Problem Set #5 – Solutions
Problem 1. Equivalence of Tree Definitions
1. A tree is connected and acyclic. A forest is an acyclic graph, and a tree is one connected component of a forest. Thus, both definitions describe the same structure.
2. If a graph is connected and acyclic, it cannot have more than V−1 edges, otherwise a cycle would appear.
3. If a graph is connected and has at most V−1 edges, it cannot contain a cycle, since any cycle requires at least V edges.
4. In a connected acyclic graph, every edge is essential. Removing any edge disconnects the graph, so a tree is minimally connected.
5. An acyclic graph with fewer than V−1 edges must be disconnected. Therefore, an acyclic graph with at least V−1 edges is connected.
6. A tree is acyclic, and adding any new edge creates a cycle. Hence, it is maximally acyclic.
7. In a tree, there is exactly one path between any two vertices. More than one path would imply a cycle.
Thus, all definitions of a tree are equivalent.
Problem 2. Sparse Representation of Graphs
Graph 1 (Undirected)
Adjacency Matrix:
[[0,1,1,0,0],
[1,0,1,1,0],
[1,1,0,1,0],
[0,1,1,0,1],
[0,0,0,1,0]]
Edges:
A-B, A-C, B-C, B-D, C-D, D-E
Graph 2 (Directed)
Adjacency Matrix:
[[0,1,1,0,0],
[0,0,0,1,1],
[0,0,0,0,0],
[0,1,0,0,1],
[0,0,0,0,0]]
Directed edges:
B→A, C→A, D→B, E→B, B→D, E→D
Unique directed cycle:
B → D → B