Tugas 12

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1. Dengan medan listrik berosilasi coba gantikan variabel ρ menjadi β untuk setiap turunan *density matrix* atau yang dikenal sebagai *bloch equations*.

Jawaban :

Dengan

$$\vec{E} = E_0 \cos \omega t = \frac{E_0}{2}\left(e^{i\omega t} + e^{-i\omega t}\right)$$

Dan

$$\beta_{21}(t) = \rho_{21}e^{i\omega t}\ , \beta_{12}(t) = \beta_{21}^* = \rho_{12}e^{-i\omega t} = (\rho_{21})^* e^{-i\omega t}$$

Maka persamaan

$$\dot{\rho}_{21} = i\frac{\vec{E}\mu_d}{\hbar}(\rho_{11} - \rho_{22}) - i\omega_{21}\rho_{21} - \frac{\rho_{21}}{T_2}$$

Dapat diubah dalam bentuk β_{21} dengan mengalikan $e^{i\omega t}$ ke seluruh ruas seperti

$$(\frac{d}{dt}\rho_{21})e^{i\omega t} = i\frac{\mu_d}{\hbar}\frac{E_0}{2}\left(e^{i\omega t} + e^{-i\omega t}\right)(\rho_{11} - \rho_{22})e^{i\omega t} - i\omega_{21}\rho_{21}e^{i\omega t} - \frac{\rho_{21}}{T_2}e^{i\omega t}$$

$$\frac{d}{dt}\beta_{21} - i\omega\rho_{21}e^{i\omega t} = i\frac{\mu_d}{2\hbar}E_0(e^{2i\omega t} + 1)(\rho_{11} - \rho_{22}) - i\omega_{21}\beta_{21} - \frac{\beta_{21}}{T_2}$$

$$\frac{d}{dt}\beta_{21} = i\frac{\mu_d}{2\hbar}E_0(\rho_{11} - \rho_{22}) + i(\omega - \omega_{21})\beta_{21} - \frac{\beta_{21}}{T_2}$$

Lalu untuk hasil persamaan berikut

$$\dot{\rho}_{11} - \dot{\rho}_{22} = 2i\frac{\vec{E}\mu_d}{\hbar}(\rho_{21} - (\rho_{21})^*) - \frac{\rho_{11} - \rho_{22} - (\rho_{11} - \rho_{22})_0}{T_1}$$

Jika dimasukkan $\vec{E}$ ke dalam persamaan di atas maka

$$\dot{\rho}_{11} - \dot{\rho}_{22} = 2i\frac{\mu_d}{\hbar}\frac{E_0}{2}\left(e^{i\omega t} + e^{-i\omega t}\right)(\rho_{21} - (\rho_{21})^*) - \frac{\rho_{11} - \rho_{22} - (\rho_{11} - \rho_{22})_0}{T_1}$$

$$= i\frac{\mu_d}{\hbar}E_0\left(\rho_{21}e^{i\omega t} - (\rho_{21})^* e^{i\omega t} + \rho_{21}e^{-i\omega t} - (\rho_{21})^* e^{-i\omega t}\right) - \frac{\rho_{11} - \rho_{22} - (\rho_{11} - \rho_{22})_0}{T_1}$$

$$= i\frac{\mu_d}{\hbar}E_0\left(\rho_{21}e^{i\omega t} - \beta_{21}^* e^{2i\omega t} + \beta_{21}e^{-2i\omega t} - \left(\rho_{21}\right)^* e^{-i\omega t}\right) - \frac{\rho_{11} - \rho_{22} - \left(\rho_{11} - \rho_{22}\right)_0}{T_1}$$

$$= i\frac{\mu_d}{\hbar}E_0(\rho_{21}e^{i\omega t} - (\rho_{21})^* e^{-i\omega t}) - \frac{\rho_{11} - \rho_{22} - (\rho_{11} - \rho_{22})_0}{T_1}$$

$$\dot{\rho}_{11} - \dot{\rho}_{22} = i\frac{\mu_d}{\hbar}E_0(\beta_{21} - \beta_{21}^*) - \frac{\rho_{11} - \rho_{22} - (\rho_{11} - \rho_{22})_0}{T_1}$$

Hasil kedua persamaan di atas merupakan *aproximation* dengan membuang bagian yang mengandung $e^{\pm 2i\omega t}$ sehingga persamaan Bloch untuk $\dot{\rho}_{11} - \dot{\rho}_{22}$ dan $\dot{\beta}_{21}$.

2. Cari solusi persamaan *bloch* dalam *steady state*.

Jawaban :

Dalam keadaan tunak, tidak ada perubahan lagi dalam *fractional population difference* $(\rho_{11}-\rho_{22})$ maka

$$\frac{d(\rho_{11}-\rho_{22})}{dt} = 0$$

Begitu juga dengan

$$\frac{d\beta_{21}}{dt} = 0$$

Maka kedua persamaan akan menjadi

$$0 = i\frac{\mu_d}{\hbar}E_0(\beta_{21}-\beta_{21}^*) - \frac{\rho_{11}-\rho_{22}-(\rho_{11}-\rho_{22})_0}{T_1}$$

$$0 = i\frac{\mu_d}{2\hbar}E_0(\rho_{11}-\rho_{22}) + i(\omega-\omega_{21})\beta_{21} - \frac{\beta_{21}}{T_2}$$

Untuk bagian bawah dapat diubah menjadi

$$i\frac{\mu_d}{2\hbar}E_0(\rho_{11}-\rho_{22}) = \left(i(\omega_{21}-\omega)+\frac{1}{T_2}\right)\beta_{21}$$

$$\beta_{21} = \Omega(\rho_{11}-\rho_{22})\frac{1}{\left(\omega_{21}-\omega-\frac{i}{T_2}\right)}$$

Dengan $\Omega = \frac{\mu_d}{2\hbar}E_0$ dan konjugatnya

$$\beta_{21}^* = \Omega(\rho_{11}-\rho_{22})\frac{1}{\left(\omega_{21}-\omega+\frac{i}{T_2}\right)}$$

Lalu persamaan berwarna oranye dapat diubah menjadi

$$i2\Omega(\beta_{21}-\beta_{21}^*) = \frac{(\rho_{11}-\rho_{22})-(\rho_{11}-\rho_{22})_0}{T_1}$$

Dengan melakukan substitusi nilai β maka didapat

$$i2\Omega^2(\rho_{11}-\rho_{22})\left(\frac{1}{\left(\omega_{21}-\omega-\frac{i}{T_2}\right)} - \frac{1}{\left(\omega_{21}-\omega+\frac{i}{T_2}\right)}\right) = \frac{(\rho_{11}-\rho_{22})-(\rho_{11}-\rho_{22})_0}{T_1}$$

$$(\rho_{11}-\rho_{22})\left(-\frac{1}{\left(\omega_{21}-\omega-\frac{i}{T_2}\right)} + \frac{1}{\left(\omega_{21}-\omega+\frac{i}{T_2}\right)} + \frac{1}{i2\Omega^2T_1}\right) = \frac{(\rho_{11}-\rho_{22})_0}{i2\Omega^2T_1}$$

$$(\rho_{11}-\rho_{22})\left(\frac{T_2}{(T_2(\omega_{21}-\omega)+i)} - \frac{T_2}{(T_2(\omega_{21}-\omega)-i)} + \frac{1}{i2\Omega^2T_1}\right) = \frac{(\rho_{11}-\rho_{22})_0}{i2\Omega^2T_1}$$

Untuk bagian biru dikerjakan terlebih dahulu sebagai berikut

$$\ldots + \frac{1}{i2\Omega^2 T_1} = \frac{T_2\big((T_2(\omega_{21}-\omega)-i)-(T_2(\omega_{21}-\omega)+i)\big)}{(T_2(\omega_{21}-\omega)+i)(T_2(\omega_{21}-\omega)-i)} + \frac{1}{i2\Omega^2 T_1}$$

$$= \frac{T_2(-2i)}{(T_2)^2(\omega_{21}-\omega)^2+1} + \frac{1}{i2\Omega^2 T_1}$$

$$= \frac{T_2(-2i)i2\Omega^2 T_1 + (T_2)^2(\omega_{21}-\omega)^2+1}{((T_2)^2(\omega_{21}-\omega)^2+1)i2\Omega^2 T_1}$$

$$\ldots + \frac{1}{i2\Omega^2 T_1} = \frac{1+(\omega_{21}-\omega)^2 T_2^2 + 4\Omega^2 T_2 T_1}{((T_2)^2(\omega_{21}-\omega)^2+1)i2\Omega^2 T_1}$$

Sehingga dengan memasukkan kembali ke persamaan sebelumnya, didapat

$$(\rho_{11}-\rho_{22})\left(\frac{1+(\omega_{21}-\omega)^2 T_2^2 + 4\Omega^2 T_2 T_1}{((T_2)^2(\omega_{21}-\omega)^2+1)i2\Omega^2 T_1}\right) = \frac{(\rho_{11}-\rho_{22})_0}{i2\Omega^2 T_1}$$

$$(\rho_{11}-\rho_{22}) = (\rho_{11}-\rho_{22})_0 \frac{1+(\omega_{21}-\omega)^2 T_2^2}{1+(\omega_{21}-\omega)^2 T_2^2 + 4\Omega^2 T_2 T_1}$$

Lalu baru dapat mencari nilai β_{21} setelah mendapat $\rho_{11}-\rho_{22}$ yaitu

$$\beta_{21} = \Omega(\rho_{11}-\rho_{22})_0 \frac{1+(\omega_{21}-\omega)^2 T_2^2}{1+(\omega_{21}-\omega)^2 T_2^2 + 4\Omega^2 T_2 T_1} \frac{1}{\left(\omega_{21}-\omega-\frac{i}{T_2}\right)}$$

$$= \Omega(\rho_{11}-\rho_{22})_0 \frac{1+(\omega_{21}-\omega)^2 T_2^2}{1+(\omega_{21}-\omega)^2 T_2^2 + 4\Omega^2 T_2 T_1} \frac{T_2}{(T_2(\omega_{21}-\omega)-i)}$$

$$= \Omega(\rho_{11}-\rho_{22})_0 \frac{(T_2(\omega_{21}-\omega)+i)(T_2(\omega_{21}-\omega)-i)}{1+(\omega_{21}-\omega)^2 T_2^2 + 4\Omega^2 T_2 T_1} \frac{T_2}{(T_2(\omega_{21}-\omega)-i)}$$

$$\beta_{21} = \Omega(\rho_{11}-\rho_{22})_0 \frac{(T_2^2(\omega_{21}-\omega)+iT_2)}{1+(\omega_{21}-\omega)^2 T_2^2 + 4\Omega^2 T_2 T_1}$$

Maka nilai real dan imajinernya menjadi

$$\text{real}(\beta_{21}) = \frac{\Omega(\rho_{11}-\rho_{22})_0 T_2^2(\omega_{21}-\omega)}{1+(\omega_{21}-\omega)^2 T_2^2 + 4\Omega^2 T_2 T_1}$$

$$\text{im}(\beta_{21}) = \frac{\Omega(\rho_{11}-\rho_{22})_0 T_2}{1+(\omega_{21}-\omega)^2 T_2^2 + 4\Omega^2 T_2 T_1}$$

Terdapat sedikit perbedaan di dalam buku dimana di buku untuk komponen $(\omega_{21}-\omega)^2$ ditulis dengan argumen $(\omega-\omega_{21})^2$. Jika coba ditelaah, hasilnya bisa dimodifikasi seperti

$$(\omega_{21}-\omega)^2 = (\omega_{21}-\omega)(\omega_{21}-\omega)$$

$$= \big(-(\omega-\omega_{21})\big)\big(-(\omega-\omega_{21})\big) = (\omega-\omega_{21})^2$$

Namun itu hanya berlaku untuk fungsi kuadratnya saja.

3. Cari relasi *complex* χ dengan ϵ.

Jawaban :

Dengan diketahui untuk linear dielektrik didapat polarisasi P sebagai berikut

$$\vec{P} = \epsilon_0 \chi \vec{E}$$

Dan perpindahan atau *displacement* D dalam material sebagai berikut

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon \vec{E}$$

$$\epsilon_0 \vec{E} + \epsilon_0 \chi \vec{E} = \epsilon \vec{E}$$

$$\frac{\epsilon}{\epsilon_0} = 1 + \chi$$

$$\epsilon_r = 1 + \chi$$

Susceptibilitas χ di sini merupakan bilangan kompleks dan ϵ_r merupakan permitivitas relatif atau *complex frequency dependent dielectric constant*. Maka jika dicari relasi kompleks dari persamaan tersebut didapat

$$\epsilon' + i\epsilon'' = (1 + \chi') + i\chi''$$