4.2 Interaction of an atom with a classical field

C_k = sebagai fungsi waktu (t)

$$|\psi(t)\rangle = \sum_k C_k(t)\, e^{-\frac{iE_k t}{\hbar}}$$

$$i\hbar \frac{\partial|\psi(t)\rangle}{\partial t} = \left(\hat{H}_0 + \hat{H}^{(1)}\right)|\psi(t)\rangle$$

$$\frac{\partial}{\partial t}\left\{\sum_k C_k e^{-\frac{iE_k t}{\hbar}}|k\rangle\right\} = -\frac{i}{\hbar}\sum_k C_k e^{-\frac{iE_k t}{\hbar}}\hat{H}_0|k\rangle - \frac{i}{\hbar}\sum_k C_k e^{-\frac{iE_k t}{\hbar}}\hat{H}^{(1)}|k\rangle$$

$$\langle l|e^{\frac{iE_l t}{\hbar}}\frac{\partial}{\partial t}\left(\sum_k C_k e^{-\frac{iE_k t}{\hbar}}|k\rangle\right) = -\frac{i}{\hbar}\sum_k \langle l|e^{-\frac{iE_k t}{\hbar}} C_k E_k|k\rangle$$

$$\langle l|e^{\frac{iE_l t}{\hbar}}\sum_k\left(\dot{C}_k e^{-\frac{iE_k t}{\hbar}} + \left(-\frac{iE_k}{\hbar}\right)e^{-\frac{iE_k t}{\hbar}}C_k\right)|k\rangle$$

$$\sum_k\left(\langle l|\,\dot{C}_k e^{-\frac{iE_k t}{\hbar}}\right)|k\rangle - \frac{i}{\hbar}\sum_k\left(\langle l|e^{-\frac{i(E_l-E_k)t}{\hbar}}E_k C_k|k\rangle\right)$$

$$\sum_k\left(\langle l|\,\dot{C}_k e^{-\frac{iE_k t}{\hbar}}\right)|k\rangle$$

$$C_k(t) = C_k^{(0)} + \lambda C_k^{(1)} + \lambda^2 C_k^{(2)} + \cdots$$

$$\dot{C}_k(t) = \dot{C}_k^{(0)} + \lambda\dot{C}_k^{(1)} + \lambda^2\dot{C}_k^{(2)} + \cdots$$

$$-\frac{i}{\hbar}\sum_l C_l(t)\,\langle k|\hat{H}^{(1)}|l\rangle\, e^{-\frac{iE_k t}{\hbar}} = -\frac{i}{\hbar}\sum_l\left(C_l^{(0)} + \lambda C_l^{(1)} + \lambda^2 C_l^{(2)}\right)\langle k|\hat{H}^{(1)}|l\rangle\, e^{-\frac{iE_k t}{\hbar}}$$

$$\dot{C}_k^{(0)} + \lambda\dot{C}_k^{(1)} + \lambda^2\dot{C}_k^{(2)} + \cdots = -\frac{i}{\hbar}\sum_l C_l^{(0)}\,\langle k|\lambda\hat{H}^{(1)}|l\rangle e^{-\frac{iE_k t}{\hbar}}$$

$$-\frac{i}{\hbar}\sum_l \lambda C_l^{(1)}\,\langle k|\lambda\hat{H}^{(1)}|l\rangle e^{-\frac{iE_k t}{\hbar}} - \frac{i}{\hbar}\sum_l \lambda^2 C_l^{(2)}\,\langle k|\lambda\hat{H}^{(1)}|l\rangle e^{-\frac{iE_k t}{\hbar}}$$

$$\dot{C}_l^{(0)} = 0$$

$$\dot{C}_k^{(1)} = -\frac{i}{\hbar}\sum_l C_l^{(0)}\,\langle k|\lambda\hat{H}^{(1)}|l\rangle e^{-\frac{iE_k t}{\hbar}}$$

$$\dot{C}_k^{(2)} = -\frac{i}{\hbar}\sum_l C_l^{(1)}\,\langle k|\lambda\hat{H}^{(1)}|l\rangle e^{-\frac{iE_k t}{\hbar}}$$

$$\dot{C}_f^{(1)} = -\frac{i}{\hbar}\sum_i C_i^{(0)}\,\langle f|\lambda\hat{H}^{(1)}|i\rangle e^{-\frac{iE_k t}{\hbar}}$$

$$= -\frac{i}{\hbar}C_l^{(0)} e^{-\frac{iE_k t}{\hbar}}\langle f|\lambda\hat{H}^{(1)}|i\rangle$$

$$\frac{dC_f^{(1)}}{dt} = -\frac{i}{\hbar}C_l^{(0)} e^{-\frac{iE_k t}{\hbar}}\hat{H}_{fi}^{(1)}$$

$$dC_f^{(1)} = \left(-\frac{i}{\hbar}C_l^{(0)} e^{-\frac{iE_k t}{\hbar}}\hat{H}_{fi}^{(1)}\right)dt$$

$$C_f^{(1)} = -\frac{i}{\hbar}\int_o^t C_l^{(0)} e^{-\frac{iE_k t}{\hbar}}\hat{H}_{fi}^{(1)}\ dt'$$

$$\dot{C}_f^{(2)} = -\frac{i}{\hbar}C_l^{(1)} e^{-\frac{iE_k t}{\hbar}}\langle f|\lambda\hat{H}^{(1)}|i\rangle$$

$$\frac{dC_f^{(2)}}{dt} = -\frac{i}{\hbar}C_l^{(1)} e^{-\frac{iE_k t}{\hbar}}\hat{H}_{fi}^{(1)}$$

$$dC_f^{(2)} = \left(-\frac{i}{\hbar}C_l^{(1)} e^{-\frac{iE_k t}{\hbar}}\hat{H}_{fi}^{(1)}\right)dt$$

$$C_f^{(2)} = -\frac{i}{\hbar}\int_o^t C_l^{(1)} e^{-\frac{iE_k t}{\hbar}}\hat{H}_{fi}^{(1)}\ dt'$$

Dengan,

$\hat{H} = \hat{V}\cos\omega t$, $V_{fi} = \langle f|\hat{V}|i\rangle$, $\cos(\omega t) = \frac{e^{-i\omega t}+e^{i\omega t}}{2}$

$$
\begin{aligned}
C_f^{(1)}(t) &= -\frac{i}{\hbar}\int_o^t \hat{H}_{fi}^{(1)}(t')\, e^{i\omega_{fi}t'}\ dt' \\
&= -\frac{i}{\hbar}\int_o^t \langle f|\hat{H}|i\rangle\ e^{i\omega_{fi}t'}\ dt' \\
&= -\frac{i}{\hbar}\int_o^t \langle f|\hat{V}\cos\omega t\,|i\rangle\ e^{i\omega_{fi}t'}\ dt' \\
&= -\frac{i}{\hbar}\int_o^t \langle f|\hat{V}\left(\frac{e^{-i\omega t}+e^{i\omega t}}{2}\right)|i\rangle\ e^{i\omega_{fi}t'}\ dt' \\
&= -\frac{i}{2\hbar}\hat{V}_{fi}\int_o^t \left(e^{i(\omega_{fi}-\omega)t\prime}+e^{i(\omega_{fi}+\omega)t\prime}\right)|i\rangle\ e^{i\omega_{fi}t\prime}\ dt' \\
&= -\frac{1}{2\hbar}\hat{V}_{fi}\left(\frac{1}{(\omega_{fi}-\omega)}\left(e^{i(\omega_{fi}-\omega)t}-1\right)+\frac{1}{(\omega_{fi}+\omega)}\left(e^{i(\omega_{fi}+\omega)t}-1\right)\right) \\
&= -\frac{1}{2\hbar}\hat{V}_{fi}\left(-\frac{e^{-i(\omega-\omega_{fi})t}-1}{(\omega-\omega_{fi})}+\frac{e^{i(\omega_{fi}+\omega)t}-1}{(\omega_{fi}+\omega)}\right) \\
&= -\frac{1}{2\hbar}\hat{V}_{fi}\left(\frac{e^{i(\omega+\omega_{fi})t}-1}{(\omega+\omega_{fi})}-\frac{e^{-i(\omega-\omega_{fi})t}-1}{(\omega-\omega_{fi})}\right)
\end{aligned}
$$

$$
\begin{aligned}
C_f^m(t) &= -\frac{\hat{V}_{fi}}{2\hbar}\left(\frac{e^{i(\omega+\omega_{fi})t}-1}{(\omega+\omega_{fi})}-\frac{e^{-i(\omega-\omega_{fi})t}-1}{(\omega-\omega_{fi})}\right) \\
&= \frac{\hat{V}_{fi}}{2\hbar}\left(\frac{e^{-i(\omega-\omega_{fi})t}-1}{(\omega-\omega_{fi})}\right)
\end{aligned}
$$

Dengan $(\omega-\omega_{fi}) = \Delta$

$$
\begin{aligned}
P_{i\to f}^{(1)}(t) &= \left|C_f^{(1)}(t)\right|^2 \\
&= \left|\frac{\hat{V}_{fi}}{2\hbar}\left(\frac{e^{-i(\omega-\omega_{fi})t}-1}{(\omega-\omega_{fi})}\right)\right|^2 \\
&= \frac{\left|\hat{V}_{fi}\right|^2}{4\hbar^2}\left(\frac{4\sin^2\left(\frac{(\omega-\omega_{fi})t}{2}\right)}{(\omega-\omega_{fi})^2}\right) \\
\left(P_{i\to f}^{(1)}(t)\right)_{max} &= \frac{\left|\hat{V}_{fi}\right|^2}{\hbar^2}\left(\frac{\sin^2(\Delta\, {}^t\!/_2)}{(\Delta)^2}\right)\frac{({}^t\!/_2)^2}{({}^t\!/_2)^2} \\
&= \frac{\left|\hat{V}_{fi}\right|^2}{\hbar^2}\frac{t^2}{4}\lim_{\Delta\, {}^t\!/_2\to 0}\frac{\sin^2(\Delta\, {}^t\!/_2)}{(\Delta\, {}^t\!/_2)^2} \\
\left(P_{i\to f}^{(1)}(t)\right)_{max} &= \frac{\left|\hat{V}_{fi}\right|^2}{\hbar^2}\frac{t^2}{4}
\end{aligned}
$$

$(e^{-ix}-1)(e^{ix}-1)$

$2-(e^{-ix}+e^{ix})$

$2-2\cos(x)$

$2(1-\cos(x))$ $\qquad$ $\cos 2x = 1-2\sin^2 x \to \cos x = 1-2\sin^2\left(\frac{x}{2}\right)$

$2\left(2\sin^2\left(\frac{x}{2}\right)\right)$ $\qquad$ $\cos x = 1-2\sin^2\left(\frac{x}{2}\right)$

$2\sin^2\left(\frac{x}{2}\right) = 1-\cos x$