Fundamental Algorithm Techniques

Problem Set #5

Review on November 08

Problem 1 (Graph and Tree Definitions, 5/10 pts)**.** *Prove that the following definitions are all equivalent:*

1. *A tree is a connected acyclic graph.*
2. *A tree is one component of a forest. (A forest is an acyclic graph.)*
3. *A tree is a connected graph with at most $V - 1$ edges.*
4. *A tree is a minimally connected graph; removing any edge disconnects the graph.*
5. *A tree is an acyclic graph with at least $V - 1$ edges.*
6. *A tree is a maximally acyclic graph; adding an edge between any two vertices creates a cycle.*
7. *A tree is a graph that contains a unique path between each pair of vertices.*

Problem 2 (Sparse representation of graphs, 5/10 pts)**.** *Given the following CSC (Compressed Sparse Column) representations for two graphs on vertices $\{A, B, C, D, E\}$ (indexed as $A \to 0$, $B \to 1$, $C \to 2$, $D \to 3$, $E \to 4$):*

Graph 1 (undirected):

```
col_pointers = [0, 2, 5, 8, 11, 12]
row_indices = [1, 2, 0, 2, 3, 0, 1, 3, 1, 2, 4, 3]
values = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]
```

Graph 2 (directed):

```
col_pointers = [0, 0, 2, 4, 5, 7]
row_indices = [0, 3, 0, 1, 2, 1, 3]
values = [1, 1, 1, 1, 1, 1, 1]
```

For each graph, reconstruct: (a) the adjacency matrix, and (b) a clear diagram of the graph (layout and style of your choice), (c) what is the unique cycle in directed graph (Describe it as e.g. X –> Y –> Z –> X).