

## Problem Set 7.

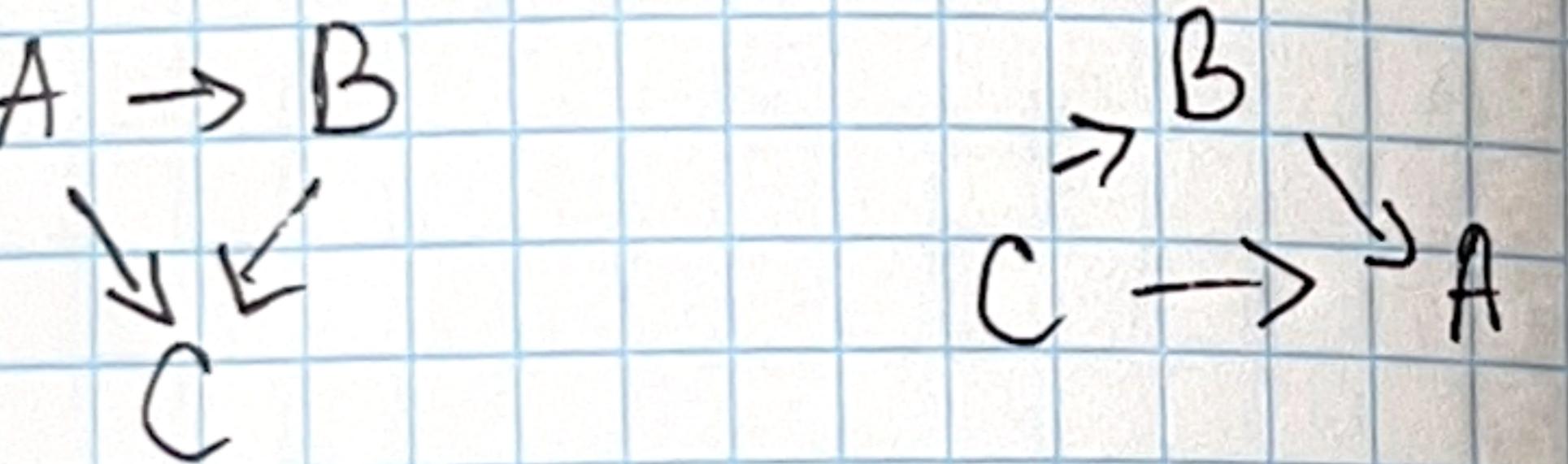
### Problem 1:

#### 1. Directed Graphs and their transposes

For a directed graph  $G = (V, E)$ , the transpose  $G^T$  is formed by reversing the direction of every edge.

Formally, if  $(u, v) \in E$ , then  $\underline{(v, u)} \in E^T$

Example 1. Original graph  $G$       Transpose  $G^T$



Example 2. Original  $H$       Transpose  $H^T$

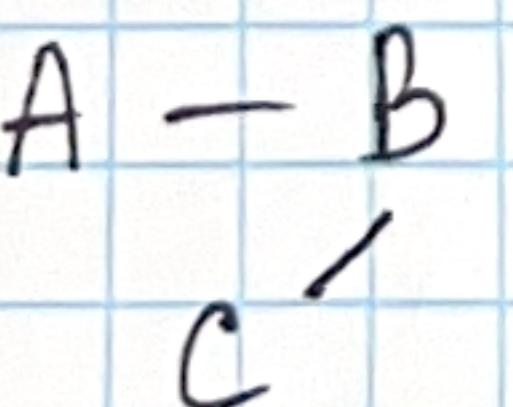


#### 2. Undirected graphs and their inverse graphs

The inverse graph of an undirected graph has an edge exactly where the original does not.

Example 1.

Original graph  $G$  on  $\{A, B, C\}$ ,  $E = \{AB, BC\}$



Inverse  $G^{-1}$  edges =  $AC$

Example 2:

$H$  on  $\{1, 2, 3, 4\}$       Edges - 12, 23, 34

Inverse  $H^{-1} = \underline{13, 14, 24}$

3. What happens if the original graph is dense for the inverse?

If the original graph is dense, meaning close to complete, its inverse becomes sparse; possibly even empty, if the original is a complete graph.

Example: If  $G$  is  $K_4$ , a complete graph on

4 vertices  $\rightarrow 6$  edges

Then the inverse has 0 edges.

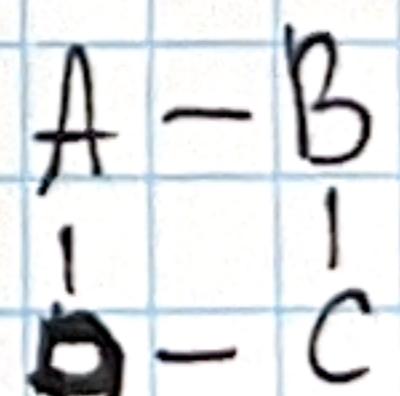
4. Simple examples of undirected graph and their dual graph

Dual graph is formed for planar embeddings:

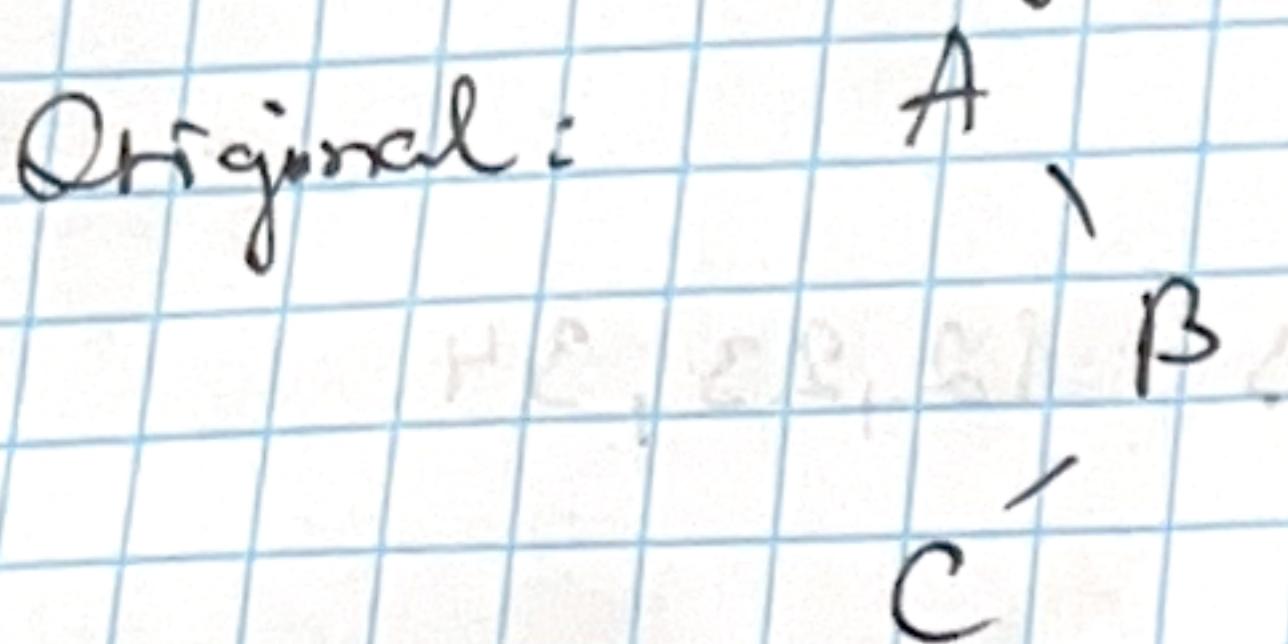
Each face becomes a vertex, and edges represent adjacency between faces:

Example: Square

Original planar graph:



Example: Triangle



faces:  $f_{in}$  = inside triangle  
 $f_{out}$  = outside

5. Why is the dual only well-defined for planar graphs

Dual graphs require:

- A clear definition of faces
- Non-planar graphs cannot be drawn without crossing edges.

Example: Non-planar graph  $K_{3,3}$

Nodes on left: {a, b, c}

Nodes on right: {x, y, z}

Edges: all bipartite connections

## Problem 2

Graph:  $V = \{A, B, C, D\}$   $E = \{AB, AC, BC, CD\}$

1. Initial Call:  $R = \emptyset$ ,  $P = \{A, B, C, D\}$ ,  $X = \emptyset$

$$R = \emptyset$$

$$P = \{A, B, C, D\}$$

$$X = \emptyset$$

2. Trace first two recursive calls leading to a maximal clique

$$R = \{A\} \quad P = \{B, C\} \quad X = \emptyset - \text{Choose } A$$

$$R = \{A, B\} \quad P = \{C\} \quad X = \emptyset - \text{Choose } B$$

$$R = \{A, B, C\} \quad P = \emptyset \quad X = \emptyset - \text{Choose } C$$

This is the first maximal clique

3. List all maximal cliques & identify maximum

All maximal cliques

1.  $\{A, B, C\}$

2.  $\{C, D\}$

Maximum clique size=3