EXO 10

Problem 1. Complexity class.

1. Polinomial time
   - findmax, linear search, shortest path in unweighted graph.
   - Sorting of list, Dijksna on non negative weights, BFS, DFS, mergesort

2. NP-complete.
   - sudoku
   - 3colaring graph, scheduling with conflicts
   - Traveling · Salesperson Problem, Hamilton Cycle, Clique
   - Crypto graphy, factoring large integers

Problem 2. Bayes

Disease affects 0.1% of people. ($P(\text{disease}) = 0.001$)

Test is 99% accurate.

Petient tests positive

Solution: $P(\text{disease} \mid \text{positive test})$

For example we have 100000 people.

People with disease: $100000 \cdot 0,001 = 100$ people

True positive $100 \cdot 0.99 = 99$ people

People without disease: $100000 \cdot 0.999 = 99900$ people

False positive: $99900 \cdot 0,01 = 999$ people.

Total positive: $99 + 999 = 1098$ people

Probability: $P(\text{disease} \mid \text{positive}) = 99 : 1098 = 0.09 \approx 9\%$

Why is low? Because the disease is very rare (0.1%) even though the test is accurate there are many more healthy people getting false positive than sick people getting true positives

Problem 3. Shannon Entropy.

$$H(X) = -\sum_{i}^{n} P_i \log_2(P_i)$$

For coin $n=2$ (H and T)

coin A : Fair coin 50%.

$P(H)=0.5 \quad P(T)=0.5$

$H(A) = -(0.5 \cdot \log(0,5) + 0,5 \cdot \log_2(0,5)) = -(0,5 \cdot -1) + 0,5 \cdot (-1) = 1 \text{ bit}$

Coin B (Baised 99% H)

$P(H)=0,99 \quad P(T)=0,01$

$H(B) = -(0,99 \cdot \log_2(0,99) + 0,01 \cdot \log_2(0,01)) = -(0,99 \cdot (-0,0145) + 0,01(-6.644))$

$= -(-0,0143 + 0,0664) = 0.08 \text{ bits}$

Coin C (Baised, 1% H)

$P(H)=0,01 \quad P(T)=0.99$

$H(C) = -(0,01 \cdot \log_2(0,01) + 0,99 \cdot \log_2(0.99)) = -(0,01 \cdot (-6,644) + 0.99 \cdot (-0,0145))$

$= 0.08 \text{ bits}$

Fair coin : 1 bit ~~We~~ truly don't know what will happen. Each flip gives you 1 full bit of information

99% baised coin : we almost always know it will be heads. When it comes up heads, you learn very little new information because you expected it.