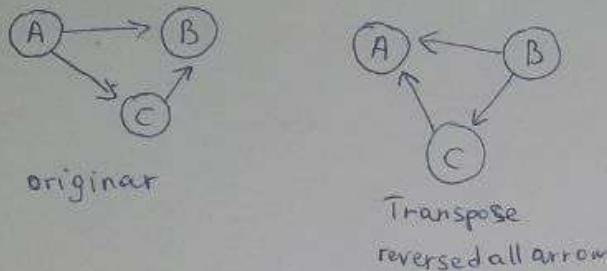


EXO 7

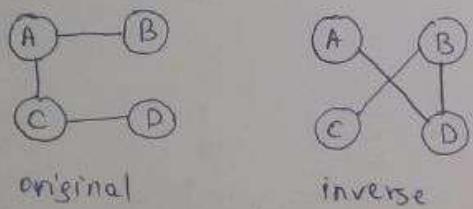
Problem 1 Draw a few undirected graphs and a few directed graphs

1. Directed graph \rightarrow transpose



Transposed graph = reverse all arrow directions. If original $A \rightarrow B$, transpose: $B \rightarrow A$
Transpose helps us find paths going backwards. If you want to know "who points to A", look at a transpose to see which vertices A points to.

2. Undirected graph \rightarrow inverse



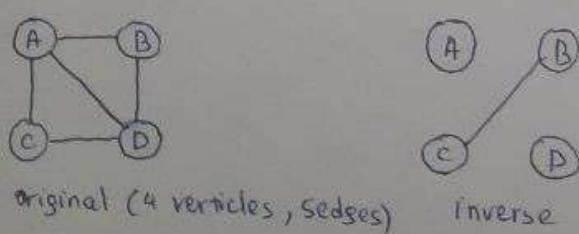
Inverse = complement, remove existing edges
add missing edges between all vertices

original A-B \Rightarrow NOT A-B

original missing B-D \Rightarrow WILL B-D

Sometimes we want to find vertices that are NOT connected. Inverse shows us their connections

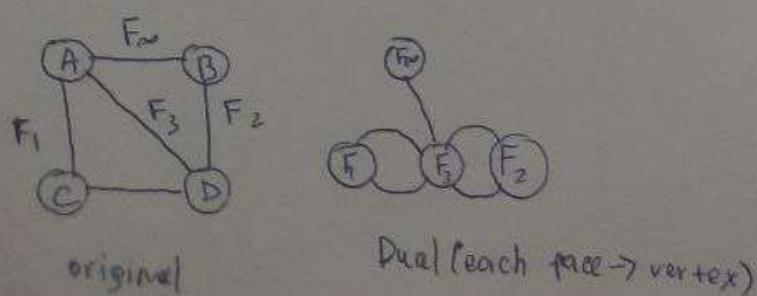
3. Dense graph \rightarrow inverse



If original is dense (many edges), the inverse graph become sparse few edges.
Dense \rightarrow Sparse

if original will have 5 vertices \rightarrow completed
Dense and Sparse are opposite. If one is big other is small, they balance each other

4. Dual graph

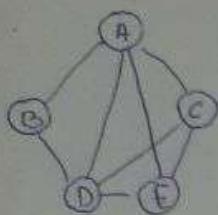


Dual graph: each face (region) become a vertex.
If two faces share an edge, connect their vertex.
Face is region (area) in graph surrounded by edges
like room with walls

Dual helps us to solve problems about regions by turning them into vertex problems

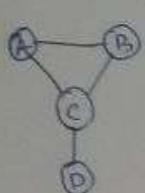
5. Why dual need plane graph?

non planer graph



non planer cannot be drawn without edges crossing, so face are not well defined
dual graph only work for planar graphs where face are well-defined
when edges cross we cannot tell which region is which
Only planer graphs can have dual graph. Non planer no.

Problem 2: Consider the following undirected graph C with vertices $V = \{A, B, C, D\}$ and edges $E = \{AB, AC, BC, CD\}$



$$\text{graph} = \{ A : [B, C], B : [A, C], C : [A, B, D], D : [C] \}$$

1. initial call to the algorithm

- R (the current clique being built) : [] - empty set.
- P - the set of potential candidates to extend the clique : [A, B, C, D]
- X - the set of excluded vertices: []

2. First recursive call:

We need to try each vertex from P and make recursive call to extend R.

We choose A from P

$$R = [A]$$

$$P = [B, C]$$

$$X = []$$

recursive 1 with A :

$$R = [A]$$

$$P \in [B, C] \Rightarrow$$

$$X = []$$

$$B:$$

$$R = [A, B]$$

$$P = [C]$$

$$X = []$$

recursive 2 with A and B :

$$R = [A, B]$$

$$P = [C]$$

$$X = []$$

$$C:$$

$$\Rightarrow R = [A, B, C]$$

P = [] no candidate

$$X = []$$

⇒ The maximal clique : [A, B, C]

3. Second recursive call from another maximal clique.

After we return from the first recursive branch, we move to other vertices from P.
After many operation all branch, we find maximum clique.

Maximal clique of :

[A, B, C]. - has 3 vertices.

[C, D].

This is simple explanation Bron-Kerbosch algorithm works