Available online at www.sciencedirect.com

ScienceDirect

Transportation Research Procedia 23 (2017) 81–100

Transportation Research Procedia

www.elsevier.com/locate/procedia

22nd International Symposium on Transportation and Traffic Theory

Effect of Information Availability on Stability of Traffic Flow: Percolation Theory Approach

Alireza Talebpour [a], Hani S. Mahmassani [b,*], Samer H. Hamdar [c]

[a] *Zachry Department of Civil Engineering, Texas A&M University, 3136 TAMU, College Station, TX 77843, United States*
[b] *Transportation Center, Northwestern University, 600 Foster St., Chambers Hall, Evanston, IL 60208, United States*
[c] *School of Engineering and Applied Science, George Washington University, 20101 Academic Way #201-I, Ashburn, VA 20147, United States*

Abstract

Connectivity and automation are expected to enhance safety and efficiency in transportation systems. Connectivity will provide information to drivers/autonomous vehicles to enhance decision-making reliability at the operational and tactical levels. Consequently, drivers are more likely to execute safe and efficient maneuvers and autonomous vehicles will have a more accurate perception of the traffic condition and an "error-free" execution of the driving maneuvers. At the operational level, ensuring string stability is a key consideration since unstable traffic flow results in shockwave propagation and possibly crashes. While several studies have examined the effects of information availability on string stability in a connected environment, most of the approaches are focused on automated driving (e.g., Cooperative Adaptive Cruise Control systems) and do not consider a mixed environment with regular, connected, and autonomous vehicles. To ensure connectivity in such a mixed environment, the correlation between communication range and connected vehicles density should be considered. To capture the effects of this correlation, this study uses the Continuum Percolation theory to determine the effects of the vehicular density and communication range on the connectivity level in telecommunications network and consequently, on the string stability of traffic flow. Based on the Continuum Percolation theory, there is a critical density of connected vehicles above which the entire system is connected. This critical density also depends on the communication range. This study presents an analytical derivation of this critical density. Moreover, this study evaluates the string stability under different communication ranges and market penetration rates of connected and autonomous vehicles. Results revealed that as communication range increases, the system becomes more stable and at high communication ranges, the system performs similar to the system with full connectivity. Moreover, results indicated the existence of an optimal communication range to maximize stability and ensure information delivery.

Peer review under responsibility of the scientific committee of the 22nd International Symposium on Transportation and Traffic Theory.

Keywords: Connected Vehicles; Autonomous Vehicles; Percolation Theory; String Stability.

2352-1465
Peer review under responsibility of the scientific committee of the 22nd International Symposium on Transportation and Traffic Theory.
10.1016/j.trpro.2017.05.006

1. Introduction

Connected and autonomous vehicles continue to promise improvements in the efficiency and safety of individual vehicle maneuvers throughout the surface transportation systems. In a connected Vehicle-to-Vehicle (V2V) and Vehicle-to-Infrastructure (V2I) communications network, a vehicle receives information from other vehicles in its vicinity and traffic management center (TMC). This information can help drivers and autonomous vehicles execute safer and more reliable and efficient maneuvers. Efficient maneuvers, on the other hand, result in different traffic patterns, which can influence communications and information availability. Unfortunately, the collective effects of this additional information on traffic flow dynamics are not sufficiently known. Preparing the surface transportation systems to maximize the benefits of these technologies requires an understanding of these possible collective effects on traffic flow dynamics. Therefore, considering such information is essential to investigate the effects of connected environment on transportation systems (Mahmassani, 2016).

At the operational driving decision-making level and car-following behavior, traffic flow stability is a key factor. A string stable system resolves shockwaves and prevent them from propagating upstream and creating severe congestion. Unfortunately, the effects of information propagation and signal interference on the stability of traffic flow have not been investigated extensively in the literature. Though a few studies have investigated the effect of telecommunications on stability, most of these assume full connectivity (Talebpour et al., 2016). However, full connectivity in a V2V communications network does not always exist due to several factors, including physical barriers and signal interference (Talebpour, 2015; Talebpour et al., 2016). Talebpour et al. (2016) illustrated the effects of signal interference on data quality by comparing the actual vehicle trajectories and the location data received by a roadside unit through V2I communications (See Figure 1). The actual vehicle trajectories are based on the NGSIM US-101 dataset (Federal Highway Administration, 2006) and data transmission is simulated using ns-3 (ns-3.25). One roadside unit is simulated in the middle of the 2100ft segment on US-101, which was used to collect the NGSIM data. The message routing protocol is the Adopted Message Routing Protocol (AODV).

In addition to imperfect communication, vehicular density and communication range are important factors in communication networks. In a real-world environment, 1-hop communication (*i.e.,* direct communication between vehicles) is not always possible and does not guarantee propagation of all relevant messages. In fact, multi-hop communication is essential to ensure that all the vehicles in immediate need of the information will receive it. To ensure this multi-hop communication, enough vehicles should be within the communication range of each vehicle. Therefore, communication range should be carefully selected in relation with the vehicular density. Without V2V communications and additional information, drivers are uncertain about their leaders' behavior and their car-following behavior is expected to be very similar to that of regular vehicles. Therefore, considering the communication range along with the vehicular density is essential to determine the effect of connectivity and automation on stability of traffic flow.

The propagation of information within vehicular networks is very similar to the behavior of a fluid spreading through a porous medium, a disease spreading among people, or information propagation in social networks. The latter physical and behavior phenomena/observations have been studied extensively in the literature and several models have been introduced to capture the underlying mechanisms. Percolation theory (Broadbent and Hammersley, 1957) is among the most reliable and accurate approaches to model these phenomena. The original Percolation theory describes the behavior of connected clusters in a random graph with infinite number of nodes and edges (Meester and Roy, 1996) and describes a critical phenomenon in which a connected path with an infinite size will almost surely appear for the first time in the original graph. Recognizing the similarity between information propagation in wireless communication networks and these phenomena, a few studies showed that the propagation of information in these networks can be captured by the Percolation theory (Ammari and Das, 2008).

Similar to the original percolation model, the information propagation problem in vehicular networks is focused on the behavior of connected cluster in the connectivity graph. Building upon the findings of Ammari and Das (2008), this paper presents an analytical derivation of the critical density above which the system is connected when percolation occurs in vehicular networks, and uses the Percolation theory to determine the effects of the vehicular density and communication range on the stability of traffic flow. The analytical investigation of stability uses Percolation theory to determine the connectivity level among vehicles, and adopts Ward's approach (Ward, 2009) to investigate string stability in a mixed platoon of connected, automated, and regular vehicles. Accordingly, the contributions of this paper consist of the following: (1) analytical derivation of the critical vehicular density beyond which communication may be secured between all vehicles in a given platoon; (2) the use of percolation theory to investigate the platoon string stability as a function of the communication range and the traffic conditions (i.e. density levels); and (3) endogenous consideration of the market penetration rates of regular, connected, and automated vehicles (i.e. information availability and vehicular types) when studying the stability of a heterogeneous mixed platoons.

The remainder of this paper is organized as follows: section 2 presents the necessary background on the principles of Percolation theory. Section 3 presents the definition of the key terms used in the paper. Section 4 discusses the analytical derivation of the critical density of connected vehicles to achieve percolation in a V2V communications network. Descriptions of the acceleration framework and analytical investigations of string stability are offered in Section 5. The paper is concluded with summary remarks and future research needs in section 6.

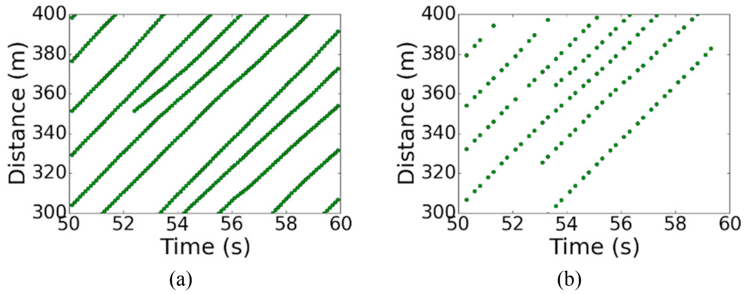

Fig. 1. Vehicle trajectory data based on NGSIM US-101 dataset (a) complete trajectories, (b) received data (simulated using ns-3) (Talebpour et al., 2016).

2. Background

The study of percolation goes back to the discrete percolation model (Broadbent and Hammersley, 1957; Meester and Roy, 1996). Assume a d-dimensional lattice in Z^d where each pair of vertices in this lattice is connected if the two vertices are one Euclidean distance from each other. These nodes are called neighbors, and there is an edge (often called "bond") between any two neighbors. Each bond can be used for communication with probability $0 > p > 1$. Let E denote the set of these bonds. The discrete percolation model investigates the probability that a cluster with an infinite size exists in this lattice. It can be shown (using Kolmogorov's 0 – 1 law) that the probability that a cluster with infinite size exists is one for all $p > p_c$ where p_c is the critical density (Meester and Roy, 1996). For $p < p_c$, this probability is zero. Discrete percolation is concerned with finding this critical density.

This concept was subsequently extended to the continuum space. Similar to the discrete percolation model, continuum percolation is concerned with finding the critical density in which a cluster with an infinite size exists. Two types of Continuum percolation models were introduced (Meester and Roy, 1996): Boolean model and Random Connection model. In this research is focus is on the Boolean model. Assume a stationary point process, X , in R^d. In the Boolean model (or Poisson blob model), each point in X (denotes by x) is the center of a circle with a random radius R. Let $C(x,r)$ denote this circle. These circles divide the space into two regions: occupied region and vacant region. A pair of nodes is connected if both nodes belong to the same occupied/vacant region. Figure 2 illustrates the occupied and vacant regions in a 2D space. In this figure, point A and point B are connected in the occupied space, while point C and point D are connected in the vacant space. In the Random Connection model, connectivity is defined based on a connection function, which is a non-increasing function ($g: R \rightarrow [0, 1]$). Note that the most common connection functions are defined as decreasing functions of the Euclidean distance between two points.

Continuum Percolation models have been adopted in the literature to study connectivity in wireless communication networks. Glauche et al. (2003) used Continuum Percolation to model connectivity in random geometric wireless ad-hoc communication networks. Their approach was based on the Random Connection model and they consider signal transmission power and signal to noise ratio to define the connection function. They found that the critical density is not sensitive to the spatial patterns of the transmission nodes. Pinto and Win (2012) investigated the percolation in secure communication networks and showed that full connectivity can be achieved at a critical density. Moreover, they estimated the critical density using Monte-Carlo simulation. Almiron et al. (2013) investigated percolation in obstructed networks. They used the grid structure of street intersections to create an obstructed wireless network and calculated the critical density based on the Percolation theory and geometric properties. Ammari and Das (2008) used Continuum Percolation to investigate combined connectivity and coverage in wireless ad-hoc networks. They proposed an analytical approach to estimate the critical density for a combined full connectivity and coverage in these networks. Later, Khanjary et al. (2015) extended the findings of Ammari and Das (2008) to fixed directional sensors and estimated the critical density for the system. Note that in general, the studies of percolation in wireless networks focus on systems that, unlike vehicular ad-hoc networks, have very limited to no mobility. The structure of vehicular ad-hoc networks is constantly changing and changes the connectivity level, which influences the available information to drivers. Therefore, understanding the effects of highly mobile vehicular ad-hoc networks on connectivity is essential to study the effects of connectivity on traffic flow dynamics. This understanding is particularly critical in evaluating string stability of traffic flow in the presence of connected and autonomous vehicle.

The effects of connectivity and automation on string stability have been explored in the literature. Several studies investigated string stability in Adaptive Cruise Control (ACC) and Cooperative Adaptive Cruise Control (CACC) systems. Naus et al. (2010) derived the necessary and sufficient criteria to achieve string stability based on a decentralized CACC control logic. Wang et al. (2013) utilized the receding horizon control concept to develop a control logic for driver assistance systems and investigated the string stability in a platoon of ACC and CACC vehicles. Bose and Ioannou (2003) utilized the Intelligent Cruise Control (ICC) model (Ioannou and Xu, 1994) to investigate the stability of a mixed traffic stream consisting of ACC and regular vehicles. They showed that ACC vehicles could improve the stability of traffic flow, reduce emissions, and improve fuel efficiency. Schakel et al. (2010) showed that a CACC system can enhance string stability in mixed traffic. Shaw and Hedrick (2007) investigated the string stability of heterogeneous vehicle strings based on the constant spacing policy and derived necessary and sufficient conditions to ensure string stability. Recently, Talebpour and Mahmassani (2016) investigated string stability in a platoon of connected and autonomous vehicles and showed that automation has more significant effect on stability than connectivity. Despite the above and several other efforts, the effects of information availability on string stability have not been explored extensively. Swaroop (1997) has investigated the effects of connectivity on the string stability of CACC and showed that the platoon head and immediate leader are the most important vehicles, and receiving information from these two vehicles can ensure string stability in CACC. Ploeg et al. (2013) investigated the effects of information loss on string stability in a CACC platoon. They assumed that without information, the CACC system reverts back to ACC. They developed a procedure to estimate the communicated desired acceleration based on the information from the leader. Using this procedure, they showed that string stability can be improved compared with using ACC only.

While these and several other studies investigated the effects of communication on string stability in CACC systems and built the foundation to investigate the effects of connectivity and automation on string stability in a broader perspective, many questions remain to be addressed, including the effects of communication on human drivers and on string stability in a platoon of regular, connected, and autonomous vehicles. This study adopts the Percolation theory to model the availability of information and evaluate the effects of communication range and connected vehicles density on string stability at different market penetration rates of connected and autonomous vehicles.

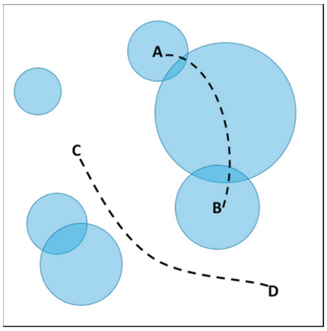

Fig. 2. A Boolean model: the blue regions are occupied regions and vacant regions are in white (Meester and Roy, 1996).

3. Definitions

This section defines the key terms used in this paper. Assume that connected vehicles along a highway form a two-dimensional homogeneous Poisson point process where density is denoted by λ .

Definition 1: String Stability of Traffic Flow (Wilson and Ward, 2010). In general, any car-following model can be formulated by coupled differential equations (Treiber and Kesting, 2013; Wilson and Ward, 2010):

$$\dot{x}_n = v_n \ , \tag{1}$$

$$\dot{v}_n = f\left(s_n, \Delta v_n, v_n\right) \tag{2}$$

where x_n is the longitudinal location of vehicle n (as x_n increases, a vehicle traverses more distance along the longitudinal roadway direction), v_n is the velocity of vehicle n, s_n is the spacing between vehicle n and the lead vehicle $n-1$, and Δv_n is the relative velocity between vehicle n and the lead vehicle $n-1$ (i.e. $v_n - v_{n-1}$); $\dot{x}_n$ is the differential of longitudinal location of vehicle n with respect to time (i.e. velocity) and $\dot{v}_n$ is the differential of velocity with respect to time (i.e. acceleration). Empirical observations suggest that there exists an equilibrium speed-spacing relationship (Treiber et al., 2000). In other words, there exist a function $v = V(s)$ such that $f(s^*, 0, V(s^*)) = 0$ for $s^* > 0$. s^* is called equilibrium spacing and $V(s^*)$ denotes the speed at equilibrium. Consider an infinite platoon of vehicles at equilibrium. If a vehicle deviates from equilibrium, the resulting perturbation will propagate to the upstream traffic. If the car-following model of interest is string stable, the perturbation will decay as it propagates upstream.

Definition 2: Homogeneous Poisson Point Process. Let $X_\lambda(L)$ denote a random variable representing the number of vehicles in a segment of the highway. Assume a fixed width for the highway segment (i.e. a fixed number of lanes along the highway segment). Assuming homogenous Poisson distribution, the probability that there exist k vehicles in this segment can be computed as:

$$P(X_\lambda(L) = k) = \frac{(\lambda SL)^k}{k!} e^{-\lambda SL} \tag{3}$$

where S and L denote the segment width and length, respectively. Note that since the highway width is fixed, its effects can be captured by L .

Definition 3: Transmission Range. Transmission range of a vehicle i located at x_i is a disk with radius R_i. Note that this study assumes an identical transmission range for all connected vehicles.

Definition 4: Communicating Vehicles (Ammari and Das, 2008). The communication function between two vehicles is defined as:

$$h(x_i,x_j)=\begin{cases}1 & if\left|x_i-x_j\right|<\min\{R_i,R_j\}\\ 0 & Otherwise\end{cases} \tag{4}$$

Definition 5: Communication Path (Ammari and Das, 2008). A sequence of n vehicles in which vehicles i and $i+1$ are communicating ($i=1,\dots,n-1$) is called a communication path (See Figure 3).

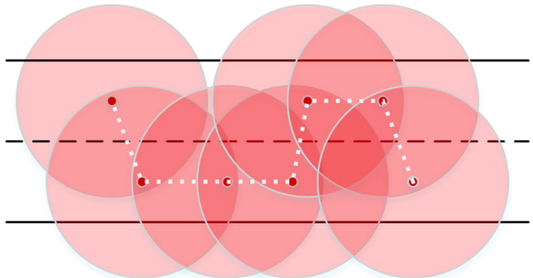

Fig. 3. An illustration of a communicating path.

Definition 6: Connected k-component (Ammari and Das, 2008). A set of communicating vehicles (*i.e.* there exists a *communication* path between any two vehicles in the set) is called a connected component if it is not a subset of another set of communicating vehicles. A set of k vehicles with these characteristics is called a connected k-component and is shown as CC_k in this paper.

Definition 7: Distance between Two Connected k-components (Ammari and Das, 2008). This study considers the *minimum* distance between pairs of vehicles (each from one connected k-component) as the minimum distance between two connected k-components.

Definition 8: Continuum Percolation in Vehicular Ad-Hoc Networks (Boolean Model). The definition of Continuum Percolation in vehicular ad-hoc network follows the original definition of the Boolean model. However, vehicular connectivity imposes more restriction on this definition as connectivity exists between two sensors (i and j) if $\left|x_i-x_j\right|<\min\{R_i,R_j\}$. Based on the continuum percolation model, there exists a critical density (λ_c) that a connected component with infinite nodes will form for the first time.

4. Percolation of Vehicular Ad-Hoc Networks

This section explores the critical density for connected vehicles above which percolation happens in the communications network. At such densities, all connected vehicles in the system can communicate information and assist drivers with operational, tactical, and strategic decisions.

4.1. Connected Components: Length Estimation

Each connected k-component of vehicles along a highway can be characterized by its length in the direction of travel (assuming a fixed width for the highway segment). Assuming a Poisson point process with density λ for the connected vehicles along a highway segment, the distribution of the connected k-components along the same segment follows a Poisson distribution with density λ_k. Assuming identical transmission range for all the vehicles (denoted by R), a connected k-component of vehicles along a highway segment can be enclosed by a rectangle Assume that the length of this rectangle is L. Moreover, no vehicle is located at a distance less than R from either ends of this segment (Figure 4). The probability that there exist only one connected k-component in this rectangular can be calculated as follows:

$$P(N_L = k) = prob\{R_I \ contains \ a \ CC_K \,|\, (R_{II} \cup R_{III}) is \ empty\} \tag{5}$$

Using (3), (5) can be rewritten as follows:

$$P(N_L = k) = \frac{\dfrac{\left(\int_{X-R}^{X+L+R} \lambda(x)dx\right)^k}{k!} e^{-\int_{X-R}^{X+L+R} \lambda(x)dx}}{e^{-\int_{X+L}^{X+L+R} \lambda(x)dx} . e^{-\int_{X-R}^{X} \lambda(x)dx}} = \frac{\dfrac{[\lambda(L+2R)]^k}{k!} e^{-\lambda(L+2R)}}{e^{-\lambda R} . e^{-\lambda R}} = \frac{[\lambda(L+2R)]^k}{k!} e^{-\lambda L} \tag{6}$$

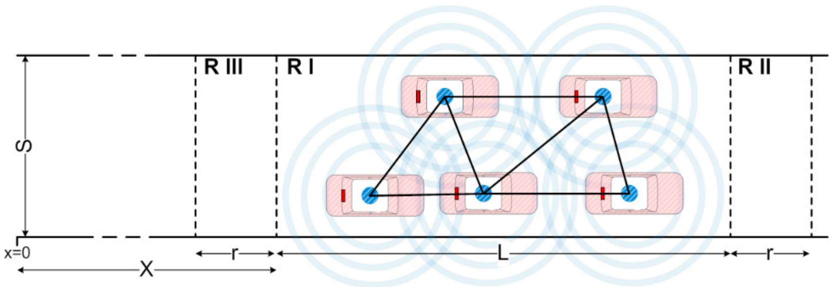

Fig. 4. Connected Component along A Highway Segment.

4.2. Connected Components: Critical Density

This section calculates the critical density of covered k-components along a highway segment. This is an essential step to calculate the connected vehicles critical density. This study follows the approach presented by Ammari and Das (2008) to calculate the critical density of covered k-components in a highway environment. Accordingly, two lemmas and their proofs are presented. Both lemmas calculate the average distance between connected k-components from two different approaches. These lemmas are then combined to calculate the critical density.

Lemma 1: Considering a homogeneous Poisson distribution for vehicles along a highway segment, average distance between connected k-components along this highway segment at percolation is given by:

$$d_{ave}^{1} = \frac{1}{\lambda_c(k)} \tag{7}$$

Proof: Assuming homogenous Poisson distribution for vehicles along a highway segment, the average distance between connected k-components can be calculated as follows,

$$E[Y] = \frac{1}{\lambda(k)} \tag{8}$$

At the percolation point, the density, $\lambda(k)$, is equal to critical density, $\lambda_c(k)$. Therefore,

$$E[Y_c] = d_{ave}^{1} = \frac{1}{\lambda_c(k)} \tag{9}$$

□

Lemma 2: Considering a homogeneous Poisson distribution for vehicles along a highway segment, average distance between connected k-components along this highway segment at percolation is given by:

$$d_{ave}^2 = \frac{1}{\lambda_c} - \left(R + \frac{1}{\lambda_c}\right)e^{-\lambda_c R} \tag{10}$$

Proof: Assuming homogenous Poisson distribution for connected vehicles along the highway, the probability that the distance between two points is less than the transmission range can be calculated as follows,

$$P(Y \le R) = 1 - e^{-\lambda R} \tag{11}$$

Therefore, the probability density function can be calculated as follows,

$$f(Y \mid Y \le R) = \lambda e^{-\lambda R} \tag{12}$$

At critical density, a connected component will span the entire system. Therefore, the distance between connected k-components at critical density belongs to the interval $[0, R]$. Therefore,

$$E[Y \mid Y \le R] = \int_0^R t \times f(Y \mid Y \le t)dt = \int_0^R t\left(\lambda e^{-\lambda t}\right)dt \tag{13}$$

Recall that $\int u dv = uv - \int v du$. Thus,

$$d_{ave}^2 = E[Y \mid Y \le R] = -te^{-\lambda t}\Big]_0^R - \int_0^R -e^{-\lambda t}dt = \frac{1}{\lambda_c} - \left(R + \frac{1}{\lambda_c}\right)e^{-\lambda_c R} \tag{14}$$

□

Combining Lemma 1 and Lemma 2, Theorem 1 calculates the critical density of connected k-components at percolation.

Theorem 1: The critical density of connected k-components at percolation is given by

$$\lambda_c(k) = \frac{\lambda_c}{1 - \left(\lambda_c R + 1\right)e^{-\lambda_c R}} \tag{15}$$

Proof: Based on Lemma 1 and Lemma 2, $d_{ave}^1 = d_{ave}^2$. Substituting (7) and (10) into this equality,

$$\frac{1}{\lambda_c(k)} = \frac{1}{\lambda_c} - \left(R + \frac{1}{\lambda_c}\right)e^{-\lambda_c R} \Rightarrow \lambda_c(k) = \frac{\lambda_c}{1 - \left(\lambda_c R + 1\right)e^{-\lambda_c R}} \tag{16}$$

□

4.3. Connected Components: Critical Length

At the percolation point, any nonempty circle with radius R should enclose a connected component. Moreover, all these vehicles within this circle form a complete graph. Theorem 2 presents another approach to calculate the density of connected k-components at the percolation point.

Theorem 2: At the percolation point, the density of connected k-components within a circle with radius R is given by,

$$\lambda_c(k) = \lambda_c \frac{[\lambda_c(3R)]^k}{k!} e^{-\lambda_c R} \tag{17}$$

Proof: Assuming a homogenous Poisson point process, connected vehicles density can be calculated as follows,

$$\lambda = \frac{N}{L} \tag{18}$$

where N is the total number of connected vehicles in the segment with length L. Using $\overline{N}_k = \lambda(k)L$,

$$\lambda(k) = \lambda \frac{\overline{N}_k}{N} \tag{19}$$

Note that $\frac{\overline{N}_k}{N}$ can be estimated by the probability of finding a connected k-component with radius R.

$$\frac{\overline{N}_k}{N} = P(rad(CC_k) = R) \tag{20}$$

This statement is equivalent to the conditional probability of finding k vehicles in a circle with radius R. Therefore, based on (6), (19) can be rearranged as follows,

$$\lambda_c(k) = \lambda_c \frac{[\lambda_c(3R)]^k}{k!} e^{-\lambda_c R} \tag{21}$$

□

The combination of (15) and (17) can characterize the critical density at the percolation point,

$$f(\lambda_c, R, k) = \left[1 - (\lambda_c R + 1)e^{-\lambda_c R}\right] \times \left[\left(\frac{[\lambda_c(3R)]^k}{k!}\right)e^{-\lambda_c R}\right] - 1 = 0 \tag{22}$$

Assuming $A_c(R) = 1 - e^{-\lambda_c R}$, (22) can be rearranged as follows,

$$f(\lambda_c, R, k) = \left[1 - (\mu + 1)e^{-\mu}\right] \times \left[\left(\frac{[3\mu]^k}{k!}\right)e^{-\mu}\right] - 1 = 0 \tag{23}$$

where $\mu = -\log(1 - A_c(R))$. Note that $A_c(R)$ belongs to $[0,1]$ and facilitates the calculations. Equation (22) forms the basis for considering connectivity level in the stability analysis calculations. Figure 5.a shows the results of Equation (23) for different values of connected k-component sizes. Results indicate that percolation first occurs at $k=3$ and $A_c(R)=0.785$. Thus, when the number of communication links of a connected vehicle is larger than two ($k \geq 3$), there is an infinite connected component that spans the entire system. Figure 5.b shows the relationship between minimum density and transmission range to ensure percolation. For any combination of the transmission range and density that falls on the left hand side of this curve, connected vehicles cannot form a connected component that spans the entire network. Therefore, some connected vehicles behave similar to regular vehicles. Moreover, as the communication range increases, the required density to reach the percolation point decreases. Note that these calculations are all based on the perfect communication assumption.

Perfect communication, however, is a strong assumption and imposes certain restrictions on the validity of those analyses. As discussed previously, many factors can result in communication loss and delay including signal interference. Talebpour et al. (2016) showed that packet drop ratio is minimal in communication ranges below 120-130m. Figure 6 shows the simulation results based on the US-101 Next Generation Simulation (NGSIM) dataset (Federal Highway Administration, 2006). Even though the simulation results could slightly change based on the network configuration, the numbers are not expected to change significantly. Moreover, they showed that the system experiences a reduction in packet delivery at communication ranges beyond 120-140m and the reduction in more significant at higher communication ranges. Combining the packet delivery ratio from Figure 6 and the communication range, for any communication range above 140m, the number of delivered packets is equivalent to its number at 140m. Thus, the effective communication range stays within the $[120,140]$ interval. Accordingly, to limit the impacts of information loss on the string stability analysis, this study limits the effective communication range to 130m and presents a sensitivity analysis on the effects of communication range on stability of traffic flow.

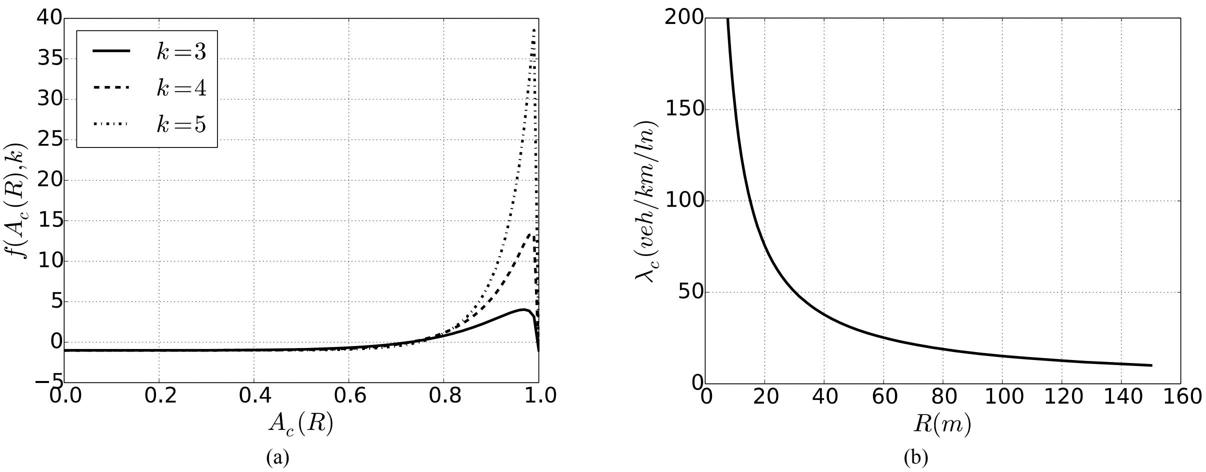

Fig. 5. (a) Percolation at critical density at $k=3$ and $A_c(R)=0.785$, and (b) critical density for percolation based on transmission range.

Note that full communication is guaranteed for any combination of connected vehicles density and effective communication range that results in $A_c(R) \geq 0.785$. When $A_c(R) < 0.785$, the system will suffer from incomplete information and its performance is expected to degrade. In these situations, connected vehicles that do not receive the information should be modelled as regular vehicles. Assuming homogenous Poisson distribution for connected vehicles along a highway segment with infinite length, the probability that there is only one point in any circle with radius R can be calculated as follows,

$$P(N_{2R}=1)=2R\lambda e^{-2R\lambda} \tag{24}$$

Figure 7 presents this probability at different combinations of density and communication range.

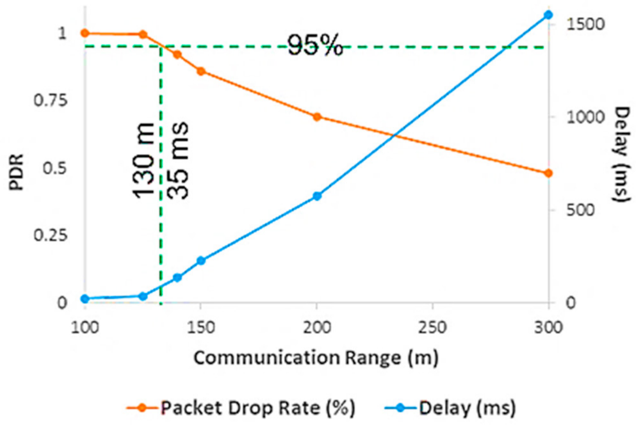

Fig. 6. Packet Delivery Ratio and Transmission Delay in V2V Communications Network for different Communication Ranges (Simulated Using ns-3 and NGSIM US-101 Dataset (Talebpour et al., 2016).

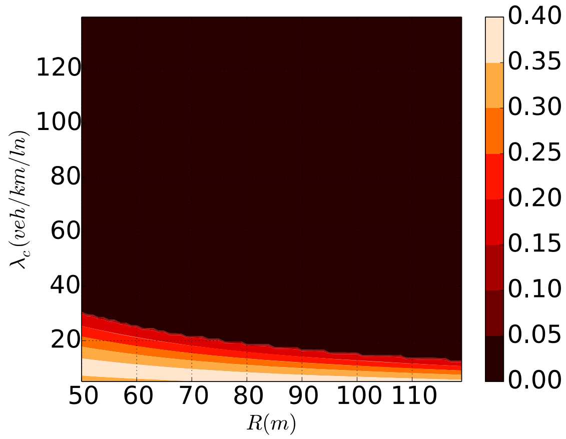

Fig. 7. Communication probability at different values of communication range and density (based on Percolation theory and Poisson distribution).

5. Analytical Investigation of String Stability

Connected and autonomous vehicles will change the dynamics of interactions among transportation system users. These changes are expected to influence the fundamentals of traffic flow as well as string stability of a platoon of regular, connected, and autonomous vehicles. Despite considerable number of studies on the string stability of simple car-following models (Treiber and Kesting, 2013; Wilson and Ward, 2010), studies on the string stability of traffic flow in the presence of connected and autonomous vehicles are limited. Moreover, those studies do not consider the effects of information availability on driver/vehicle behavior. This is particularly critical since without connectivity, drivers of connected vehicles will not have access to the information and their behavior should be modeled similarly to that of the drivers in regular vehicle drivers.

Accordingly, the main motivation of this study is to combine the Percolation theory with linear string stability analysis to consider the effects of different connectivity levels on string stability. The key element in incorporating the results of the Percolation theory into string stability analysis is that connected vehicles without information behave similar to regular vehicles and should be modeled the same way. This section presents an analytical investigation of string stability of traffic flow in a platoon of regular, connected, and autonomous vehicles under different market penetration rates of connected and autonomous vehicles.

Following the approach presented by Talebpour and Mahmassani (2016), this study adopts the findings of Ward's study (Ward, 2009) to investigate string stability of a heterogeneous traffic flow. Considering small perturbations in headway and speed of a vehicle in a platoon of infinite length, $s_l = s^* + \bar{s}_l$ and $v_l = V(s^*) + \bar{v}_l$, and linearizing Equations (1) and (2) about the equilibrium, Ward found the following instability condition for a heterogeneous traffic flow,

$$\sum_n \left[\frac{{f_v^n}^2}{2} - f_{\Delta v}^n f_v^n - f_s^n \right] \left[\prod_{m \neq n} f_s^m \right]^2 < 0 \tag{25}$$

where n denotes different vehicle types and the expansion coefficients are

$$f_s^n = \left. \frac{\partial f(s_n, \Delta v_n, v_n)}{\partial s_n} \right|_{(s^*, 0, V(s^*))}, \; f_{\Delta v}^n = \left. \frac{\partial f(s_n, \Delta v_n, v_n)}{\partial \Delta v_n} \right|_{(s^*, 0, V(s^*))}, \; f_v^n = \left. \frac{\partial f(s_n, \Delta v_n, v_n)}{\partial s_v} \right|_{(s^*, 0, V(s^*))}.$$

Following Equation (24), the stability of different combinations of regular, connected, and autonomous vehicles are investigated in the following sections. Note that the n subscript is dropped in the following sections unless otherwise necessary.

5.1. Acceleration Model Formulation

Acceleration behavior has been studied extensively in the literature and several models with different levels of complexity have been introduced to model the acceleration decision making. Several of these models could be potentially used to model regular, connected, and autonomous vehicles. However, accurate modeling requires actual observations of the real-world implementation of these systems. Therefore, the focus of this study is on providing a platform for analyzing the effects of connected and autonomous vehicles on stability of traffic flow rather than introducing an acceleration framework. The model selection is based on the current state-of-the-art in modeling these vehicles and the following factors are considered:

- *Regular Vehicles:* The behavior of regular vehicles is probabilistic in nature and accordingly, crashes are possible due to drivers' misjudgment of the situation and driving environment. The Hamdar et al. (2015) model adapted Kahneman and Tversky's prospect theory for the evaluation of perceived gains and losses while driving, thereby allowing risk-taking maneuvers when drivers are uncertain of the leader's future behavior. Accordingly, accidents would be possible and no artificial constraints are needed to preclude them. This study adopts this framework to model the driving behavior of regular vehicles.
- *Connected Vehicles:* Connected vehicles technology will provide accurate information about surrounding traffic condition and is expected to enhance drivers' strategic, tactical, and operational decision making, which will result in less risk-taking maneuvers and more deterministic behavior. Accordingly, the Intelligent Driver Model (IDM) (Kesting et al., 2010b), a deterministic, accident free model, is selected to reflect the possible changes in driving behavior in a connected environment.
- *Autonomous vehicles:* The behavior of the autonomous vehicles is fully deterministic and their perception of surrounding traffic condition is based on the information they received from on-board sensors. Accordingly, this study simulates the sensors input and uses an optimal-velocity (another deterministic model) to model the car-following behavior of autonomous vehicles.

5.1.1. Modeling Vehicles with No Communication Capability (Regular Vehicles)

The drivers of these vehicles only rely on their own perception of the surrounding driving environment. Therefore, their acceleration behavior is probabilistic and lack of accurate information can result in high-risk maneuvers and crashes. Avoiding crashes is an extremely important factor in drivers' decision making because of its severe consequences. Hamdar et al. (2008) presented an acceleration model that avoids (most) crashes by specifying behavioral mechanism based on Kahneman and Tversky's prospect theory (Kahneman and Tversky, 1979). This study adopts their framework to model the behavior of regular vehicles. The value function in this model has the following form:

$$U_{PT}(a_n) = \frac{\left[w_m + (1-w_m)(\tanh\left(\frac{a_n}{a_0}\right)+1) \right]}{2} \left[\frac{\left(\frac{a_n}{a_0}\right)}{(1+\left(\frac{a_n}{a_0}\right))^{\gamma}} \right] \tag{26}$$

where U_{PT} denotes the value function. $\gamma > 0$ and w_m are parameters to be estimated. γ is the sensitivity exponents of the generalized utility and w_m asymmetry factor for negative utilities. Once the expected value function is calculated, total utility function of acceleration can be formulated as follows:

$$U(a_n) = (1-p_{n,i})U_{PT}(a_n) - p_{n,i} w_c k(v,\Delta v) \tag{27}$$

where $p_{n,i}$, w_c, and $k(v,\Delta v)$ denote the crash probability, weighing factor for crashes, and crash seriousness term, respectively. Finally, to reflect the stochastic response adopted by the drivers, the logistic functional is used to calculate the probability density function (Hamdar et al., 2015):

$$f(a_n) = \begin{cases} \dfrac{e^{\beta_{PT}U(a_n)}}{\int_{a_{\min}}^{a_{\max}} e^{\beta_{PT}U(a')} da'} & a_{\min} < a_n < a_{\max} \\ 0 & \textit{Otherwise} \end{cases} \tag{28}$$

where β_{PT} reflects the sensitivity of choice to the utility $U(a_n)$. Talebpour and Mahmassani (2016) calculated the partial derivatives of Equation (24) for this model.

$$f_s = \frac{2}{\tau_{\max}^{\ 2}} \tag{29}$$

$$f_{\Delta v} = \frac{-2}{\tau_{\max}} \tag{30}$$

$$f_v = \frac{2\alpha z_e^*}{\tau_{\max}} + \frac{2\alpha v_e}{\tau_{\max}} \left[\frac{1}{\sqrt{2} v_e} \left(\ln\left(\frac{w_c \tau_{\max}}{2\sqrt{2\pi}\alpha v_e} \right) \right)^{-1/2} \right] \tag{31}$$

where $\tau_{\max}$ denotes maximum anticipation time horizon, α denotes velocity uncertainty variation coefficient, v_e is the equilibrium speed, and z_e^* is denoted by the following equation:

$$z_e^* = \sqrt{2\ln\left(\frac{w_c \tau_{\max}}{2\sqrt{2\pi}\alpha v_e}\right)} \tag{32}$$

5.1.2. Modeling Communication-Ready Vehicles (Connected Vehicles)

Communication provides additional information to the drivers and enhances their decision making abilities. In fact, all aspects of drivers' operational, tactical, and strategic decisions are expected to change. However, as discussed previously, information about other vehicles might not be available at all times. Since this study only focuses on vehicle-to-vehicle (V2V) communications, the following two scenarios can be defined depending on the availability of V2V communications.

5.1.2.1. Active Vehicle-to-Vehicle Communications

Information availability increases the drivers' awareness about their surrounding and provides information on other drivers' behavior. Therefore, a more deterministic behavior is expected from connected vehicles and high-risk maneuvers are less likely to happen. Following the approach of Talebpour and Mahmassani (2016), this paper utilizes the Intelligent Driver Model (IDM) (Kesting et al., 2010a) to capture the behavior of connected vehicles. While capturing different congestion dynamics, this model provides greater realism than most of the deterministic acceleration modeling frameworks.

IDM specifies a following vehicle's acceleration as a continuous function of the vehicle's current speed, the ratio of the current spacing to the desired spacing, and the difference between the leading and the following vehicles' velocities. Perceptive parameters such as desired acceleration, desired gap size, and comfortable deceleration are considered in this model (Kesting et al., 2010a; Treiber et al., 2000):

$$a_{IDM}^n(s_n, v_n, \Delta v_n) = \bar{a}_n\left[1 - \left(\frac{v_n}{v_0^n}\right)^{\delta_n} - \left(\frac{s^*(v_n, \Delta v_n)}{s_n}\right)^2\right], \tag{33a}$$

$$s^*(v_n, \Delta v_n) = s_0^n + T_n v_n + \frac{v_n \Delta v_n}{2\sqrt{\bar{a}_n \bar{b}_n}} \tag{33b}$$

where δ_n, T_n, $\bar{a}_n$, $\bar{b}_n$, s_0^n, and v_0^n denote free acceleration exponent, desired time gap, maximum acceleration, desired deceleration, jam distance, and desired speed, respectively. Note that the braking term in the IDM is designed to preclude crashes in the simulation. Talebpour and Mahmassani (2016) calculated the partial derivatives of Equation (24) for this model.

$$f_s^n = \frac{2\bar{a}}{s_e}\left(\frac{s_0 + Tv_e}{s_e}\right)^2, \tag{34}$$

$$f_{\Delta v}^n = -\frac{v_e}{s_e}\sqrt{\frac{\bar{a}}{\bar{b}}}\left(\frac{s_0 + Tv_e}{s_e}\right), \tag{35}$$

$$f_v^n = \frac{-\bar{a}\delta}{v_0}\left(\frac{v}{v_0}\right)^{\delta-1} - \sqrt{\frac{\bar{a}}{\bar{b}}}\left(\frac{s_0 + Tv_e}{s_e}\right)\left(\frac{2T\sqrt{\bar{a}\bar{b}}}{s_e}\right) \tag{36}$$

where v_e and s_e denote the equilibrium speed and spacing, respectively. Note that $\Delta v_e = 0$.

5.1.2.2. Inactive Vehicle-to-Vehicle Communications

When connected vehicles do not receive information through V2V communications, drivers' only source of information is their perception of surrounding traffic condition. Drivers' behavior in this case can be modeled similar to the case that vehicles have no communication capability (see Section 5.1.1 for more detail).

5.1.3. Modeling Autonomous Vehicles

Autonomous vehicles constantly monitor their surrounding traffic at a high frequency through their sensors. Therefore, they are certain about other drivers'/vehicles' behavior. Moreover, these vehicles reaction time is only limited by sensing and mechanical delays. Considering these factors, a deterministic acceleration modeling framework is suitable for modeling the movements of these vehicles. Following Talebpour and Mahmassani (2016), this study adopts the models of Van Arem et al. (2006) and Reece and Shafer (1993) to capture the movements of autonomous vehicles. Moreover, individual sensors are simulated in order to create the input data for the acceleration model. Siz SMS Automation Radars (UMRR-00 Type 30) with 90m±2.5% detection range and ±35 degrees horizontal FOV are simulated to provide a 360° coverage around the vehicle. Each sensor updates the sensing information every 50ms and can track up to 64 objects. Note that autonomous vehicles can only detect objects within their sensor range. Therefore, it is critical for these vehicles to be able to react to any unexpected event happening beyond their detection range. This is equivalent to assume that there is a vehicle at a complete stop right outside of the sensors detection range, which cannot be spotted by the sensors at the time of decision making. Moreover, their speed should be low enough to allow them to react to a leader's behavior. Considering the maximum possible deceleration for the autonomous vehicle and its leader, Talebpour and Mahmassani (2016) calculated the maximum safe speed as follows,

$$\Delta x_1 = \left(x_{i-1} - x_i - l_{i-1}\right) + v_i\tau + \frac{v_{i-1}^2}{2a_{i-1}^{decc}}, \tag{37}$$

$$\Delta x = \min\left(\Delta x_1, Sensor\,Detection\,Range\right), \tag{38}$$

$$v_{\max} = \sqrt{-2a_i^{decc}\Delta x} \tag{39}$$

where i and $i-1$ present the automated vehicle and its leader, respectively. x_i is the location of vehicle i, l_i is the length of vehicle i, v_i is the speed of vehicle i, τ is the reaction time of vehicle i, and a_i^{decc} is the maximum deceleration of vehicle i. Once the safety criteria is evaluated, autonomous vehicles car-following behaviour is captured based on the model by Van Arem et al. (2006),

$$a_i^d(t) = k_a a_{i-1}(t) + k_v(v_{i-1}(t) - v_i(t)) + k_d(s_i(t) - s_{ref}) \tag{40}$$

where a_i^d is the acceleration of vehicle i and k_a, k_v, and k_d are model parameters. s_i is the spacing and s_{ref} is the minimum between minimum distance ($s_{\min}$), following distance based on the reaction time (s_{system}), and safe following distance (s_{safe}). In this study, minimum distance is set at 2.0 m and s_{system} and s_{safe} are calculated as follows:

$$s_{safe} = \frac{v_{i-1}^2}{2}\left(\frac{1}{a_i^{decc}} - \frac{1}{a_{i-1}^{decc}}\right), \tag{41}$$

$$s_{system} = v_i \tau \tag{42}$$

Finally, the acceleration of the autonomous vehicle can be calculated using the following equation:

$$a_i(t) = \min\left(a_i^d(t), k(v_{\max} - v_i(t)\right) \tag{43}$$

where k is a parameter. In this study, based on the recommendation of Van Arem et al. (2006), $k = 1.0$, $k_a = 1.0$, $k_v = 0.58$, and $k_d = 0.1$. Talebpour and Mahmassani (2016) calculated the partial derivatives of Equation (24) for this model. Note that they assumed that the acceleration of the leader is zero during the estimation time. This assumption simplifies the calculation of partial derivatives.

$$f_s = k_d, \tag{44}$$

$$f_{\Delta v} = k_v, \tag{45}$$

$$f_v = -k_d \tau \tag{46}$$

5.2. Results and Discussion

This section investigates the linear string stability of a platoon of regular, connected, and autonomous vehicles at different market penetration rates. The key factor in this investigation is the consideration of the effects of communication on the string stability of traffic flow. Note that the string stability investigation utilizes the analytical formulation presented earlier which built on the research performed by Talebpour and Mahmassani (2016); accordingly, this is not a simulation-based investigation.

Based on the findings of the previous section from Percolation theory, any combination of connected vehicles density and effective communication range that results in $A_c(R) \geq 0.785$ ensures full communication. When $A_c(R) < 0.785$, this study uses the Poisson distribution to model the communication between vehicles. Note that connected vehicles without information are modeled as regular driven vehicles. In the remainder of this section, the effective market penetration rate of connected vehicles (*i.e.,* the proportion of connected vehicles with active communication) is determined based on Figure 7. To magnify the effects of connectivity and automation on stability of traffic flow, parameters of regular vehicles are adjusted to create a very unstable traffic flow in a platoon of regular vehicles. Tables 1 and 2 present the model parameters and their respective values used in this section for connected and regular vehicles, respectively.

Figure 8 shows the results of sensitivity analyses for different values of effective communication range. Figure 8.a shows the stability results with the full communication assumption. This figure shows the effects of different market penetration rates of connected and autonomous vehicles on string stability of traffic flow. The colors represent the speed above which the magnitude of any perturbation will grow as the shockwave propagates upstream in traffic (*i.e.,* string unstable traffic flow). Note that at low market penetration rates of autonomous vehicles, stability increases as market penetration rate of autonomous vehicles increases. Moreover, at low market penetration rates of connected vehicles, system stability does not improve significantly until the market penetration rate of autonomous vehicles reaches 70%. Note that Talebpour and Mahmassani (2016) conducted a sensitivity analysis on model parameters and reported similar findings.

Table 1. Acceleration model parameters and their typical values for car-following model of Hamdar et al. (2015).

Parameters	Typical Value
Sensitivity Exponents of the Generalized Utility	$\gamma = 0.2$
Asymmetry Factor for Negative Utilities	$w_m = 2.0$
Velocity Uncertainty Variation Coefficient	$\alpha = 0.08$
Weighing Factor for Accidents	$w_c = 10000.0$
Maximum Anticipation Time Horizon	$\tau_{\max} = 4.0s$
Logit Uncertainty Parameter (Intra-Driver Variability)	$\beta = 5.0$
Maximum Acceleration	$a_{\max} = 4m/s^2$
Minimum Acceleration	$a_{\min} = -8m/s^2$

Table 2. Acceleration model parameters and their typical values for IDM (Kesting et al., 2010a).

Parameters	Typical Value
Free Acceleration Exponent	$\delta = 4.0$
Desired Time Gap	$T = 4.5s$
Jam Distance	$s_0 = 2.0m$
Maximum Acceleration	$\overline{a} = 1.4m/s^2$
Desired Deceleration	$\overline{b} = -2.0m/s^2$

Figures 8.b through 8.f show the effects of communication range and partial connectivity on the string stability of traffic flow. Comparing Figure 8.a with Figures 8.b through 8.f, as effective communication range increases, stability increases (instabilities are observed at higher speeds). Note that when effective communication range reaches 130m, the system performs similar to the full connectivity assumption. The improvement in the stability, however, is insignificant for communication ranges above 90m. Overall, the effects are minor with the typical set of parameter values for connected vehicles.

Figure 9 shows that a slight increase in connected vehicles acceleration domain ($\overline{a} = 3.0m/s^2$ and $\overline{b} = -4.0m/s^2$), magnifies the effects of communication range on string stability of traffic flow and better indicates the importance of considering connectivity in studying the effects of connectivity and automation on string stability. Figure 9.b through 9.f presents the effects of communication range on string stability of traffic flow. This figure indicates that at any market penetration rate of automated vehicles, as communication range increases, the system becomes more stable (maximum speed to ensure string stability increases as communication range increases for any combination of penetration rates of connected and automated vehicles). For instance, with no automated vehicles on the roadway (market penetration rate = 0), when the communication range is 50m, the critical speed threshold changes from 4.5 to 6.0m/s at 55% connected vehicles. This threshold changes at 50% and 40% at 70m and 90m communication ranges, respectively.

Moreover, these figures indicate that the effects of communication range on string stability are insignificant at high market penetration rates of connected vehicles. This is mainly due to the high density of connected vehicles (see Figures 5 and 7 for more details on the combined effects of density and communication range on critical density). In contrast, at low market penetration rates of connected vehicles, string stability improves as communication range increases. Note that at low communication ranges, the effects are significant even at high market penetration rates of autonomous vehicles. Moreover, Figures 8 and 9 indicate that there is an optimal communication range to ensure string stability and prevent signal interference and communication loss. For the selected model parameter values, this range is 70m. However, this finding is exploratory and accurate estimation of the optimal communication range requires actual observations of real-world implementation of these systems.

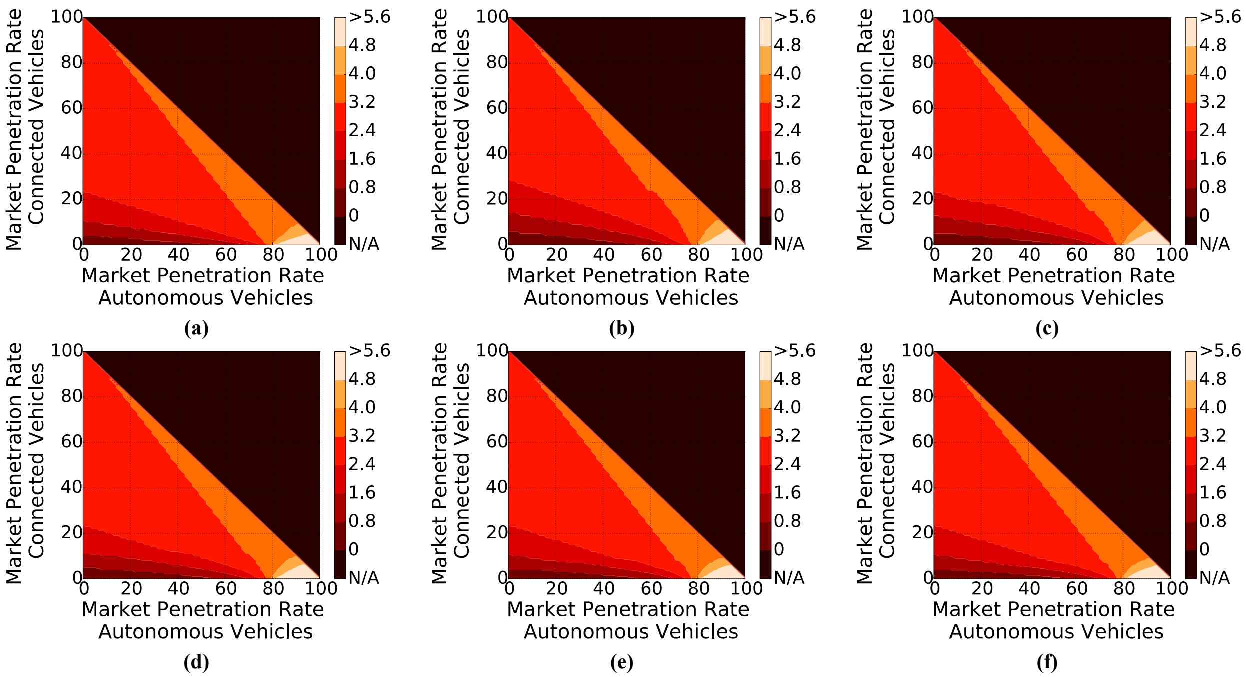

Fig. 8. Critical Speed (m/s) at different market penetration rates of connected and autonomous vehicles for a platoon of regular, connected, and autonomous vehicles with infinite length (any speed value above the critical speed results in an unstable traffic flow; model parameters are based on Tables 1 and 2): (a) full connectivity assumption, (b) 50m communication range, (c) 70m communication range, (d) 90m communication range, (e) 110m communication range, and (f) 130m communication range

6. Conclusion

Connectivity and automation are expected to influence string stability in a mixed driving environment with regular, connected, and autonomous vehicles. Most previous studies related to string stability of connected vehicle systems have assumed full connectivity in their analyses. This paper adopts the Continuum Percolation theory to determine the connectivity level at different communication ranges and uses the results to explore the string stability under different communication ranges and market penetration rates of connected and autonomous vehicles.

By employing the Continuum Percolation theory, this study calculates the critical density of connected vehicles to ensure percolation in vehicular ad-hoc networks. Moreover, this study determines the relationship between critical density and communication range. Utilizing these findings and adopting the modeling framework of Talebpour and Mahmassani (2016), this paper offers an evaluation of string stability under different communication ranges and market penetration rates of connected and autonomous vehicles. The analytical studies reveal that as communication range increases, the system becomes more stable. When the communication range reaches 130m, the system performs very similar to the system with full connectivity assumption. Note that several models could be potentially used to model regular, connected, and automated vehicles. The model selection in this paper was based on the current state-of- the-art in modeling these vehicles. Therefore, the results presented in this paper are illustrative and exploratory, and future evaluation of the effects of these technologies on transportation systems requires actual observations of real-world implementations of these systems. Such test deployments coupled with detailed measurement are essential to further advance the state of the art in this domain.

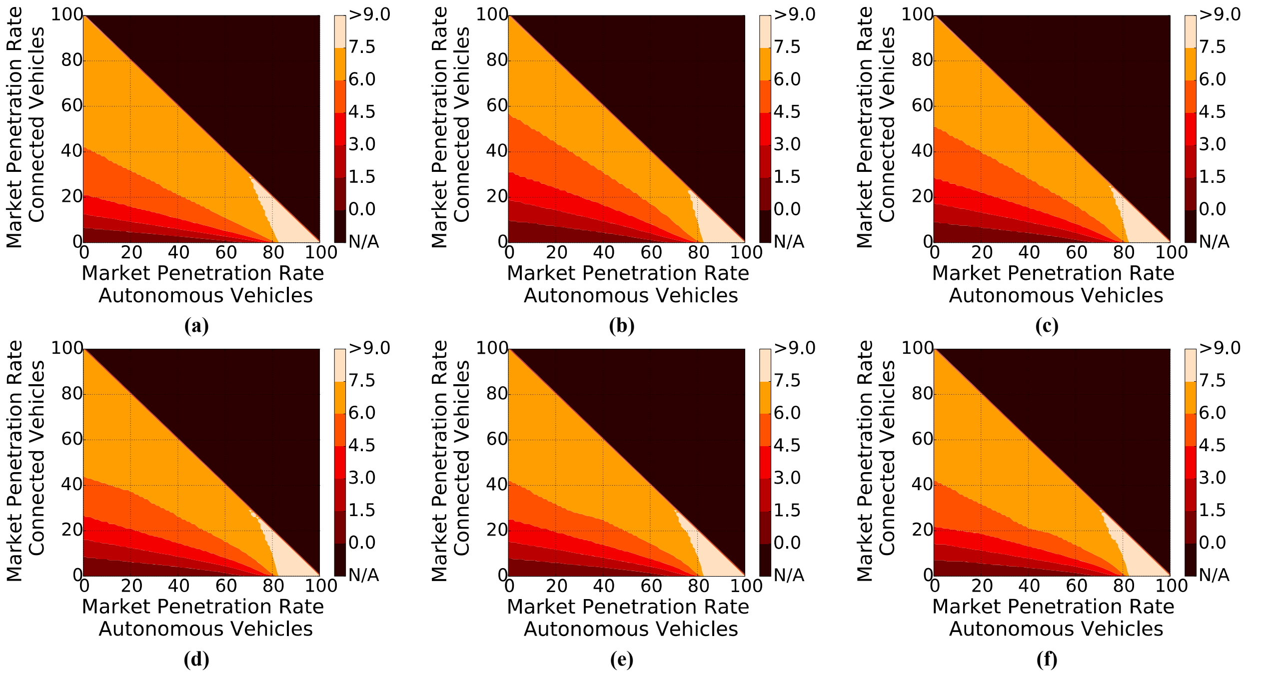

Fig. 9. Critical Speed (m/s) at different market penetration rates of connected and autonomous vehicles for a platoon of regular, connected, and autonomous vehicles with infinite length (any speed value above the critical speed results in an unstable traffic flow; model parameters are adjusted to create more heterogeneous connected vehicles $\overline{a} = 3.0 m/s^2$ and $\overline{b} = -4.0 m/s^2$): (a) full connectivity assumption, (b) 50m communication range, (c) 70m communication range, (d) 90m communication range, (e) 110m communication range, and (f) 130m communication range

7. References

Almiron, M.G., Goussevskaia, O., Loureiro, A.A.F., Rolim, J., 2013. Connectivity in obstructed wireless networks: from geometry to percolation, Proceedings of the fourteenth ACM international symposium on Mobile ad hoc networking and computing. ACM, Bangalore, India, pp. 157-166.

Ammari, H.M., Das, S.K., 2008. Integrated Coverage and Connectivity in Wireless Sensor Networks: A Two-Dimensional Percolation Problem. Computers, IEEE Transactions on 57(10), 1423-1434.

Bose, A., Ioannou, P.A., 2003. Analysis of traffic flow with mixed manual and semiautomated vehicles. Intelligent Transportation Systems, IEEE Transactions on 4(4), 173-188.

Broadbent, S.R., Hammersley, J.M., 1957. Percolation processes. Mathematical Proceedings of the Cambridge Philosophical Society 53(03), 629-641.

Federal Highway Administration, 2006. Next Generation Simulation: US-101 Highway Dataset.

Glauche, I., Krause, W., Sollacher, R., Greiner, M., 2003. Continuum percolation of wireless ad hoc communication networks. Physica A: Statistical Mechanics and its Applications 325(3–4), 577-600.

Hamdar, S., Treiber, M., Mahmassani, H., Kesting, A., 2008. Modeling Driver Behavior as Sequential Risk-Taking Task. Transportation Research Record: Journal of the Transportation Research Board 2088(-1), 208-217.

Hamdar, S.H., Mahmassani, H.S., Treiber, M., 2015. From behavioral psychology to acceleration modeling: Calibration, validation, and exploration of drivers' cognitive and safety parameters in a risk-taking environment. Transportation Research Part B: Methodological 78, 32-53.

Ioannou, P., Xu, Z., 1994. Throttle And Brake Control Systems For Automatic Vehicle Following. I V H S Journal 1(4), 345-377.

Kahneman, D., Tversky, A., 1979. Prospect Theory: An Analysis of Decision under Risk. Econometrica 47(2), 263-291.

Kesting, A., Treiber, M., Helbing, D., 2010a. Enhanced intelligent driver model to access the impact of driving strategies on traffic capacity. Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences 368(1928), 4585-4605.

Kesting, A., Treiber, M., Helbing, D., 2010b. Enhanced intelligent driver model to access the impact of driving strategies on traffic capacity. Philosophical Transactions of the Royal Society of London A: Mathematical, Physical, and Engineering Sciences 368(1928), 4585-4605.

Khanjary, M., Sabaei, M., Reza Meybodi, M., 2015. Critical density for coverage and connectivity in two-dimensional fixed-orientation directional sensor networks using continuum percolation. Journal of Network and Computer Applications 57, 169-181.

Mahmassani, H.S., 2016. Autonomous Vehicles and Connected Vehicle Systems: Flow and Operations Considerations. Transportation Science 50(4), 1140-1162.

Meester, R., Roy, R., 1996. Continuum Percolation. Cambridge University Press.

Naus, G.J.L., Vugts, R.P.A., Ploeg, J., van de Molengraft, M.J.G., Steinbuch, M., 2010. String-Stable CACC Design and Experimental Validation: A Frequency-Domain Approach. Vehicular Technology, IEEE Transactions on 59(9), 4268-4279.

ns-3.25, ns-3, http://www.nsnam.org/.

Pinto, P.C., Win, M.Z., 2012. Percolation and Connectivity in the Intrinsically Secure Communications Graph. IEEE Transactions on Information Theory 58(3), 1716-1730.

Ploeg, J., Semsar-Kazerooni, E., Lijster, G., Wouw, N.v.d., Nijmeijer, H., 2013. Graceful degradation of CACC performance subject to unreliable wireless communication, 16th International IEEE Conference on Intelligent Transportation Systems (ITSC 2013), pp. 1210-1216.

Reece, D.A., Shafer, S.A., 1993. A computational model of driving for autonomous vehicles. Transportation Research Part A: Policy and Practice 27(1), 23-50.

Schakel, W.J., Arem, B.v., Netten, B.D., 2010. Effects of Cooperative Adaptive Cruise Control on traffic flow stability, Intelligent Transportation Systems (ITSC), 2010 13th International IEEE Conference on, pp. 759-764.

Shaw, E., Hedrick, J.K., 2007. String Stability Analysis for Heterogeneous Vehicle Strings, 2007 American Control Conference, pp. 3118-3125.

Swaroop, D.V.A.H.G., 1997. String Stability of Interconnected Systems: An Application to Platooning in Automated Highway Systems. University of California Berkeley, Institure of Transportation Studies, p. 120.

Talebpour, A., 2015. Modeling Driver Behavior in a Connected Environment: An Integration of Microscopic Traffic Simulation and Telecommunication Systems, Ph.D. Dissertation, Civil and Environmental Engineering. Northwestern University, Evanston, Illinois, p. 161.

Talebpour, A., Mahmassani, H.S., 2016. Influence of connected and autonomous vehicles on traffic flow stability and throughput. Transportation Research Part C: Emerging Technologies 71, 143-163.

Talebpour, A., Mahmassani, H.S., Bustamante, F.E., 2016. Modeling Driver Behavior in a Connected Environment: Integrated Microscopic Simulation of Traffic and Mobile Wireless Telecommunication Systems, Transportation Research Record: Journal of the Transportation Research Board 2560, 75-86.

Treiber, M., Hennecke, A., Helbing, D., 2000. Congested traffic states in empirical observations and microscopic simulations. Physical Review E 62(2), 1805-1824.

Treiber, M., Kesting, A., 2013. Traffic Flow Dynamics: Data, Models and Simulation. Springer.

Van Arem, B., van Driel, C.J.G., Visser, R., 2006. The Impact of Cooperative Adaptive Cruise Control on Traffic-Flow Characteristics. Intelligent Transportation Systems, IEEE Transactions on 7(4), 429-436.

Wang, M., Treiber, M., Daamen, W., Hoogendoorn, S.P., van Arem, B., 2013. Modelling supported driving as an optimal control cycle: Framework and model characteristics. Transportation Research Part C: Emerging Technologies 36(0), 547-563.

Ward, J.A., 2009. Heterogeneity, Lane-Changing and Instability in Traffic: A Mathematical Approach, Department of Engineering Mathematics. University of Bristol, Bristol, United Kingdom, p. 126.

Wilson, R.E., Ward, J.A., 2010. Car-following models: fifty years of linear stability analysis – a mathematical perspective. Transportation Planning and Technology 34(1), 3-18.