Matrix-Optimization

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I. Problem Statement

Let $\mathbf{f}(\mathbf{x}) = x^2 + \frac{1}{x^2}$ and $\mathbf{g}(\mathbf{x}) = x - \frac{1}{x}$, x∈R-(-1,0,1). If $\mathbf{h}(\mathbf{x}) = \frac{f(x)}{g(x)}$, then local minimum value of $\mathbf{h}(\mathbf{x})$ is

II. Construction

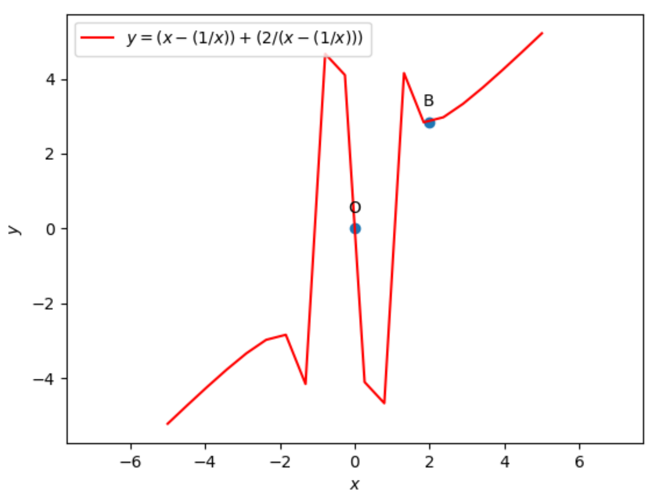

Fig. 1. $\mathbf{h}(\mathbf{x}) = \frac{(x-\frac{1}{x})^2+2}{x-\frac{1}{x}}$

Symbol	Value
$\mathbf{f}(\mathbf{x})$	$x^2 + \frac{1}{x^2}$
$\mathbf{g}(\mathbf{x})$	$x - \frac{1}{x}$
$\mathbf{h}(\mathbf{x})$	$\frac{\mathbf{f}(\mathbf{x})}{\mathbf{g}(\mathbf{x})}$
x	∈ R-(-1,0,1)

TABLE I
Given Parameters

III. Solution

Using **Gradient descend Algorithm**, we get local minimum is 2.83

IV. Proof

According to given conditions $\mathbf{h}(\mathbf{x})$ is expressed as,

$$\mathbf{h}(\mathbf{x}) = \frac{x^2 + \frac{1}{x^2}}{x - \frac{1}{x}} \tag{1}$$

Eq-1 can be expressed as

$$\mathbf{h}(\mathbf{x}) = \frac{(x - \frac{1}{x})^2 + 2}{x - \frac{1}{x}} \tag{2}$$

i.e,

$$\mathbf{h}(\mathbf{x}) = x - \frac{1}{x} + \frac{2}{x - \frac{1}{x}} \tag{3}$$

Differentiating on both sides,

$$\mathbf{h}'(\mathbf{x}) = 1 + \frac{1}{x^2} - \frac{2}{(x - \frac{1}{x})^2}(1 + \frac{1}{x^2}) \tag{4}$$

Yielding,

$$\mathbf{h}'(\mathbf{x}) = \frac{1 + x^2}{(x - \frac{1}{x})^2 x^4}(x^4 + 1 - 4x^2) \tag{5}$$

$\mathbf{h}(\mathbf{x})$ has a local minimum at $\mathbf{h}'(\mathbf{x}) = 0$

Therefore,

$$(1 + x^2)(x^4 + 1 - 4x^2) = 0 \tag{6}$$

Since, x∈ R-(-1,0,1)

Yielding,
x = ±1.932
Thus,

$$\mathbf{h}(1.932) = 2.83$$

Therefore,
The minimum value of $\boldsymbol{h(x)}$ is $\mathbf{2.83}$