Problem Set 1: Set Theory, Calculus, and Limits

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Summer, 2019

For all problems, ensure that you show your work. Ensure you use the definitions, and present an argument, if needed, to prove your answer is correct. For example, if I ask you to find a relation that maps the even numbers in $A = \{1,2,3,4,5,6,7,8\}$ to it's subsequent odd number in $B = \{1,2,3,4,5,6,7\}$, your answer would look like this:

We know the relation is defined with the first element being an even number in A and the second element being the next odd number in B. Let $E = \{x|x \in A \text{ and } x \text{ is even}\}$ and let $D = \{y|y \in B \text{ and } y \text{ is odd}\}$. Then $E = \{2,4,6\}$ and $D = \{1,3,5,7\}$. So, the relation that maps the even to the next odd in these sets would be $R = \{(x,y)|x \in E \land y \in D \land y = x+1\} = \{(2,3),(4,5),(6,7)\}$.

1. What set would represent all points, (x,y), on a standard 2-dimensional plane? Explain why.

2. Using Demorgan's Laws and the Laws of Sets (Theorems 1 - 5 in the notes), simplify the following expression:

$$C \cap ((A \cup B)^c \cap (B \cup A))^c$$

3. Let A be a countable set. Show that for subset $B \subset A$, B is also a countable set.

4. Show that the set $[5,10]$ has the same cardinality as $\mathbb{R}$

5. Suppose an investor would like to turn his initial investment of $100 into $10,000 in 365 days of trading on the stock market. If we assume that the investor earns the same percentage interest rate every single day on the market, what would this rate be? Make sure to show your entire derivation.

6. If $f(x) = x^2 - 2x + 1$ and $g(x) = \frac{1}{x}$, and $h(x) = e^{-2x+5}$, then what is the function $h(g(f(x)))$? Again, make sure to show your entire derivation.

7. Let $f(x) = -x^3 + x^2 + 5x$. Using the limit approach, select 10 values of x close to and to the left of $\frac{5}{3}$, compute an approximation to the derivative at $\frac{5}{3}$, and explain why the maximum of $f(x)$ for $x > 0$ is at the point $x = \frac{5}{3}$

8. Suppose a truck leaves a facility and within 60 minutes continuously picks up products and subsequently delivers them. Suppose for the first 30 minutes that the truck continuously picks up products, while the last 30 minutes the truck continuously drops off products.

It costs the company \$0.20 per minute to hold a single product in the truck. The number of items in the truck (decimals are allowed) at time t is found using the equation $f(t) = -\frac{1}{10}t^2 + 6t$. Use the rectangle approach to approximate the total cost incurred by the company for holding the product over the time from $t \in [0, 60]$.

9. Using the limit laws, compute the limit $\lim\limits_{x \to 3} \frac{x^2 + 3x + 2}{x^2 - 2x - 3}$. If this limit does not exist, please explain why.

10. Using the limit laws, compute the limit $\lim\limits_{x \to \infty} \frac{x^3 + 3x^2 - x + 3}{2x^3 - 5x + 2}$. If the limit does not exist, please explain why.