Solution Long Quiz #4.2 (30-Apr): CSC-2259: Discrete Structures, Sp 2020
Your answers must be to the point. Total = 50; marks for each question is shown in [ ].

LastName: FirstName

1. Fill the table below when X = #(H in 3 tosses of a coin) and p = Prob(H) in a toss. [8]

Value i of X	Associated sample points	Prob($X = i$)
0	TTT	q^3
1	HTT, THT, TTH	$3pq^2$
2	HHT, HTH, THH	$3p^2q$
3	HHH	p^3

Verify that the sum of probabilities above equals 1; show details. [2]

$q^3 + 3pq^2 + 3p^2q + q^3 = (p+q)^3 = 1$

For what values of p, Prob($X = 2$) will be the larger than the probability of other values? [2]

$2/4 < p < 3/4$

2. Give all details of the computation of $E(X)$ when X has a Binomial probability distribution (for general $n \geq 1$). [5]

$$\begin{aligned} E(X) &= \sum_{0\leq i\leq n} i.\,C(n,\,i)p^i q^{n-i} &&= \sum_{1\leq i\leq n} i.\,C(n,\,i)p^i q^{n-i} \\ &= np.\sum_{1\leq i\leq n} C(n-1,\,i-1)p^{i-1}q^{n-i} &&= np.\sum_{0\leq j\leq n-1} C(n-1,\,j)p^j q^{(n-1)-j} \\ &= np.\,(p+q)^{n-1} = np. \end{aligned}$$

3. Consider 3 tosses of a coin with Prob(H) = 2/3. If every H gives a gain of 2 and every T gives a loss of 1, i.e., gain of −1, what would be the expected net gain? Show your computations by filling the table below. [9]

#(H)	Probability	Total net gain	Contribution to Expected net gain	Expected net gain
0	1/27	−3	−3/27	
1	6/27	0	0	3
2	12/27	3	36/27	
3	8/27	6	48/27	

4. Complete the sentence below and give an example of "the things" to justify the statement. [2+2]

Probability Theory finds the things that are certain even in presence of uncertainties. The sum-rule or complement-rule.

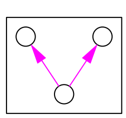
5. Assune R to be the $n\times n$ relation-matix of a partial order. Give an efficient way to determine whether R gives a linear order or not. Also, an efficient way to determine from R all maximal items in the partial order. [4+4]

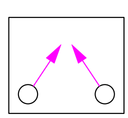
(a) Determine #(1) in R and if that equals $n(n+1)/2$ then R is a linear order; otherwise, it is not a linear order.

*** Another possible answer: if for any $i,\ j$ we have $R[i][j] + R[j][i] = 0$ then it is not a linear order.

(b) Determine for each row #(1); if it equals 1 then the corresponding item is a maximal item.

6. Give the structure of Hasse-diagrams of all parital orders on 3 items that are NOT linear orders. Also, give #(parital orders) for each structure. [4+4]


#(partial orders) = 3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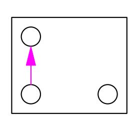
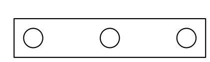

#(partial orders) = 6


#(partial orders) = 1

7. When do we say a relation is anti-symmetric? [2]

For each $x \neq y$, at most one of $(x,\ y)$ and $(y,\ x)$ can be in R, i.e., if $(x,\ y) \in R$ then $(y,\ x) \notin R$.

8. In what way an equivalence relation differs from a partial order? [2]

An equivalence relation is symmetric and a partial order is anti-symmetric.