

c) $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \dots + \frac{1}{\sqrt{n-1}+\sqrt{n}}$ với n là số tự nhiên khác 0

$$= \frac{\sqrt{1}-\sqrt{2}}{(\sqrt{1}+\sqrt{2})(\sqrt{1}-\sqrt{2})} + \frac{\sqrt{2}-\sqrt{3}}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})} + \frac{\sqrt{3}-\sqrt{4}}{(\sqrt{3}+\sqrt{4})(\sqrt{3}-\sqrt{4})} + \dots + \frac{\sqrt{n-1}-\sqrt{n}}{(\sqrt{n-1}+\sqrt{n})(\sqrt{n-1}-\sqrt{n})}$$

$$= \frac{\sqrt{1}-\sqrt{2}}{-1} + \frac{\sqrt{2}-\sqrt{3}}{-1} + \frac{\sqrt{3}-\sqrt{4}}{-1} + \dots + \frac{\sqrt{n-1}-\sqrt{n}}{-1}$$

$$= \frac{1-\sqrt{n}}{-1} = \sqrt{n}-1$$

d) $\frac{1}{\sqrt{1}-\sqrt{2}} - \frac{1}{\sqrt{2}-\sqrt{3}} + \frac{1}{\sqrt{3}-\sqrt{4}} - \dots - \frac{1}{\sqrt{24}-\sqrt{25}}$

Bài 5. Chứng minh

a) $\sqrt{27} + \sqrt{6} > \sqrt{48}$

b) $\frac{2002}{\sqrt{2003}} + \frac{2003}{\sqrt{2002}} > \sqrt{2002} + \sqrt{2003}$

c) $\left(\frac{\sqrt{5}+1}{1+\sqrt{5}+\sqrt{3}} + \frac{\sqrt{5}-1}{1+\sqrt{3}-\sqrt{5}} \right) \left(\sqrt{3}-4\sqrt{\frac{1}{3}}+2 \right) \sqrt{0,2}-\sqrt{1,01} > 0$

d) $\frac{\sqrt{2}+\sqrt{3}-1}{2+\sqrt{6}} + \frac{\sqrt{2}-\sqrt{3}}{2\sqrt{6}} \left(\frac{\sqrt{3}}{2-\sqrt{6}} + \frac{\sqrt{3}}{2+\sqrt{6}} \right) - \frac{1}{\sqrt{2}} + \sqrt{3-\sqrt{2}} > 0$

Bài 6. Cho số $S = \frac{1}{3(1+\sqrt{2})} + \frac{1}{5(\sqrt{2}+\sqrt{3})} + \frac{1}{7(\sqrt{3}+\sqrt{4})} + \dots + \frac{1}{97(\sqrt{48}+\sqrt{49})}$.

So sánh S với $\frac{3}{7}$