

CSE-321 Homework #2

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1-) Array = { 6, 5, 3, 11, 7, 5, 2 }

checkValue = 5

6	5	3	11	7	5	2
---	---	---	----	---	---	---

↑
Index

Compare 6 and 5,
5 is smaller so replace
index with 6

6	6	3	11	7	5	2
---	---	---	----	---	---	---

↑
Index

We are in first position
replace first element with
checkValue(5)

checkValue = 3

5	6	3	11	7	5	2
---	---	---	----	---	---	---

↑
Index

Compare 3 and 6,
3 smaller so replace 3 with 6

5	6	6	11	7	5	2
---	---	---	----	---	---	---

↑
Index

Compare 5 and 6,
5 smaller so replace 6 with 5

5	5	6	11	7	5	2
---	---	---	----	---	---	---

↑
Index

We are in first position, just
replace first element with
checkValue(3)

checkValue = 11

3	5	6	11	7	5	2
---	---	---	----	---	---	---

↑
Index

Compare 11 and 6,
11 is greater so don't
change anything.

Check Value = 7

3	5	6	11	7	5	2
---	---	---	----	---	---	---

↑
index

Compare 7 and 11,
11 greater so replace 7
with 11.

3	5	6	11	11	5	2
---	---	---	----	----	---	---

↑
index

Compare 6 and 7.
6 smaller than 7 so
replace index with 7.

Check Value = 5

3	5	6	7	11	5	2
---	---	---	---	----	---	---

↑
index

Compare 5 and 11.
11 greater than 5 so
replace 5 with 11.

3	5	6	7	11	11	2
---	---	---	---	----	----	---

↑
index

Compare 7 and 5
7 greater than 5 so
replace index with 7

3	5	6	7	7	11	2
---	---	---	---	---	----	---

↑
index

Compare 6 and 5
6 greater than 5 so
replace index with 6

3	5	6	6	7	11	2
---	---	---	---	---	----	---

↑
index

Compare 5 and 5
5 is not greater than 5 so
replace index with 5

3	5	5	6	7	11	2
---	---	---	---	---	----	---

checkValue = 2

3	5	5	6	7	11	2
---	---	---	---	---	----	---

↑
index

Compare 11 and 2
11 greater than 2 so
replace index with 11

3	5	5	6	7	11	11
---	---	---	---	---	----	----

↑
index

Compare 7 and 2
7 greater than 2 so
replace index with 7

3	5	5	6	7	7	11
---	---	---	---	---	---	----

↑
index

Compare 6 and 2
6 greater than 2 so
replace index with 6

3	5	5	6	6	7	11
---	---	---	---	---	---	----

↑
index

Compare 5 and 2
5 greater than 2 so
replace index with 5

3	5	5	5	6	7	11
---	---	---	---	---	---	----

↑
index

Compare 5 and 2
5 greater than 2 so
replace index with 5

3	5	5	5	6	7	11
---	---	---	---	---	---	----

↑
index

Compare 3 and 2
3 greater than 2 so
replace index with 3

3	3	5	5	6	7	11
---	---	---	---	---	---	----

↑
index

We are in the first pos.
So just replace first pos with
checkValue(2)

2	3	5	5	6	7	11
---	---	---	---	---	---	----

Other loop reach end of array
so our algorithm completed
and array sorted.

③

2-)

a) function (int n) {

if (n == 1) return;

for (int i = 1; i <= n; i++) { $O(n)$

for (int j = 1; j <= n; j++) {

printf("*");

break;

$O(1)$

}

}

}

Firstly if n is equal 1 this function time complexity will be $O(1)$ because it just enter first if statement and return.

So, best case for this algorithm $T_{\text{Best}}(n) = O(1)$

If n is not equal 1 time complexity will be $O(n)$ because first loop run n times but inner loop only run 1 time because it will break out after every first iteration. Also print and break statements are constant. So total time complexity will be $O(n)$

$$T_{\text{worst}}(n) = T_{\text{Average}}(n) = O(n)$$

So we can say:

$$T(n) = O(n)$$

b) Void function(int n) {
 int count=0;

① for(int i=n/3; i<=n; i++)
 ② for(int j=1; j<=n/3; j++)
 ③ for(int k=1; k<=n; k=k*3)
 count++ — constant
 }

for loop ③

k
 1
 3
 9
 ⋮
 3^k

$$3^k = n \Rightarrow k = \log_3 n$$

so this loop $O(\log_3 n)$

for loop ②

$$\sum_{j=1}^{\frac{2n}{3}} 1$$

for(int j=1; j<=n/3; j++) This loop runs $\frac{2n}{3}$ times
 after discarding constants we can see its time complexity $O(n)$

for loop ①

$$\sum_{i=n/3}^n 1$$

for(int i=n/3; i<=n; i++) This loop runs $n - \frac{n}{3} + 1 = \frac{2n}{3} + 1$
 After ignore constants we find its time complexity $O(n)$

So; After combine this 3 loop we find our complexity as;

$$\underline{O(n^2 \log_3 n)}$$

3-)

Procedure func($L[1:n]$, desired-num)

$L.sort()$

$i = 1$

$j = n$

prev-pair = null

while $i < j$

value = $L[i] * L[j]$

if (value = desired-num)

if (prev-pair = null or prev-pair $\neq (L[i], L[j])$)

yield ($L[i], L[j]$)

prev-pair = ($L[i], L[j]$)

end if

$i++$

$j--$

end if

else if (value < desired-num)

$i++$

end else if

else

$j--$

end else

end while

end procedure

- In my algorithm, firstly it sort given list. For sorting we can use merge, quick or heap sort. Python sort function use merge sort. So we can sort array in $O(n \log n)$ complexity. After that I declare 2 Variable i and j . i starts from the first element of list and j starts from the last element of list and while $i < j$ check $L[i] * L[j]$

If this value smaller than our desired number just increment i by 1

If this value greater than our desired number just decrement j by 1

Also I declare a variable called `prev-pair`. This variable hold last returned pair. So with using this variable I check new pair is same as the previous pair or not. If they not same I return this pair. So because of that I don't return same pairs multiple times.

So, If value and desired num equals and previous pair and new pair is different yield this new pair and set `prev-pair` with this pair. Then increment i by 1 and decrement j by 1.

Lets show this algorithm with an example;

Lets our list = $\{1, 2, 3, 6, 5, 4\}$ and desired-num = 6

first we sort this list and it becomes $[1, 2, 3, 4, 5, 6]$

this sorting take $O(n \log n)$

Now we are in while loop

$[1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6]$
 $\uparrow \quad \quad \quad \uparrow$
 $i \quad \quad \quad j$

$L[i] * L[j] = \text{desired-val}$
So yield $(L[i], L[j])$
and increment i by 1 and
decrement j by 1

$[1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6]$
 $\uparrow \quad \quad \quad \uparrow$
 $i \quad \quad \quad j$

$L[i] * L[j] > \text{desired-val}$
So just decrement j by 1.

$[1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6]$
 $\uparrow \quad \quad \quad \uparrow$
 $i \quad \quad \quad j$

$L[i] * L[j] > \text{desired-num}$
So just decrement j by 1

[1 2 3 4 5 6]
 ↑ ↑
 i j

$L[i] + L[j] = \text{desired value}$
 so yield $(L[i], L[j])$
 and increment i by 1 and decrement
 j by 1.

[1 2 3 4 5 6]
 ↑ ↑
 j i

now $i > j$ so break loop and
 end function.

So, this while loop just traverse list. Its complexity is $O(n)$
 But in the beginning of algorithm we have $O(n \log n)$ complexity
 sorting.

Hence, this algorithm time complexity;

$$T(n) = O(n \log n + n) \equiv \underline{O(n \log n)}$$


```

4-) procedure mergeBST(BST Tree1, BST Tree2)
    List Tree1-list;
    List Tree2-list;
    inorder(Tree1, Tree1-list) //create a sorted list with tree1 elems
    inorder(Tree2, Tree2-list) //create a sorted list with tree2 elems

    List m-list;
    merge-list(Tree1-list, Tree2-list, m-list)

    Merged-Tree = create-merge-tree(m-list, 0, m-list.length)
    return merged-tree.
end procedure

```

```

procedure inorder(Tree, list)
    if (Tree != null)
        inorder(Tree.left, list) →  $T(\frac{n}{2})$ 
        list.add(Tree.data) → 1
        inorder(Tree.right, list) →  $T(\frac{n}{2})$ 
    end if
end procedure

```

```

procedure merge-list(L1, L2, L3)
    i=0, j=0, k=0
    while i < L1.length and j < L2.length
        if (L1[i] < L2[j])
            L3[k++] = L1[i++]
        end if
        else
            L3[k++] = L2[j++]
        end else
    end while
    while i < L1.length
        L3[k++] = L1[i++]
    end while
    while j < L2.length
        L3[k++] = L2[j++]
    end while
end procedure

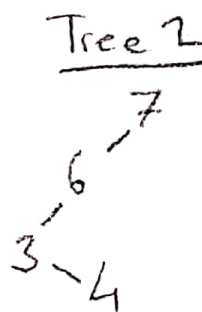
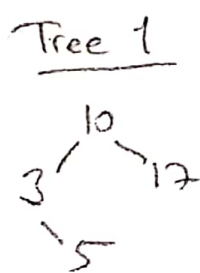
```

```

procedure create-merge-tree(list, first, last)
  if (first > last)
    return null
  end if
  mid = (first + last) / 2
  node = new Node(list[mid])
  node.left = create-merge-tree(list, first, mid - 1)
  node.right = create-merge-tree(list, mid + 1, last)
  return node
end procedure

```

In my algorithm, mergeBST create 2 list and I fill them with inorder traversal values. So because of inorder traversal of BST these lists will be ordered. for example lets our trees like these:



After inorder traversal for tree 1 List 1 will be $[3, 5, 10, 17]$

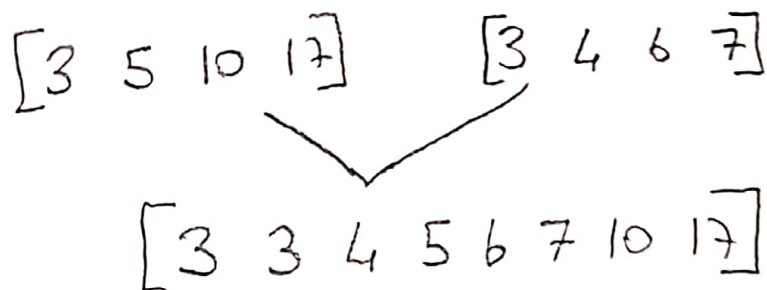
After inorder traversal for tree 2 List 2 will be $[3, 4, 6, 7]$

And our inorder traverse algorithm has $T(n) = 2T(\frac{n}{2}) + 1$

So, inorder traverse part has $O(n)$ time complexity

After that I merge this 2 sorted list with merge-list function. This function traverse both list and we know our both lists has same a length.

So, this function has $O(n+n) = \underline{O(n)}$ time complexity.



Then, we create our merged tree with turning this list to a BST. Also my create-merge-tree algorithm

$T(n) = 2T(\frac{n}{2}) + C$ so it has $\underline{O(n)}$ time complexity.

So, My merge algorithm has $O(n+n+n) = \underline{O(n)}$ time complexity.

5-) This problem can be solved by brute force. We can search every smaller array element in the big array. But this method has $O(n^2)$ complexity. For linear time complexity we can use a hashmap. Firstly we can add smaller array elements to the our hashmap then just iterate bigger array and check 'our hashmap' has these elements or not. This method has linear time complexity. Lets see this on pseudo code;

```
procedure find-elements(arr-1, arr-2)
    my-map = new HashMap()
    i = 0
    while i < arr-1.length
        my-map.add(arr-1[i])
    end while
    j = 0
    while j < arr-2.length
        if (my-map.contains(arr-2[j]))
            yield arr-2[j]
        end if
    end while
end procedure
```

} $O(m)$

} $O(n)$

As we know, hashmap has constant time complexity for add and contains methods. So, if our smaller array length is m first while loop complexity will be $O(m)$. And if bigger array length is n , second while complexity will be $O(n)$.

So, this algorithm complexity will be $O(m+n) = \underline{O(n)}$