ch5

5.1 Convergence in Probability

5.1.3. Let W_n denote a random variable with mean μ and variance b/n^p, where $p > 0$, μ, and b are constants (not functions of n). Prove that W_n converges in probability to μ.
Hint: Use Chebyshev's inequality.

5.1.5. Let $X_1, \ldots, X_n$ be iid random variables with common pdf

$$f(x) = \begin{cases} e^{-(x-\theta)} & x > \theta \quad -\infty < \theta < \infty \\ 0 & \text{elsewhere.} \end{cases} \qquad (5.1.3)$$

This pdf is called the **shifted exponential**. Let $Y_n = \min\{X_1, \ldots, X_n\}$. Prove that $Y_n \to \theta$ in probability, by first obtaining the cdf of Y_n.

5.1.7. For Exercise 5.1.5, obtain the mean of Y_n. Is Y_n an unbiased estimator of θ? Obtain an unbiased estimator of θ based on Y_n.

5.2 Convergence in Distribution

5.2.2. Let Y_1 denote the minimum of a random sample of size n from a distribution that has pdf $f(x) = e^{-(x-\theta)}$, $\theta < x < \infty$, zero elsewhere. Let $Z_n = n(Y_1 - \theta)$. Investigate the limiting distribution of Z_n.

5.2.4. Let Y_2 denote the second smallest item of a random sample of size n from a distribution of the continuous type that has cdf $F(x)$ and pdf $f(x) = F'(x)$. Find the limiting distribution of $W_n = nF(Y_2)$.

5.2.5. Let the pmf of Y_n be $p_n(y) = 1$, $y = n$, zero elsewhere. Show that Y_n does not have a limiting distribution. (In this case, the probability has "escaped" to infinity.)

5.2.11. Let the random variable Z_n have a Poisson distribution with parameter $\mu = n$. Show that the limiting distribution of the random variable $Y_n = (Z_n - n)/\sqrt{n}$ is normal with mean zero and variance 1.

5.2.18. Let $Y_1 < Y_2 < \cdots < Y_n$ be the order statistics of a random sample (see Section 5.2) from a distribution with pdf $f(x) = e^{-x}, 0 < x < \infty$, zero elsewhere. Determine the limiting distribution of $Z_n = (Y_n - \log n)$.

5.3 Central Limit Theorem

5.3.2. Let $\overline{X}$ denote the mean of a random sample of size 128 from a gamma distribution with $\alpha = 2$ and $\beta = 4$. Approximate $P(7 < \overline{X} < 9)$.

5.3.3. Let Y be $b(72, \frac{1}{3})$. Approximate $P(22 \leq Y \leq 28)$.

5.3.5. Let Y denote the sum of the observations of a random sample of size 12 from a distribution having pmf $p(x) = \frac{1}{6}$, $x = 1, 2, 3, 4, 5, 6$, zero elsewhere. Compute an approximate value of $P(36 \leq Y \leq 48)$.
Hint: Since the event of interest is $Y = 36, 37, \ldots, 48$, rewrite the probability as $P(35.5 < Y < 48.5)$.

5.3.12. Let $X_1, X_2, \ldots, X_n$ be a random sample from a Poisson distribution with mean μ. Thus, $Y = \sum_{i=1}^{n} X_i$ has a Poisson distribution with mean $n\mu$. Moreover, $\overline{X} = Y/n$ is approximately $N(\mu, \mu/n)$ for large n. Show that $u(Y/n) = \sqrt{Y/n}$ is a function of Y/n whose variance is essentially free of μ.