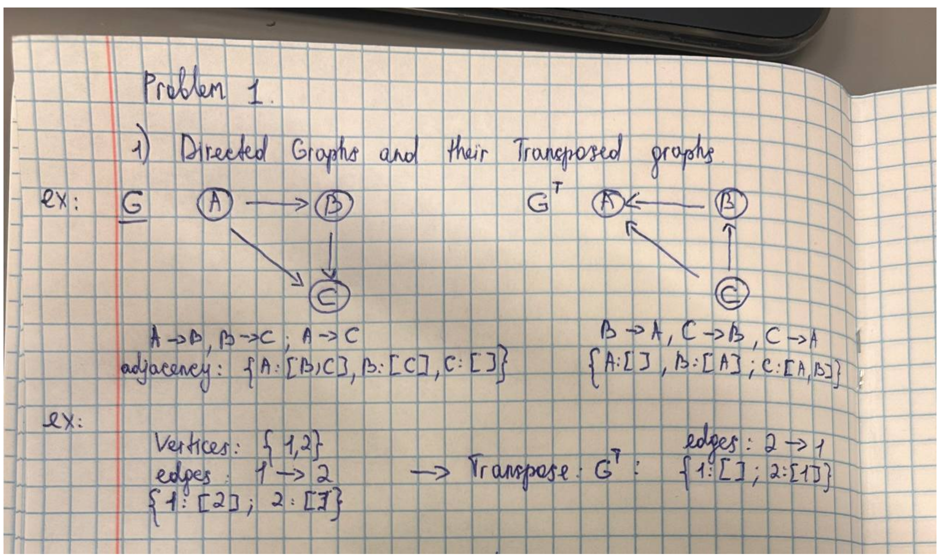
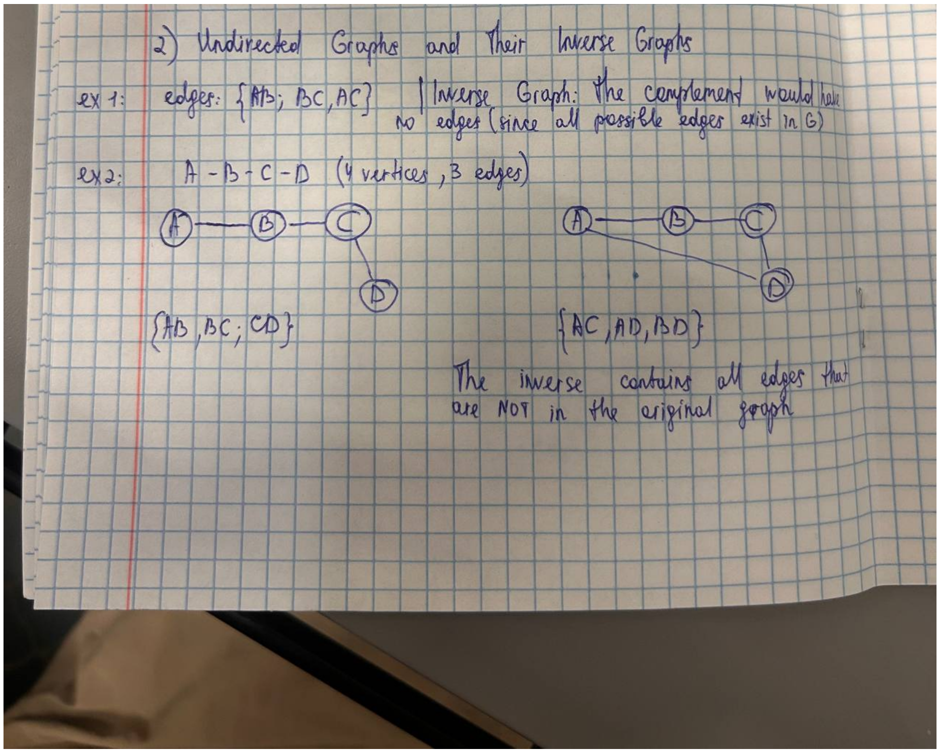
Exo - 7

Problem 1

1 — Directed graphs and their transposes

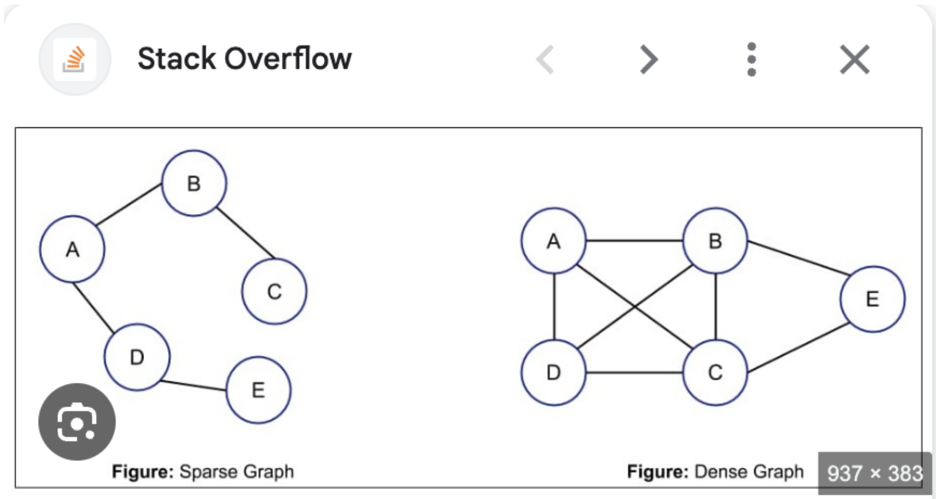


2 — Undirected graphs and their inverse graphs



3 — What happens if the original graph is dense?

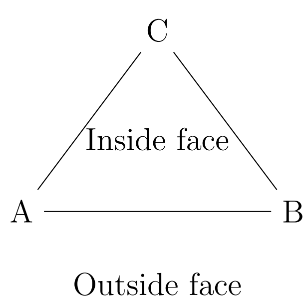
If G is **dense** (many edges), then the complement $\overline{G}$ will be **sparse** (few edges). Similarly, if G is sparse, $\overline{G}$ is dense.



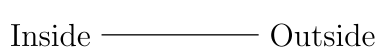
4 — Simple Examples of Undirected Graphs and Their Dual Graphs

Example 1: Triangle (Cycle C_3)

Original Graph

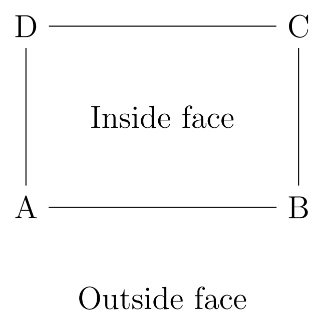


Dual Graph

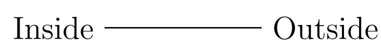


Example 2: Square (Cycle C_4)

Original Graph

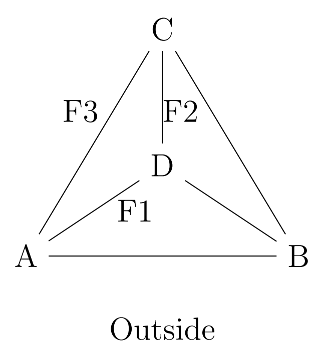


Dual Graph

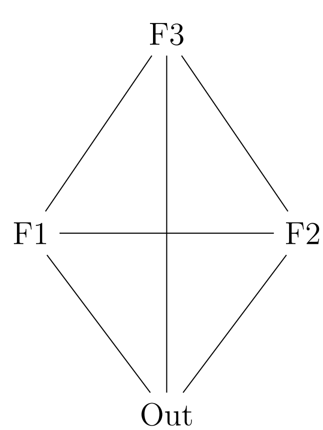


Example 3: Graph with Three Interior Faces

Original Graph



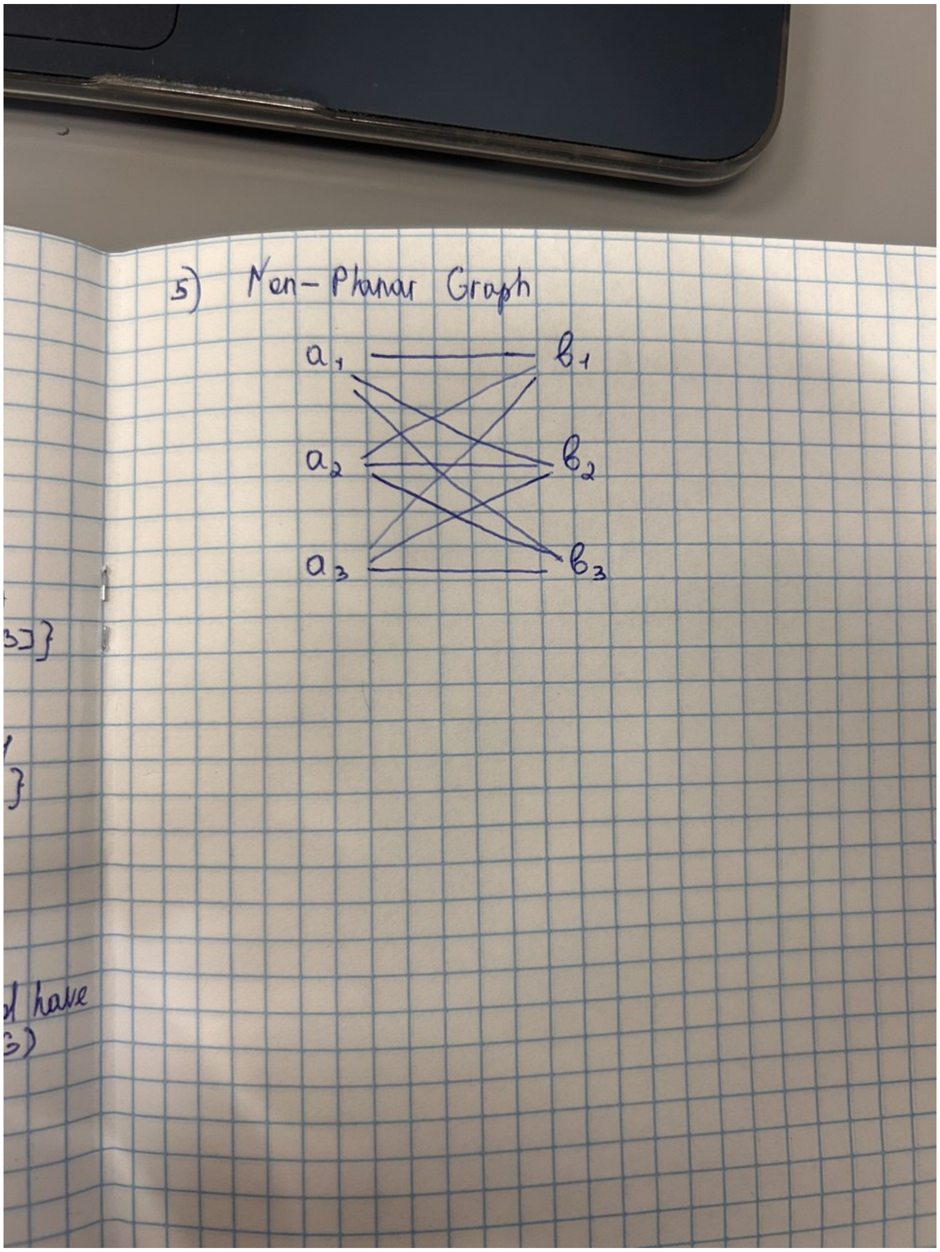
Dual Graph



5 — Non Planar Graph

A dual graph requires a **planar embedding** where all faces are clearly defined. Non-planar graphs cannot be drawn without crossings, so **faces do not exist**, making the dual graph impossible to construct.

Ex: Non-planar graph



This graph cannot be drawn without crossings. Thus, faces are not well-defined ⇒ no dual graph possible.;

Problem 2 — Bron–Kerbosch Algorithm

In computer science, the Bron–Kerbosch algorithm is an enumeration algorithm for finding all maximal cliques in an undirected graph.

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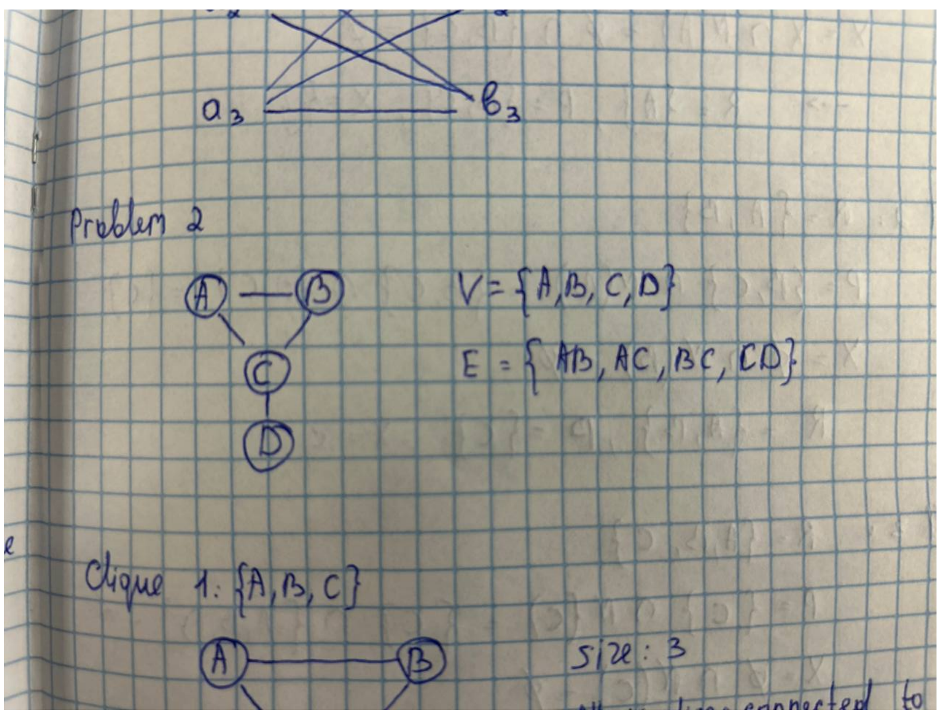
algorithm BronKerbosch1(R, P, X) is
    if P and X are both empty then
        report R as a maximal clique
    for each vertex v in P do
        BronKerbosch1(R ∪ {v}, P ∩ N(v), X ∩ N(v))
        P := P \ {v}
        X := X ∪ {v}
```

I took it from wikipedia (without Pivoting)

Graph:

$$V = \{A, B, C, D\}, \quad E = \{AB, AC, BC, CD\}$$



$R = \emptyset \quad P = \{A, B, C, D\} \quad X = \emptyset$

Call 1: $R = \{A\}$

$P = P \cap N(A) = \{A, B, C, D\} \cap \{B, C\} = \{B, C\}$

$X = X \cap N(A) = \emptyset \cap \{B, C\} = \emptyset$

$\longrightarrow \quad R = \{A\},\ P = \{B, C\},\ X = \{\}$

Call 2: $R = \{A, B\}$

$P = \{B, C\} \cap N(B) = \{B, C\} \cap \{A, C\} = \{C\}$

$X = \emptyset \cap N(B) = \emptyset$

$R = \{A, B\},\ P = \{C\},\ X = \emptyset$

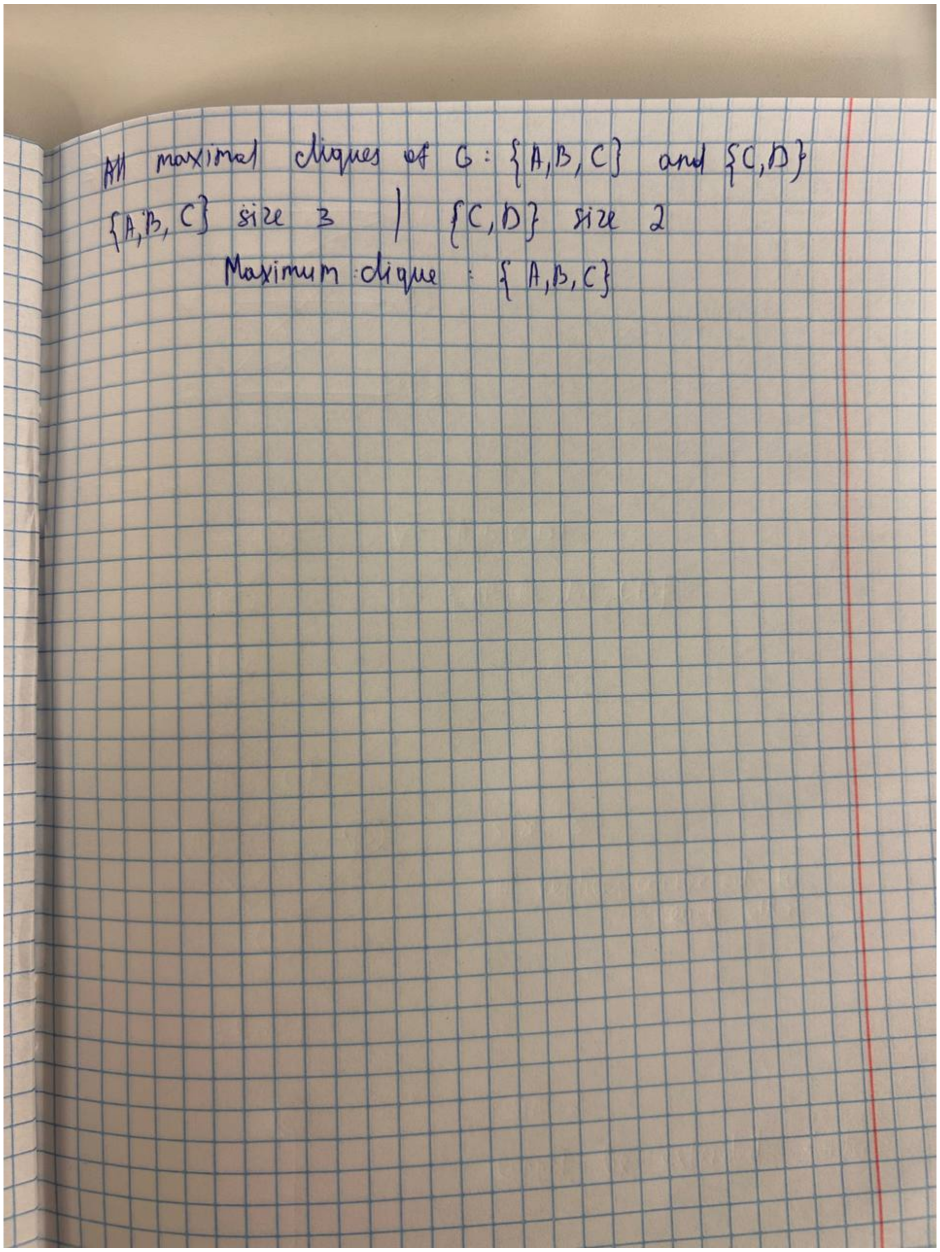
Call 3: $R = \{A, B, C\}$

$P = \{C\} \cap N\{C\} = \{C\} \cap \{A, B, D\} = \emptyset$

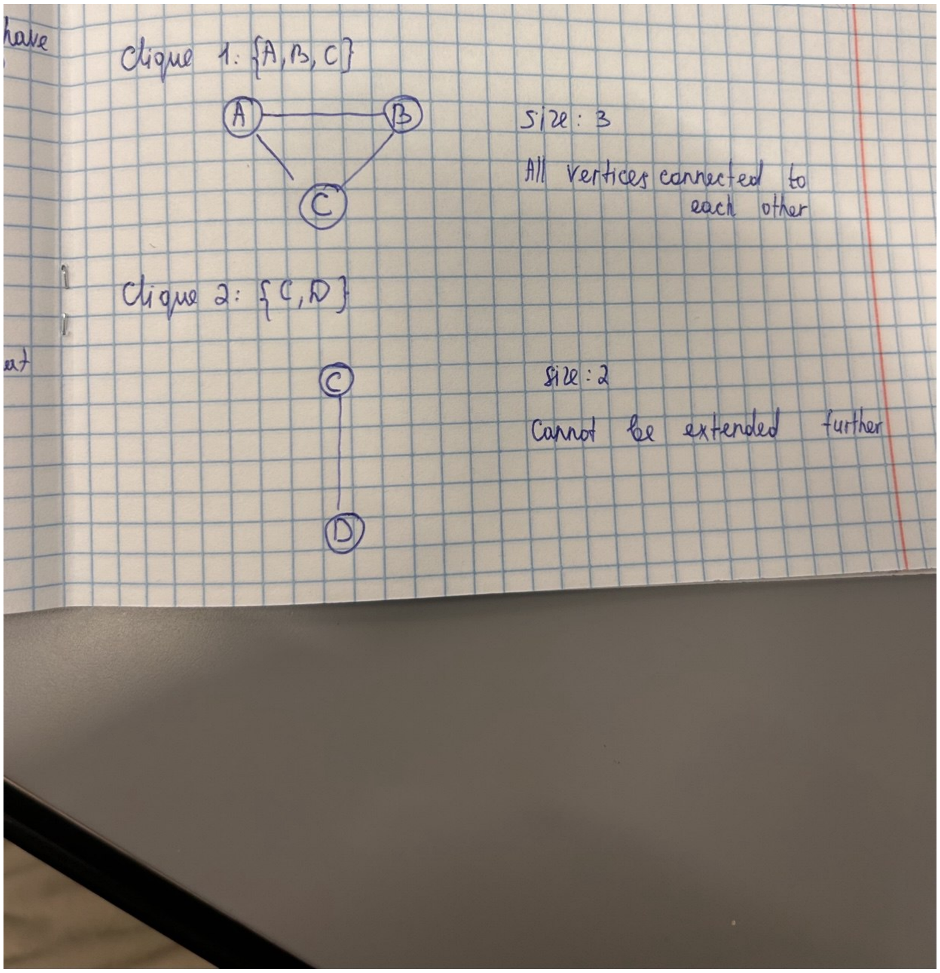
$X = \emptyset \cap N(C) = \emptyset$

New $P = \emptyset \quad X = \emptyset \quad \longrightarrow R = \{A, B, C\}$ - maximum

All maximal cliques


All maximal cliques of G : {A, B, C} and {C, D}
{A, B, C} size 3 | {C, D} size 2
Maximum clique : {A, B, C}

$$\{A, B, C\}, \quad \{C, D\}$$



Maximum clique

$$\{A, B, C\}$$