Problem 1. Graph play

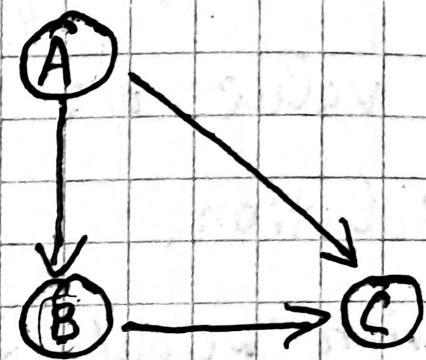
1) $G=(V,E)$, the transpose G^T is the graph with the same vertices and all edges reversed

$(u,v)\in E \Longleftrightarrow (v,u)\in E^T$

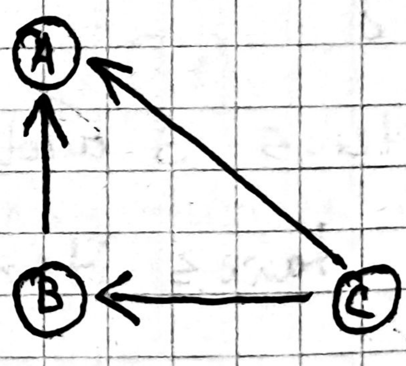
Example 1.

$V=\{A,B,C\}$

$E=\{AB, BC, AC\}$



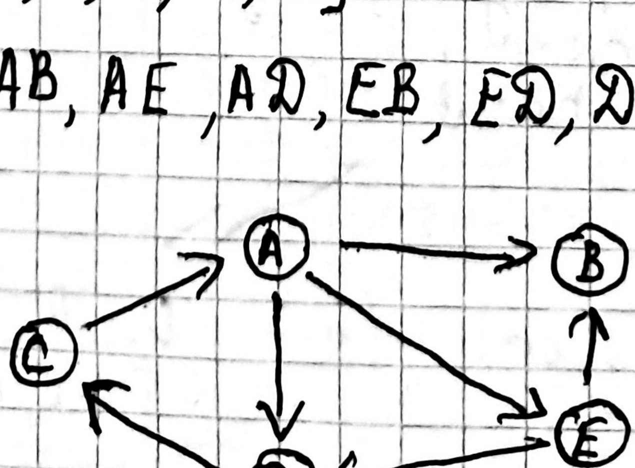
$G^T \searrow$ $E=\{BA, CB, CA\}$



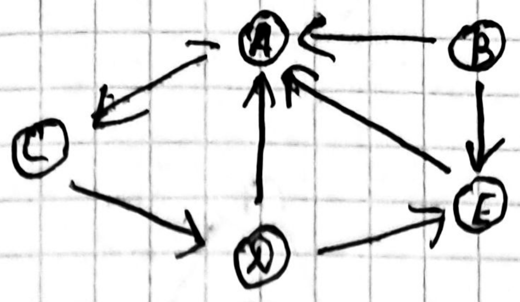
Example 2.

$V=\{A,B,C,D,E\}$

$E=\{AB, AE, AD, EB, ED, DC, CA\}$



$E = \{BA, EA, DA, BE, DE, CD, AC\}$



2) For every pair $u \neq v$

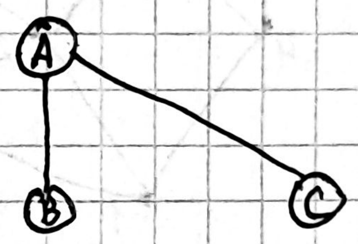
if $uv \in E$ in the original graph, then $uv \notin E^{inv}$

if $uv \notin E$ in the original graph, then $uv \in E^{inv}$

Example

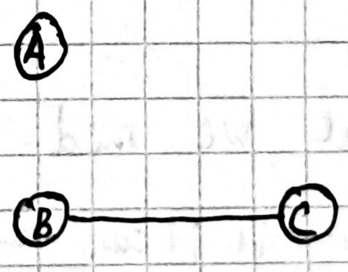
$V = \{A, B, C\}$

$E = \{AB, AC\}$



G^{inv}

$\Rightarrow E = \{BC\}$



3) Dense graph has almost all possible edges

On n vertices, a complete graph K_n has $\frac{n(n-1)}{2}$ edges

Dense $G \rightarrow$ Sparse G^{inv}

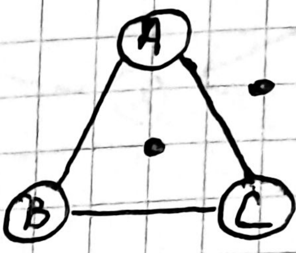
Sparse $G \rightarrow$ Dense G^{inv}

The inverse/complement G^{inv} contains edges exactly where the original has no edge.

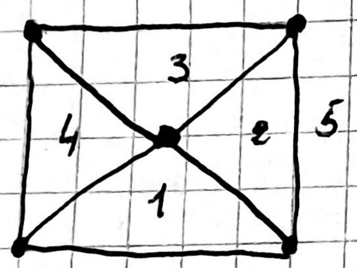
4)

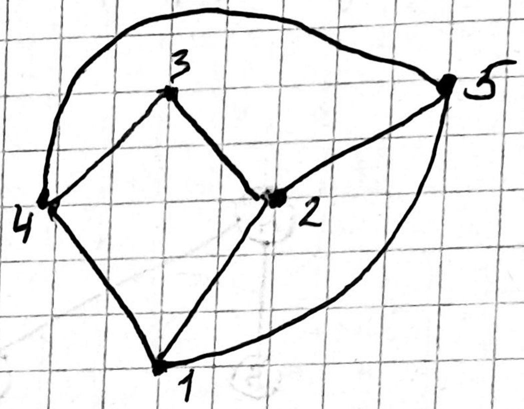
$V = \{A, B, C\}$

$E = \{AB, BC, CA\}$



Dual graph: A single vertex that represents the single enclosed face

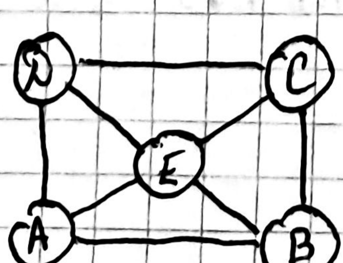




5) To define a dual, we need faces of a planar embedding

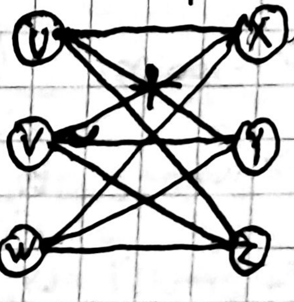
A graph is planar if it can be drawn in the plane without any edge crossings.

If the graph is non-planar, you cannot draw it without crossings so faces are not well-defined in a consistent way



planar graph

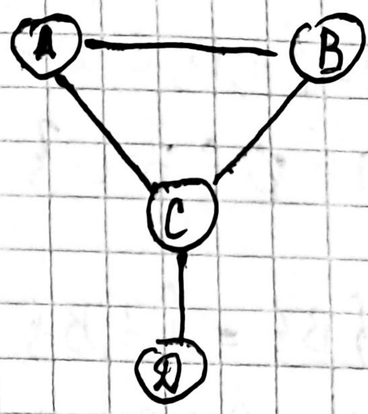
Non-planar



Problem 2.

$V = \{A, B, C, D\}$

$E = \{AB, AC, BC, CD\}$



Neighbors

$N(A) = \{B, C\}$

$N(B) = \{A, C\}$

$N(C) = \{A, B, D\}$

$N(D) = \{C\}$

initial Call

$R = \emptyset \quad P = \{A, B, C, D\} \quad X = \emptyset$

Clique $\{A, B, C\}$

1) $V = A$

$R = \{A\} \quad P = \{B, C\} \quad X = \emptyset$

2) $V = B$

$R = \{A, B\} \quad P = \{C\} \quad X = \emptyset$

3) $V = C$

$R = \{A, B, C\} \quad P = \emptyset \quad X = \emptyset$

1) $R_1 = R \cup \{A\} = \{A\}$

$P_1 = P \cap N(A) = \{A, B, C, D\} \cap \{B, C\} = \{B, C\}$

$X_1 = X \cap N(A) = \emptyset \{B, C\} = \emptyset$

2) $R_2 = \{A\} \cup \{B\} = \{A, B\}$

$P_2 = P \cap N(B) = \{B C\} \cap \{A, C\} = \{C\}$

$X_2 = X \cap N(B) = \emptyset \cap \{A, C\} = \emptyset$

3) $R_3 = \{A, B\} \cup \{C\} = \{A, B, C\}$

$P_3 = P \cap N\{C\} = \{C\} \cap \{A, B, D\} = \emptyset$

$X_3 = X \cap N(C) = \emptyset \cap \{A, B, D\} = \emptyset$

Call 4 inside call 1

$R = \{A\}, P = \{C\}, X = \{B\}$

$R_4 = \{A\} \cup \{C\} = \{A, C\}$

$P_4 = P \cap N(C) = \{C\} \cap \{A, B, D\} = \emptyset$

$X_3 = X \cap N(C) = \{B\} \cap \{A, B, D\} = \{B\}$

Clique $\{C, D\}$

$R = \emptyset \quad P = \{B, C, D\} \quad X = \{A\}$

Call 5, $v = B$

$R = \{B\}$

$P' = P \cap N(B) = \{B, C, D\} \cap \{A, C\} = \{C\}$

$X' = X \cap N(B) = \{A\} \cap \{A, C\} = \{A\}$

$\underline{R = \{B\} \quad P = \{C\} \quad X = \{A\}}$

$v \in P \quad v = C$

Call 6 inside call 5 $\quad v = C$

$R = \{B, C\}$

$P' = \{C\} \cap N(C) = \{C\} \cap \{A, B, D\} = \emptyset$

$X' = \{A\} \cap N(C) = \{A\} \cap \{A, B, D\} = \{A\}$

$\underline{R = \{B, C\} \quad P = \emptyset \quad X = \{A\}}$

$P = \{C\} \setminus \{C\} = \emptyset$

$X = \{A\} \cup \{C\} = \{A, C\}$

$v = B$

$\downarrow$

$P = \{B, C, D\} \setminus \{B\} = \{C, D\}$

$X = \{A\} \cup \{B\} = \{A, B\}$

Call 7 $\quad v = C$

$R = \emptyset \quad P = \{C, D\} \quad X = \{A, B\}$

$R = \{C\}$

$P' = P \cap N(C) = \{C, D\} \cap \{A, B, D\} = \{D\}$

$X' = X \cap N(C) = \{A, B\} \cap \{A, B, D\} = \{A, B\}$

7) $\underline{R = \{C\} \quad P = \{D\} \quad X = \{A, B\}}$

Call 8 inside call 7 $\quad V = D$

$R = \{C, D\}$

$P' = \{D\} \cap N(D) = \{D\} \cap \{C\} = \emptyset$

$X' = \{A, B\} \cap N(D) = \{A, B\} \cap \{C\} = \emptyset$

8) $\underline{R = \{C, D\} \quad P = \emptyset \quad X = \emptyset}$

P and X empty so $\{C, D\}$ as a max clique

This is second reported max clique

in call 7 $\quad V = D$

$P = \{D\} \setminus \{D\} = \emptyset$

$X = \{A, B\} \cup \{D\} = \{A, B, D\}$

$V = C$

$P = \{C, D\} \setminus \{C\} = \{D\}$

$X = \{A, B\} \cup \{C\} = \{A, B, C\}$

next v in root $V = D$

$R = \{D\}$

$P' = \{D\} \cap N(D) = \{D\} \cap \{C\} = \emptyset$

$X' = \{A, B, C\} \cap N(D) = \{A, B, C\} \cap \{C\} = \{C\}$

9) $\underline{R = \{D\} \quad P = \emptyset \quad X = \{C\}}$

Maximal cliques of G are

$\{A, B, C\}$ $\quad |\{A, B, C\}| = 3$

$\{C, D\}$ $\quad |\{C, D\}| = 2$

Maximum clique is

$\{A, B, C\}$ size 3