MATHEMATICS 2090
Practice Test I

1. Find a solution to the initial value problem $y' = \dfrac{x}{y^2}$, $y(0) = 1$.
2. Find the general solution to the differential equation $xy' + 4y = \dfrac{\ln x}{x^5}, \quad x > 0$.
3. Solve the differential equation $(y^2 - 2x)dx + 2xydy = 0$.
4. Show that $u(x) = x^{-1}$ is an integrating factor for the equation $(xy - 1)dx + x^2dy = 0$, and then find its general solution.
5. If $A = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & 2 \end{pmatrix}, B = \begin{pmatrix} 2 & -1 & 0 \\ 0 & 1 & -1 \\ -2 & 0 & 1 \end{pmatrix}$. Please compute the following if possible.
   a). AB b). BA^T c). A^TB d). $tr(B)$
6. Use elementary row operations to reduce the following matrix to a reduced row echelon form, and find its rank.

$$\begin{pmatrix} 2 & 3 & 1 \\ 1 & 0 & 2 \\ 0 & 3 & -1 \end{pmatrix}$$