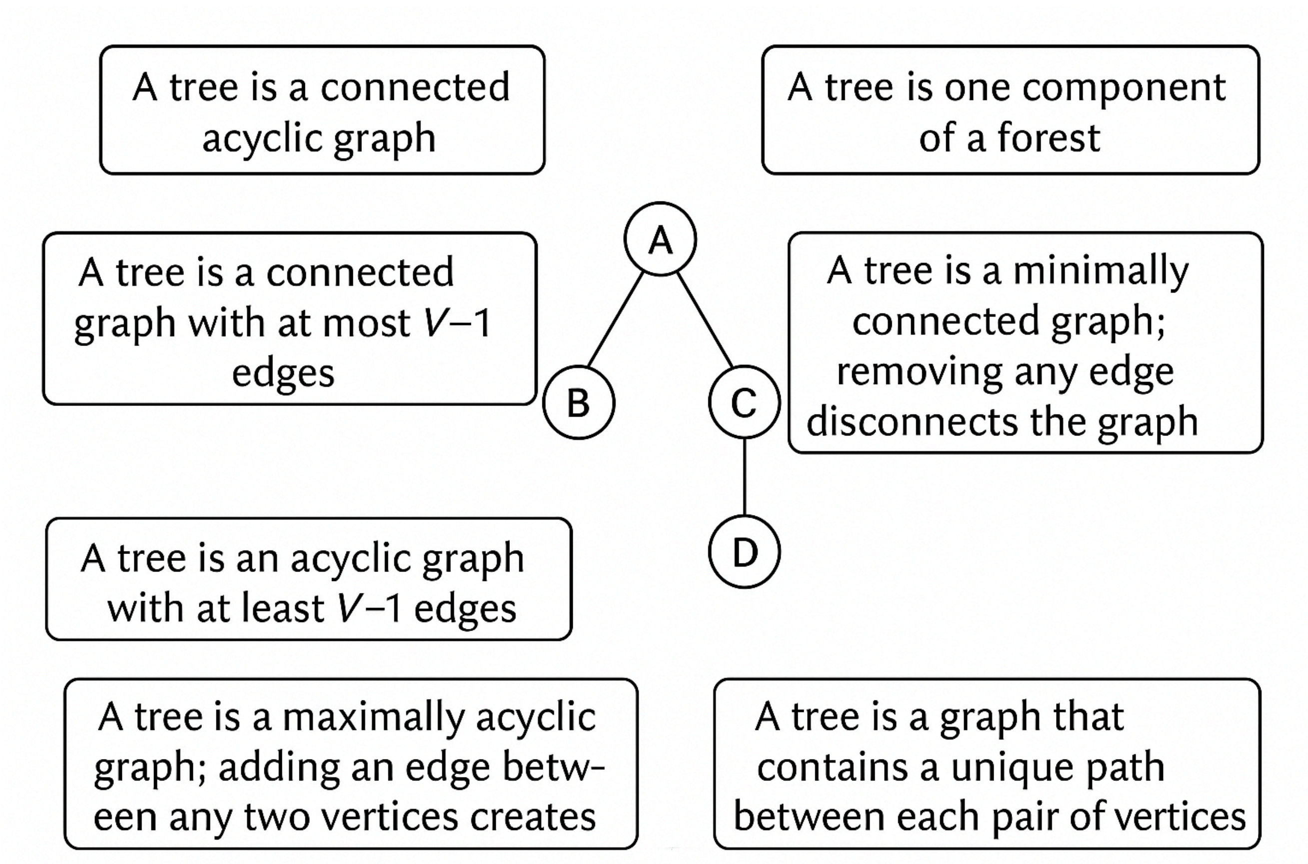
1. **Connected and acyclic is component of a forest**
   - A forest is a graph with no cycles.
   - If a graph is connected and has no cycles, it is one part (component) of a forest.
2. **Connected and acyclic is V-1 edges**
   - A connected graph without cycles and with V vertices always has exactly V-1 edges.
   - So “connected with at most V-1 edges” also describes a tree.
3. **V-1 edges is minimally connected**
   - If a graph has exactly V-1 edges and is connected, removing any edge will make it disconnected.
   - This is called “minimally connected.”
4. **Acyclic with ≥ V-1 edges is maximally acyclic**
   - A graph without cycles cannot have more than V-1 edges.
   - Therefore, if it has exactly V-1 edges, adding any new edge will create a cycle.
   - This is called “maximally acyclic.”
5. **Connected and acyclic is unique path between any two vertices**
   - If there were two paths between the same pair of vertices, a cycle would exist.
   - So in a tree, every two vertices are connected by exactly one path.
6. **Unique path is connected and acyclic**

- If there is exactly one path between each pair of vertices, the graph is connected and has no cycles.

Conclusion

All seven definitions describe the same structure. The main idea is:

- The graph is **connected**.
- The graph has **no cycles**.
- From these, other properties (V-1 edges, unique paths, minimal/maximum properties) follow automatically.



problem 2