Problem Set #7 – Solutions
Problem 1. Graph Play
1. Directed graphs and transposed graphs.
Example:
Original: A -> B, B -> C
Transposed: B -> A, C -> B
In a transposed graph, all edge directions are reversed.
2. Undirected graphs and inverse graphs.
The inverse (complement) graph contains exactly the edges that are missing in the original graph.
Example:
Vertices: A, B, C
Original edge: A - B
Inverse edges: A - C, B - C
3. Dense graphs and their inverse.
If the original graph is dense, its inverse is sparse.
If the original graph is sparse, its inverse is dense.
4. Dual graphs.
For planar graphs, each face becomes a vertex in the dual graph.
Example:
A triangle has a dual graph that is also a triangle.
5. Why the dual is only defined for planar graphs.
Dual graphs require a planar embedding with well-defined faces.
Non-planar graphs do not have a unique set of faces.
Example:
The graph K3,3 is non-planar and therefore has no dual graph.
Problem 2. Bron–Kerbosch Algorithm
Graph:
Vertices: A, B, C, D
Edges: AB, AC, BC, CD
Adjacency list:
A: B, C
B: A, C
C: A, B, D
D: C
1. Initial call:
R = {}
P = {A, B, C, D}
X = {}
2. First recursive calls:
R = {A}, P = {B, C}, X = {}
R = {A, B}, P = {C}, X = {}
R = {A, B, C}, P = {}, X = {}
This reports the maximal clique {A, B, C}.
3. Maximal cliques:
{A, B, C}
{C, D}
Maximum clique:
{A, B, C}