

Advanced Quantum Mechanics: Lecture 1

Mengnan Chen

Department of physics

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Overview

① Introduction

② Fundamental Concepts

- Kets, Bras and Operators
- Base kets and matrix representations
- Measurements, observables, and the uncertainty relations
- The uncertainty relation
- Change of Basis
- Position, Momentum, and Translation
- The density matrix

Recommend Readings

Text Book:

J. J. Sakurai, *Modern Quantum Mechanics*, Cambridge University Press (3rd edition, 2020).

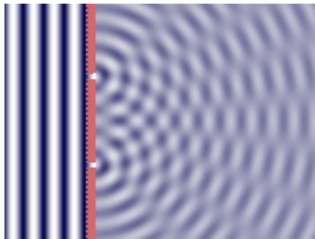
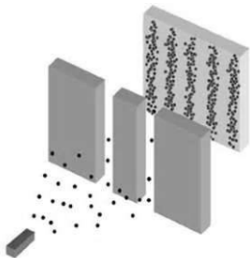
Additional reading materials:

1. P. A. M. Dirac, *Principles of Quantum Mechanics*.
2. Guang-Jong Ni and Su-Qing Chen, *Advanced Quantum Mechanics*, Fudan University Press. (In Chinese)
3. R. Shankar, *Principles of Quantum Mechanics*, Plenum Press.
4. Li Zhang and Molin Ge, *Frontier Problems in Quantum Mechanics*.

The essence of quantum theory

The very notion of existence is changed:

- The existence is both **particles** and **waves**
- The existence is neither **particles** nor **waves**
 \Rightarrow New quantum existence: particle-wave duality



- Particle at location $|x_1\rangle$ or $|x_2\rangle$ or ...

Superposition of different location states $\sum_i \psi(x_i, t) |x_i\rangle$

Wave function

$\psi(x, t)$: complex number represents the Wave function

Born's statistical interpretation of the wave function

probability density: $|\psi(x_i, t)|^2$ at position x_i and time t

Normalized property

$$\sum_i |\psi(x_i, t)|^2 = 1 \quad \text{or} \quad \int dx |\psi(x, t)|^2 = 1$$

\Rightarrow Wave $\psi(x, t) \propto e^{ikx - i\omega t}$, $k \propto$ momentum ($p = \hbar k$), $\omega \propto$ energy ($E = \hbar\omega$)

The relation between ω and k is named as the **dispersion relation**.

In **condensed matter physics**, the **dispersion relation** is also known as the **electronic band structure**.

The quantum mechanics is basic mathematical framework to describe the microscopic objects.