EC504 Homework Assignment 5 (due November 22 at 4pm)

1. In the programming language of your choice, code up the Ford-Fulkerson algorithm that we discussed in class. When finding $s-t$ augmenting paths in the residual graph, you can use either BFS or DFS; in either case, you will have to write your own implementation. The data structure for storage of the graph is up to you (you will have to appropriately store and represent the capacities, as well as the flow values).

   Use your implementation to determine if there is a perfect matching in the graph contained in the file `hw5test.txt`. This file contains edge information for a bipartite graph. Each line in the file corresponds to an edge between a node in X and a node in Y. Nodes in X are numbered 1 to 100, and nodes in Y are numbered 101 to 200. Determine the size of the largest bipartite matching in this graph, and print it out.

2. Statistically, the arrival of spring typically results in increased accidents, and increased need for emergency medical treatment, which often requires blood transfusions. Consider the problem faced by a hospital that is trying to evaluate whether their blood supply is sufficient.

   The basic rule for blood donation is the following. A person's own blood supply has certain *antigens* present (we can think of antigens as a kind of molecular signature); and a person cannot receive blood with a particular antigen if their own blood does not have this antigen present. Concretely, this principle underpins the division of blood into four *types*: A, B, AB, and O. Blood of type A has the A antigen, blood of type B has the B antigen, blood of type AB has both, and blood of type O has neither. Thus, patients with type A can receive only blood types A or O in a transfusion, patients with type B can receive only B or O, patients with type O can receive only O, and patients with type AB can receive any of the four types.[1]

   (a) Let s_O, s_A, s_B and s_{AB} denote the supply in whole units of the different blood types on hand. Assume that the hospital knows the projected demand for each blood type d_O, d_A, d_B and d_{AB} for the coming week. Give a polynomial time algorithm to evaluate if the blood on hand would suffice for the projected need.

   (b) Consider the following example. Over the next week, they expect to need at most 100 units of blood. The typical distribution of blood types in US patients is 45% type O, 42% type A, 10% type B, and 3% type AB. The hospital wants to know if the blood supply they have on hand would be enough if 100 patients arrive with the expected type distribution. There is a total of 105 units of blood on hand. The table below gives these demands, and the supply on hand.

blood type:	O	A	B	AB
supply:	50	36	11	8
demand:	45	42	10	3

   Is the 105 units of blood on hand enough to satisfy the 100 units of demand? Solve this in two ways: first, use an argument based on minimum capacity cut to show why not all patients can receive blood. Second, find the maximum flow by running the corresponding flow network for

[1] The Austrian scientist Karl Landsteiner received the Nobel Prize in 1930 for his discovery of the blood types A, B, O, and AB.

this example through your implementation of Ford-Fulkerson. Verify that the results match what you determined by hand.

Also, provide an explanation for this fact that would be understandable to the clinic administrators, who have not taken a course on algorithms.

3. Solve problem 26-1 in CLRS (page 760).