

Graph Theory Bondy - Solutions Manual

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Chapter 1

Graphs

1.1 Graphs and Their Representation

Exercise 1.1.1

m is the number of edges. $\binom{n}{2}$ is the number of pairs of vertices. In a simple graph, every pair of vertices share at most one edge. Hence, $m \leq \binom{n}{2}$.

Exercise 1.1.2

(a) Each of the r vertices in X can link to at most s vertices in Y . It cannot link to any other vertex. The maximum number of edges is thus rs . Hence $m \leq rs$.

(b) $r = \frac{n}{2} + k$ and $s = \frac{n}{2} - k$ for some k . For verification, notice that $r + s = n$. Then,

$$\begin{aligned} rs &= \left(\frac{n}{2} + k\right) \left(\frac{n}{2} - k\right) \\ &= \frac{n^2}{4} - k^2 \leq \frac{n^2}{4} \end{aligned}$$

Of course, this only works if n is even, but the odd case follows a similar procedure.

(c) The equality is strict when $k = 0$, i.e. both sides have the same number of vertices.

Exercise 1.1.3

(a) Simply alternate sides.

(b) For every edge that goes to the other side, the next edge must come back. This creates pairs of back-and-forth edges. Since the number of edges is even, then the number of vertices is even as well.

Exercise 1.1.4

It's pretty obvious. $d(v_i) \geq \delta(G)$ for all i . Hence

$$\sum_{i=1}^n d(v_i) \geq n\delta(G)$$

$$\frac{\sum_{i=1}^n d(v_i)}{n} = d(G) \geq \delta(G)$$

A similar procedure is used for $\Delta(G)$.

Exercise 1.1.5

When $k = 0$, every vertex is isolated. For $k = 1$, the vertices are grouped in pairs, hence the graphs must have an even number of vertices. For $k = 2$, the graphs consist of disjoint cycles of arbitrary length.

Exercise 1.1.6

(a)