Advanced Quantum Mechanics

Axia

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Advanced Quantum Mechanics: Lecture 1

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September 14, 2023

Overview

1. Introduction
2. Fundamental Concepts
   - Kets, Bras and Operators
   - Base kets and matrix representations
   - Measurements, observables, and the uncertainty relations
   - The uncertainty relation
   - Change of Basis
   - Position, Momentum, and Translation
   - The density matrix

Recommend Readings

Text Book:
J. J. Sakurai, *Modern Quantum Mechanics*, Cambridge University Press (3rd edition, 2020).

Additional reading materials:

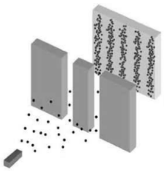
1. P. A. M. Dirac, *Principles of Quantum Mechanics*.
2. Guang-Jong Ni and Su-Qing Chen, *Advanced Quantum Mechanics*, Fudan University Press. (In Chinese)
3. R. Shankar, *Principles of Quantum Mechanics*, Plenum Press.
4. Li Zhang and Molin Ge, *Frontier Problems in Quantum Mechanics*.

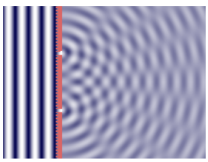
The essence of quantum theory

The very notion of existence is changed:

- The existence is both particles and waves
- The existence is neither particles nor waves

  $\Longrightarrow$ New quantum existence: particle-wave duality





- Particle at location $|x_1\rangle$ or $|x_2\rangle$ or ...

  Superposition of different location states $\sum_i \psi(x_i,t)|x_i\rangle$

Wave function

$\psi(x,t)$: complex number represents the Wave function

Born's statistical interpretation of the wave function

probability density: $|\psi(x_i,t)|^2$ at position x_i and time t

Normalized property

$$\sum_i |\psi(x_i,t)|^2 = 1 \quad \text{or} \quad \int \mathrm{d}x |\psi(x,t)|^2 = 1$$

$\Rightarrow$Wave $\psi(x,t) \propto \mathrm{e}^{\mathrm{i}kx-\mathrm{i}\omega t}$, $k \propto$ momentum ($p = \hbar k$), $\omega \propto$ energy ($E = \hbar\omega$)

The relation between ω and k is named as the **dispersion relation**.

In **condensed matter physics**, the **dispersion relation** is also known as the electronic band structure.

The quantum mechanics is basic mathematical framework to describe the microscopic objects.

Advanced Quantum Mechanics: Lecture 2

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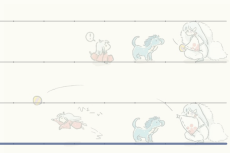
October 11, 2023

Overview

1. Time Evolution and the Schrödinger Equation
   - Time Evolution Operator
   - Energy Eigenkets
   - Time Dependence of Expectation Values
2. Correlation Amplitude and the Energy-Time Uncertainty Relation
3. The Schrödinger Versus the Heisenberg Picture
   - Unitary Operators
   - The Heisenberg Equation of Motion
4. Construction of the path integral
   - Analytic continuation to imaginary time
5. The WKB Approximation
6. Potentials and Gauge Transformations

Time Evolution