

$$\hat{B} = \int B_p(\alpha, \alpha^*) |\alpha\rangle \langle \alpha| d^2\alpha \quad (3.110)$$

$$\begin{aligned} \langle \hat{B} \rangle &= Tr(\hat{B} \hat{\rho}) \\ &= Tr \left(\int B_p(\alpha, \alpha^*) |\alpha\rangle \langle \alpha| d^2\alpha \sum_n |n\rangle \langle n| \hat{\rho} \right) \\ &= \int \sum_n B_p(\alpha, \alpha^*) |\alpha\rangle \langle \alpha| \hat{\rho} |n\rangle \langle n| d^2\alpha \\ &= \int \sum_n B_p(\alpha, \alpha^*) \langle \alpha| \hat{\rho} |n\rangle \langle n| \alpha \rangle d^2\alpha \\ &= \int B_p(\alpha, \alpha^*) \sum \langle \alpha| \hat{\rho} |n\rangle \langle n| \alpha \rangle d^2\alpha \\ &= \int B_p(\alpha, \alpha^*) \langle \alpha| \hat{\rho} | \alpha \rangle d^2\alpha \end{aligned} \quad (3.111)$$

$$Q(\alpha) = \frac{\langle \alpha| \hat{\rho} | \alpha \rangle}{\pi} \quad (3.112)$$

$$\int Q(\alpha) d^2\alpha = 1 \quad (3.113)$$

$$\begin{aligned} \langle \hat{B} \rangle &= \hat{1} \cdot \hat{\rho} \cdot \hat{1} \\ &= \sum_n |n\rangle \langle n| \cdot \hat{B} \cdot \sum_m |m\rangle \langle m| \\ &= \sum_n \sum_m |n\rangle \langle n| \hat{B} |m\rangle \langle m| \\ &= \sum_n \sum_m |n\rangle B_{nm} \langle m| \end{aligned}$$

$$\begin{aligned} B_Q(\alpha, \alpha^*) &= \langle \alpha| \hat{B} | \alpha \rangle \\ &= \langle \alpha| \left\{ \sum_n \sum_m |n\rangle B_{nm} \langle m| \right\} | \alpha \rangle \\ &= \sum_n \sum_m \langle \alpha| n \rangle B_{nm} \langle m| \alpha \rangle \\ &= \sum_n \sum_m e^{-\frac{1}{2}|\alpha|^2} \sum_k \frac{(\alpha^*)^k}{\sqrt{k!}} \langle k| n \rangle B_{nm} \langle m| e^{-\frac{1}{2}|\alpha|^2} \sum_k \frac{(\alpha)^k}{\sqrt{k!}} |k\rangle \\ &= e^{-\frac{1}{2}|\alpha|^2} \sum_n \frac{(\alpha^*)^n}{\sqrt{n!}} \delta_{nn} B_{nm} \delta_{mm} \sum_m \frac{(\alpha^*)^m}{\sqrt{m!}} \\ &= e^{-|\alpha|^2} \sum_n \sum_m \frac{(\alpha^*)^n (\alpha)^m}{\sqrt{n! m!}} B_{nm} \\ &= e^{-|\alpha|^2} \sum_n \sum_m \frac{B_{nm}}{(n! m!)^{1/2}} (\alpha^*)^n (\alpha)^m \end{aligned} \quad (3.114)$$

$$\begin{aligned} \langle \hat{B} \rangle &= Tr(\hat{B} \hat{\rho}) \\ &= Tr \int \hat{B} P(\alpha) |\alpha\rangle \langle \alpha| d^2\alpha \\ &= Tr \int P(\alpha) |\alpha\rangle \hat{B} \langle \alpha| d^2\alpha \\ &= \int P(\alpha) \langle \alpha| \hat{B} | \alpha \rangle d^2\alpha \\ &= \int P(\alpha) B_Q(\alpha, \alpha^*) d^2\alpha \end{aligned} \quad (3.115)$$