Project Problem & Instructions

AE 410: Navigation and Guidance
AE 641: Introduction to Navigation and Guidance

Total Points: 100

Submission Deadline: November 24, 2024

(Group 20)

Instructions :

- A common report (only one per group) for every group needs to be submitted online in portable document format (pdf) on Moodle. Handwritten reports should be scanned in good quality to generate the pdf file. All the plots and figures need to be legible (with suitable font size).
- Please submit the codes for generating the necessary figures for your report. The filename for the report should be Team-number-Report. For instance, the final file of Group-01 must be named like Group-01-Report.

Problem 1: [50] Consider the longitudinal dynamics of an aircraft (*Section* 4.4, [1]),

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{w} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{bmatrix} = \underbrace{\begin{bmatrix} X_u & X_w & 0 & -g \\ Z_u & Z_w & u_0 & 0 \\ M_u + M_{\dot{w}} Z_u & M_w + M_{\dot{w}} Z_w & M_q + M_{\dot{w}} u_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{bmatrix} + \underbrace{\begin{bmatrix} X_{\delta_e} \\ Z_{\delta_e} \\ M_{\delta_e} + M_{\dot{w}} Z_{\delta_e} \\ 0 \end{bmatrix}}_{\mathbf{b}} \Delta\delta_e$$

where $\mathbf{x} = \begin{bmatrix} \Delta u & \Delta w & \Delta q & \Delta\theta \end{bmatrix}^T$ is the state vector and $\Delta\delta_e$ is the control input.

1. Utilize the aerodynamic coefficients of the aircraft model Fighter Aircraft: F104-A referring Appendix B [1] and determine the stability derivatives using Table 4.2 [1]. Assume X_{δ_e} to be zero.
2. Compute the eigenvalues of the open-loop system and design an optimal feedback control $\Delta\delta_e = \mathbf{k}^T\mathbf{x}$ referring LQR control design discussed in Lecture 9, Slides 5-10 of AE 305 Course.
3. Tune the gain vector $\mathbf{k}$ appropriately so that the matrix $\left(\mathbf{A} + \mathbf{b}\mathbf{k}^T\right)$ is Hurwitz, and obtain the closed-loop expression as below,
$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{b}\Delta\delta_e \\ &= \left(\mathbf{A} + \mathbf{b}\mathbf{k}^T\right)\mathbf{x}, \end{aligned}$$
which can be converted to a discrete-time linear system as:
$$\mathbf{x}(k+1) = \mathbf{F}\mathbf{x}(k).$$
While creating the discrete-time linear system, select a suitable sampling interval such that the absolute value of each eigenvalue of the matrix $\mathbf{F}$ is strictly less than 1.
4. To generate **true states**, the process noise should be added after the integration step, i.e.:
$$\mathbf{x}(k+1) = \mathbf{F}\mathbf{x}(k) + \mathbf{w}(k), \quad \mathbf{y}(k) = \mathbf{x}(k) + \mathbf{v}(k)$$

and take the covariance matrices $\mathbf{Q}$ and $\mathbf{R}$ corresponding to $\mathbf{w}(k)$ and $\mathbf{v}(k)$ with a standard deviation to be a small percentage $(1-5\%)$ of the corresponding state value.

5. It is advised to put the command "randn('state', α)" in the beginning of your code, where α is some positive integer. This will initialize the random number generator (randn in this case) to the same value every time you run the code, and thus ensure that your results are repeatable.

6. Design Kalman Filter (KF) to estimate the **true states**. Plot the true states and their estimation and write the observations.

Problem 2: [50] Consider an engagement scenario, shown in Figure 24, where the missile is 20 km away in the radial direction from the target line-of-sight (LOS) of 30°. Assume the speed of the missile (denoted by V_M) is equal to 1000 m/s and that of the target (denoted by V_T), is two-fifth of V_M, respectively. The target has a flight path angle of 120°. The missile's autopilot is assumed to be perfect.

1. Design at least three kinds of guidance laws taught in the lecture. Plot the time evolution of the missile and target trajectories and the required guidance command for each guidance law designed.

2. Compare the plots of trajectories and guidance commands for each of the guidance laws and write your observations.

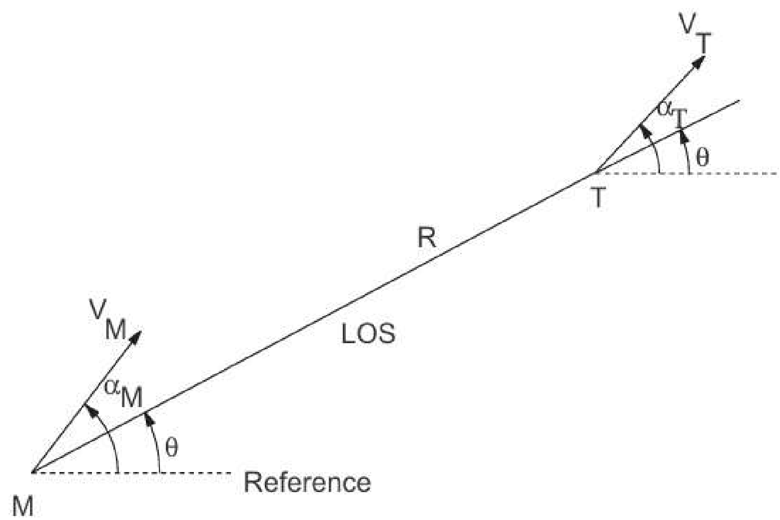

Figure 18: Missile-Target Engagement Geometry

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References

[1] Nelson, Robert C. Flight stability and automatic control. Vol. 2. New York: WCB/McGraw Hill, 1998.