

Fundamental Algorithm Techniques

Problem Set #10

Review on December 14

Problem 1 (P, NP, NP-complete, NP-hard, 4/10). *Classify the problems below into their corresponding class or classes (each line has same class).*

1. find max, linear search, shortest path in unweighted graph, matrix multiplication
2. sorting of list, Dijkstra on non-negative weights, BFS, DFS, merge sort, quicksort
3. sudoku
4. 3 coloring of graph, scheduling with conflicts
5. Traveling Salesperson Problem, Hamiltonian Cycle, Clique
6. Cryptography, factoring large integers
7. Halting Problem ¹, busy beaver

Problem 2 (Intro to Bayes, 3/10 pts). *You are a doctor. A deadly disease affects 0.1% of people. Your test is 99% accurate (both sensitivity and specificity). A patient tests positive.*

What is the probability he actually has the disease?

Answer is 9%! why?

Problem 3 (Intro to Shannon Entropy, 3/10). *Three coins:*

Coin	$P(\text{Heads})$	Surprise when heads appears
A	50%	Medium
B	99%	Tiny
C	1%	HUGE!

Use the Shannon Entropy to discuss the bits required to describe each above coins:

$$H(X) = - \sum_i^n p_i \log_2 p_i,$$

whereas n is the number of all possible outcome². Why is a fair coin “worth” 1 bit, but a 99% biased coin only 0.08 bits?

¹Halting Problem: a program that can determine, for any given program and its input, whether that program will eventually stop or run forever.

²For alphabet it would be ≈ 26 , here it is 2.