

1-) Because of our text contains n zeros, pattern 0010 always done 3 comparisons, because third bit(1) will fail after comparison and brute force algorithm don't check last 0 bit. And this operation continues for $n-3$ times

For example if $n=5$ and we check 0010 pattern

$n=5 \rightarrow$	00000	00000	
	0010	0010	
	<u> </u>	<u> </u>	Total 6 times compare
	3 time	3 time	
	compare	compare	

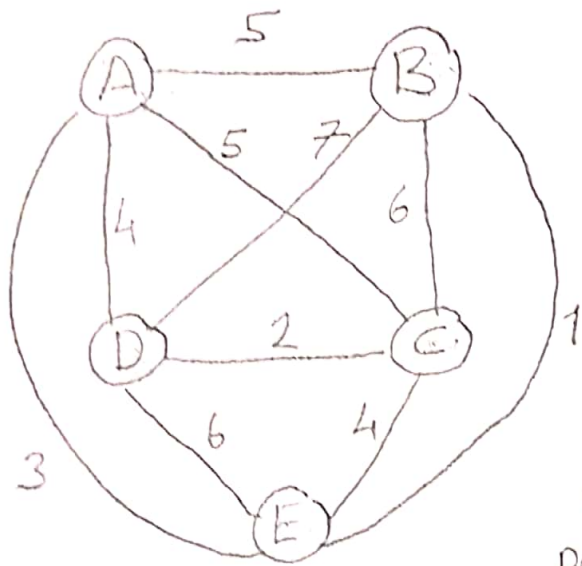
So, we compare 3 bit each $n-3$ times, general formula

$$\underline{3 \cdot (n-3) \text{ compare}}$$

What is the worst case input pattern of length 3(3bits) for BF?

For brute force algorithm, worst case 3 bit pattern is "001" because its always do 3 comparison and last comparison will fail because of 1.

2-) Apply brute-force algorithm for the travelling salesman problem.



In travelling salesman problem, we have to find minimum cost travelling route that visit every node once and return starting one.

by using brute-force algorithm we can do this by checking every possible route and found min. one.

Brute force all possible routes: Lets start from A because it doesn't matter

ABCDEA \rightarrow 22

ACBDEA \rightarrow 27

ADBCEA \rightarrow 25

ABDCEA \rightarrow 21

ABCEDA \rightarrow 25

ABECDCA \rightarrow 16 \leftarrow

ABEDCA \rightarrow 19

ABDECA \rightarrow 25

ACBEDA \rightarrow 22

ACEBDA \rightarrow 21

ACDBEA \rightarrow 18

ADCBEA \rightarrow 16 \leftarrow

We don't need write reverse routes too, because they have same cost.

So, minimum route has 16 cost

3) Design a decrease-by-half algorithm for computing $\log n$ (base 2) calculate its time efficiency.

```
function log(n):
```

```
    if (n < 2):
```

```
        return 0;
```

```
    else
```

```
        return 1 + log( $\lfloor \frac{n}{2} \rfloor$ );
```

Because we divide half every iteration, this algorithm has $O(\log n)$ time complexity.

4-) This is same as fake coin problem that we saw in class.

For solving this by a decrease-and-conquer algorithm, we can divide bottles 2 part and check their weights. If one of them weight is wrong, it means incorrect bottle in these part. If bottle count is an odd number just select a bottle and divide other bottles 2 part again. If those 2 part have same weight, it means selected bottle is wrong one. Otherwise wrong bottle is in part that has incorrect weight.

In best case, if bottle number is odd and after selecting one bottle and divide other bottles 2 part and those part balanced, it means selecting bottle is wrong one. This take $O(1)$ time complexity.

But, in worst case and average case because we every time divide 2 part this algorithm take $O(\log n)$ time

5-) Merging arrays first is forbidden, so we can sort both array firstly with quick sort or merge sort algorithm. After sorting 2 array we can use this algorithm for finding x^{th} element.

```
def Algorithm(arr1, arr2, x)
```

```
    if(len(arr1) == 0): return arr2[k]
```

```
    if(len(arr2) == 0): return arr1[k]
```

```
    mid-1 = len(arr1) / 2
```

```
    mid-2 = len(arr2) / 2
```

```
    if((mid-1 + mid-2) < x):
```

```
        if(arr1[mid-1] > arr2[mid-2]):
```

```
            return Algorithm(arr1, arr2[mid-2+1:], k-mid-2-1)
```

```
        else:
```

```
            return Algorithm(arr1[mid-1+1:], arr2, k-mid-1-1)
```

```
    else:
```

```
        if arr1[mid-1] > arr2[mid-2]:
```

```
            return Algorithm(arr1[:mid-1], arr2, k)
```

```
        else:
```

```
            return Algorithm(arr1, arr2[:mid-2], k)
```

```
end
```

In this algorithm, I take middle indices of arr1 and arr2, Lets assume $arr1[mid-1] < x$, then clearly the elements after mid-2 cannot be the required element. Then, we set the last element of arr2 to $arr2[mid-2]$. In this way we reduce the problem size to half of the arrays so complexity is $O(\log(m) + \log(n))$ in worst case. But sorting part complexity $O(n \log n)$