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number state. → non classic

α = continue. complex

$$\hat{\rho} = \sum_i p_i |\psi_i\rangle\langle\psi_i| \qquad Tr(\hat{\rho}) = \sum_i p_i = 1$$

$$\hat{\rho} = \hat{1}\,\hat{\rho}\,\hat{1} \quad \leftarrow \quad 1 = \sum_n |n\rangle\langle n|$$

$$\hat{\rho} = \sum_n \{|n\rangle\langle n|\}\,\hat{\rho}\,\sum_m \{|m\rangle\langle m|\}$$

$$\hat{\rho} = \sum_n \sum_m |n\rangle\left[\langle n|\hat{\rho}|m\rangle\right]\langle m|$$

$$\hat{\rho} = \sum_n \sum_m |n\rangle\,\hat{\rho}_{nm}\,\langle m|$$

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coherent state.

$$\int |\alpha\rangle\langle\alpha| \frac{d^2\alpha}{\pi} = 1$$

$$\hat{\rho} = \hat{1}\,\hat{\rho}\,1$$

$$\hat{\rho} = \int |\alpha'\rangle\langle\alpha'| \frac{d^2\alpha'}{\pi}\,\hat{\rho}\,\int |\alpha''\rangle\langle\alpha''| \frac{d^2\alpha''}{\pi}$$

$$\hat{\rho} = \iint \frac{d^2\alpha'}{\pi}\frac{d^2\alpha''}{\pi}\,|\alpha'\rangle\left[\langle\alpha'|\hat{\rho}|\alpha''\rangle\right]\langle\alpha''|$$

$$\hat{\rho} = \iint \langle\alpha'|\hat{\rho}|\alpha''\rangle\,|\alpha'\rangle\langle\alpha''|\,\frac{d^2\alpha'\,d^2\alpha''}{\pi^2}$$

$$\hat{\rho} = \int p(\alpha)\,|\alpha\rangle\langle\alpha|\,d^2\alpha$$

tapi bisa di short cut. → langsung ?

$$Tr\{|a\rangle\langle b|\} = \langle b|a\rangle$$

$$Tr(|a\rangle\langle b| = \sum_n \langle\phi_n|a\rangle\langle b|\phi_n\rangle$$

$$= \sum_n \langle b|\phi_n\rangle\langle\phi_n|a\rangle$$

$$= \langle b|\left[\sum_n |\phi_n\rangle\langle\phi_n|\right]|a\rangle$$

$$=$$

$$Tr\,\hat{\rho} = Tr\left(\int p(\alpha)\,\hat{1}\,|\alpha\rangle\langle\alpha|\,d^2\alpha\right) \qquad 1 = \sum_n |n\rangle\langle n|$$

$$= Tr\left\{\int \sum_n p(\alpha)\,|n\rangle\langle n|\alpha\rangle\langle\alpha|\,d^2\alpha\right\}$$

$$= \int \sum_n p(\alpha)\,\langle n|\alpha\rangle\langle\alpha|n\rangle\,d^2\alpha$$

$$= \int p(\alpha)\sum_n \langle\alpha|n\rangle\langle n|\alpha\rangle\,d^2\alpha$$

$$= \int p(\alpha)\,\langle\alpha|\alpha\rangle\,d^2\alpha$$

$$= \underbrace{\int p(\alpha)\,d^2\alpha = 1}_{\text{phase-space prob.}}$$

$$(\ldots(1-u \quad |u\rangle\ ) \longrightarrow \hat{\rho} = \int P(\alpha)\ |\alpha\rangle\langle\alpha|\ d^2\alpha$$

$$\langle -u|\hat{\rho}|u\rangle = \int P(\alpha)\ \langle -u|\alpha\rangle\langle\alpha|u\rangle\ d^2\alpha$$

$$= \int P(\alpha)\ \langle -u|\alpha\rangle\langle\alpha|u\rangle\ d^2\alpha \qquad \langle\beta|\alpha\rangle = e^{(-\frac{1}{2}|\beta|^2 - \frac{1}{2}|\alpha|^2 + \beta^*\alpha)}$$

$\alpha^* u - \alpha u^* = 2i(x'y - xy')$

$$= \int P(\alpha)\ e^{(-\frac{1}{2}|u|^2 - \frac{1}{2}|\alpha|^2 - u^*\alpha)}\ e^{(-\frac{1}{2}|\alpha|^2 - \frac{1}{2}|u|^2 + \alpha^* u)}\ d^2\alpha \qquad = e^{-\frac{1}{2}|\beta|^2} e^{-\frac{1}{2}|\alpha|^2 + \beta^*\alpha}$$

$\alpha = x + iy$

$u = x_1 + iy_1$

$$\langle -u|\hat{\rho}|u\rangle = \cancel{\int P(\alpha)}\ e^{-|u|^2} \int P(\alpha)\ e^{-|\alpha|^2}\ e^{\alpha^* u - u^*\alpha}\ d^2\alpha$$

$$= e^{-|u|^2} \int P(\alpha)\ e^{2i(x'y - xy')}\ d^2\alpha$$

$\Rightarrow$ hanya menunjukkan dapat melalui transform fourier *!

$$e^{|u|^2}\langle -u|\hat{\rho}|u\rangle = \int P(\alpha)\ e^{\alpha^* u - u^*\alpha}\ d^2\alpha$$

$$e^{|u|^2}\langle -u|\hat{\rho}|u\rangle = g(u)\ \cancel{\quad}$$

$$f(\alpha) = \frac{1}{\pi^2}\int g(u)\ e^{u^*\alpha - u\alpha^*}\ d^2u$$

$$f(\alpha)\ e^{-|\alpha|^2} = \frac{1}{\pi^2}\int e^{|u|^2}\langle -u|\hat{\rho}|u\rangle\ e^{u^*\alpha - u\alpha^*}\ d^2u.$$

$$P(\alpha) = \frac{e^{|\alpha|^2}}{\pi^2}\int e^{|u|^2}\langle -u|\hat{\rho}|u\rangle\ e^{u^*\alpha - u\alpha^*}\ d^2u.$$

pure coherent state $\rightarrow \hat{\rho} = |\beta\rangle\langle\beta|$

$$\langle -u|\hat{\rho}|u\rangle = \langle -u|\beta\rangle\langle\beta|u\rangle \qquad = e^{\frac{1}{2}(\beta^* u - \beta u^* - |\beta - u|^2)}$$

$$\underbrace{\langle -u|\beta\rangle}_{1}\ \langle\beta|u\rangle \qquad = e^{\frac{1}{2}(-|\beta|^2 - |u|^2) + \beta^* u}$$

$$= e^{\frac{1}{2}((-u)^*\beta - (-u)\beta^* - |-u-\beta|^2}$$

$$= e^{\frac{1}{2}(-|\beta|^2 - |u|^2) - u^*\beta}$$

$$\langle -u|\hat{\rho}|u\rangle = e^{-|\beta|^2}\ e^{-|u|^2}\ e^{\beta^* u - u^*\beta}$$

$$P(\alpha) = \frac{e^{|\alpha|^2}}{\pi^2}\int e^{|u|^2}\langle -u|\hat{\rho}|u\rangle\ e^{u^*\alpha - u\alpha^*}\ d^2u$$

$$= \frac{e^{|\alpha|^2}}{\pi^2}\int e^{|u|^2}\ e^{-|\beta|^2}\ e^{-|u|^2}\ e^{\beta^* u - u^*\beta}\ d^2u$$

$$= \frac{e^{|\alpha|^2 - |\beta|^2}}{\pi^2}\int e^{u^*(\alpha-\beta)}\ e^{-u(\alpha^* - \beta^*)}\ d^2u.$$

$$\frac{1}{\pi^2}\int \exp[u^*(\alpha-\beta) - u(\alpha^* - \beta^*)]\ d^2u$$

$$= \frac{1}{\pi^2}\int \exp[ipy - iqx]\ dp\ dq$$

$$= \frac{1}{\pi}\int e^{ipy}\ dp\ \frac{1}{\pi}\int e^{-iqx}\ dq$$

$$= \delta(y)\ \delta(x)$$

$$= \delta^2(\alpha - \beta)$$

$$\delta^2(\alpha-\beta) = \delta(\mathrm{Re}[\alpha] - \mathrm{Re}[\alpha])\ \delta(\mathrm{Im}[\alpha] - \mathrm{Im}[\alpha])$$

misal $x = \mathrm{Re}[\alpha] - \mathrm{Re}[\beta]$, $y = \mathrm{Im}[\alpha] - \mathrm{Im}[\beta]$

def $u = \frac{p}{2} + i\frac{q}{2}$

$$u^*(\alpha-\beta) - u(\alpha^* - \beta^*) = (\tfrac{p}{2} - i\tfrac{q}{2})(x+iy) - (\tfrac{p}{2} + i\tfrac{q}{2})(x-iy)$$

$$= ipy - iqx$$