

# Fundamental Algorithm Techniques

## Problem Set #10

Review on December 14

**Problem 1** (P, NP, NP-complete, NP-hard, 4/10). Classify the problems below into their corresponding class or classes (each line has same class).

1. find max, linear search, shortest path in unweighted graph, matrix multiplication
2. sorting of list, Dijkstra on non-negative weights, BFS, DFS, merge sort, quicksort
3. sudoku
4. 3 coloring of graph, scheduling with conflicts
5. Traveling Salesperson Problem, Hamiltonian Cycle, Clique
6. Cryptography, factoring large integers
7. Halting Problem<sup>1</sup>, busy beaver

**Problem 2** (Intro to Bayes, 3/10 pts). You are a doctor. A deadly disease affects 0.1% of people. Your test is 99% accurate (both sensitivity and specificity). A patient tests positive.

What is the probability he actually has the disease?

Answer is 9%! why?

**Problem 3** (Intro to Shannon Entropy, 3/10). Three coins:

Coin	$P(\text{Heads})$	Surprise when heads appears
A	50%	Medium
B	99%	Tiny
C	1%	HUGE!

Use the Shannon Entropy to discuss the bits required to describe each above coins:

$$H(X) = - \sum_i^n p_i \log_2 p_i,$$

whereas  $n$  is the number of all possible outcome<sup>2</sup>. Why is a fair coin “worth” 1 bit, but a 99% biased coin only 0.08 bits?

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<sup>1</sup>Halting Problem: a program that can determine, for any given program and its input, whether that program will eventually stop or run forever.

<sup>2</sup>For alphabet it would be  $\approx 26$ , here it is 2.