Fundamental Algorithm Techniques

Problem Set #9

Review on December 6

Problem 1 (Finite Function on the computer, 3/10 pts)**.** *Finite functions can be described on the computer as:*

$$F : \{0,1\}^n \longrightarrow \{0,1\}^m,$$

Can you show that for:

- *an output* $\{0,1\}$ *there are* 2^{2n} *possible such functions to realise all the possibilities.*
- *an output* $\{-1,0,1\}$, 3^{2n}
- *an output* $\{0,1\}^m$, $2^{m \cdot 2^n}$

It could help to work with a decision trees model (n, m *leaves and labels resp.).*

Problem 2 (Equivalence NAND $\Rightarrow$ AND, OR & NOT, 4/10 pts)**.** *Draw 3 linear codes to generate AND, OR, NOT from NAND (or* $\uparrow$*).*

A	B	$A \uparrow B$
0	*0*	*1*
0	*1*	*1*
1	*0*	*1*
1	*1*	*0*

This motivates the following Theorem: NAND is a universal operation. For each circuit C *of* n *components, there is a NAND equivalent using at most* $3n$ *components.*

Problem 3 (Universality of Boolean Circuits, 3/10 pts)**.** *Consider*

$$F : \{0,1\}^n \longrightarrow \{0,1\},$$

For all $x \in \{0,1\}^n$, *use that there exists a function* $\delta_x \Rightarrow \{0,1\}$. *Explain that the function can be realised by a circuit of size* $\mathcal{O}(n)$ *and therefore,* F *is computable using* $\mathcal{O}(n \cdot 2^n)$ *circuits.*