

Matrix-Optimization1

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I. PROBLEM STATEMENT

Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}$, $x \in \mathbb{R} - (-1, 0, 1)$. If $h(x) = \frac{f(x)}{g(x)}$, then local minimum value of $h(x)$ is

II. CONSTRUCTION

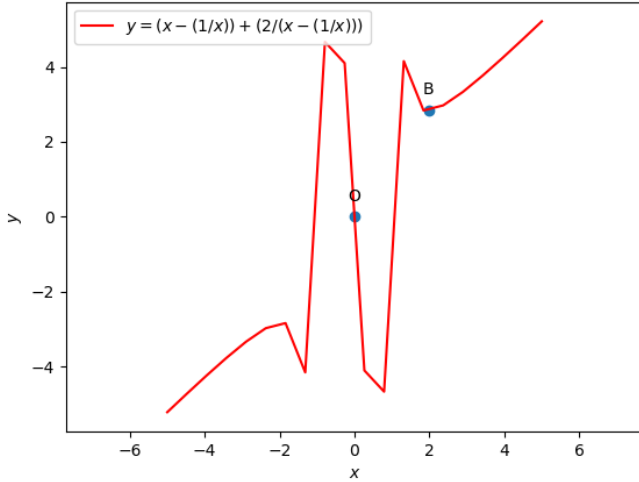


Fig. 1. $h(x) = \frac{(x - \frac{1}{x})^2 + 2}{x - \frac{1}{x}}$

Symbol	Value
$f(x)$	$x^2 + \frac{1}{x^2}$
$g(x)$	$x - \frac{1}{x}$
$h(x)$	$\frac{f(x)}{g(x)}$
x	$\in \mathbb{R} - (-1, 0, 1)$

TABLE I
GIVEN PARAMETERS

III. SOLUTION

Using **Gradient descend Algorithm**, we get local minimum is 2.83

IV. PROOF

According to given conditions $h(x)$ is expressed as,

$$h(x) = \frac{x^2 + \frac{1}{x^2}}{x - \frac{1}{x}} \quad (1)$$

Eq-1 can be expressed as

$$h(x) = \frac{(x - \frac{1}{x})^2 + 2}{x - \frac{1}{x}} \quad (2)$$

i.e.,

$$h(x) = x - \frac{1}{x} + \frac{2}{x - \frac{1}{x}} \quad (3)$$

Differentiating on both sides,

$$h'(x) = 1 + \frac{1}{x^2} - \frac{2}{(x - \frac{1}{x})^2} (1 + \frac{1}{x^2}) \quad (4)$$

Yielding,

$$h'(x) = \frac{1 + x^2}{(x - \frac{1}{x})^2 x^4} (x^4 + 1 - 4x^2) \quad (5)$$

$h(x)$ has a local minimum at $h'(x) = 0$

Therefore,

$$(1 + x^2)(x^4 + 1 - 4x^2) = 0 \quad (6)$$

Since, $x \in \mathbb{R} - (-1, 0, 1)$

Yielding,

$$x = \pm 1.932$$

Thus,

$$h(1.932) = 2.83$$

Therefore,

The minimum value of $h(x)$ is **2.83**