Performance Evaluation Method - Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)

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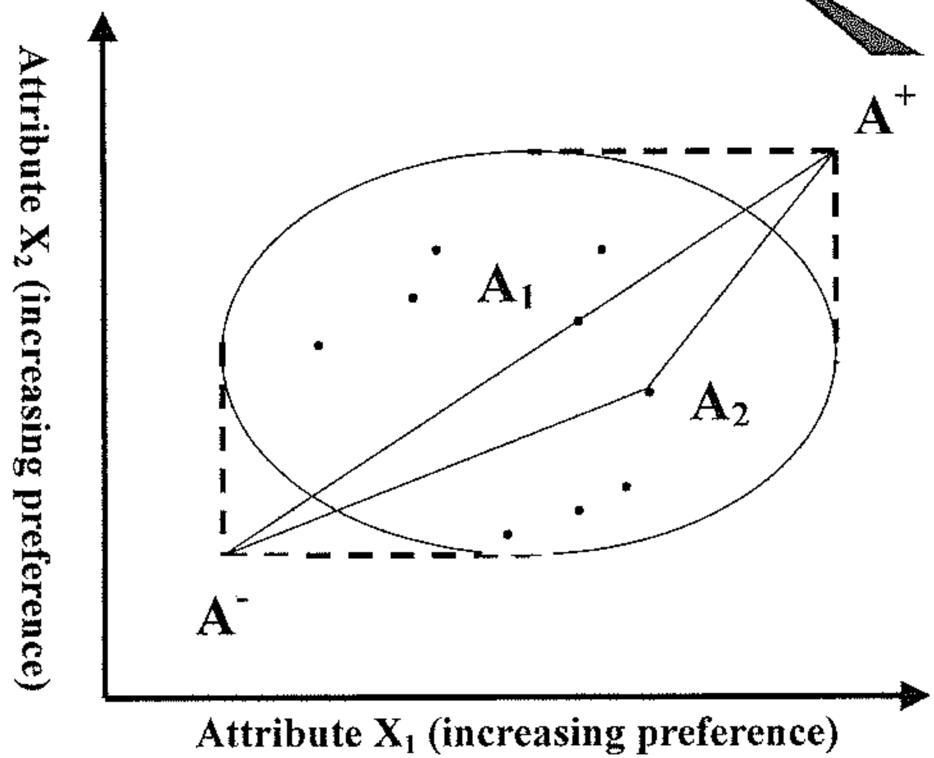
Origin and History

- 1980: development by Kwangsun Yoon and Hwang Ching-Lai
  - Yoon, K., "System Selection by Multiple Attribute Decision Making," Ph. D. Dissertation, Kansas State University, Manhattan, Kansas, 1980.
  - Yoon, K. and C. L. Hwang, "TOPSIS (Technique for Order Preference by Similarity to Ideal Solution)- A Multiple Attribute Decision Making," a paper to be published, 1980.

Basic Concept-1

- the chosen alternative should have the shortest distance from the ideal solution and the farthest from the negative-ideal solution

Basic Concept-2



It is very difficult to justify the selection of A_1

Decision Matrix

- m alternative, n attributes (or criteria)

$$
D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ \cdot \\ \cdot \\ \cdot \\ A_m \end{array}
\begin{array}{c} \begin{array}{ccccccc} x_1 & x_2 & x_3 & \cdot & \cdot & \cdot & x_n \end{array} \\
\begin{bmatrix}
x_{11} & x_{12} & x_{13} & \cdot & \cdot & \cdot & x_{1n} \\
x_{21} & x_{22} & x_{23} & \cdot & \cdot & \cdot & x_{2n} \\
x_{31} & x_{32} & x_{33} & \cdot & \cdot & \cdot & x_{3n} \\
\cdot & \cdot & \cdot & & & & \cdot \\
\cdot & \cdot & \cdot & & & & \cdot \\
\cdot & \cdot & \cdot & & & & \cdot \\
x_{m1} & x_{m2} & x_{m3} & \cdot & \cdot & \cdot & x_{mn}
\end{bmatrix} \end{array}
$$

choices

attributes

m choices by n attributes

Hypothesis-1

- each attribute in the decision matrix takes either monotonically increasing or monotonically decreasing utility

Hypothesis-2

- a set of weights for the attributes is required

Hypothesis-3

- any outcome which is expressed in a non-numerical way should be quantified through the appropriate scaling technique

Steps-1

- Construct the normalized decision matrix
  - to transform the various attribute dimensions into non-dimensional attributes, which allows comparison across the attributes

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^{2}}}$$

Steps-2

- Construct the weighted normalized decision matrix

$$V = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1j} & \cdots & v_{1n} \\ \vdots & \vdots & & \vdots & & \vdots \\ v_{i1} & v_{i2} & & v_{ij} & & v_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ v_{m1} & v_{m2} & \cdots & v_{mj} & \cdots & v_{mn} \end{bmatrix} = \begin{bmatrix} w_1 r_{11} & w_2 r_{12} & \cdots & w_j r_{ij} & \cdots & w_n r_{1n} \\ \vdots & \vdots & & \vdots & & \vdots \\ w_1 r_{i1} & w_2 r_{i2} & & w_j r_{ij} & & w_n r_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ w_1 r_{m1} & w_2 r_{m2} & \cdots & w_j r_{mj} & \cdots & w_n r_{mn} \end{bmatrix}$$

Steps-3

- **Determine ideal and negative-ideal solutions**

$$A^{+} = \{(\max_i v_{ij} \mid j \in J), (\min_i v_{ij} \mid j \in J') \mid i = 1,2,\ldots m\}$$
$$= \{v_1^{+}, v_2^{+}, \ldots, v_j^{+}, \ldots, v_n^{+}\}$$

$$A^{-} = \{(\min_i v_{ij} \mid j \in J), (\max_i v_{ij} \mid j \in J') \mid i = 1,2,\ldots m\}$$
$$= \{v_1^{-}, v_2^{-}, \ldots, v_j^{-}, \ldots, v_n^{-}\}$$

$$\textit{where} \quad J = \left\{ j = 1,2,\ldots,n \mid j \quad \textit{associated} \quad \textit{with} \quad \textit{benefit} \quad \textit{criteria} \right\}$$
$$J' = \left\{ j = 1,2,\ldots,n \mid j \quad \textit{associated} \quad \textit{with} \quad \cos t \quad \textit{criteria} \right\}$$

Steps-4

- **Calculate the separation measure**
  - **ideal separation**

$$s_i^+ = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^+\right)^2} \qquad i = 1,2,\ldots,m$$

  - **negative-ideal separation**

$$s_i^- = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^-\right)^2} \qquad i = 1,2,\ldots,m$$

Steps-5

- Calculate the relative closeness to the ideal solution

$$c_i^* = \frac{s_i^-}{(s_i^+ + s_i^-)}, \quad 0 < c_i^+ < 1, \quad i = 1,2,\dots,m$$

$$c_i^* = 1 \quad if \quad A_i = A^+$$

$$c_i^* = 0 \quad if \quad A_i = A^-$$

Steps-6

- Rank the preference order
  - A set of alternatives can now be preference ranked according to the descending order of c_i^*

Numerical Example

- **decision matrix**

$$
D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}
\begin{array}{c}
\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\
\begin{bmatrix}
2.0 & 1500 & 20000 & 5.5 & 5 & 9 \\
2.5 & 2700 & 18000 & 6.5 & 3 & 5 \\
1.8 & 2000 & 21000 & 4.5 & 7 & 7 \\
2.2 & 1800 & 20000 & 5.0 & 5 & 5
\end{bmatrix}
\end{array}
$$

Numerical Example

- **normalized decision matrix**

$$
\begin{array}{cc}
 & \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\
R = & \begin{bmatrix}
0.4761 & 0.3662 & 0.5056 & 0.5063 & 0.4811 & 0.6708 \\
0.5839 & 0.6591 & 0.4550 & 0.5983 & 0.2887 & 0.3727 \\
0.4204 & 0.4882 & 0.5308 & 0.4143 & 0.6736 & 0.5217 \\
0.5139 & 0.4392 & 0.5056 & 0.4603 & 0.4811 & 0.3727
\end{bmatrix}
\end{array}
$$

.20 .11 .10 .14 .20 .30

w .20 .10 .10 .10 .20 .30

Numerical Example

- **weighted decision matrix**

$$
\begin{array}{cc}
 & \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\
V = & \begin{bmatrix} 0.0934 & 0.0366 & 0.0506 & 0.0506 & 0.0962 & 0.2012 \\ 0.1168 & 0.0659 & 0.0455 & 0.0598 & 0.0577 & 0.1118 \\ 0.0841 & 0.0488 & 0.0531 & 0.0414 & 0.1347 & 0.1565 \\ 0.1028 & 0.0439 & 0.0506 & 0.0460 & 0.0962 & 0.1118 \end{bmatrix}
\end{array}
$$

Numerical Example

- **the ideal and negative-ideal solutions**

$$A^{+} = (0.1168, 0.0659, 0.0531, 0.0414, 0.1347, 0.2012)$$

$$A^{-} = (0.0841, 0.0366, 0.0455, 0.0598, 0.0577, 0.1118)$$

Numerical Example

- **separation measures**

$$S_i^{+} = \sqrt{\sum_{j=1}^{6} \left(v_{ij} - v_j^{+}\right)^2} \qquad i = 1, 2, 3, 4$$

$$S_1^{+} = 0.0545 \quad ; \quad S_2^{+} = 0.1197$$

$$S_3^{+} = 0.0580 \quad ; \quad S_4^{+} = 0.1009$$

$$S_i^{-} = \sqrt{\sum_{j=1}^{6} \left(v_{ij} - v_j^{-}\right)^2} \qquad i = 1, 2, 3, 4$$

$$S_1^{-} = 0.0983 \quad ; \quad S_2^{-} = 0.0439$$

$$S_3^{-} = 0.0920 \quad ; \quad S_4^{-} = 0.0458$$

Numerical Example

- **the relative closeness to the ideal solution**

$$c_1^* = \frac{s_1^-}{(s_1^+ + s_1^-)} = 0.643$$

$$c_2^* = 0.268 \quad ; \quad c_3^* = 0.613 \quad ; \quad c_4^* = 0.312$$

- **rank the preference order**

$$A_1 \succ A_3 \succ A_4 \succ A_2$$