Fundamental Algorithm Techniques
Problem Set #10

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1 Introduction

This report presents solutions to all problems from **Problem Set #10** in the course *Fundamental Algorithm Techniques*. Each task is explained clearly and concisely, with emphasis on correct classification, probabilistic reasoning, and information-theoretic interpretation.

The report consists of three parts:

- classification of computational problems,
- application of Bayes' rule,
- calculation of Shannon entropy.

2 Problem 1: Classification of Problems

In this task, different computational problems are classified into complexity classes such as P, NP-complete, NP-hard, and undecidable.

Line 1

find maximum, linear search, shortest path in an unweighted graph, matrix multiplication

Class: P

All listed problems admit polynomial-time algorithms. For example, linear search and finding the maximum run in linear time, while shortest paths in unweighted graphs can be solved using BFS.

Line 2

sorting, Dijkstra (non-negative weights), BFS, DFS, merge sort, quicksort

Class: P

These are classical algorithmic problems with efficient solutions. All have well-known polynomial-time implementations.

Line 3

Sudoku

Class: NP-complete

While small Sudoku instances are easy to solve, the general problem of solving Sudoku of arbitrary size is NP-complete.

Line 4

3-coloring, scheduling with conflicts

Class: NP-complete

Graph 3-coloring and conflict-based scheduling are standard NP-complete decision problems.

Line 5

Traveling Salesperson (decision version), Hamiltonian Cycle, Clique

Class: NP-complete

These problems belong to the core set of NP-complete problems used in many reductions in complexity theory.

Line 6

cryptography, factoring large integers

Class: NP-hard / unknown

Integer factorization is not known to be solvable in polynomial time, nor is it proven NP-complete. Many cryptographic systems rely on the assumed hardness of such problems.

Line 7

Halting Problem, Busy Beaver

Class: Undecidable

No algorithm can solve these problems for all possible inputs. They are proven to be undecidable.

Summary

- Lines 1–2: P
- Lines 3–5: NP-complete
- Line 6: NP-hard / unknown
- Line 7: Undecidable

3 Problem 2: Bayes' Rule and the 9% Result

We are given the following information:

- Disease prevalence: $P(D) = 0.001$ (0.1%),
- Test sensitivity: $P(+|D) = 0.99$,
- Test specificity: $P(-|\neg D) = 0.99$.

Thus, the false positive rate is:

$$P(+|\neg D) = 0.01.$$

Using Bayes' theorem, we compute the probability that a person has the disease given a positive test result:

$$P(D|+) = \frac{P(+|D) \cdot P(D)}{P(+|D) \cdot P(D) + P(+|\neg D) \cdot P(\neg D)}.$$

Substituting the values:

$$P(D|+) = \frac{0.99 \cdot 0.001}{0.99 \cdot 0.001 + 0.01 \cdot 0.999} = \frac{0.00099}{0.01098} \approx 0.09.$$

Interpretation

Even though the test is highly accurate, the disease itself is very rare. As a result, most positive test results come from healthy individuals. Therefore, the actual probability of having the disease after a positive test is only about 9%.

4 Problem 3: Shannon Entropy of Three Coins

The Shannon entropy of a random variable X is defined as:

$$H(X) = -\sum_i p_i \log_2 p_i.$$

Each coin has two outcomes: Heads (H) and Tails (T).

Coin A

$$P(H) = 0.5, \quad P(T) = 0.5$$

$$H = -0.5 \log_2(0.5) - 0.5 \log_2(0.5) = 1 \text{ bit}.$$

A fair coin is maximally unpredictable and therefore carries one full bit of information.

Coin B

$$P(H) = 0.99, \quad P(T) = 0.01$$

$$H \approx -0.99 \log_2(0.99) - 0.01 \log_2(0.01) \approx 0.08 \text{ bits}.$$

Since the outcome is almost always Heads, very little information is gained from each toss.

Coin C

$$P(H) = 0.01, \quad P(T) = 0.99$$

$$H \approx -0.01 \log_2(0.01) - 0.99 \log_2(0.99) \approx 0.08 \text{ bits}.$$

This coin is also highly biased, leading to low entropy.

Conclusion

A fair coin has maximum entropy because its outcomes are equally likely. Highly biased coins produce much less uncertainty, so their entropy values are close to zero.

5 Conclusion

In this report:

- computational problems were classified into standard complexity classes,
- Bayes' theorem was applied to explain a counterintuitive medical test result,
- Shannon entropy was used to measure uncertainty in random experiments.

These problems highlight key concepts in algorithms, probability, and information theory.