

1. Diketahui:

$$a_1 = \left(\frac{\sqrt{3}}{2}, \frac{1}{2} \right) a, \quad a_2 = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2} \right) a$$
$$Q_i \cdot b_j = 2\pi \delta_{ij}$$

Dapatkan vektor-vektor kisi resiprok untuk graphene:

$$\vec{a}_1 \cdot \vec{b}_1 = 2\pi$$
$$\begin{bmatrix} \frac{\sqrt{3}}{2} & 1 \\ \frac{1}{2} & 2 \end{bmatrix} \begin{bmatrix} b_{1x} \\ b_{1y} \end{bmatrix} = 2\pi \frac{\sqrt{3}}{4\pi}$$
$$\frac{\sqrt{3}}{2} b_{1x} + \frac{1}{2} b_{1y} = \frac{1}{2} \sqrt{3}$$
$$\sqrt{3} b_{1x} + b_{1y} = \sqrt{3}$$

$$\vec{a}_2 \cdot \vec{b}_2 = 2\pi$$
$$\begin{bmatrix} \frac{\sqrt{3}}{2} & -1 \\ \frac{1}{2} & 2 \end{bmatrix} \begin{bmatrix} b_{2x} \\ b_{2y} \end{bmatrix} = 2\pi \frac{\sqrt{3}}{4\pi}$$
$$\frac{\sqrt{3}}{2} b_{2x} - \frac{1}{2} b_{2y} = \frac{1}{2} \sqrt{3}$$
$$\sqrt{3} b_{2x} - b_{2y} = \sqrt{3}$$

$$\vec{a}_1 \cdot \vec{b}_2 = 0$$
$$\begin{bmatrix} \frac{\sqrt{3}}{2} & 1 \\ \frac{1}{2} & 2 \end{bmatrix} \begin{bmatrix} b_{2x} \\ b_{2y} \end{bmatrix} = 0$$
$$\frac{\sqrt{3}}{2} b_{2x} + \frac{1}{2} b_{2y} = 0$$
$$\sqrt{3} b_{2x} + b_{2y} = 0$$
$$b_{2y} = -\sqrt{3} b_{2x}$$

$$\vec{a}_2 \cdot \vec{b}_1 = 0$$
$$\begin{bmatrix} \frac{\sqrt{3}}{2} & -1 \\ \frac{1}{2} & 2 \end{bmatrix} \begin{bmatrix} b_{1x} \\ b_{1y} \end{bmatrix} = 0$$
$$\frac{\sqrt{3}}{2} b_{1x} - \frac{1}{2} b_{1y} = 0$$
$$\sqrt{3} b_{1x} - b_{1y} = 0$$
$$b_{1y} = \sqrt{3} b_{1x}$$

$$\begin{aligned}
\sqrt{3}b_{1x} + b_{1y} &= \sqrt{3} \\
\sqrt{3}b_{1x} + \sqrt{3}b_{1x} &= \sqrt{3} \\
\sqrt{3}(b_{1x} + b_{1x}) &= \sqrt{3} \\
\sqrt{3} \, 2b_{1x} &= \sqrt{3} \\
b_{1x} &= \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
\sqrt{3}b_{1x} - b_{1y} &= 0 \\
\frac{\sqrt{3}}{2} &= b_{1y}
\end{aligned}$$

$$\begin{aligned}
\sqrt{3}b_{2x} - b_{2y} &= \sqrt{3} \\
\sqrt{3}b_{2x} - (-\sqrt{3}b_{2x}) &= \sqrt{3} \\
\sqrt{3} \, 2b_{2x} &= \sqrt{3} \\
b_{2x} &= \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
b_{2y} &= -\sqrt{3}b_{2x} \\
b_{2y} &= -\frac{\sqrt{3}}{2}
\end{aligned}$$

$$\therefore \vec{b_1} = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \frac{4\pi}{\sqrt{3}a} \quad \vec{b_2} = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right) \frac{4\pi}{\sqrt{3}a}$$