Chapter 3

ANALYSIS AND CORRECTION OF HEAT FLOW ON LAND

> . . . a very large correction must be applied for the convexity of the ground; for it is evident that a point in the ground vertically under a steep crest is more exposed to the cooling influence of the air than a point at the same depth beneath an extensive level surface.
>
> J.D. Everett, 1871

As a result of the processes described in the previous chapter, each site of measurement yields two sets of data to be combined into a single measurement of heat flow. One is a series of temperatures at points on a vertical profile and the other is a series of conductivities on the same vertical profile. This chapter describes how this combination is achieved, the complexities that may arise, and the corrections that may be required owing to physical characteristics of the site.

3.1 CALCULATION OF HEAT FLOW

Heat flow Q in the vertical dimension z is given by the equation

$$Q = -K(\mathrm{d}v/\mathrm{d}z)$$

where K is conductivity and v is temperature. It is immediately apparent that the temperature appears in the form of a gradient, whereas the measurements are of absolute temperature. Just as temperature has a value at every point, the first differential, or gradient, of temperature also has a value at every point, and neither quantity may be expected to be uniform in the earth's crust. Although temperature is a continuously varying function, temperature gradient is discontinuous wherever sudden changes in conductivity occur. Two temperatures measured at two different points allow the calculation of an average gradient given by

$$\frac{\mathrm{d}v}{\mathrm{d}z}\,(\mathrm{ave}) = \frac{v_2 - v_1}{z_2 - z_1}$$

but it is improbable that this single figure for a gradient is valid over the entire distance between the two measurement points. Two temperature measurements give no more than the average or integral of the gradient over the distance between the two points

$$v_2 - v_1 = \int_{z_1}^{z_2} (\mathrm{d}v/\mathrm{d}z)\mathrm{d}z$$

In contrast, each measurement of conductivity gives the conductivity of only the piece of rock that is measured. One piece of rock 1 cm thick in each 10 m length of borehole is only 0.1% of the material to be evaluated, and its measurement is required to represent the entire 10 m. This small sample is most unlikely to reveal the average conductivity over a length a thousand times its own, but in practice this may be the maximum sample that can be obtained and measured. It is often possible to see, from the lithological logs and from the form of the temperature profile, which parts of the vertical section might be expected to have a reasonably uniform conductivity, and one can often derive some guidance in the selection of conductivity samples from these sources. It is usually necessary to reach some compromise between a fully adequate number of samples and the number that can reasonably be measured.

Several methods have been used to combine the temperature and conductivity data to give a heat flow. Which method is best depends upon the circumstances. Probably the most popular is the "thermal depth method" or the "Bullard method" as it is often named, after the first user, Sir Edward Bullard (1939). This method probably best meets the problems presented by non-uniform temperature gradient and conductivity. Thermal depth is given by the integral of the physical depth z divided by the conductivity K, and is calculated for each temperature measurement point in the profile by

$$w = \int_{0}^{z} (1/K)\ \mathrm{d}z$$

conductivity being a function of depth. Since the conductivity is known only at a series of discrete points, which we shall denote by a series of number pairs z_n, K_n, this equation must be modified to enable real data to be used. Usually each value of conductivity is assumed to represent the depth interval that reaches half way to the next value on each side, i.e. the conductivity K represents the depth interval from $z_{n-1,n}$ to $z_{n,n+1}$, where

$$z_{n,n+1} = (z_n + z_{n+1})/2$$

except that $z_{0,1} = 0$. The point denoted here by z_0 does not necessarily mean the surface of the ground; it can be any point arbitrarily chosen. The thermal depth at any point z, falling within the nth interval, is given by

$$w = (z - z_{n-1,n})/K_n + \sum_{r=1}^{n-1} (z_{r,r+1} - z_{r-1,r})/K_r$$

and, by means of this equation, the thermal depth of each temperature measurement point may be calculated. The heat flow is the gradient of temperature with thermal depth

$$Q = \mathrm{d}v/\mathrm{d}w$$

and is usually calculated by the method of least squares.

A second method of calculating heat flow has been used extensively in circumstances where the geological column can be divided easily into formations of different rock type and consequently different conductivities. Each formation has its own average conductivity from the samples measured, and an average temperature gradient can be calculated by the least-squares method from the temperature data within the formation. If the average temperature gradient is multiplied by the average conductivity, a heat-flow value is obtained for each formation. An extension of this method, which has been used where division into formations is not relevant, consists of the division of the vertical profile into several arbitrary sections and the calculation of an average conductivity, temperature gradient and heat flow in each of these sections.

There seems little point in applying the restriction of division into fixed sections when the physical situation does not require it, and a very useful third method, which needs a computing system, consists of the division of the temperature data into overlapping sets of some suitable number of consecutive data. The first set of a fixed number of temperature data, starting from the top of the profile, is analysed for average temperature gradient, and the average conductivity within the same depth interval. Gradient and conductivity are then multiplied to give the heat flow. The second set of data begins with the second temperature point, and one more point is added on to the end to give the same number of data in the section. The calculation is repeated for this and each succeeding set. This process of discarding the upper data point and adding an extra one to the lower end is repeated until the bottom of the profile is reached. The number of temperature data in a set must be chosen carefully: too few will produce excessive scatter due to conductivity sampling errors, but too many will produce excessive smoothing of heat-flow variation. The heat flow from each set of data is associated with the mid-point of the set of points of measurement. A distribution of heat flow with depth is thus produced, which may reveal disturbances by water flow and other departures from an equilibrium conductive system. Sometimes variations of apparent heat flow due to conductivity sampling errors are revealed. This method is very powerful for showing the individual characteristics of any data set, for detecting disturbances, or for verifying the quality of the measurement, but it involves much more computation than other methods. It will be referred to as the "interval method".

A fourth method has recently been developed to obtain reasonable estimates of heat flow from logs of oil and gas wells. Where a series of logs is run in a well, there may be a corresponding series of bottom-hole temperatures, each with the time lapse since the end of circulation of drilling fluid. The method of estimating equilibrium temperature from such data is described later in this chapter. Where an equilibrium temperature may be derived at each of three or more depths, there are at least two independent temperature intervals for calculation of gradient. A count of the net rock content of these intervals and an assignment of conductivity for each rock type permit the calculation of a total thermal resistance in each temperature interval, and the combination of the two quantities gives the heat flow. The disadvantage of using estimated conductivity for each rock type is substantially compensated by the accurate assessment of rock content. This method has been found to give varied results, errors being introduced both by the quality of the temperature data and by the estimation of conductivity. However, the degree of agreement between the two independent heat-flow estimates gives some indication of the value of the final average result (Reiter and Tovar, 1982).

A fifth method, which was used in the early days of heat-flow measurement, consists of the determination of one temperature at the bottom of a borehole, the estimation from climatic data of another temperature at the top of the hole, and the averaging of some conductivity measurements between the two. An average temperature gradient is obtained by dividing the difference of the temperatures by the distance between them, and this figure is multiplied by the average conductivity of the samples measured. This method gives one figure for heat flow but gives no indication of possible errors or disturbances. It gives no indication of any variation of heat flow with depth for any reason, it neglects any difference between air and ground temperature, and it is inadequate to yield anything but a rough approximation.

3.2 SELECTION OF METHOD OF ANALYSIS

Except for rejecting the last, it is impossible to recommend any one of these methods over the others in all circumstances, since each set of data from a borehole has its own characteristics and these sometimes become apparent only during the course of analysis. It is important to try more than one method of analysis in order to provide the maximum chance to detect the problems in any data set. It is also impossible to say just how many temperature and conductivity data are required to ensure a good measurement of heat flow. These numbers depend entirely on the circumstances and the choice must be left to the operator. Unfortunately, the operator does not always have the freedom of choice that would be desirable, and sometimes lack of core or other restrictions make it impossible to obtain the data that are required for a good measurement. On such occasions one has to be satisfied with what is available. Because temperature data have an integrating effect whereas the conductivity data apply to the individual piece of rock only, the number of conductivity data should exceed the number of temperature data, but closely spaced temperature data can reveal conductivity contrasts or distortion by water movement.

A few examples will serve to show some of the pitfalls that can be avoided with adequate analysis. Fig. 16 shows the temperature, the conductivity and the heat flow, calculated by the interval method, from a site at Oldham, Nova Scotia (Jessop and Judge, 1971). The temperature gradient is quite uniform below a depth of about 200 m, but the conductivities are scattered. The rocks penetrated consisted of interbedded slates and quartzites. The conductivity of the quartzite was considerably higher than that of the slate, so that the two rock types can be identified by their conductivity alone. The two types of rock were mixed fairly uniformly throughout the column, but in the sampling a bias was introduced and most of the slate samples were chosen from the lowest 30% of the core. The effect on the heat-flow profile is very clear. If each section is analysed for slate and

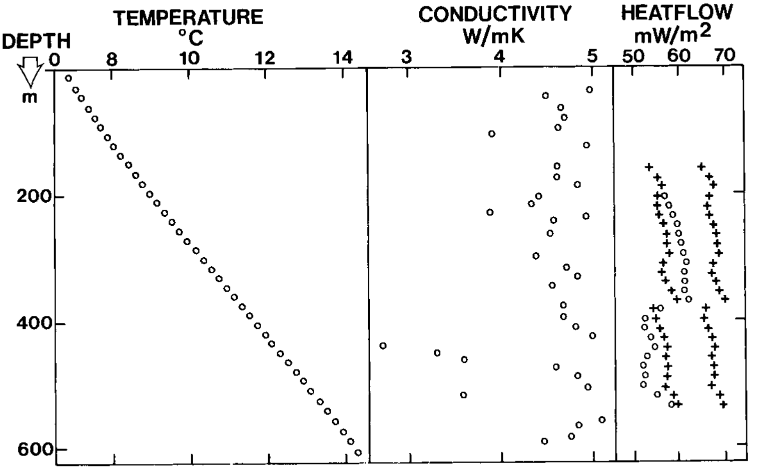

Fig. 16. Temperature, conductivity and heat flow, calculated by the interval method, at Oldham, Nova Scotia. Circles show uncorrected data; crosses show heat flow corrected first for conductivity sampling bias, and second for climatic disturbance to temperature. The final heat flow is 67 mW/m^2. Diagram: Earth Physics Branch, Ottawa.

quartzite content and a correction is applied to allow for the bias one way or the other, the corrected heat-flow values are obtained. The corrected values show much less scatter than the uncorrected values but are still not as uniform as might be expected. More conductivity measurements could have been made, and the bias between the rocks could have been corrected by this means, but it is unlikely that the final value of heat flow would have been significantly different.

Fig. 17 shows the data and analysis of a site on Nielsen Island in Hudson Bay (Jessop, 1968). The upper 400 m of core consists of meta-quartzite and chert, and the lower part consists of gneiss of the Canadian Shield. The strong conductivity contrast causes a clear change in temperature gradient, but the heat flow, calculated by the interval method, is uniform apart from some random variation, as shown on the right-hand side of the diagram. The plot of temperature against thermal depth is shown for comparison with the plot against real depth, and the resulting heat flow is also shown with the heat flow calculated by the interval method. The Bullard plot is a straight line, showing no effect of the conductivity contrast, and the resulting heat flow agrees very well with the results of the interval method. This example reveals the powerful character of the Bullard method when conductivity contrasts are present.

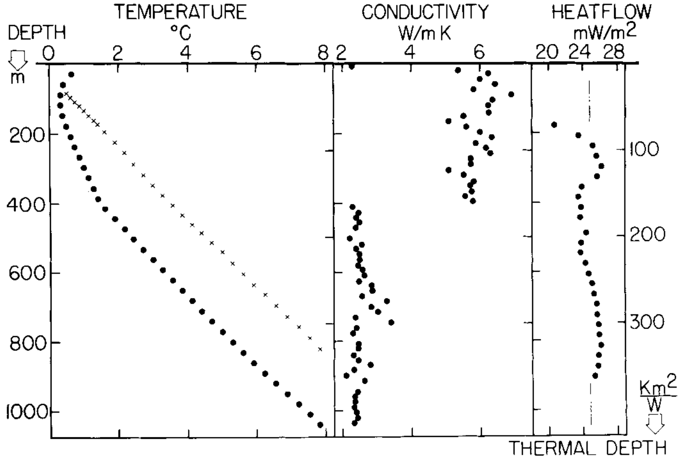

Fig. 17. Temperature, conductivity and heat flow at Nielsen Island, Hudson Bay. Solid circles show data plotted against physical depth and heat flow calculated by the interval method. A change in temperature gradient and conductivity at 400 m is readily apparent. Crosses show temperature plotted against thermal depth, and lines at top and bottom indicate heat flow calculated by the Bullard method. Diagram: Earth Physics Branch, Ottawa.

3.3 CORRECTION OF MEASUREMENTS

The measurement and calculation of terrestrial heat flow, as described above, assumes a perfectly conductive heat flow in a vertical direction. For ideal heat-flow measurement we need a spherical earth free of irregularities of elevation and free of any lateral variation of temperature or any change with time. The earth's surface is not perfectly spherical; it contains mountains and valleys, rivers and lakes, all of which serve to create a spatially variable temperature. Furthermore, these features are not permanent but are regularly changed by sedimentation and erosion and occasionally by more violent occurrences. The temperature at any point on the surface is not constant, owing to daily, seasonal and climatic changes. Variations in surface temperature promote changes in the subsurface temperature at progressively greater depths as time passes, and a temperature history of the surface is recorded in the rocks below.

The disturbing influences mentioned above clearly belong at the surface of the earth. They create disturbances to the measured heat flow that are not due to any subterranean cause and so may legitimately be removed by some form of correction. There are many other factors below the surface that also influence the heat flow and that may or may not be regarded as disturbances according to the purpose of the investigator. Deviations from the truly vertical flow of heat, due to contrasts of conductivity between steeply dipping formations or between laterally adjacent bodies of rock, are usually thought of as disturbances to be removed by correction. Conductive heat flow may be severely disturbed by water movement through fractures or permeable formations. If the purpose is the examination of the lower crust and mantle these should be treated as disturbances requiring correction, but the water movements revealed by temperature patterns may provide information about the rock. This topic will be examined in Chapter 10. Lateral variations of heat flow due to the cooling of recent volcanic intrusives are usually thought of as providing information about the cause and not as a requirement for correction. There is no real dividing line between a correction and an interpretation, and the individual scientist must decide which he is doing in accordance with his purpose.

Disturbances to heat flow due to time changes in surface temperature, usually known as "climatic corrections", and disturbances due to local variations of elevation, usually known as "topographic corrections", are described below. A climatic correction should be considered for all measurements of heat flow, and a topographic correction is necessary for many. Other forms of correction or interpretation depend upon the local circumstances of the individual borehole and cannot all be dealt with in detail in one book. Since each site has its own characteristics different from any other site, each must be dealt with according to its own peculiar character.

Most of these corrections must be made on the temperature data rather than on the calculated heat flow, so that we must step back to consider the sequence of corrections that must be carried out before the final calculation is made.

3.4 CORRECTION FOR DRILLING DISTURBANCE

The process of drilling a hole introduces temporary disturbances to the temperature of the surrounding rock, both by the friction of drilling and by heat exchange with the drilling fluid. The heat capacity of the drilling fluid normally absorbs the heat of friction, and it is the temperature of the fluid that determines whether the rock surrounding the hole is warmed or cooled. The complete picture is rather complex, but in general the lower part of the hole is cooled because the drilling fluid starts off at a lower temperature, and the upper part of the hole is warmed because the fluid returning from greater depths is warmer than the surrounding rock. At the end of drilling, after the circulation of fluid is stopped, the temperature of the fluid left in the hole quickly adjusts to that of the immediately adjacent rock. However, the disturbance has by this time penetrated into the rock over all but the bottom few metres, and recovery to the original rock temperature takes place over a much longer time. As a rough approximation the recovery is completed to within the accuracy of most measurements in a period that is ten to twenty times greater than the time of drilling (Bullard, 1947).

It is possible to make very approximate calculations of the amounts of these disturbances, but it is preferable to make a series of measurements in the completed hole over a sufficient time interval to determine the equilibrium temperature. For a diamond-drilled hole this time period is normally no longer than about a year, but for a large rotary-drilled hole the return to equilibrium temperature may take many years, and a system of extrapolation of observed data is essential. This extrapolation is usually achieved by plotting the observed temperature against T, a logarithmic function of the time elapsed since end of drilling and the time of duration of the drilling, as shown in Fig. 18. The time function is given by

$$T = \ln((t + t_1)/t)$$

where t_1 is the duration of drilling and t is the time elapsed since completion. The function T becomes zero when the time elapsed is very large, and thus the temperature extrapolated to the axis gives the equilibrium temperature of the well and of the rock around it (Bullard, 1947). Fig. 18 shows a long sequence of data, from the well Reindeer D-27, in the Mackenzie Delta of northern Canada, plotted linearly against time, on the left, and against time function T on the right (Judge et al., 1979). The straight-line relation of temperature to the logarithmic function, derived by least-squares analysis for each of the four depths illustrated,

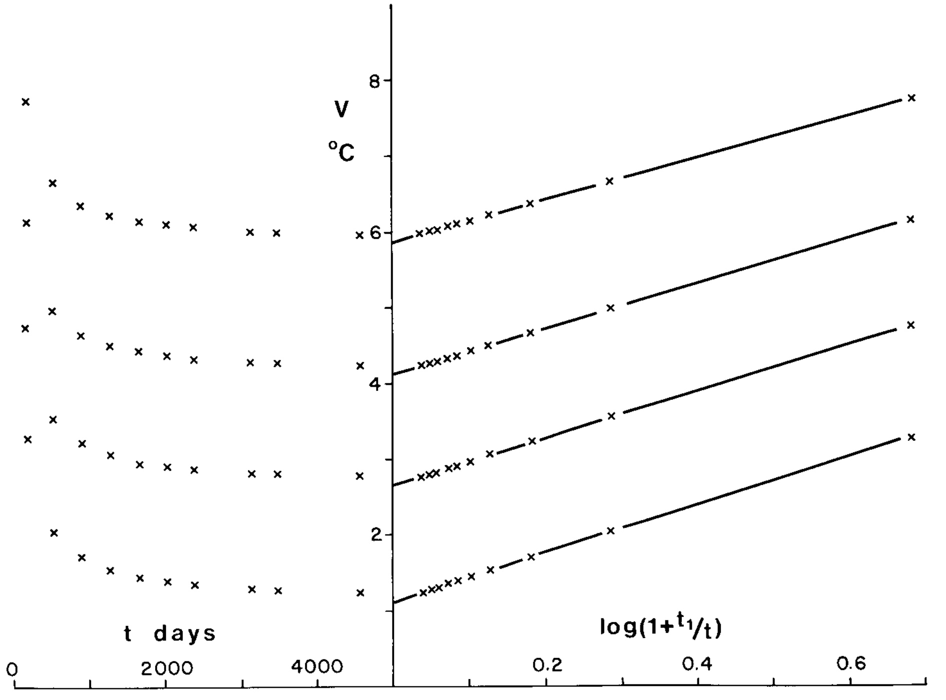

Fig. 18. Temperature data from the well Reindeer D-27, plotted against time t since end of drilling, on the left, and against a logarithmic function of time t and the period of drilling t_1, on the right. The lines are fitted to each of the four data seqeuences by the method of least squares. Each of the data sequences refers to one level in the well.

is also shown. These data were acquired by means of a permanent multi-thermistor cable, giving accurately repeated depths on each measurement occasion, and they cover a period of 12 years, which is much longer than is normally available. Calculation of the successive estimates of equilibrium temperature shows that estimates available after three years differ from those available after 12 years by less than 0.02 K.

Where the well penetrates solid rock the cooling effect of the drilling fluid spreads by conduction, and after drilling it dissipates by conduction. Where water is present in the rock, either in pores or fractures, this process may be modified, and this will be described in detail in Chapter 10. Where water is present in the form of ice, and where the drillers use warm fluids to maintain circulation, the latent heat of the phase-change between ice and water may play a major role.

Fig. 19 shows a series of temperature logs from which Fig. 18 is derived. This was one of the first wells to be drilled in northern Canada, and the operators did not realise the possible extent of permafrost or the consequences of using hot circulating mud. Much of the upper 300 m of the ground was ice-rich unconsolidated sediment, and the drilling process thawed the walls of the well and caused many problems with cave-ins. Eventually the well was completed and a thermistor cable was installed in the upper 600 m about 6 months later. The first log derived from this cable showed an isothermal section from the surface to 330 m, with temperatures between -0.09 °C and -0.42 °C, with a significant jump in temperature at 330–380 m, followed by a linear gradient of 24 mK/m. In later years the isothermal section has adjusted to an approximately linear gradient of 18 mK/m, the temperature step has disappeared, and the lower part has increased its gradient to 26 mK/m while generally declining in temperature.

The stabilised temperature profile conforms to the pattern expected from a conductive system with a higher conductivity in the permafrost section. Since ice has a higher conductivity than water this is to be expected. The strange behaviour of the well between the time of drilling and the approach to stable temperature

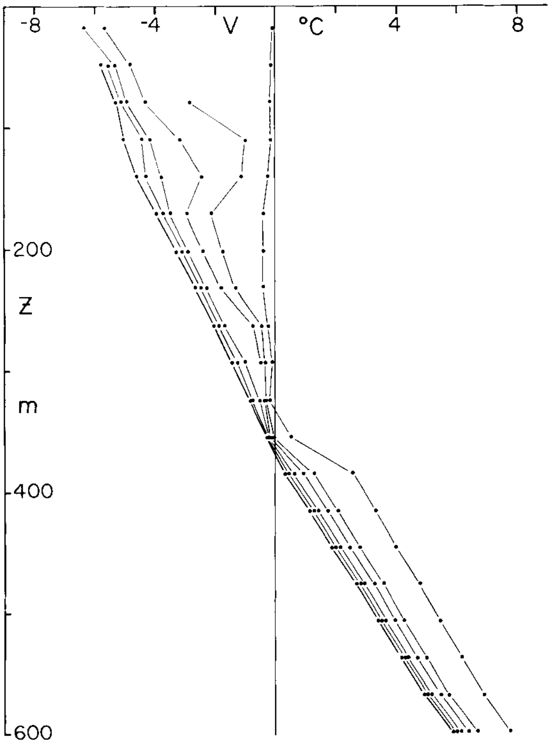

Fig. 19. A series of temperature logs from the well Reindeer D-27, taken over a period of 12 years following the completion of drilling. These data provide the plots of temperature against time in Figs. 18 and 20.

is interpreted as follows. In the section below permafrost the hot mud warmed the walls of the well by several kelvins. The well returned towards equilibrium as shown in Fig. 18, the decline after the time of the first log being about 2 K. In the permafrost section also warming occurred, thawing the ice-rich sediments and destroying their strength and ability to form a wall to the well. After casing was installed the thawed material was constrained behind it, but the temperature rise was delayed by the requirement of the ice for latent heat of fusion. Thus the temperature rise in the permafrost section was slower, smaller and less regular than in the section below, as shown in Fig. 20. Comparison of Fig. 18 with Fig. 20 shows that, whereas temperature stabilisation in the zone of positive temperature follows the logarithmic pattern very well, in the zone of negative temperature the return to equilibrium is erratic and different from point to point. In particular, many of the temperature paths start from a point just below 0 °C, reflecting the refreezing of the saturated sediments behind the well casing. One of the most useful and revealing results of this irregular behaviour is a step in temperature at the permafrost boundary, which gradually disappeared during the first few years after drilling, as shown in Fig. 19. This step has since been observed in other wells and has become one of the indicators of the depth extent of permafrost.

Fortunately, permafrost is of limited extent, so that the problems of freezing and thawing are not usually encountered. In many wells the thermal disturbance

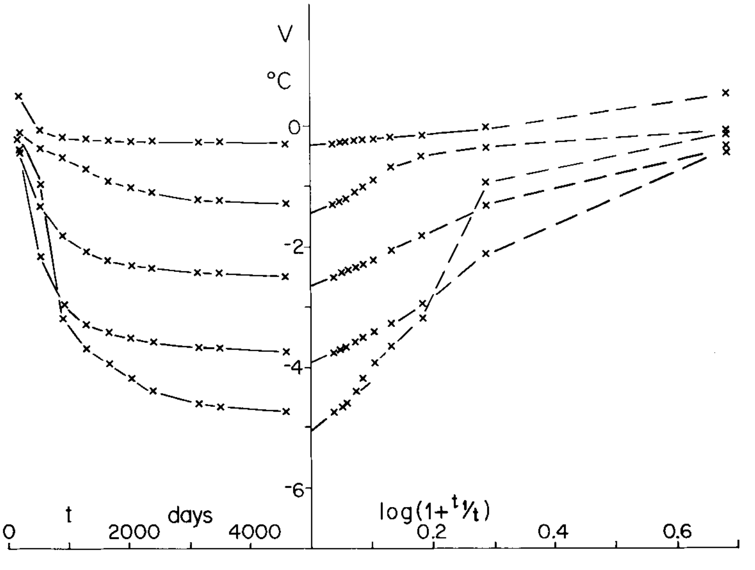

Fig. 20. Temperature data from Reindeer D-27, plotted against time in the same way as in Fig. 18. These data are taken from the upper part of the well, where thawing and refreezing of the surrounding sediments has a strong thermal effect.

of drilling is small enough to cause no problem, particularly in holes drilled by diamond bits, but it is always preferable to take more than one log, with a time interval at least equal to several times the drilling time between the end of drilling and the last log.

3.5 CORRECTION FOR INCLINATION OF THE BOREHOLE

Where a borehole is not vertical but is inclined at an angle, the depths of measurement, recorded as length along the bore, must be adjusted to give a true vertical depth below the surface. The data that permit this correction are usually given as a series of angles at specific points in the hole. Although the inclination is often given as the angle below the horizontal, it is more convenient to think of it as the departure from the vertical, and this version is the one used in the following analysis. The simplest assumption that may be made is that the angle varies linearly from one measured value to the next. If the length–angle pairs are represented by h_n, θ_n the gradient of angle is given by

$$g_n = (\theta_{n+1} - \theta_n)/(h_{n+1} - h_n)$$

Each small section of length dh contributes a small depth element of dz, given by

$$dz = dh \cos(\theta)$$

If the depth at point n is z_n, the depth at any point at length h_a between points n and $n + 1$ is

$$z = z_n + \int_{h_n}^{h_a} \cos[\theta_n + g_n (h - h_n)]dh$$

which gives

$$z = z_n + (2/g_n) \cos(\theta_n + \Delta\theta) \sin(\Delta\theta)$$

where

$$\Delta\theta = g_n (h_a - h_n)/2$$

This calculation must be done for each data point, of both temperature and conductivity, before the computation of heat flow. However, a borehole must be 8° from the vertical before the temperature gradient will incur an error of

1%, and so the correction is unnecessary for many holes that are close to vertical. Holes often deviate more as depth increases, and since the lower parts of a hole often give the best heat flow measurement, the inclination should always be checked.

3.6 CORRECTION FOR CLIMATIC CHANGE

Every part of the earth's surface is subject to change of temperature from time to time. This may be the result of extensive glaciation, such as occurred during the Pleistocene period, or of smaller changes in climate, such as have occurred in the last few thousand years and are continuing today. Alternatively, movements of shorelines of lakes or seas, which drain or inundate an area, the drying of swamps or changes in groundwater flow may cause local temperature to be controlled by a different environment. Such changes may be natural or man-made, and other human activities such as clearing forests or building cities may also change the temperature of the surface of the earth.

By the "temperature of the surface of the earth" one really means the temperature of the solid material immediately below the surface. Each temperature disturbance propagates a change of temperature down into the ground, and the depth of penetration of a measurable change depends on both the amplitude and the duration of the disturbance at the surface. Hence the effects of the Pleistocene ice-ages, which lasted over several hundred thousand years, extend to a few thousand metres, the effects of climatic changes in the last thousand years extend to 500 m, the effects of the annual cycle of seasonal fluctuations extend to a few tens of metres, and the effects of daily heating and nightly cooling extend to a very few metres. In heat-flow measurement we are not concerned with the upper few metres, and it is customary to choose some arbitrary depth, often 20 m, below which the seasonal fluctuation is insignificant, and to accept that temperature as the "average surface temperature". This does not mean that the temperature and heat flow at any lower point are in equilibrium with this value, and it is often convenient to extrapolate to the surface from the part of the temperature profile that seems to be in reasonable equilibrium and to accept this as the average surface temperature. Of course, the term "reasonable equilibrium" is itself subject to definition because it depends on the depth to which measurements have been made. In practice a reasonable extrapolation can usually be made from temperatures below a depth of about 200 m.

In theory it is possible to correct any temperature profile for any history of surface temperature, but in practice it is difficult to obtain adequate historical data. There are many diverse opinions about the extent and duration of the great ice sheets of the Pleistocene, and there is little point in making calculations on the basis of any history other than a simple approximation. The simplest possible approximation of surface temperature changes due to an ice-age consists of two step functions of temperature: a negative step at the beginning of the ice-age and a positive step of the same amplitude at the close. Such steps are unrealistic, but if they are placed roughly in the middle of the time interval in which the temperature was rising or falling the results will be adequate.

The magnitude of the temperature changes at the beginning and end of each glaciation depends on the figure that can be assumed to represent the temperature of the ground immediately below the ice. This probably varied from place to place and also with time. Under thick ice the temperature was probably close to the freezing point under the existing pressure, at about -1 °C to -2 °C, as recorded at Byrd Station (Gow et al., 1968). Under thin ice the temperature could have been much lower during the coldest times. The value of -13 °C found at Camp Century in Greenland (Hansen and Langway, 1966) under 1387 m of ice does not fit the idea of freezing-point temperatures in the ice-base, but the temperature profile, as it is described, suggests a rapid accretion rate and a severe non-equilibrium thermal state. If this result were typical of glacial periods, some indication of it would probably have been detected in existing heat-flow profiles.

Ground temperatures outside the area of glaciation were doubtless lower than at present, but it is difficult to determine by how much. It may be reasonable to assume that the fall in ground temperature was similar to the fall in air temperature, particularly if it can be shown that precipitation, drainage and the extent of vegetation cover were not substantially changed. The nature of the vegetation may yield clues to the air temperature and other climatic conditions.

The times of the beginning and end of the major Pleistocene glaciations have been estimated by many writers, each on the basis of indirect evidence. Birch (1948) summarised the best estimates available and chose his own sequence from among them. The figures used in Table 6 are taken from the work of Emiliani (1961). In Canada, where the glacial history is particularly important, a map has been published of the time of final retreat of the Wisconsin ice sheet (Prest, 1969). This date is the most important of all, since it is the most recent, and dissipation of the effect of this event is the least advanced.

It is convenient, for the purposes of calibration, to assume a uniform thermal diffusivity of the rock. The temperature at any depth z due to a single ice-age is given by

$$v = V_0 + gz + V_1 \{\mathrm{erf}[z/2\sqrt{(st_1)}] - \mathrm{erf}[z/2\sqrt{(st_2)}]\}$$

where V_0 is the present average surface temperature, V_1 is the temperature step and is negative for an ice-age, s is the diffusivity, and t_1 and t_2 are the times of

TABLE 6

Corrections to temperature gradient for Pleistocene climate (expressed in mK/m)

Depth (m)	Glacial period (end ky B.P. start ky B.P.) Wisconsin 12 65	Illinoian 100 125	Kansan 175 200	Nebraskan 265 300
0	2.387	0.153	0.071	0.054
500	1.894	0.146	0.069	0.053
1000	0.790	0.127	0.063	0.050
1500	−0.217	0.098	0.055	0.045
2000	−0.734	0.064	0.044	0.040
2500	−0.819	0.027	0.032	0.033
3000	−0.691	−0.006	0.019	0.025
3500	−0.513	−0.033	0.006	0.017
4000	−0.353	−0.053	−0.005	0.009
4500	−0.230	−0.064	−0.015	0.002
5000	−0.142	−0.068	−0.022	−0.005
6000	−0.046	−0.059	−0.031	−0.016
7000	−0.012	−0.040	−0.031	−0.022
8000	−0.003	−0.023	−0.026	−0.024
9000		−0.011	−0.019	−0.022
10000		−0.005	−0.013	−0.019

Temperature gradient calculated from Eqn. 1.
Diffusivity s = 1.2 mm^2/s.
Temperature step V_1 = 5 K.

the end and beginning of the ice-age. A table of values of the error function erf*(x)* is given in Appendix 1. By differentiation one can show that the temperature gradient at any depth z is given by

$$dv/dz = g + V_1 \left[(\pi s t_1)^{-1/2} \exp(-z^2/4st_1) - (\pi s t_2)^{-1/2} \times \exp(-z^2/4st_2)\right] \qquad (1)$$

and at the surface this reduces to

$$(dv/dz)_{z=0} = g + V_1 \left[(\pi s t_1)^{-1/2} - (\pi s t_2)^{-1/2}\right]$$

Any number of ice-ages or other periods of changed temperature can be simulated in the same way and the corrections for them can be added together to make up a single temperature correction for each depth.

When dealing with temperature profiles from boreholes of depth less than about 1000 m, it is the last ice-age, the Wisconsin or Wurm glaciation, that contributes the major part of the correction. The other ice-ages are less important at these depths but become comparatively more significant in deeper boreholes. It is essential when dealing with the ice-ages to deal with each one as a whole and not to assume that the beginning was sufficiently far in the past that it may be neglected. Unless one takes into account both the end and the beginning of each ice-age, one will generate serious errors. Table 6 shows the magnitude of the corrections to the measured temperature gradient due to a moderate surface temperature differential of 5 °C. The total corrections amount to 2.7, 2.2 and 1.0 mK/m at the surface and depths of 500 and 1000 m respectively. With a moderate conductivity of 3.0 W/mK, these gradients give heat-flow corrections of 8.1, 6.6 and 3.0 mW/m^2, or 5–10% of the average heat flow.

The smaller temperature changes that have occurred since the ending of the last glaciation may also be of importance in correcting heat-flow measurements, particularly those made in holes less than 300 m deep (Beck, 1977). It is not always easy to obtain sufficient information on the short-term Holocene climate changes to permit reasonable estimates of the errors. However, in certain temperature profiles from boreholes in rocks having a very uniform diffusivity and conductivity, it is sometimes possible to detect the effects of these temperature changes and to derive information about the past from the temperature record in the rocks. A study by Cermak (1971) of data from two boreholes in northern Ontario has demonstrated a very good correlation with climatic variations that are already well documented. The changes in question were the warm period from about 900 to 1100 A.D. and the cold period from about 1600 to 1750 A.D.

The most recent major climatic change, the rapid warming of 1880–1940 A.D., has caused a temperature inversion in many parts of North America, Europe and Australia, at depths of 50–100 m, depending on the thermal properties of the rock. This inversion is seen in many temperature logs, and Cermak's data illustrate it well, as shown in Fig. 21. The temperature at depth z below a surface where the temperature is linearly increasing from an initial value in equilibrium with a constant geothermal gradient is given by

$$v = 4kt\, i^2\mathrm{erfc}[z/2\sqrt{(st)}]$$

where

$$v_0 = kt$$

and

$$i^2\mathrm{erfc}(x) = (1/4)[(1 + 2x^2)\mathrm{erfc}(x) - (2/\pi^{1/2})\, x \exp(-x^2)]$$

The effect of a warming of finite duration may be calculated by subtracting a second term based on a later starting time. The physical constants from the Kapuskasing data have been used to set up a model, the results of which are

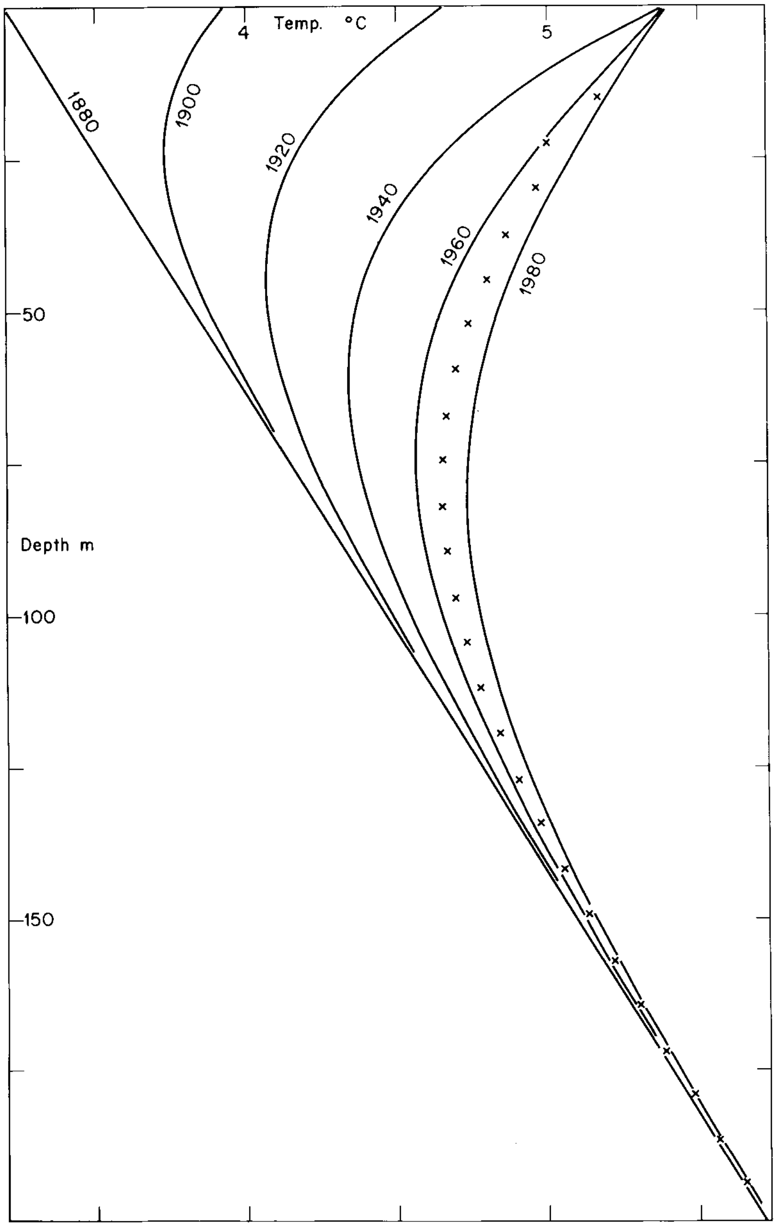

Fig. 21. Temperature profile in the upper 200 m of a borehole, calculated from a climatic model of linear temperature rise from 1880 to 1940. Points show temperatures measured in 1970. The borehole that is modelled and from which the data were obtained is near Kapuskasing, Canada.

shown in Fig. 21. Assuming an initial surface temperature of 3.2 °C in 1880 A.D., increasing to 5.4 °C in 1940 A.D., and a uniform diffusivity of 1.06 mm^2/s, temperature profiles at 20-year intervals may be calculated. The data from the log of 1970 A.D. lie approximately mid-way between the curves for 1960 and 1980 A.D., showing that climatic warming is one possible explanation for the observed temperature inversions. Since the inversions are so widely observed, the cause must also be of a widespread nature, and a general warming must be preferred to local effects such as land clearing or urban development.

Climatic correction has produced a great deal of argument concerning its legitimacy, both on the question of principle and on the question of ability to achieve a suffiently accurate result. In the matter of principle, little that is new has been contributed since the paper by Birch in 1948. This paper was written in response to some large and erroneous corrections that had been made by earlier writers. Birch put forward the method of calculating the correction, which is summarised above, and concluded that the correction to temperature gradient would rarely exceed 3 mK/m. This figure was not intended to be a fixed upper limit, nor did Birch argue against the use of climatic correction: he was attempting to ensure that such corrections would be made in a defensible manner. The problem of assembling sufficient data from which to calculate a correction of sufficient accuracy, in comparison with the possible error in the total result, is the only legitimate obstacle to the application of the correction.

Cermak's work has shown without a doubt that disturbances to underground temperatures due to climatic effects are real and must be taken into account when interpreting geothermal measurements in terms of deeper causes. On the other hand, the information on which the correction is based is often of a low accuracy, and the correction itself will not be precise. Since the correction will not usually amount to more than 20% of the total heat flow and often much less, errors of 25% of the correction can be tolerated. It is better to have a correction of this accuracy than none at all.

To give further observational support to these calculated corrections it would be useful to find a deep hole, free of other thermal disturbances and with sufficient core for good conductivity measurement, so that the climatically induced variation of heat flow with depth might be detected. No suitable hole has yet been found. Although measurements have been made in several holes to a depth of 3 km, none has given a conclusive demonstration of the presence or absence of glacial disturbance. A promising hole was drilled at Flin Flon in central Canada, and heat flow has been reported by Sass et al. (1971a), but very little core was taken. Conductivity measurements were made on rock fragments produced by the drill, but, as pointed out in Chapter 2, this cannot provide high accuracy. Furthermore, since this area is one of marginal permafrost and ground temperatures are similar to those under the ice sheet, only a very small climatic disturbance would be expected. The fact that no variation of heat flow with depth was observed was predictable, but the ground temperature of the site, extrapolated from the log, was unusually high and remains unexplained. The results must be regarded as inconclusive.

3.7 CORRECTION FOR TOPOGRAPHIC RELIEF

When heat is flowing upwards towards the surface of the earth, a mountain or other high land is an extra insulation at the surface, and consequently heat flow tends to become concentrated towards the valleys. Heat flow measured in a valley will thus be erroneously high, and heat flow measured on a mountain top will be correspondingly low. The ground surface on a mountain is generally cooler than the ground surface in the valleys, but there is no simple relation between ground temperature and elevation. A gradient in air temperature exists, and its amplitude is known approximately in many places, but this is no better than a rough indication of the distribution of ground temperature. Factors such as the direction of the slope relative to the sun, the nature of the vegetation cover and the mineral surface, the sediment in the bottom of the valley, and the movements of groundwater all combine to create a complex pattern. The gradient of ground temperature is considerably less than the normal geothermal gradient, and it is usually assumed, for want of a better assumption, that the air-temperature gradient, or "lapse rate", is applicable.

The real system, for which a solution is required, may be represented by a three-dimensional half-space, having a surface of irregular form and temperature, and having a uniform heat flow towards the surface at a distance from it that is large compared with the relief. The problem is to determine this uniform heat flow from the heat flow measured at some point near enough to the surface to be a function of the irregularity of the surface. All real surfaces present an impossible task of mathematical analysis, but there are a few special surfaces for which a solution is available. Alternatively it is possible to calculate the effects of a plane surface of non-uniform temperature. These two options provide the two ways of tackling the problem: either one may approximate the real surface to one of the special surfaces and calculate an exact solution, or one may derive an approximate temperature distribution on a horizontal plane from a knowledge of the real topography and calculate a solution from that. The first method produces an exact calculation from approximate surface conditions, while the second method produces an approximate calculation from exact surface topography.

The second method is the more generally useful, except when severe relief occurs near to the borehole. This method is also useful when water bodies or swamps are nearby, since these usually provide a different surface temperature from that on dry land. The method was first described by Jeffreys in 1937, and is usually named after its author.

The temperature at the collar of the borehole is obtained from the measured data and is denoted by V_0. All elevation is measured relative to this point and is denoted by h as shown in Fig. 22. A uniform lapse rate is assumed to exist, but data may be difficult to obtain. A normal value is from 2 to 5 mK/m and is denoted by g'. The temperature at every point on the surface is now given by

$$v = V_0 - g'h$$

since g' is expressed as a positive number and h is considered to be a positive elevation, i.e. in the opposite sense to the coordinate z. The temperature gradient beneath the surface is everywhere similar to the measured temperature gradient at the borehole, subject to variation due to conductivity contrast and to the topographic disturbance that we are trying to remove. As a first approximation it is assumed that this temperature gradient is uniform and equal to g. The temperature at any point on a horizontal plane through the collar can now be written as

$$V = V_0 + (g - g')h \tag{2}$$

and this temperature is a function of the coordinates in the horizontal plane because the elevation h is such a function.

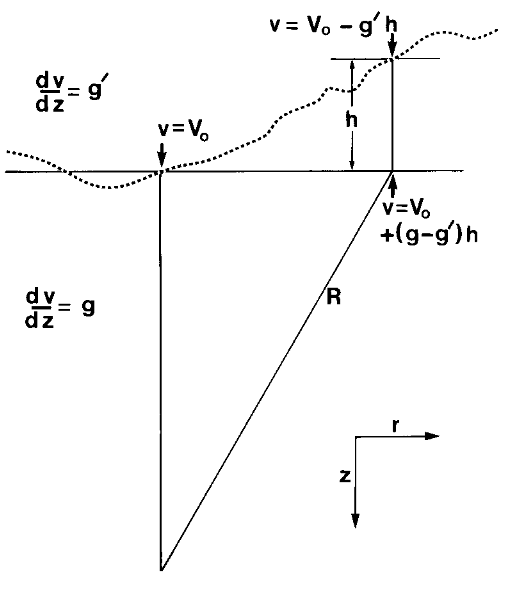

Fig. 22. The Jeffreys method of correction for topography. The temperature at each point on the real surface is calculated from a knowledge of the lapse rate. The temperature at the point directly above or below, on the horizontal plane through the collar of the borehole, is then calculated on the assumption of a uniform temperature gradient.

The theory depends on the mathematical principle that the temperature at any point in a semi-infinite space, near a boundary plane of non-uniform temperature, depends on the plane temperature in proportion to the solid angle from the point. This may be written as

$$v = (1/2\pi) \int_0^{2\pi} V \,\mathrm{d}\Omega$$

or, in polar coordinates

$$v = (1/2\pi) \int_0^{2\pi} \int_0^{\infty} VzrR^{-3}\mathrm{d}r \,\mathrm{d}\theta \qquad (3)$$

where

$$R^2 = z^2 + r^2$$

For the purposes of computation, elevations on a polar grid of finite dimensions are required, and these may be obtained by reading spot elevations from a topographic map. It is most convenient to obtain the average elevations on a series of concentric circles so that an average temperature on circle m may be calculated from a series of points on radii 1 to N

$$\bar{V}_m = (1/2\pi) \int_0^{2\pi} V \,\mathrm{d}\theta$$

which is replaced by

$$\bar{V}_m = (1/N) \sum_{n=1}^{N} V_n \qquad \text{for } \theta = \theta_1, \theta_2, \theta_3, \ldots, \theta_N$$

where V_n is calculated from Eqn. 2. Substituting this into Eqn. 3 yields

$$v = \int_0^{\infty} \bar{V}zrR^{-3}\mathrm{d}r$$

which reduces to

$$v = \sum_{m=1}^{\mathrm{M}} \bar{V}_m z r_m R_m^{-3} \,\delta r \quad \text{for } r = r_1, r_2, r_3, \ldots r_M$$

where

$$R_m^2 = z^2 + r_m^2$$

The increments of radius and angle are chosen to give a compromise between excessive computation and reasonable representation of the topography. The solid angle of a disc of radius r, as seen by a point at distance z from its centre and on its axis, is given by

$$\Omega = 2\pi\,(1 - z/r)$$

Thus it is possible to cover 90% of the half-sphere as seen from the bottom of the hole if one extends the calculations to a distance of ten times the borehole depth. This should normally be sufficient, but the person making the calculations must satisfy himself in this regard.

In practice it is important to choose a map of sufficient scale and with sufficient contours to provide elevation at any point without a great deal of interpolation. The points may be located either by carefully drawing them directly onto the map or by placing over the map a polar grid drawn on tracing paper or other transparent material.

3.8 CORRECTION FOR NEARBY BODIES OF WATER

Since the system just described of correcting for irregularities of topography works by reducing the problem to that of a plane of varying temperature, the system also provides a convenient way of dealing with errors due to the proximity of nearby bodies of water or to any other factor that might cause local anomalies in surface temperature. The temperatures at the bottom of a lake or a shallow sea are controlled by the water and will often be different from the temperatures of the land surface close by, particularly in far northern areas where the land surface will be very much colder than the bottom of a lake that does not freeze to its total depth. In order to calculate a correction, the method of the preceding section is followed with the exception that the temperature at each point on the surface is determined by a different method. The summing of a grid of points is then carried out exactly as before.

There is a further complication that some lakes and sea-shores have been in their present position for a relatively short time. This is dealt with by inserting a factor to account for these changes, as follows

$$v = (1/2\pi)\int_0^{2\pi}\int_0^{\infty} VzrR^{-3}\,\mathrm{erf}\,\{R/[2\sqrt{(st)}]\}\,\mathrm{d}r\,\mathrm{d}\theta$$

where s is diffusivity and t is the time since formation of the features. These corrections should be considered for any measurements made in boreholes near to bodies of water. A well-known example of such correction has been determined for shallow boreholes drilled in the early 1950s at Resolute Bay in the Canadian Arctic Islands. The measured heat flow of 121 mW/m^2, first measured and published by Misener (1955), was reduced to 60 mW/m^2 by Lachenbruch (1957) by means of the appropriate correction technique. Since the correction is so large and is only an approximation, the resulting heat flow is subject to a large uncertainty.

3.9 PLANE SLOPES

When relief occurs within a horizontal distance from the borehole equivalent to its own height or when the collar of the borehole is within an inclined area, the Jeffreys method may give substantial errors. In such circumstances it is preferable to revert to the first method as described above. It is then necessary to choose one of the special surfaces for which a solution is available and which bears some resemblance to the real surface. Two classes of surface have been presented in the literature, but solutions may be obtainable for others. The more adaptable class consists of various combinations of inclined planes to form ridges, valleys, ramps and benches. The analysis was provided by Lachenbruch in two publications: a practical exposition (1968a) and a detailed presentation (1968b). The mathematical expressions of the solution are long and complex and will not be reproduced here.

All the planes have parallel edges so that variation in one horizontal direction is eliminated and the geometry is reduced to two dimensions. The basic system consists of an incline between two horizontal sections of different elevation. Lachenbruch gives the solution in the form of graphs and tables. Fig. 23 shows the data expressed as a function of position relative to the horizontal extent of the slope. This illustration also includes an indication of the heat-flow distribution calculated by the Jeffreys method. It may be seen that the differences are less than about 5%, provided the slope does not exceed 9°, and that greater slopes may be tolerated as the distance from the point of interest is increased.

The Jeffreys approximation has the advantage that corrections calculated for two separate topographic features may be added together directly, but the exact solution for individual plane slopes does not have this attractive property. Lachenbruch developed the solution of the single slope into solutions for pairs of slopes forming valleys, ridges or terraces. The results are in the form of upper and lower limits to the heat flow, and except in unfavourable situations the difference between these limits is less than 10% of the regional heat flow.

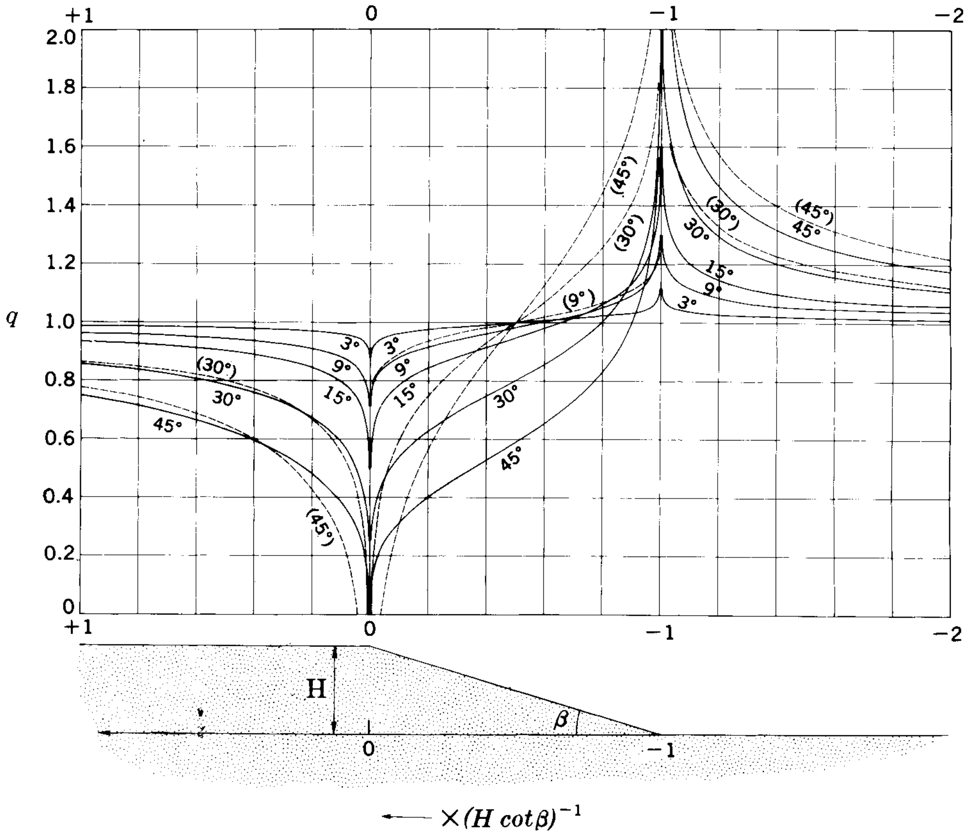

Fig. 23. The heat flow, normalised to a regional value of unity, varying laterally over a plane slope and the level regions behind the brink and beyond the foot. Dashed lines show the Jeffreys approximation. Diagram: Lachenbruch (1968a).

Taken so far, Lachenbruch's papers provide a useful process for estimating the errors in heat flow at the surface when the topography can be approximately related to plane slopes and when the surface temperature can be assumed to be uniform. Lachenbruch extended the system to deal with more complex systems of slopes and to the question of calculating the heat-flow distribution below the surface. There is no substitute for the examination, by the potential user, of the original papers.

3.10 LEES HILLS

A second solution to the steady-state heat-conduction equation within a two-dimensional half-space, used by Lees (1910) and later by Jaeger and Sass (1963), takes the form

$$v = V_0 + gz + \frac{b(z + a)}{x^2 + (z + a)^2} \tag{4}$$

where V_0 is the temperature of the surface at elevation $z = 0$, g is geothermal gradient, and a and b are constants governing the shape of the hill. The surface temperature is assumed to be dependent on the lapse rate g' and elevation

$$v = V_0 + g'z$$

and these two equations may be combined to give the equation of the surface

$$(g' - g)\, z = \frac{b(z + a)}{x^2 + (z + a)^2} \tag{5}$$

If the height of the hill is represented by H, and the width at half its height is $2w$, then the constants a and b are

$$a = H + (H^2/4 + w^2)^{1/2} \tag{6}$$

$$b = (g - g')\, H\, (H^2/4 + w^2)^{1/2} \tag{7}$$

The cross-section of a family of hill forms is shown in Fig. 24.

Eqn. 4, with the constants a and b calculated from Eqns. 6 and 7, may be used to calculate a correction to temperature observed in a borehole or tunnel. In the ideal case of a vertical borehole x is also a constant. The temperature gradient may be derived by differentiation of Eqn. 4

$$\frac{\partial v}{\partial z} = g + \frac{b[x^2 - (z + a)^2]}{[x^2 + (z + a)^2]^2}$$

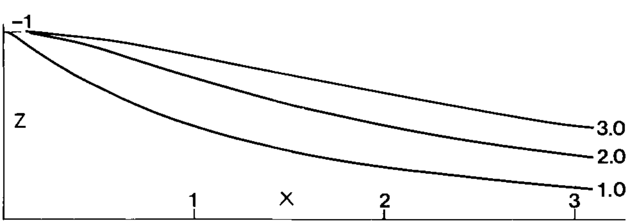

Fig. 24. The cross-section of a family of Lees hills, calculated from Eqn. 5. The numbers shown indicate the half-widths of the hill in terms of its height. Only one half of each hill is shown: the complete hills are symmetrical about the vertical through the summit.

This solution can be extended to cover the situation of a series of parallel ridges located at positions c_n by replacing Eqn. 4 with

$$v = V_0 + gz + \sum_{n=1}^{N} \frac{b_n (z + a_n)}{(x + c_n)^2 + (z + a_n)^2}$$

and this can be further extended to give the solution of a monocline by integration

$$v = V_0 + gz + b'(z + a) \int_0^{\infty} \frac{d\phi}{(x + \phi)^2 + (z + a)^2}$$

from which it follows that

$$v = V_0 + gz + (1/2)(g - g')H\{1 + (2/\pi)\mathrm{atan}[x/(z + a)]\} \tag{8}$$

has the value

$$v = V_0 + g'z$$

on the surface

$$x = (z + a)\cot(\pi z/H) \tag{9}$$

provided that

$$b' = (g - g')H/\pi$$

and that the surface passes through the point $x = 0$, $z = -H/2$.

Fig. 25 shows the family of curves generated by Eqn. 9, each curve having a different value of a. In order to maintain uniform direction of slope, a is limited to values greater than unity. The temperature gradient may be derived by differentiation of Eqn. 8

$$\frac{\partial v}{\partial x} = g - (g - g')\frac{H}{\pi}\frac{x}{x^2 + (z + a)^2}$$

and it may be calculated for any point below or on the hill surface.

The method of Lees hills has the advantage that it can be used to evaluate data from non-vertical holes that have been drilled into hillsides for mineral exploration. Eqns. 4 and 8 include both vertical and horizontal coordinates, and

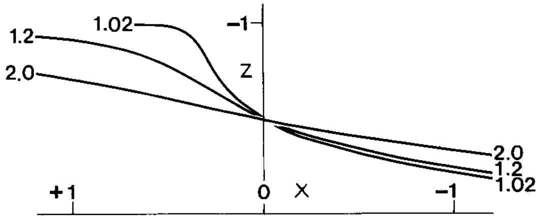

Fig. 25. The cross-section of a family of monoclines, calculated from Eqn. 9. Numbers indicate the values of the parameter a. All hills have elevation $z = 0$ at large negative x, and elevation $z = -1$ at large positive x.

this permits the calculation of a relation between surface temperature V_0 and temperature gradient g for each measuring point. On the assumption of a uniform conductivity, these equations can be solved for the best values of these two variables.

3.11 THE SEQUENCE AND VALUE OF CORRECTIONS

There is a logical order to the sequence of corrections but, since the corrections themselves are often of an approximate nature, it is probably not important in most cases. In general, the most immediate and local disturbances should be removed first. Thus the first corrections to be made are those for angle of the hole and for disturbance of temperature due to drilling fluid circulation. Since the first of these acts on the depth and the other acts on the temperature as a function of depth, it is important to correct for angle first.

After this one usually applies the correction for topographic relief and then the correction for climate, since the climate is of more widespread and general concern. In reality these factors cannot be separated. We have seen how the influence of recent shoreline changes may be incorporated into the topographic correction, but the present form of much of the terrain we see today dates only from the end of the last ice-age. Thus the two corrections are inextricably connected, and the separate treatment described here is at best a gross simplification. We could make more comprehensive detailed corrections, but the data on which to base the calculations are not usually available.

Other corrections are sometimes made, to remove the effects of refraction by contrast in conductivity or for sedimentation and uplift, to mention two examples. Such special situations must be dealt with as they arise, and the individual researcher must decide on the methods to be used.

As was pointed out earlier in the chapter, the boundary between correction and interpretation is indistinct. The purpose of correction is to remove the effects that may hide the evidence that the researcher is seeking. Since the usual purpose is research into the interior of the earth, all corrections related to surface features must be made, but the level of effort should not be allowed to exceed that justified by the data. Whatever corrections are made, the reporter should carefully state the methods used and the assumptions made for the benefit of those who will wish to evaluate the results and reinterpret the data in company with others.

3.12 PRACTICAL CONSIDERATIONS

The preceding chapters are an attempt to describe the ideal way to make measurements of conducted heat flow on land. The question of convected heat flow and water flow in boreholes will be dealt with in Chapter 10. In the early days many of the requirements for a good measurement free of disturbances were not realised, and many of the results are unreliable by current standards. Even now most measurements are still made in holes that were drilled originally for other purposes, such as mining exploration or oil and gas exploration, and these sites are often unsuitable for geothermal purposes. Unfortunately such sites are the only ones available to most people, and, on the basis that a poor measurement is better than no measurement, these sites are continually used. One can spend a great deal of time visiting mining camps on the word of company officials that holes are available, only to find that surface casing has been removed and the holes are hopelessly lost, or the holes have caved at shallow depths, or there is water flowing from the collars. Even when a good temperature profile is obtained, one still has to contend with the question of conductivity measurements. Some companies split the entire core and so ruin it for the purpose of cutting discs. Other companies examine their core in a geological sense and then discard it. Others abbreviate the core to a small fraction of the total and are then understandably reluctant to part with specimens. At the opposite extreme, the author remembers talking to a mining company official who had the complete core from a 1200 m borehole and who refused to part with 10 cm lengths at intervals of 20 m for conductivity measurement. Although only the bottom 100 m was of any commercial interest, he maintained that the entire core was of value to the company geologists. Either he has changed his mind by now or the company concerned spends a great deal of money on core storage. When rival mining companies are working in the same area, industrial secrecy occasionally prevents a geologist from releasing core samples that he would otherwise be happy to supply. Despite these adverse comments it is only honest to acknowledge that the majority of company geologists are extremely cooperative and that many of them will go to considerable trouble to ensure that holes are preserved and to guide visiting scientists to the locations when required.

Despite the cooperation of the mining personnel, the practical problems of using industrial boreholes remain. The best measurements can be made only in those holes that are drilled under the control of the scientist at sites carefully chosen to avoid anomalous areas and problems of terrain.

In view of the difficulties mentioned, it is up to the scientists to make sure that the measurements are as good as they can be. It is also necessary that the reporters should give sufficient detail of the measurement for users to be able to assess whether the result is reliable and what sort of errors may be present. Limits of probable error have been specified in several different ways, and they are of very little value unless the method of computation is revealed. It would be a great help to all concerned if heat-flow results were reported in the detail that is included in the *World Heat-Flow Data Collection — 1975* (Jessop et al., 1976). Specifications are to be found in Appendix 2.