

Graph Theory Bondy - Solutions Manual

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Chapter 1

Graphs

1.1 Graphs and Their Representation

Exercise 1.1.1

m is the number of edges. $\binom{n}{2}$ is the number of pairs of vertices. In a simple graph, every pair of vertices share at most one edge. Hence, $m \leq \binom{n}{2}$.

Exercise 1.1.2

- (a) Each of the r vertices in X can link to at most s vertices in Y . It cannot link to any other vertex. The maximum number of edges is thus rs . Hence $m \leq rs$.
- (b) $r = \frac{n}{2} + k$ and $s = \frac{n}{2} - k$ for some k . For verification, notice that $r + s = n$. Then,

$$rs = \left(\frac{n}{2} + k\right) \left(\frac{n}{2} - k\right)$$

$$= \frac{n^2}{4} - k^2 \leq \frac{n^2}{4}$$

Of course, this only works if n is even, but the odd case follows a similar procedure.

- (c) The equality is strict when $k = 0$, i.e. both sides have the same number of vertices.

Exercise 1.1.3

- (a) Simply alternate sides.
- (b) For every edge that goes to the other side, the next edge must come back. This creates pairs of back-and-forth edges. Since the number of edges is even, then the number of vertices is even as well.

Exercise 1.1.4

It's pretty obvious. $d(v_i) \geq \delta(G)$ for all i . Hence

$$\sum_{i=1}^n d(v_i) \geq n\delta(G)$$

$$\frac{\sum_{i=1}^n d(v_i)}{n} = d(G) \geq \delta(G)$$

A similar procedure is used for $\Delta(G)$.

Exercise 1.1.5

When $k = 0$, every vertex is isolated. For $k = 1$, the vertices are grouped in pairs, hence the graphs must have an even number of vertices. For $k = 2$, the graphs consist of disjoint cycles of arbitrary length.

Exercise 1.1.6

(a)