Lecture 09 – HW2 and Intro to Krylov Methods (CG)

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NERS/ENGR 570 - Methods and Practice of Scientific Computing (F22)

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Outline

- 10,000 ft view
- Krylov Methods Alphabet Soup
- Conjugate Gradient
- HW2

Learning Objectives: By the end of Today's Lecture you should be able to

- (*Skill*) Decide what Krylov method to use for a particular problem
- (Knowledge) explain what defines a Krylov method
- (Knowledge) implement the CG algorithm
- (Knowledge) have an intuition about CG converges
- (*Skill*) Complete HW 2

The 10,000’ View

Overview of Solution Methods

$Ax = b$

Direct $O(n^3)$

Iterative $O(n^2) \times m$ (m: # iterations)

SpMV

$O(n)$ - Fast

Direct (L7)

- LU (Gaussian Elimination)
- QR
- Cholesky
- Singular Value Decomposition
- Eigendecomposition

Iterative

- Non-Stationary (L9)
  - Conjugate gradient
  - Lanczos
  - Arnoldi
  - Krylov
  - 1950's, 1970's, 1990's
- Stationary (L8)
  - Jacobi
  - Gauss-Seidel
  - SOR / SSOR
  - Multigrid
  - ~1800's, 1950's

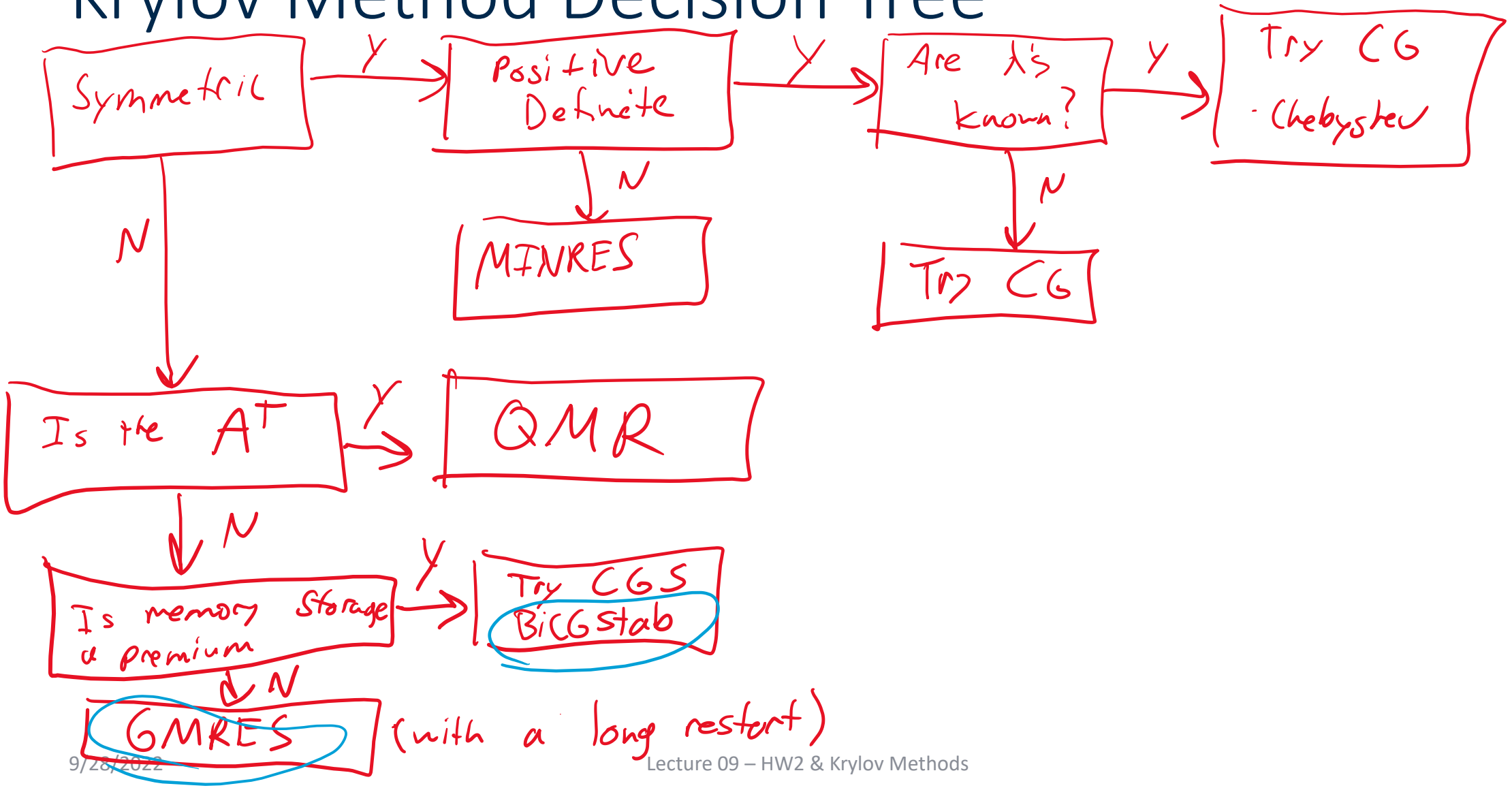
Alphabet Soup

CG,SYMMLQ,MINRES,GMRES,BICGTSAB,QMR,…

Krylov Method Categorization

	$Ax = b$	$Av = \lambda v$
$A = A^T$	Conjugate Gradient (CG) - A is s.p.d. MINRES (Minimized Resig.) SYMMLQ	Lanczos * implicitly restarted * Jacobi-Davidson
$A \neq A^T$	CGNE/LSQR $\Rightarrow A^TAx = A^Tb$ GMRES (Generalized Minimum Residual) QMR, Bi-Conjugate Grad (BiCG) CGS, Bi-Conjugate Gradient-Stabilized BiCGstab $\rightarrow$ 1992	Arnoldi * implicitly restarted * Jacobi-Davidson (JD)

Krylov Method Decision Tree



The Method of Conjugate Gradients

History of CG

Krylov (subspace) Methods

What CG does

How does CG work?

The actual procedure

The polynomial aspect

Convergence of CG