Control Theory (AE 308)

Course Project Report

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27 November 2023

Contents

Introduction

Given the open-loop transfer function of the system:

$$G(s) = \frac{k}{s(s+3)(s+7)}$$

The system is in a unity feedback structure, meaning that closed-loop transfer function T(s) is given by.

$$T(s) = \frac{G(s)}{1+G(s)}$$

Performance parameters of the uncompensated system subject to suitable test signals are as follows

- Settling Time (T_s):
  The settling time is the required time for the system's response to reach and stay within a certain percentage (usually 5%) of its final value. We can calculate the settling time by observing the system's step response which is 65 seconds.
- Peak Overshoot (%OS):
  Peak Overshoot is the maximum percentage by which the response exceeds the steady-state value. It can be obtained by observing the step response, which vanishes to zero signifying no damping.

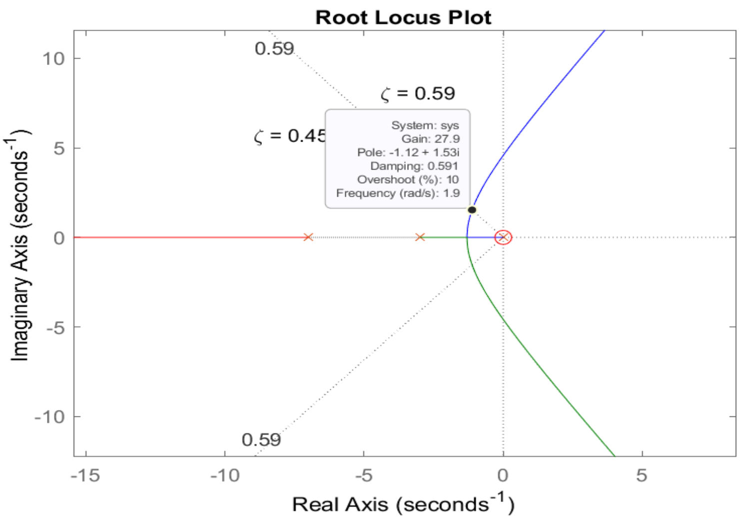
Computations related to uncompensated system:

The given system is a third-order type 1 system. Now, we will calculate the value of K using the first requirement with respect given percent overshoot 10 percent.

We get the damping ratio corresponding to it as

$$e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} = \%\,OS$$

$$\zeta = 0.519$$



The dominant poles that we get from the root locus corresponding to the zeta value are p = -1.12+1.53i, -1.12-1.53i ,

And for these poles gain k= 27.9

For uncompensated steady state error constant

$K = \lim_{n\to\infty} G(s) = k/(3)(7) = 1.32$

Control Objective

Given:

Open loop transfer system

$$G(s) = \frac{K}{s(s+3)(s+7)}$$

Form this G(s) we get.

Poles are: 0,-3,-7. Number of Poles: 3

Zeros are: 0; Number of zeros: 0

The controller needs to be designed such that the steady-state error constant is 4 such that dominant poles are significantly intact.

Controller Design:

from the above computations and the root locus of the uncompensated system we get:

dominant poles : -1.12+1.53i, -1.12-1.53i

gain: k= 27.9

steady state error constant: K = 1.32

let open loop transfer function for compensator be

$$G_c(s) = \frac{\beta(Ts+1)}{(\beta Ts+1)}$$

Then according to the given condition

Steady state error constant for compensated system is 4.

$$\lim_{s\to 0} sG(s)G_C(s) = 4$$

$$\lim_{s \to 0} \frac{s\beta(Ts+1)k}{s(\beta Ts+1)(s+3)(s+7)} = 4$$

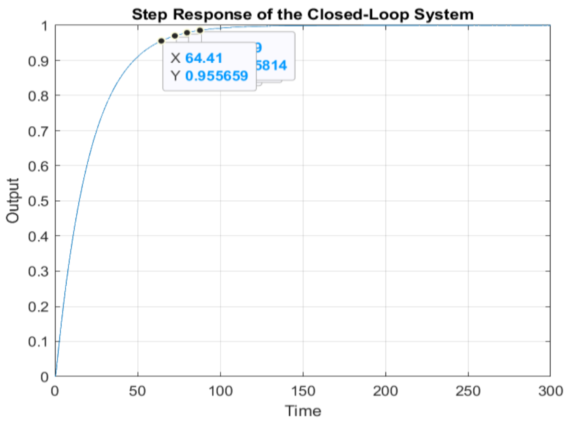
$$\beta = 3.01$$

Furthermore, the poles and zeros added by the compensator must nullify the angle contribution so that no extra phase is added the dominant poles are intact.

$$T = 0.01$$

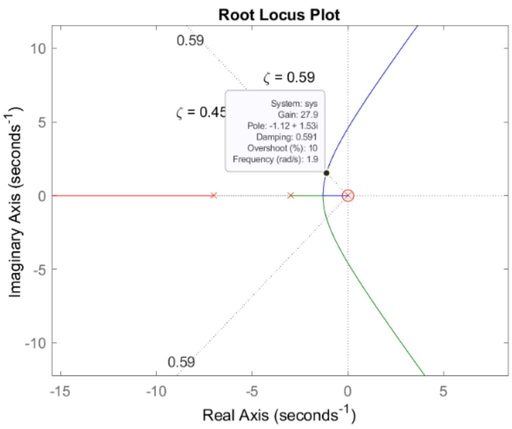
Simulation Results:

unit step response for uncompensated system:



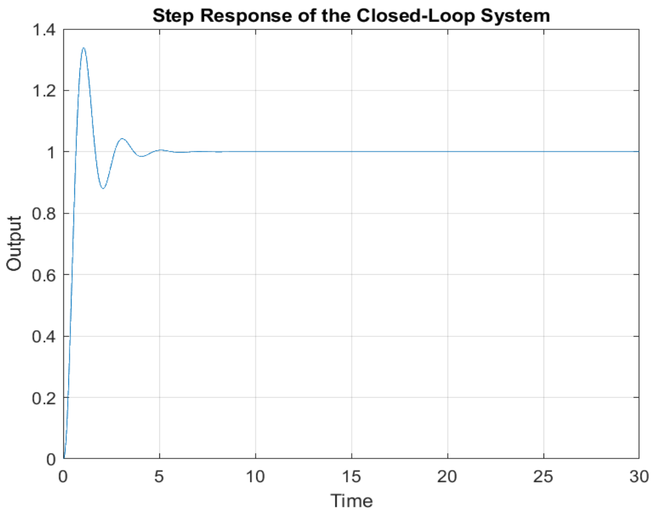

```
numerator = 1;
denominator = conv([1 0], conv([1 3], [1 7]));

% Create the transfer function G(s)
G = tf(numerator, denominator);

% Create a unity feedback system
H = 1;  % Unity feedback

% Create the closed-loop transfer function T(s) = G(s) / (1 + G(s)H(s))
T = feedback(G, H);

% Time vector for the step response plot
t = 0:0.01:300;  % Adjust the time vector as needed

% Compute the step response of the closed-loop system
[y, time] = step(T, t);

% Plot the step response
figure;
plot(time, y);
grid on;
title('Step Response of the Closed-Loop System');
xlabel('Time');
ylabel('Output');
```

root locus for uncompensated system:



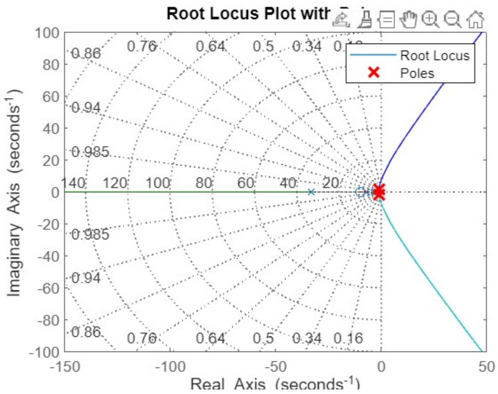

```
num = [0 0 0 1];
den = [1 10 21 0];
rlocus(num, den)
v = [-10 10 -10 10];
p1 = -1.12 + 1.53i;
p2 = -1.12 - 1.53i;
sgrid(0.519,[])
gtext('\zeta = 0.519')
```

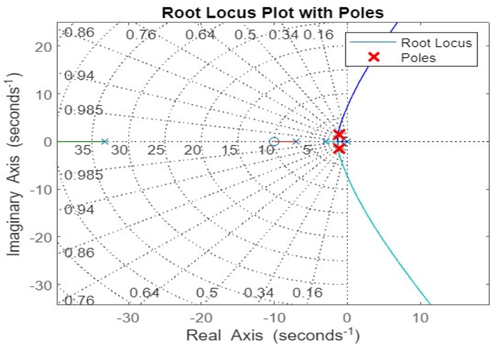
Unit response for compensated system:

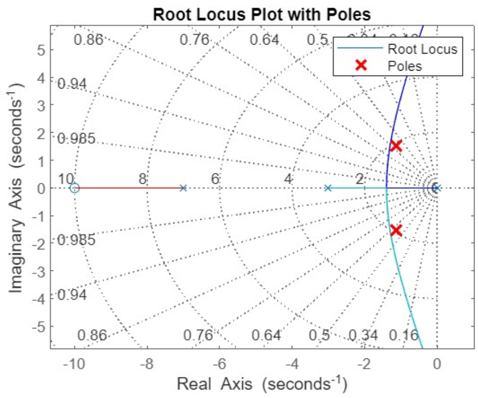



```
numerator = 27.9*3.01*[0.1 1];
denominator = conv([1 0], conv([1 3], conv([1 7],[0.0301 1])));

% Create the transfer function G(s)
G = tf(numerator, denominator);

% Create a unity feedback system
H = 1;  % Unity feedback

% Create the closed-loop transfer function T(s) = G(s) / (1 + G(s)H(s))
T = feedback(G, H);

% Time vector for the step response plot
t = 0:0.01:30;  % Adjust the time vector as needed

% Compute the step response of the closed-loop system
[y, time] = step(T, t);

% Plot the step response

figure;
plot(time, y);
grid on;
title('Step Response of the Closed-Loop System');
xlabel('Time');
ylabel('Output');
```

root locus for compensated system:








```
numerator = [174.27 * 0.1, 174.27 * 1];
denominator = conv([1 0], conv([1 3], conv([1 7], [0.0301 1])));

% Create the transfer function
sys = tf(numerator, denominator);

% Plot the root locus
figure;
rlocus(sys);
hold on;

% Plot the specific points
s1 = -1.12 + 1.53i;
s2 = -1.12 - 1.53i;

plot(real(s1), imag(s1), 'rx', 'MarkerSize', 10, 'LineWidth', 2);
plot(real(s2), imag(s2), 'rx', 'MarkerSize', 10, 'LineWidth', 2);

grid on;
title('Root Locus Plot with Poles');
xlabel('Real Axis');
ylabel('Imaginary Axis');

legend('Root Locus','Poles');
```

Conclusion :

The compensated transfer function of the system comes out to be

$$\frac{3.01 * 27.9(0.1s + 1}{s(s + 3)(s + 7)(3.01 * 0.01 + 1)} = G(s)$$

Satisfying dominant poles criteria with given steady state error constant producing and overshoot in the unit response.