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OPINION PAPER

Use of the correct heat conduction–convection equation as basis for heat-pulse sap flow methods in anisotropic wood

Maurits W. Vandegehuchte* and Kathy Steppe

Laboratory of Plant Ecology, Faculty of Bioscience Engineering, Ghent University, Coupure links 653, 9000 Gent, Belgium

* To whom correspondence should be addressed. E-mail: maurits.vandegehuchte@ugent.be

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Abstract

Heat-pulse methods to determine sap flux density in trees are founded on the theory of heat conduction and heat convection in an isotropic medium. However, sapwood is clearly anisotropic, implying a difference in thermal conductivity along and across the grain, and hence necessitates the theory for an anisotropic medium. This difference in thermal conductivities, which can be up to 50%, is, however, not taken into account in the key equation leading to the currently available heat-pulse methods. Despite this major flaw, the methods remain theoretically correct as they are based on derivations of the key equation, ruling out any anisotropic aspects. The importance of specifying the thermal characteristics of the sapwood according to axial, tangential or radial direction is revealed as well as referring to and using the proper anisotropic theory in order to avoid confusion and misinterpretation of thermal properties when dealing with sap flux density measurements or erroneous results when modelling heat transport in sapwood.

Key words: Heat capacity, heat dissipation, sap flow, thermal conductivity, thermal diffusivity.

Introduction

What errors can arise from applying the isotropic theory to model heat transport in sapwood? If heat-pulse-based sap-flow methods are founded on a theory only applicable for isotropic materials, can they be accurate for an anisotropic material such as sapwood? This opinion paper will answer these pertinent questions by summarizing and comparing the heat-pulse theory for both isotropic and anisotropic media.

Sap flow measurement methods applying heat as a tracer are based on the principles of heat conduction–convection in the sapwood. The equation for heat flow in sapwood has been developed by Marshall (1958), inspired by the work of Carslaw and Jaeger (1947). This worldwide used equation is based on the analytical solution of the partial differential equation for combined conduction and convection of heat in a specified isotropic medium. If this equation can be solved for the convection term, the velocity by which heat propagates through the medium, can be determined and from this, sap flux density in the medium can be calculated.

For the application of an instantaneous line source of heat along the z-axis in an isotropic medium, Marshall (1958) proposed the following analytical solution:

$$\Delta T = \frac{Q}{4\pi Dt} exp\left[-\frac{(x - V_{\mathrm{h}}t)^2 + y^2}{4Dt}\right] \quad (1)$$

with ΔT (K) the difference between the temperature at position (x, y) before application of the heat-pulse and a time t (s) after application of the heat pulse, Q (K m^2) defined as the temperature to which the amount of heat liberated per unit length of the line would raise a unit volume of the substance, D the thermal diffusivity (m^2 s^{-1}), and V_{h} the heat pulse velocity (m s^{-1}). This heat pulse velocity is directly proportional to sap flux density (for a list of the symbols used, see Table 1).

In this equation, thermal diffusivity is defined by:

$$D = K(\rho c)^{-1} \quad (2)$$

Table 1. List of symbols

Symbol	Unit	Explanations
D	$m^2\ s^{-1}$	Thermal diffusivity
K	$W\ m^{-1}\ K^{-1}$	Thermal conductivity
c	$J\ kg\ K^{-1}$	Heat capacity
ρ	$kg\ m^{-3}$	Density
ρc	$J\ m^{-3}\ K^{-1}$	Volumetric heat capacity
D_{ax}	$m^2\ s^{-1}$	Axial thermal diffusivity
D_{tg}	$m^2\ s^{-1}$	Tangential thermal diffusivity
K_{ax}	$W\ m^{-1}\ K^{-1}$	Axial thermal conductivity
K_{tg}	$W\ m^{-1}\ K^{-1}$	Tangential thermal conductivity
Q	$K\ m^2$	The temperature to which the amount of heat liberated per unit length of the line would raise a unit volume of the substance
q	$J\ m^{-1}$	Heat given off per unit length of the heat source
t	s	Time after application of the heat pulse
x	m	Distance between the heater needle and the axial needle
y	m	Distance between the heater needle and the tangential needle
ΔT	K	Temperature difference between the temperature during or after application of the heat pulse at a time t and the temperature before application of the heat pulse at distance (x,y) from the heater
V_h	$m\ s^{-1}$	Heat velocity
ΔT_{down}	K	Temperature difference between the temperature during or after application of the heat pulse at a time t and the temperature before application of the heat pulse at distance×downflow of the heater
ΔT_{up}	K	Temperature difference between the temperature during or after application of the heat pulse at a time t and the temperature before application of the heat pulse at distance×upflow of the heater
T_{down}	K	Temperature at distance×downflow
T_{up}	K	Temperature at distance×upflow
T_{tg}	K	Temperature at distance y
MC		Moisture content (moisture to dry weight)
ρ_d	$kg\ m^{-3}$	Density of dry wood
t_m	s	Time from the start of the heat pulse till the maximal temperature rise
P	$W\ m^{-3}$	External heat input

with K the thermal conductivity ($W\ m^{-1}\ K^{-1}$), ρ the density ($kg\ m^{-3}$), and c the heat capacity ($J\ kg^{-1}\ K^{-1}$) of the material. Multiplying Q with the volumetric heat capacity (ρc) of the material results in the amount of heat liberated by the heater per unit length of the heater, q ($J\ m^{-1}$).

It is these equations that have been used by Marshall (1958) and many others (Cohen *et al.*, 1981; Swanson and Whitfield, 1981; Swanson, 1983; Green and Clothier, 1988; Jones *et al.*, 1988; Becker, 1998; Burgess *et al.*, 2001) as the basis for the development of heat-pulse based sap flow models and methods.

Assumption of isotropic medium versus actual anisotropic sapwood

Marshall (1958) stated that dry wood is not isotropic, but has a greater thermal conductivity along the grain (K_{ax}) than across the grain (K_{tg}). He also acknowledged that wet wood might also be anisotropic, although probably to a lesser extent. However, for fresh wood segments with a water content greater than the fibre saturation point, it is known that the axial thermal conductivity (K_{ax}) can be up to two times larger than the radial (K_{rad}) or tangential (K_{tg}) conductivity, depending on wood species (Maku, 1954; Steinhagen, 1977). Moreover, when applying the equations as mentioned in Swanson (1983) based on the work of Turrell *et al.* (1967) and Siau (1971) for the calculation of K_{ax} and K_{tg}, it is clear that K_{tg} is remarkably smaller across all dry wood densities and moisture contents with K_{tg}, on average, 54±7.5% of K_{ax} (Fig. 1). The equations of Swanson (1983) were also applied by Burgess *et al.* (2001) in the Heat Ratio Method, but only the axial thermal diffusivity ($D_{ax}=K_{ax}(\rho c)^{-1}$) is considered.

When considering anisotropy, the correct analytical solution of the partial differential equation for combined conduction and convection of heat becomes:

$$\Delta T = \frac{\rho c Q}{4\pi\sqrt{K_{ax}K_{tg}t}} exp\left[-\frac{\rho c}{4t}\left(\frac{(x-V_h t)^2}{K_{ax}}+\frac{y^2}{K_{tg}}\right)\right] \qquad (3)$$

And it is, hence, this equation (equation 3) that should be the key equation for sap flow measurements based on pulsed heating by an ideal line heater instead of equation 1 which has, so far, been used as reference equation.

Implications of anisotropy for sap flow methodology

Applying equations 1 and 3 for heat transport in sapwood with a given dry wood density and moisture content and the

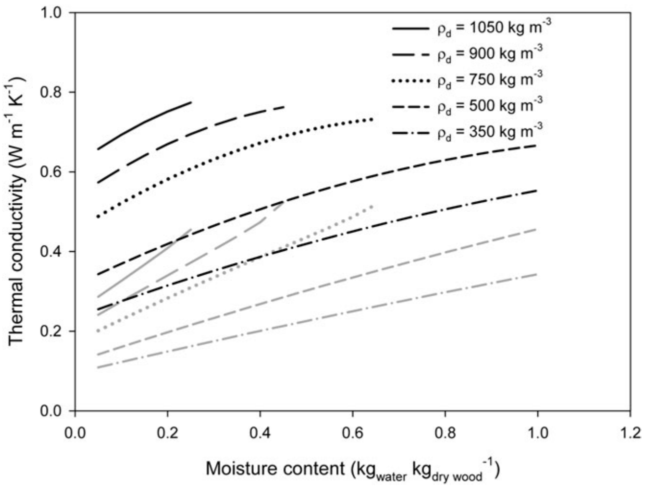

Fig. 1. Axial (black lines) and tangential (grey lines) thermal conductivities for different moisture contents and dry wood densities (ρ_d) as calculated according to Turrell *et al*. (1967) based on Siau (1971).

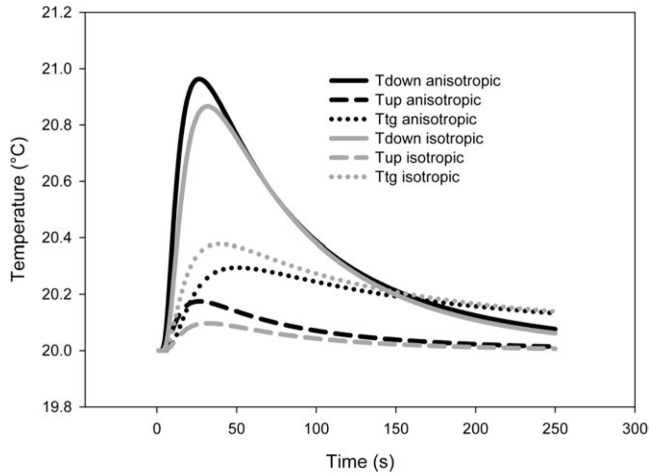

Fig. 2. Temperatures at 6 mm axially downstream, upstream and tangentially from the heater for a moisture content of 70% and dry wood density of 550 kg m^{-3}. For the isotropic case, the thermal diffusivity D was taken as the geometric mean of D_{ax} and D_{tg} of the anisotropic case.

same heat input, clearly leads to different temperature patterns in the wood (Fig. 2) due to the differences in thermal diffusivity, with the isotropic diffusivity taken as the geometric mean of the axial and tangential diffusivity (Fig. 3). Note that for higher moisture contents, these differences becomes smaller. These differences also affect the parameters used in the methods based on equation 1 to determine sap flux density, such as the Tmax Method (Cohen *et al.*, 1981) (equations 4 and 5) and the Heat Ratio Method (HRM) (Burgess *et al.*, 2001) (equation 6).

$$D = x^2(4t_m)^{-1} \tag{4}$$

$$V_h = \left(x^2 - 4Dt_m\right)^{-1/2}(t_m)^{-1} \tag{5}$$

with D the thermal diffusivity of the sapwood (m^2 s^{-1}), V_h the heat velocity (m s^{-1}), t_m the time at which ΔT is maximal (s), and x the distance between the measurement point and the heater (m).

$$V_h = (D/x)\ln\left(\Delta T_{down}/\Delta T_{up}\right) \tag{6}$$

with V_h the heat velocity (m s^{-1}), x the distance between the measurement points and the heater (m), and ΔT_{down} and ΔT_{up} the temperature differences before and after the heat pulse at x m downstream and upstream of the heater, respectively. For the HRM, thermal diffusivity is determined according to Swanson (1983).

For the Tmax Method, the difference in temperature field results in a difference in t_m (Fig. 4), while for the Heat Ratio Method, the ratio ($\Delta T_{down}/\Delta T_{up}$) is clearly influenced (Fig. 5). The risk exists that confusion in parameters can lead to erroneous results. If, for instance, thermal diffusivity is determined based on Swanson (1983), a distinction is made between axial and tangential diffusivity. If then this anisotropic axial thermal diffusivity is coupled to temperatures modelled according to the isotropic theory, for which

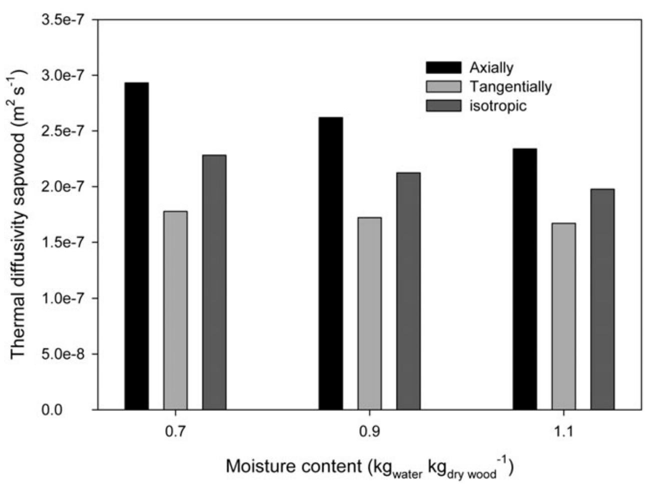

Fig. 3. Thermal diffusivities for both the isotropic and anisotropic case for different moisture contents. The dry wood density was taken as 550 kg m^{-3}. The isotropic thermal diffusivity was taken as the geometric mean of D_{ax} and D_{tg} of the anisotropic case.

both axial and tangential diffusivity are the same (for instance the geometric mean of the actual axial and tangential diffusivity), sap flux density overestimations for the Heat Ratio Method and underestimations and loss of sensitivity for the lower sap flux densities for the Tmax Method will be obtained (Fig. 6). If the anisotropic theory is applied, but thermal diffusivity considered isotropically as the geometric mean of tangential and axial diffusivity, underestimations for the HRM and overestimations for the Tmax Method would be obtained (Fig. 6). Hence, in modelling studies, it should be made clear which equation is used and for which purpose. Jones *et al.* (1988) correctly stated that all heat-pulse methods could benefit from the use of curve-fitting procedures, but implemented equation 1, thereby neglecting the anisotropy of the sapwood. This

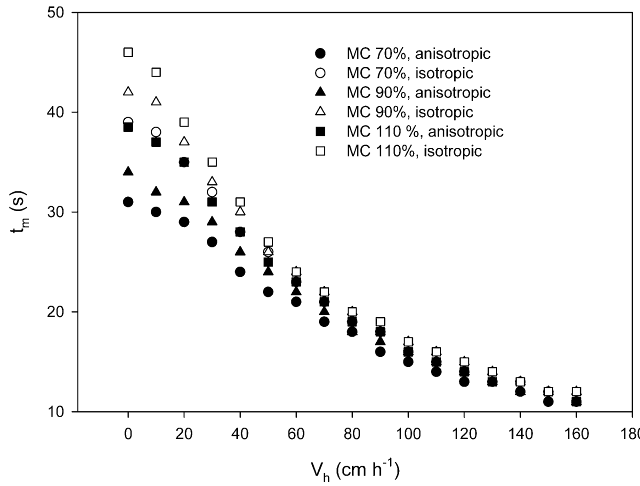

Fig. 4. t_m values for different moisture contents of sapwood, both anisotropic and isotropic. The isotropic thermal diffusivity D was taken as the geometric mean of D_{ax} and D_{tg} of the anisotropic case.

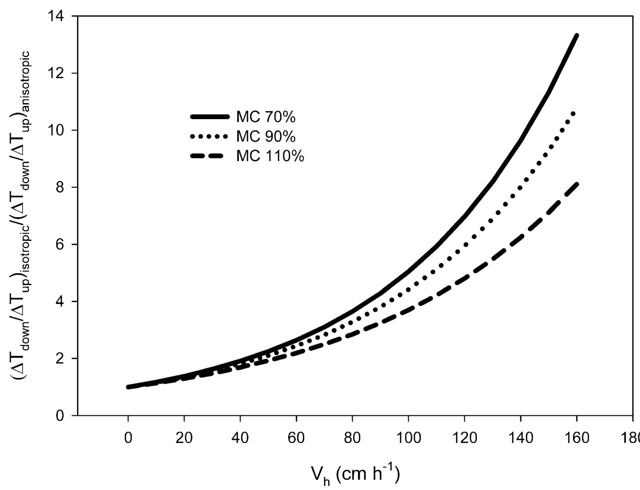

Fig. 5. Ratio of the HRM ratio ($\Delta T_{down}/\Delta T_{up}$) for isotropic and anisotropic sapwood with different moisture contents. The isotropic diffusivity was taken as the geometric mean of D_{ax} and D_{tg} of the anisotropic case.

error was also made by Becker (1998) for a theoretical example and perhaps others who have implemented models based on equation 1.

If equation 1 is not valid for sapwood with anisotropic thermal conductivity and leads to differences in temperature field and heat-pulse method parameters, does this imply that these methods are not applicable in the field? Not necessarily, because most methods are based on derivations from equation 1. Marshall (1958) based his measurements of V_h and D on the substitution of two points (t_1, T_1) and (t_2, T_2) on the temperature-time curve according to equation 1:

$$\ln\left(\frac{\Delta T_1 t_1}{\Delta T_2 t_2}\right) = -\frac{(x - V_h t_1)^2}{4Dt_1} - \frac{y^2}{4Dt_1} + \frac{(x - V_h t_2)^2}{4Dt_2} + \frac{y^2}{4Dt_2} \tag{7}$$

If the measurement point is now chosen at an axial distance x (with y=0), this becomes:

$$ln\left(\frac{\Delta T_1 t_1}{\Delta T_2 t_2}\right) = -\frac{(t_1 - t_2)^2}{4Dt_1 t_2}\left(x^2 - V_h^2 t_1 t_2\right) \tag{8}$$

When applying this equation to two other points t_3 and t_4, with $t_1+t_4=t_2+t_3=2t_2$, a similar equation is obtained, resulting in two equations with two unknown variables: D and V_h. These equations can easily be solved to obtain expressions for both D and V_h. If the same mathematics are applied in equation 3, the same result is obtained which makes it clear that the thermal diffusivity as calculated by Marshall (1958), based on the incorrect isotropic theory, is actually the axial thermal diffusivity when conducting measurements in sapwood (Fig. 7).

Hence, by locating the measurement needles only in the axial direction and choosing specific data points on the temperature–time curve, the difference introduced by anisotropy between equations 1 and 3 is ruled out. This is also the case for the Heat Ratio Method. Here, the same distances of the axial measurement points upstream and downstream from the heater needle were applied which also

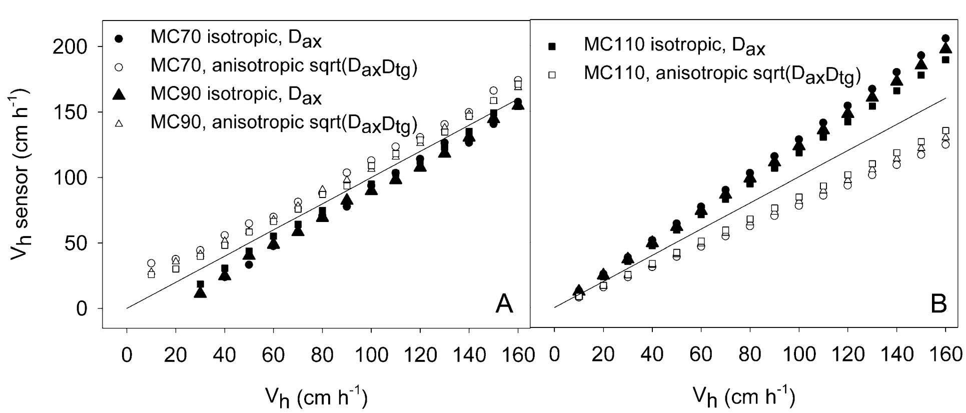

Fig. 6. Heat velocity V_h according to the Tmax Method (A) and the Heat Ratio Method (B) for different moisture contents and based on the isotropic theory with application of the axial thermal diffusivity (isotropic, D_{ax}) or the anisotropic theory with application of the isotropic diffusivity (anistropic, square root of ($D_{ax}D_{tg}$).

negates the difference between axial and tangential conductivity (Fig. 8). This also holds for the Tmax Method (Fig. 9). Even though thermal diffusivity is mentioned by Cohen *et al.* (1981) as D in equation 1, in practice, D_{ax} is being measured. By using the derivative of equation 1 for a measurement needle located axially from the heater, correct equations for both D_{ax} during conditions of zero flow and V_h are obtained:

$$D_{ax} = x^2(4t_m)^{-1} \tag{9}$$

$$V_h = \left(x^2 - 4D_{ax}t_m\right)^{-1/2}(t_m)^{-1} \tag{10}$$

with t_m the time at which ΔT is maximal. D_{tg} can be determined when measuring the temperature curve at a tangential distance (0, y) from the heater needle (Fig. 9):

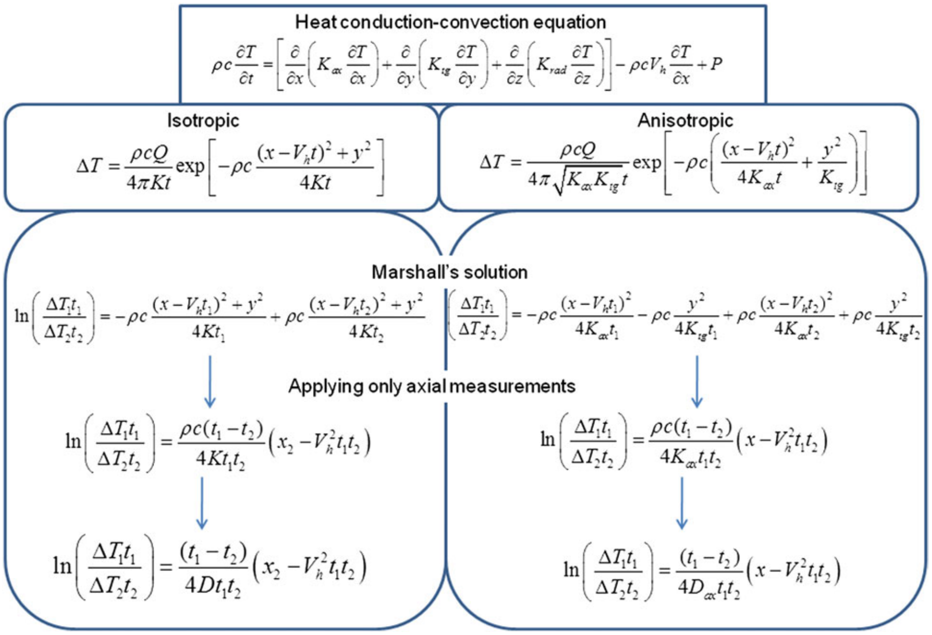

Fig. 7. Marshall's solution of equation 1 to determine V_h and D_{ax} (Marshall, 1958). By using only axial measurements and substituting specific points of the temperature-time curve, the influence of anisotropy is ruled out.

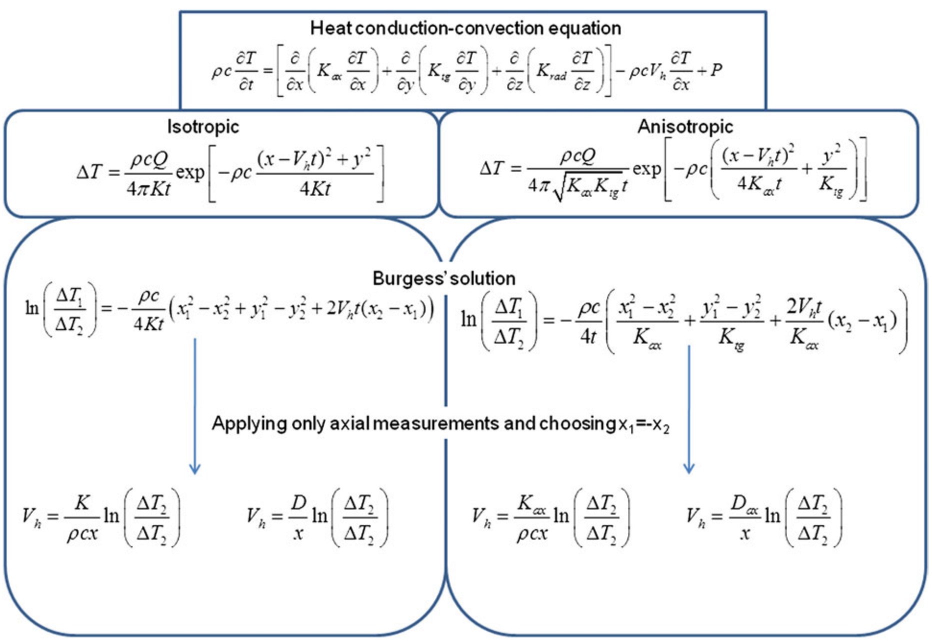

Fig. 8. Burgess' solution of equation1, based on Marshall (1958), to determine V_h. By measuring specific distances upstream and downstream of the heater (x_1= $-x_2$), the actual axial thermal conductivity is applied, even when starting from the isotropic theory.

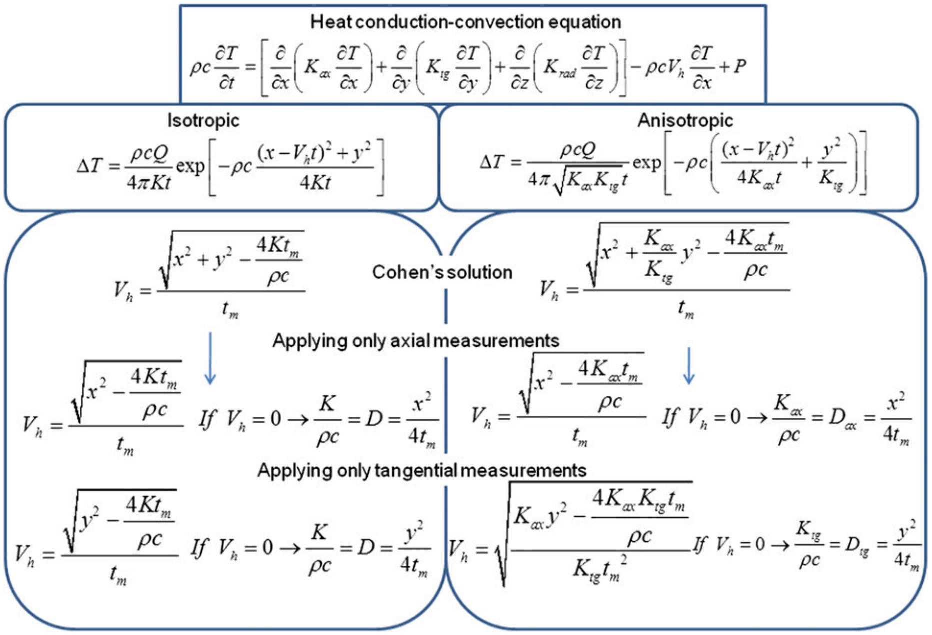

Fig. 9. By differentiating equation 1 to determine the time at which the maximal ΔT occurs, a solution for V_h is obtained. When applying only axial measurements, the actual axial thermal conductivity is used in these calculations.

$$D_{tg} = y^2(4t_m)^{-1} \tag{11}$$

Note that also for the Compensation Heat Pulse Method (Swanson and Whitfield, 1981), the isotropic and anisotropic theory lead to the same heat velocity results as the intersection point between ustream and downstream temperature profiles remains constant (Fig. 10). Hence, it can be concluded that, currently, existing heat-pulse sap flux density measurement methods remain valid, even though they are based on an inaccurate theory, as by mathematical manipulations the same results are obtained as if the correct anisotropic theory would have been applied. It is now also clear that these methods determine the actual axial thermal diffusivity and not a mean of axial and tangential or overall thermal diffusivity as is sometimes mistakenly thought.

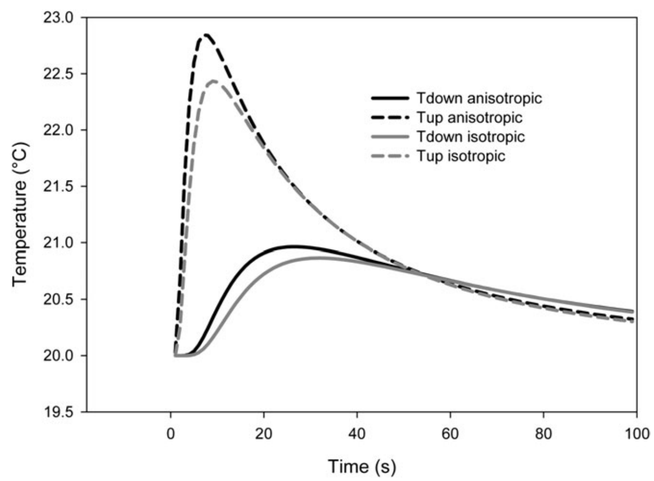

Fig. 10. Upstream and downstream temperature profiles for the CHPM both for the anisotropic and the isotropic case. The isotropic thermal diffusivity D was taken as the geometric mean of D_{ax} and D_{tg} of the anisotropic case.

Conclusion

Throughout the literature on heat pulse-based sap-flow measurement methods, there seems to be an inconsistency when referring to the thermal diffusivity of sapwood. Despite clear evidence that sapwood is an anisotropic material for which D_{ax} and D_{tg} are markedly different, thermal diffusivity is often addressed as an overall diffusivity for the whole material, based on equation 1. Nevertheless, the existing heat-pulse based sap flow methods are still theoretically correct and yield actual D_{ax} values due to their reliance on derivations of equation 1 and the use of only axial needle positioning. Other methods apply the method of mixtures mentioned in Turrell *et al.* (1967) in combination with the dry wood thermal conductivity determinations of Siau (1971) which was specifically derived for the axial and tangential direction. However, when applying equation1 for numerical modelling or for other purposes for which the mathematical derivations do not cancel out the isotropic effect, this will lead to errors in both thermal diffusivity and sap flux density calculations.

To avoid further confusion and to prevent mistakes in future research, it is suggested that equation 3 is mentioned when referring to the development of heat-pulse sensors and, consequently, to discriminate between axial or tangential thermal diffusivity and/or thermal conductivity.

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