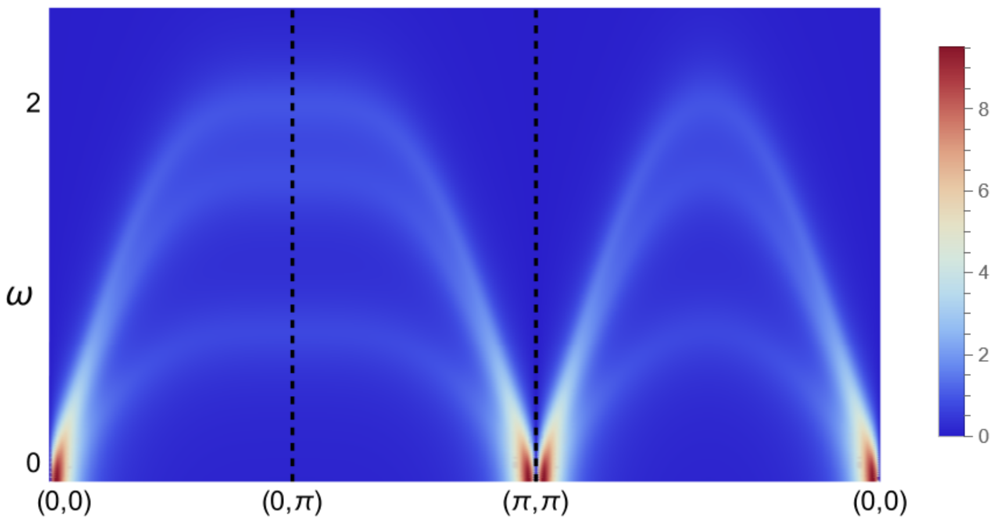
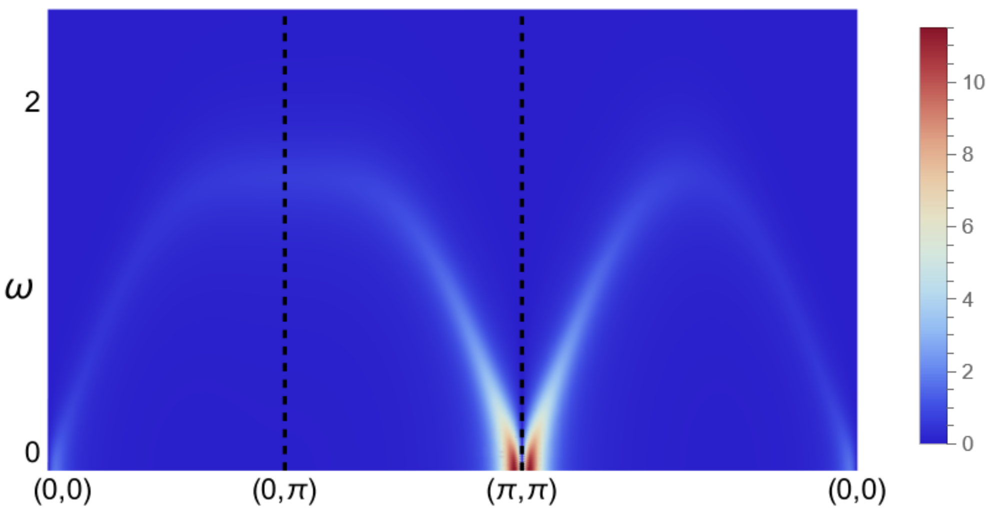
1. Gapless points: The dispersion relation Eq.(6.8.1) becomes gapless when $J_{xy} \geq J_z$. The gapless points are $Q = (0,0)$ and (π, π).



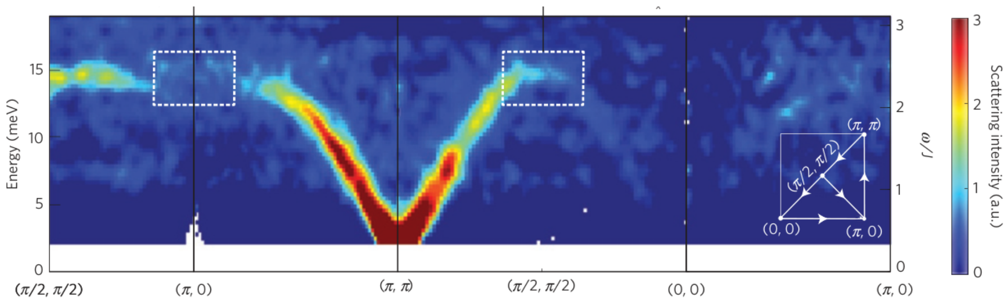
FIGURES 6.3: AFM spin wave DSF from a single sublattice G_{AA} at different magnetizations, from top to bottom: $m_z = 0.4, 0.2, 0.1$. For $m_z = 0.1$, the strength is amplified by a factor of 2.



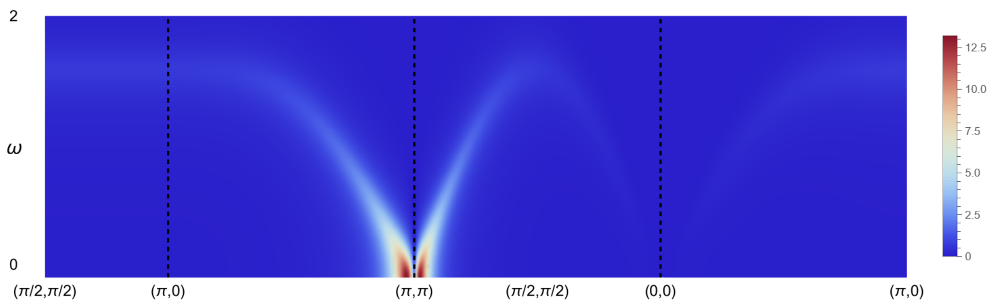
FIGURES 6.4: AFM spin wave DSF summed over all sublattices $S_{AA} + S_{BB} - S_{AB} - S_{BA}$. (note: it should all be plus sign. for some reason, I find it only fit experiment with minus sign for $S_{AB} = S_{BA}$.)

6.8.5 The two-spin problem by GFs

As a third example of an application, we wish to treat a model system with interactions, whose partition function can still be calculated exactly, so that all the interesting correlation functions are known in principle. This thus opens up the possibility of comparing the results of the Green's function method with the exact solutions.



FIGURES 6.5: experimental data from Ref. Piazza et al. [2014]



FIGURES 6.6: AFM spin wave DSF summed over all sublattices $S_{AA}+S_{BB}-S_{AB}-S_{BA}$ for direct comparison with 6.4.

The model system in question consists of two spins of magnitudes

$$S_1 = S_2 = \frac{1}{2},$$

which are coupled to each other via an exchange interaction J and are presumed to be acted upon by a homogeneous magnetic field. We describe them in terms of the correspondingly simplified Heisenberg model (2.221):

$$H = -J\left(S_1^+S_2^- + S_1^-S_2^+ + 2S_1^zS_2^z\right) - b\left(S_1^z + S_2^z\right),$$

where

$$b = \frac{1}{\hbar}g_J\mu_B B_0.$$

The limitation to $S_1 = S_2 = 1/2$ allows some simplifications:

$$S_i^{\mp}S_i^{\pm} = \frac{\hbar^2}{2} \mp \hbar S_i^z$$
$$S_i^{\pm}S_i^z = -S_i^zS_i^{\pm} = \mp\frac{\hbar}{2}S_i^{\pm}$$
$$\left(S_i^+\right)^2 = 0; \quad \left(S_i^z\right)^2 = \frac{\hbar^2}{4}$$

For our further discussion, we require several commutators:

$$\left[S_1^-, H\right]_- = -J\left[S_1^-, S_1^+\right]_- S_2^- - 2J\left[S_1^-, S_1^z\right]_- S_2^z - b\left[S_1^-, S_1^z\right]_- =$$
$$= 2\hbar J\left(S_1^zS_2^- - S_1^-S_2^z\right) - \hbar b S_1^-$$