Tugas 2

Buktikan bahwa Baker-Haursdoff-Campbell Formula menjadi seperti ini

$$e^{A}Be^{-A} = B + [A,B] + \frac{1}{2!}[A,[A,B]] + \frac{1}{3!}\big[A,[A,[A,B]]\big] + \cdots$$

Dengan menggunakan deret taylor maka

$$e^{A} = 1 + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \frac{A^4}{4!} + \cdots$$

$$e^{-A} = 1 - A + \frac{A^2}{2!} - \frac{A^3}{3!} + \frac{A^4}{4!} + \cdots$$

Maka

$$\begin{aligned}
e^{A}Be^{-A} &= \left(1 + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots\right)B\left(1 - A + \frac{A^2}{2!} - \frac{A^3}{3!} + \cdots\right)\\
&= \left(1 + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots\right)\left(B - BA + B\frac{A^2}{2!} - B\frac{A^3}{3!} + \cdots\right)\\
&= B + AB + \frac{A^2B}{2!} + \frac{A^3B}{3!} - BA - ABA - \frac{A^2BA}{2!} - \frac{A^3BA}{3!} + \frac{BA^2}{2!} + \frac{ABA^2}{2!}\\
&\quad + \frac{A^2BA^2}{2!\,2!} + \frac{A^3BA^2}{3!\,2!} - \frac{BA^3}{3!} - \frac{ABA^3}{3!} - \frac{A^2BA^3}{2!\,3!} - \frac{A^3BA^3}{3!\,3!}\\
&= B + [A,B] + \frac{1}{2!}(A^2B - 2ABA - BA^2)\\
&\quad + \frac{1}{3!}(A^3B - 3A^2BA + 3ABA^2 - BA^3) + \cdots\\
&= B + [A,B] + \frac{1}{2!}(AAB - ABA - ABA + BAA)\\
&\quad + \frac{1}{3!}(AAAB - AABA - 2(AABA - ABAA) + ABAA - BAAA) + \cdots\\
&= B + [A,B] + \frac{1}{2!}(A[A,B] + [A,A]B - B[A,A] - [A,B]A\ )\\
&\quad + \frac{1}{3!}([A,A]AB + A[A,A]B + AA[A,B]\\
&\quad - 2([A,A]BA + A[A,B]A + AB[A,A]) + [A,B]AA + B[A,A]A\\
&\quad + BA[A,A]) + \cdots\\
&= B + [A,B] + \frac{1}{2!}([A,AB - BA]) + \frac{1}{3!}[A,AAB - 2ABA + BAA] + \cdots\\
&= B + [A,B] + \frac{1}{2!}[A,[A,B]] + \frac{1}{3!}\big[A,[A,[A,B]]\big] + \cdots
\end{aligned}$$