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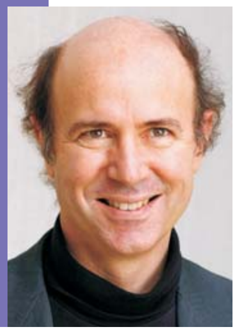

Reasonably effective: II. Devil's advocate

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Is the success of mathematics in natural science miraculous? Eugene Wigner famously claimed that it is. In my previous column (PHYSICS TODAY, November 2006, page 8), I explored some alternative, naturalistic explanations. By training and temperament, I try to be fair and balanced. For a real argument, we should bring in a lawyer.

Indeed, before accepting the validity of purported miracles, it's traditional to subject them to the cross-examination of a devil's advocate.

Let's listen now as the devil's advocate tries to stir up some reasonable doubt. (Of course, we don't expect *her* to be fair and balanced.)

"It's no miracle," she begins, "that clear thinking can clarify things. Nobody, I think, would call the success of mathematics in, say, accounting unreasonable, much less miraculous. It's just clear thinking applied to money. It's useful and important, of course, but there's nothing surprising about its success.

"What seems unreasonable, and can even appear miraculous, is that sometimes mathematics is highly leveraged. When a discovery or innovation can be conveyed in a few bits of information, but its purely mathematical elaboration comes to embrace an ever-widening circle of phenomena, it appears that magic is at work.

"Natural science provides many examples. Isaac Newton's laws of motion and gravity can be written in a few lines, but they allow us to track the planets, precess the equinoxes, discover Neptune, plan space voyages, and more. Werner Heisenberg's commutation relation can be written in a small portion of one line (to wit: $i[p_k, q_l] = \hbar$), but it allows us to master subatomic worlds. It's as if great trees could sprout from tiny seeds, or beautiful minds from tiny eggs.

"We're tempted to proclaim such occurrences to be miracles. But of course great trees *do* sprout from tiny seeds, and beautiful minds *do* grow from tiny eggs. And biologists are explaining how these things happen, step by step, without invoking miracles."

Leverage without mathematics

"Before ascribing special, miraculous status to the leverage of mathematics, we should consider whether it is unique. Can nonmathematical ideas have enormous scientific leverage? Indeed they can. For example,

- ▶ Charles Darwin's theory of evolution by natural selection is not a mathematical theory. *The Origin of Species* contains not a single equation. Yet its hypotheses, which can be stated in a few lines of prose, explain a multitude of surprising facts.
- ▶ The atomic theory of Democritus, John Dalton, and Dmitri Mendeleyev used no math beyond simple arithmetic but gave brilliant guidance to chemistry.

"And although it is not an idea in the conventional sense, the information that differentiates the genomes of *Homo habilis* and modern humans is a few bits (well, maybe a few megabytes). Yet that information is leveraged, by biological and historical processes that in no way resemble calculations, into the difference between extinct semi-monkeys and readers of PHYSICS TODAY."

Mathematics without leverage

The advocate continues playing to the jury:

"Not only can nonmathematics have lots of scientific leverage; often mathematics has little.

"There's a scene I love in *Raiders of the Lost Ark*. Maybe you'll remember it. Before an impending showdown, a sword-wielding assassin confronts Indiana Jones with a fearsome display of his prowess. In response, Indiana coolly pulls out a pistol and shoots the guy. Virtuoso displays of mathematical gymnastics that are outdone by straightforward calculations remind me of that showdown.

"Compare the three-body problem of celestial mechanics. It engaged some of the greatest mathematicians, from Newton to Henri Poincaré and beyond. (The latter part of Edmund Whittaker's classic text is devoted to this problem.[1]) Newton declared that his intense work on the problem, as it arose in connection with the Moon, was the only problem that ever "caused my head to ache." Much later, correcting his own faulty solution of a general form of the problem (stability of the solar system), Poincaré got ideas that are central to modern chaos theory. But today, if you're really interested in predicting the positions of planets and satellites or planning space voyages, the best available method is basically to grind the answer out, solving the equations using either numerical integration or high-order perturbation theory.

"Similarly, the challenge of calculating hadron masses and properties using quantum chromodynamics has stimulated many ingenious analytical investigations and discoveries; but if we really want to get concrete answers with decent precision, there's no real alternative to the relatively straightforward approach of discretization and numerical solution—lattice gauge theory.

"Over the past 20 years, nowhere in science has more dazzling and extensive mathematical work been done than in superstring theory." Feigning humility, the advocate smiles, "No doubt it passes over my head, but where I live, down to earth and close to the ground, I haven't seen much leverage." (One might conjecture that the advocate resides close to her client.)

Reverse leverage: Past

"In the extreme case," the advocate continues, "mathematical prejudice can even erect barriers to scientific understanding. Here are some important examples from history:

▶ The Pythagoreans went into denial about 'irrational' numbers, and the ancient Greeks, for all their genius, never came to terms with the concept of a numerical—as opposed to a geometric—continuum.
▶ In astronomy, insistence on the mathematical perfection of circular motion led to the epicycles that Johannes Kepler had to sweep away.
▶ It took Michael Faraday, a self-taught experimenter and mathematical ignoramus, to discover electromagnetic fields. The learned men around him instead described electric and magnetic phenomena using the established ideas of action-at-a-distance forces.

"In each of these cases, existing mathematics steered thought in the wrong direction. Instead, concepts that Nature proved necessary were brought into mathematics and greatly stimulated it. René Descartes' numerical model of space, Newton's concept of differential equations, and James Clerk Maxwell's pioneering vector field theory grew from the 'antimathematical' revolutions I just mentioned, and eventually led to new and better mathematics."

I blush for the advocate's cartoonish history, but she's making a lawyerly case.

"My client's favorite example of such reverse flow, from worldly considerations to mathematics, comes from the penumbra of science. It was the pursuit of vice—the queries of gamblers—that initiated probability theory."

The advocate sums up: "So, is the success of mathematics in science miraculous? Before you reach your verdict, consider all the evidence, and put it in perspective. I've shown you that there can be great success in science without mathematics, great success in spite of mathematics, and great success in the reverse direction, flowing from science to mathematics. A miracle worthy of the name should be more consistent. Doesn't it seem *reasonable* to doubt Wigner's 'miracle'?"

You be the jury.

Reverse leverage: Future

Niels Bohr distinguished ordinary truths from profound ones. The opposite of an ordinary truth is a falsehood; the opposite of a profound truth is another profound truth. The advocate's arguments persuade me that the success of mathematics in science is a great truth. The needs of science—broadly construed—have often stimulated the growth of mathematics into essentially new directions. Science has had (unreasonable?) success in mathematics.

It's fun to speculate: What will happen in the future?

John von Neumann fathered mathematical game theory and had a big hand in launching theoretical computer science—surely two major recent examples of science fertilizing mathematics. So his thoughts on our question carry great weight.

In an essay called "The Mathematician," von Neumann sermonized memorably in terms that the advocate could approve of:

> As a mathematical discipline travels far from its empirical source, or still more, if it is a second and third generation only indirectly inspired by ideas coming from "reality," it is beset with very grave dangers. It becomes more and more purely aestheticizing, more and more purely *l'art pour l'art*. This need not be bad, if the field is surrounded by correlated subjects, which still have closer empirical connections, or if the discipline is under the influence of men with an exceptionally well-developed taste. But there is a grave danger that the subject will develop along the line of least resistance, that the stream, so far from its source, will separate into a multitude of insignificant branches, and that the discipline will become a disorganized mass of details and complexities. In other words, at a great distance from its empirical source, or after much "abstract" inbreeding, a mathematical subject is in danger of degeneration. At the inception the style is usually classical; when it shows signs of becoming baroque, then the danger signal is up.[2]

As he was dying of cancer at the age of 53, von Neumann prepared (though he could not deliver) the Silliman Lectures for 1955. His notes formed the basis of a short book, *The Computer and the Brain*.[3] There he speaks of a mathematics he might have created, had he been given the time:

> Thus the outward forms of *our* mathematics are not absolutely relevant from the point of view of evaluating what the mathematical or logical language *truly* used by the central nervous system is. However, the above remarks about reliability and logical and arithmetical depth prove that whatever the system is, it cannot fail to differ considerably from what we consciously and explicitly consider as mathematics.

Of course, a great deal has happened since 1955. The capabilities of computers have expanded exponentially. (Here for once that usage is accurate, according to Moore's law!) The specifics of neurobiology and the broad base of knowledge in molecular genetics and biological development have also leapt forward. But I don't think that von Neumann's vision of a new mathematics synthesizing and guiding these advances has yet been realized. Were his vision realized, it might allow engineers to build into their designs the virtues of self-assembly, fault tolerance, and exploitation of rich interconnectedness that are so striking in biological information processing, and take computational intelligence to new levels.

In the preceding column, I quoted Richard Feynman's dream of a "great awakening of human intellect [that] may well produce a method of understanding the *qualitative* content of equations." It may be unnatural, or even impossible, to separate the task of understanding the qualitative content of *equations* from the task of recognizing the qualitative content of *patterns* in general. That may sound terribly vague, and it is, but a substantial discipline is evolving around this problem, which includes many connections to physics, as I've discovered through David MacKay's remarkable book.[4] Maybe this new discipline will become the new mathematics von Neumann predicted.

Just as the flow of insight from mathematics to empirical reality is not unidirectional, neither is the flow from physics. Those mature, highly elaborated sciences can avoid self-satisfied degeneration by remaining open to new challenges posed by the external world in its fullness—including technology, life, and mind.

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