AN ANALYTICAL SOLUTION OF THE TEMPERATURE RESPONSE IN GROUND HEAT EXCHANGERS WITH GROUNDWATER ADVECTION

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ABSTRACT

A ground heat exchanger is devised for extraction or injection of thermal energy from/into the ground. In order to investigate the impact of groundwater advection on the performance of geothermal heat exchangers in ground-coupled heat pump systems, a governing equation of conduction-advection is established, and an analytical solution is obtained for a line heat source in an infinite porous medium by means of the Green function analysis. Being clear and concise, the solution of this transient two-dimensional problem has never been found in literature. On this basis an explicit expression has also been derived of the mean temperature on circles around the heat source. Dimensionless criteria that dictate the process are then summarized; influence of the groundwater flow on the heat transfer in ground heat exchangers is discussed accordingly. The analytical solution has provided a theoretical basis and practical tool for design and performance simulation of the ground heat exchangers.

INTRODUCTION

Utilizing the ground as a heat source/sink, ground-coupled heat pump (GCHP) systems have been gaining increasing popularity for space conditioning in both residential and commercial buildings due to their reduced energy and maintenance costs as compared to conventional systems (Bose et al., 2002). The efficiency of GCHP systems is inherently higher than that of air source heat pumps because the ground maintains a relatively stable temperature. This system is environment-friendly, causing less CO_2 emission than their conventional alternatives. The ground heat exchanger (GHE) is devised for extraction or injection of thermal energy from/into the ground (Kavanaugh and Rafferty, 1997). In vertical borehole systems the GHE consists of a number of boreholes, each containing a U-tube pipe. In practice, the boreholes of GHEs may penetrate several geologic strata. The heat dissipation from the pipes in boreholes to far-field ground is a transient process involving a large domain and complicated geometry. The groundwater flow makes the heat transfer process even more intricate. Water movement in actual “streams” in underground channels is rare and confined to specific geological situations. The subsurface, however, generally has porosity (voids and fractures). Below the water table, water is held and moves between the grains of geologic formations in response to hydraulic gradients. Thus, the heat dissipation in aquifers may be regarded as a coupled process of heat conduction through the solid matrix and water in its pores and heat advection by moving groundwater.

Heat transfer between a GHE and its surrounding soil/rock is difficult to model for the purpose of sizing the exchanger or energy analysis of the system. The complications in its structural and geometrical configuration as well as the long durations and variable load it involves pose a great challenge to application of numerical schemes such as FDM or FEM in GHE designs. Thus, it is of great importance to work out appropriate and validated tools, by which the thermal behavior of GCHP systems can be assessed and then, optimized in technical and economical aspects. A lot of efforts have been made to understand and formulate the heat transfer process in the GHEs. On this basis a few tools are commercially available for design and/or performance simulation of the GHEs with varying sophistications (Bose et al., 1985; Caneta Research Inc., 1995). All of the design tools, however, are based simply on principles of heat conduction, and do not consider the implications of groundwater flow in carrying away heat. The reasons for such simplification are the difficulties encountered both in modeling and computing the convective heat transfer and in learning about the actual groundwater flow in engineering practice. As a consequence, the effect of the groundwater advection is usually assumed negligible in GHE designs so far.

In general, groundwater flow is beneficial to the thermal performance of GHEs since it has a moderating effect on borehole temperatures in both heating and cooling modes. Besides, the thermal load of commercial and institutional buildings is often dominated by cooling requirement that means the system rejects more heat into the ground than it extracts from it. For building in cold climate, a much larger heating load may also occur, then, a net heat extraction may be accumulated in the lifetime operation of the system. A moderate groundwater advection, however, is expected to make notable difference in alleviating the possible heat buildup around the borehole over time. As a result, it is desirable to account for the groundwater flow in the heat transfer model to avoid over-sizing of the ground heat exchangers.

Among the few reports on the effect of the groundwater flow found in literature Eskilson (1987) has discussed the problem based on a steady-state analytical solution given by Carslaw and Jeager (1959). Because the heat dissipation in the GHEs with or without water advection is characterized as a transient process covering a long duration, the steady-state analysis provides only limited insight into the groundwater advection impacts. Chiasson et al. (2000) have made a preliminary investigation of the effects of groundwater flow on the heat transfer of vertical borehole heat exchangers. A finite element numerical model was used in solving the transient two-dimensional combined heat transfer. Thermal performances of a single borehole and a 4×4 borehole field were simulated in various geological conditions. No general conclusions and correlations among the influencing parameters were obtained due to the discrete nature of the approach.

The work reported here tries to study the combined heat transfer of conduction and advection in the vertical GHEs by an analytical approach. A two-dimensional model similar to Chiasson's has been solved analytically, and an explicit expression of the temperature response has been derived describing correlation among various factors, which have impacts on this process.

FORMULATION OF COMBINED HEAT TRANSFER

As mentioned above, heat transfer in GHEs is complicated and dependent of multiple factors, some of which are hard to grasp accurately in engineering practice. It is inevitable to make assumptions and simplifications in the study so that impacts of certain factors can be highlighted and analyzed in detail. In following discussions the ground around the boreholes is assumed to be a homogeneous porous medium saturated by groundwater.

Heat is transported through a saturated porous medium in a combined mechanism: by conduction through its solid matrix and liquid in its pores as well as by convection of the moving liquid. By applying the law of conservation of energy to a control volume, an equation for heat transfer in the saturated porous medium can be expressed as:

$$\rho c \frac{\partial t}{\partial \tau} + \rho_w c_w \vec{V} \cdot \nabla t = \nabla \cdot (k \nabla t) \qquad (1)$$

were k denotes the effective thermal conductivity of the porous medium;ρc is the volumetric specific heat of the porous medium, including both the solid matrix and water in its pores, and $\rho_w c_w$ the volumetric specific heat of water. The average linear groundwater velocity $\vec{V}$ over a cross-section of the medium may be determined by the hydraulic head distribution according to the Darcy's law if the hydraulic conductivity of the medium is known.

Following the Eskilson's model, a further approximation is accepted that the groundwater velocity is uniform in the whole domain concerned and parallel to the ground surface. The term "advection" is often used to describe such a flow. Define the velocity u is in the direction of the x-coordinate. Then, on the assumption of constant thermal properties Equation (1) reduces to

$$\frac{\partial t}{\partial \tau} + U \frac{\partial t}{\partial x} = a \nabla^2 t \qquad (2)$$

where $U = u \rho_w c_w / (\rho c)$, and the effective thermal diffusivity $a = k/(\rho c)$.

And for the steady-state situations Eskilson studied, the heat transfer equation is further simplified to

$$U \frac{\partial t}{\partial x} = a \nabla^2 t \qquad (3)$$

Solution of the Transient Problem

In order to discuss the effect of groundwater advection on heat transfer of the borehole heat exchangers the authors have derived an analytical solution of the transient problem under following assumptions:

1. The ground is regarded as homogeneous and semi-infinite medium, and its physical properties do not change with temperature;
2. The effects of the ground surface as a boundary and the finite length of the borehole are neglected. Then, the problem may be simplified as two-dimensional;
3. As commonly accepted in models for heat transfer in vertical GHEs, the borehole is approximated by a line heat source while determining the temperature rise on the borehole wall and beyond;
4. The medium has a uniform initial temperature, t_0;
5. As a basic case of study, the heating rate per length of the source, q_l, is constant since a starting instant, $\tau = 0$.

Equation (2) of conduction-advection is the same in mathematical formulation as the conduction equation with a moving heat source when the equivalent velocity U takes the place of the moving speed of the heat source. The studies on conduction with moving heat sources deal with either problems in which heat sources move through a fixed medium, or cases of fixed heat sources past that a uniformly moving medium flows. Some monographs (Carslaw and Jeager, 1959; Eckert and Drake, 1972) on heat transfer have discussed such problems, and presented the solution of a moving line source on steady conditions. Following the same approach, the authors have derived the solution on transient conditions (Diao et al., 2003). When a single line source is deployed at the origin of the coordinates, the temperature rise in the medium, $\theta = t - t_0$, of the discussed case can be obtained simply by means of the Green's function method, and written as

$$\theta(x,y,\tau) = \frac{q_l}{4\pi k}\exp\left(\frac{Ux}{2a}\right)\int_0^{\frac{r^2}{4a\tau}}\frac{1}{\eta}\exp\left[-\frac{1}{\eta}-\frac{U^2r^2\eta}{16a^2}\right]\mathrm{d}\eta \tag{4}$$

where $r = \sqrt{x^2+y^2}$.

Equation (4) reduces to the solution of the steady condition while the time approaches infinity, that is

$$\theta_s(x,y) = \frac{q_l}{2\pi k}\exp\left(\frac{Ux}{2a}\right)\mathrm{K}_0\left[\frac{Ur}{2a}\right] \tag{5}$$

where $\mathrm{K}_0(z)$ is the modified Bessel function of the second kind of order zero.

On the other hand, for the specific instance when the advection velocity u, and then $U = u\rho_w c_w/(\rho c)$, is zero, Equation (4) reduces to the common temperature response of the line source in an infinite medium:

$$\theta_0(r,\tau) = -\frac{q_l}{4\pi k}\mathrm{Ei}\left(-\frac{r^2}{4a\tau}\right) \tag{6}$$

where $\mathrm{Ei}(z)$ is the exponential integral function. As this expression indicates, the temperature in the medium will keep rising and no steady condition will be reached when the time lasts to infinity.

Select $L = a/U = k/(u\rho_w c_w)$ as a characteristic length, and $T = a/U^2$ as a characteristic time of the problem. We define the dimensionless temperature excess $\Theta = 2\pi k\theta/q_l$, the non-dimensional radial coordinate $R = r/L = Ur/a$ and non-dimensional time $Fo = \tau/T = U^2\tau/a$. Another dimensionless parameter involved is the polar angle, φ. Thus, Equation (4) can be rewritten in a dimensionless form as

$$\Theta(R,\varphi,Fo) = \exp\left(\frac{R\cos\varphi}{2}\right)\int_0^{R^2/(4Fo)}\frac{1}{2\eta}\exp\left[-\frac{1}{\eta}-\frac{R^2\eta}{16}\right]\mathrm{d}\eta \tag{7}$$

COMPUTATION AND DISCUSSION

The temperature responses of above equations can be computed readily by appropriate integration schemes, which take much less computing time and assures more reliable precision than numerical solutions of the governing equation by the finite element or finite difference methods.

Temperature Isotherms

Equation (6) indicates that the temperature response to the line source heating is symmetric about the origin in a medium without advection. When the water advection plays a role in the heat transfer, the temperature distributions become two-dimensional with a downstream bias of the temperature rise. An example of the isotherms is shown in Fig. 1.

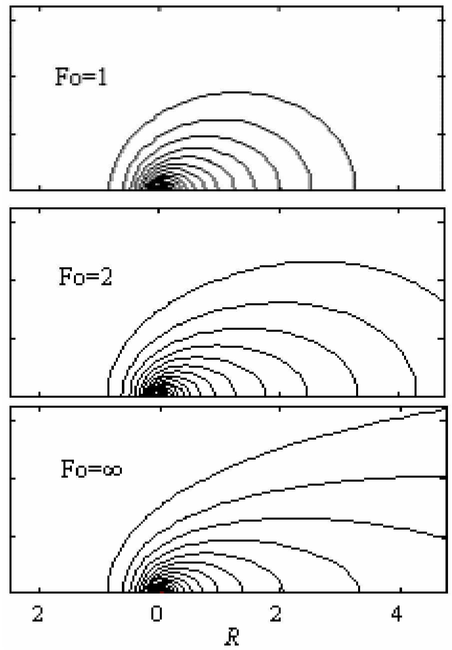

Fig. 1 Isotherms of a line-source in an infinite medium with water advection

Mean Temperature on a Circle

The temperatures on a circle around the heat source are no longer identical due to the heat carried downstream by water advection. It is desirable sometimes to know the mean temperature on such circles, for example, as the borehole wall. The mean temperature can be defined as an integral average on the circle, which turns out to be

$$\overline{\Theta}(R,Fo) = \frac{1}{\pi}\int_0^{\pi}\Theta(R,\varphi,Fo)\mathrm{d}\varphi = \mathrm{I}_0\left(\frac{R}{2}\right)\int_0^{R^2/(4Fo)}\frac{1}{2\eta}\exp\left[-\frac{1}{\eta}-\frac{R^2\eta}{16}\right]\mathrm{d}\eta \tag{8}$$

where $\mathrm{I}_0(z)$ is the modified Bessel function of the first kind of order zero.

Comparison between Equations (7) and (8) shows that the ratio of the temperature rise at a certain location to the mean value on the circle at the same distance from the source is independent of time, that is

$$\frac{\Theta(R,\varphi,Fo)}{\overline{\Theta}(R,Fo)}=\frac{\exp\left(\frac{R}{2}\cos\varphi\right)}{\mathrm{I}_0\left(\frac{R}{2}\right)} \tag{9}$$

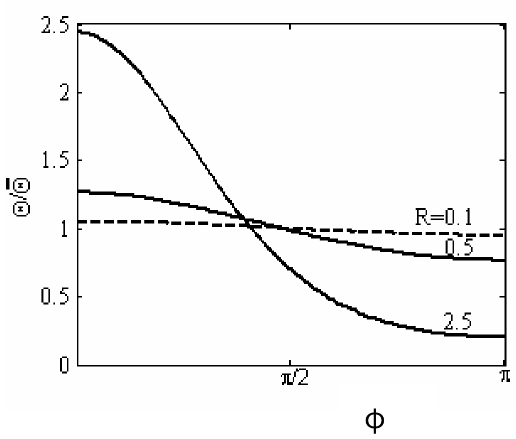

Fig. 2 The temperature rise on a circle around the heat source

Results from Equation (9) are plotted in Fig. 2, showing variations in the temperature rise along the circle. It is clear that the temperature rise in the downstream direction (φ=0) is higher than that in the upstream direction (φ=π) due to the heat transfer by advection. It can also be seen that the difference in the temperature around the circle diminishes with decrease in *R*. Calculations indicate that the ratio $\Theta/\overline{\Theta}$ is restricted between 0.99 and 1.01 as long as *R* is less than 0.005. In this sense, the non-dimensional parameter *R*, which is proportional to the advection velocity, may serve as a quantitative index in characterizing the impact of the groundwater advection; and *R*<0.005 is recommended to be a criterion that the impact of water advection on the heat transfer may be neglected.

Temperature Response Over Time

The temperature responses are computed according to Equation (7) and plotted in Fig. 3 against the dimensionless parameter $Fo/R^2 = a\tau/r^2$ so as to facilitate comparison between the models with and without advection.

It is noticeable that the dimensionless parameter *R*, which may be used to characterize the effect of groundwater advection, makes significant difference in the temperature responses. The temperature responses at upstream and downstream locations also differ notably especially when *R* is large enough, say, *R*>0.1. The temperatures at downstream locations (φ=0) rise above that of the reference case of pure conduction (*R*=0) at the early stage of the response, but later drop below the reference temperature and finally turn to steady temperatures**.**

CONCLUSION

The analytical solution results in some dimensionless parameters, facilitates both qualitative and quantitative analyses of the process. Besides, the analytical expression can be readily incorporated into existing models and software for design and performance simulation of vertical GHEs since it is much more convenient for computation than any numerical solutions of the problem.

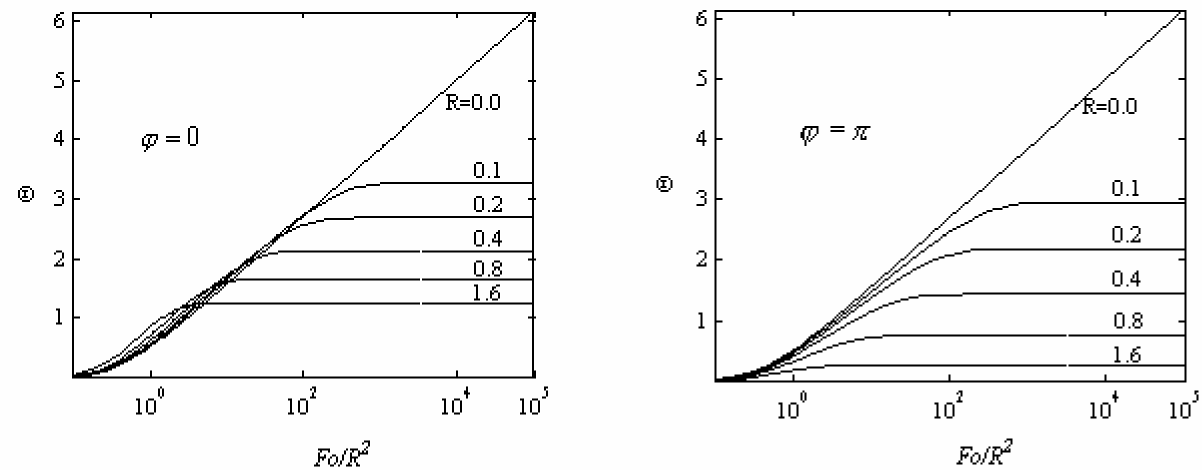

Fig. 3 Temperature responses to the line-source heating with and without water advection

The analytical solution, Equation (4), has been derived of the temperature response around a line source in an infinite porous medium with uniform water advection. The non-dimensional correlation, Equation (7), reveals that the normalized temperature rise is a function of non-dimensional coordinates and time only. Compared with the conventional Kelvin's line-source model, which makes no account of the water advection, this solution indicates that the impact of moderate groundwater flow on the heat transfer process may be prominent. The actual magnitude of the impact, however, depends mainly on the flow rate, which is characterized by the non-dimensional parameter R. This explicit and concise expression can provide an appropriate footing for qualitative and quantitative analysis of this impact for vertical ground heat exchangers in GCHP systems.

The mean temperature on a circle around the line source is also derived as a simple function of the radial distance and time, which is significant in determining the temperature on borehole walls for GHE applications. It is also noticeable that the ratio of the temperature rises at certain locations of the same distance from the source is independent of time.

By means of the superimposition principle (Fang et al., 2001) this analytical solution can be employed readily in determining the temperature rise in a borehole field with variable heating rates, and then, included into existing software for GHE design and simulation.

Finally, although a solution is obtained in this paper, proper assessment of the impact of water advection is often hindered in engineering practice by lack of sufficient information on groundwater flow. Most GHE designers have not generally found it feasible or cost-effective to obtain the hydrological data required for the assessment. Consequently, more efforts should be made in understanding the implications of practical geological and hydraulic conditions in engineering practices.

NOMENCLATURE

a	Thermal diffusivity (m^2 s^{-1})
c	Specific heat (J kg^{-1} K^{-1})
Fo	$=U^2\tau/a$, Non-dimensional time
k	Thermal conductivity (W m^{-1} K^{-1})
L	$L = k/(u\rho_w c_w)$, Characteristic length (m)
q_l	Heat flow per unit length of the line source (W m^{-1})
r	Radial coordinate (m)
R	$=Ur/a$, Non-dimensional coordinate
t	Temperature (K)
t_0	Initial temperature (K)
T	$=k\rho c/(u\rho_{ww}c_w)^2$, Characteristic time (s)
u	Advection velocity (m s^{-1})
U	$= u\rho_w c_w/(\rho c)$ (m s^{-1})
x	Coordinate (m)
y	Coordinate (m)

Greek symbols

η	Integration parameter
ρ	Density (kg m^{-3})
θ	$= t - t_0$, Temperature rise (K)
Θ	$= 2\pi k\theta/q_l$, Dimensionless temperature
φ	Polar angle
τ	Time (s)

Subscripts

0	Without advection
s	Steady-state
w	Water

Superscript

$\bar{}$	Average

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