Fundamental Algorithm Techniques

Exo#7

Aidana Muratbekova

Problem 1. Draw a few undirected graphs and a few directed graphs

1) Directed graph and its transposed graph

Let V = {A, B, C} E = {AB, BC}

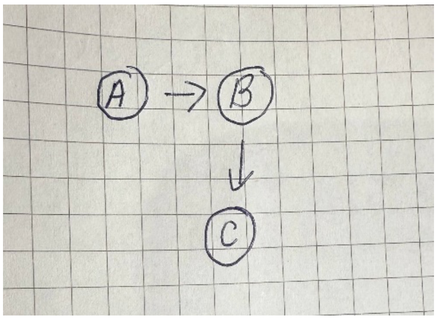


Figure 1. Directed graph

Let V = {A, B, C} E = {AB, BC}

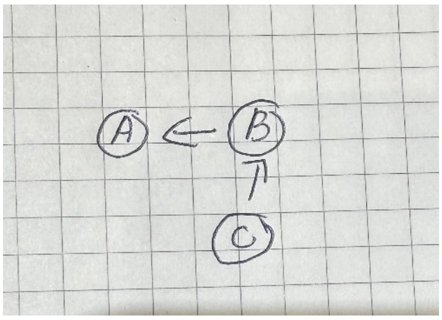


Figure 2. Transposed graph

The transposed graph is obtained by reversing the direction of all edges

2) Undirected graph and its inverse graph

Let V = {A, B, C} E = {AB, AC}

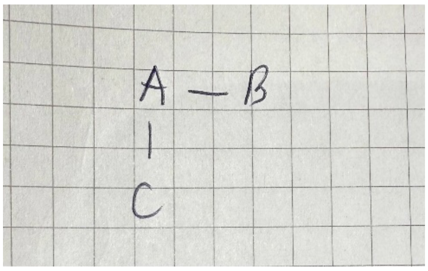


Figure 3. Undirected graph

For undirected graphs, the inverse graph is the same as the original graph.

3) What happens if the original graph is dense for the inverse?

If the original graph is dense, the inverse (transposed) graph is also dense, because the number of edges does not change, only their direction.

4) Undirected graph and its dual graph

Let V = {A, B, C} E = {AB, AC, BC}

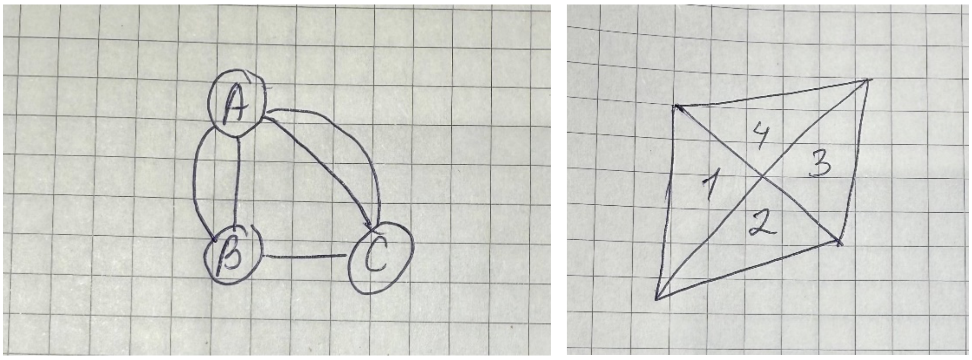


Figure 5. Planar undirected graph and Dual graph

Each vertex in the dual graph represents a face of the planar graph

5) Why is the dual graph only well-defined for planar graphs?

The dual graph is only well-defined for planar graphs because only planar graphs have clearly defined faces.

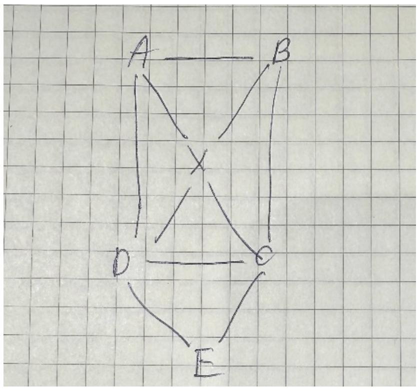
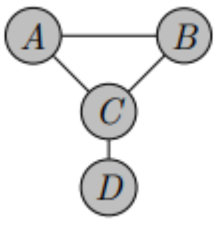


Figure 6. Non-planar graph (K_5)

This graph is non-planar and cannot be drawn without edge crossings. Therefore, its faces are not well-defined and a dual graph does not exist.

Problem 2. Consider the following undirected graph G with vertices V = {A, B, C, D} and edges: E = {AB, AC, BC, CD}



The graph can be represented as an adjacency list:

graph = { "A": ["B", "C"], "B": ["A", "C"], "C": ["A", "B", "D"], "D": ["C"] }

1) Initial call (starting values of R, P, X)

Bron–Kerbosch starts with:

- $R = \emptyset$
- $P = \{A, B, C, D\}$
- $X = \emptyset$

So the initial call is:

$$BK(R = \emptyset,\ P = \{A, B, C, D\},\ X = \emptyset)$$

2) Trace the first two recursive calls that lead to reporting a maximal clique

(We iterate vertices in order: A, B, C, D.)

Start $(R, P, X) = (\emptyset, \{A, B, C, D\}, \emptyset)$

Choose v = A (1st recursive call)

$$\begin{aligned} R_1 &= R \cup \{A\} = \{A\} \\ P_1 &= P \cap N(A) = \{A, B, C, D\} \cap \{B, C\} = \{B, C\} \\ X_1 &= X \cap N(A) = \emptyset \end{aligned}$$

So: $(\{A\}, \{B, C\}, \emptyset)$

Choose v = B (2nd recursive call)

$$\begin{aligned} R_2 &= \{A\} \cup \{B\} = \{A, B\} \\ P_2 &= \{B, C\} \cap N(B) = \{B, C\} \cap \{A, C\} = \{C\} \\ X_2 &= \emptyset \end{aligned}$$

So: $(\{A, B\}, \{C\}, \emptyset)$

Next choose v = C → report clique

$$\begin{aligned} R_3 &= \{A, B, C\} \\ P_3 &= \{C\} \cap N(C) = \{C\} \cap \{A, B, D\} = \emptyset \\ X_3 &= \emptyset \end{aligned}$$

Now $P = \emptyset$and $X = \emptyset$, so we **report a maximal clique**: $\boxed{\{A, B, C\}}$

3) List all maximal cliques of G. Which one(s) are maximum?

All **maximal cliques** in this graph are:

- $\{A, B, C\}$(triangle)
- $\{C, D\}$(edge CD)

Maximum clique (largest size) is:

- $\boxed{\{A, B, C\}}$because its size is 3 (bigger than 2)