

CSC 2259 2/20/20

$H=W$
 for (int i: 0; i < n; i++)
 { if ... return (false);
 return (true);
 } $H[i] = W[i]$

m iterations
 $1 \leq m \leq n$
 n iterations true $H[i] = W[i]$
 $\#((H, W) \text{ pairs}) = 2^n$

spiel about # (cases)
 spiel about average

geometric series

$$1 + x + x^2 + \dots + x^n = \frac{1 - x^{n+1}}{1 - x} = S$$

$$xS = x + x^2 + x^3 + \dots + x^{n+1}$$

$$xS - S = x^{n+1} - 1$$

$$S(x-1) = x^{n+1} - 1$$

$$S = \frac{x^{n+1} - 1}{x - 1} = \frac{1 - x^{n+1}}{1 - x}$$

$$S = 1 + 2x + 3x^2 + \dots + nx^{n-1}$$

$$xS = x + 2x^2 + 3x^3 + \dots + nx^n$$

$$S - xS = 1 + x + x^2 + \dots + x^{n-1} - nx^n$$

$$S(1-x) = \frac{1-x^n}{1-x} - nx^n$$

$$S = \frac{1-x^n}{(1-x)^2} - n \cdot \frac{x^n}{1-x}$$

$$1 \cdot 2 \cdot 4^{n-1} + 2 \cdot 2^2 \cdot 4^{n-2} + \dots + n \cdot 2^n + n \cdot 2^n$$

$$2 \cdot 4^{n-1} \left[1 + 2 \cdot \frac{2}{4} + 3 \cdot \frac{2^2}{4^2} + \dots + n \cdot \frac{2^{n-1}}{4^{n-1}} \right] = 2 \cdot 4^{n-1}$$

$$x = \frac{1}{2}$$

$$2 \cdot 4^{n-1} \left[4 \left(1 - \left(\frac{1}{2} \right)^n \right) - n \left(\frac{1}{2} \right)^n \right]$$

$$2 \left(1 - \left(\frac{1}{2} \right)^n \right)$$

$$\frac{2(2^{n-1})}{4^{n-1}} = 2\left(\frac{1}{2}\right)^{n-1}$$

$$\frac{2^n}{4^{n-1}} = 2 \cdot \left(\frac{1}{2}\right)^{n-1} \quad \left(\frac{1}{2}\right)^{-1} \left(\frac{1}{2}\right)^{n-1}$$

$$= \left(\frac{1}{2}\right)^{n-2} \cdot \frac{1}{2^{n-2}}$$

$$\frac{2^n}{2^{2n-2}}$$

$$\frac{2^n}{2^{2n} \cdot 2^{-1}} = \left(\frac{1}{2}\right)^n \cdot 2^{-2}$$

$$\frac{2 \cdot 2^{n-1}}{4^{n-1}} = 2\left(\frac{1}{2}\right)^{n-1} \quad 2\left(\frac{1}{2}\right)^{n-1}$$

$$\frac{2^n}{4^{n-1}} = \frac{2^n}{2^{2n-2}} = \frac{2^n}{2^{2n} \cdot 2^{-2}}$$

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