Combinatorics Cameron - Solutions Manual

Mingruifu Lin

September 2025

Contents

Chapter 1

On Numbers and Counting

Exercise 1

The assumption that there is a largest natural number is wrong.

Exercise 2

Fuh naw.

Exercise 3

(a) Clearly, the base case works: $1 > \frac{1}{e}$. Now suppose

$$n! > \left(\frac{n}{e}\right)^n$$

Then

$$(n+1)n! > (n+1)\left(\frac{n}{e}\right)^n$$

We derive the right hand side:

$$= (n+1)\left(\frac{n}{n+1}\right)^n \left(\frac{n+1}{e}\right)^n$$

Using the fact that $(1+\frac{1}{n})^n < e \quad \Rightarrow \quad \frac{1}{(1+\frac{1}{n})^n} > \frac{1}{e} \quad \Rightarrow \quad \left(\frac{n}{n+1}\right)^n > \frac{1}{e}$, we replace the middle term to create the inequality

$$> (n+1)\frac{1}{e}\left(\frac{n+1}{e}\right)^n$$

$$= \left(\frac{n+1}{e}\right)^{n+1}$$

Hence induction step completed.

(b) The first inequality is equivalent to

$$\sqrt[n]{n!} < \frac{n+1}{2}$$

The left side is the geometric mean from 1 to n. The right side is the arithmetic mean from 1 to n. The theorem tells us that the geometric mean is smaller than the arithmetic mean.

Once again, we use the same strategy as (a).

$$\begin{aligned} n! &< \left(\frac{n+1}{2}\right)^n \\ &= \left(\frac{n}{n}\right)^n \left(\frac{n+1}{2}\right)^n \\ &= \left(\frac{n+1}{n}\right)^n \left(\frac{n}{2}\right)^n \\ &= \left(1+\frac{1}{n}\right)^n \left(\frac{n}{2}\right)^n \\ &< e\left(\frac{n}{2}\right)^n \end{aligned}$$

Hence proven.

Exercise 4

No idea.

Exercise 5

Ambiguously phrased.

Exercise 6

Verified.

Exercise 7

Verified.

Exercise 8

There are 2^n configurations of inputs. Let's list them on a single row. Then, for each of the 2^n configurations, we have 2 options, either true or false, hence 2^{2^n} boolean functions. This is akin to choosing from the subset of 2^n configurations those who bear the value "true", and labelling the rest "false".

Exercise 9

I guess they can start from the existence of the empty set, which gives 0. Then work their way up using this axiom.

Exercise 10

We can easily use L'Hopital to deduce that $\lim_{n\to\infty} \frac{g(n)}{f(n)} \geq \lim_{n\to\infty} \frac{ab^n}{bn^d} = \infty$. This means that it is unbounded, hence we can find $C = 1$ such that $\frac{g(n)}{f(n)} > C = 1$, hence $g(n) > f(n)$. Too lazy to find the equality point.

Exercise 11

For the first, since b subsets each with k elements, then the combined total count of elements of bk. For the second, there are v elements, each found in exactly r subsets, which adds up to vr, which is also the combined total count of elements in subsets. Too lazy for examples.

Exercise 12

(i) Suppose $c = ab$. If a is even, then

$$c = \frac{a}{2}(2b)$$

If a is odd, then

$$\begin{aligned} c &= ab - b + b \\ &= (a-1)b + b \\ &= \frac{a-1}{2}(2b) + b \end{aligned}$$

At the end of the algorithm after n steps, we have $c = 1b_n + b_{i_1} + b_{i_2} + \cdots + b_{i_m}$, where, as per the instructions, we have $a_n = 1$. The b_i come from the shedding of the odd a's. Hence proven.
(ii) Idk, too lazy.
(iii) Fuh naw.

Exercise 13

Yeah, Buddha is dumb af.

Exercise 14

Number of books:

$$B = 25^{410\cdot 40\cdot 80}$$

Number of rooms:

$$R = \frac{B}{20\cdot 35}$$

Exercise 15

Hell nah.