MATHEMATICS 2090 Section 1
Practice 3

1. Find all eigenvalues and eigenvectors of $A = \begin{pmatrix} 2 & 0 & 2 \\ 1 & 2 & 2 \\ 0 & 0 & 1 \end{pmatrix}$ and determine whether A is defective.
2. Solve the initial value problem $y'' - 8y' + 16y = 0$, $y(0) = 2, y'(0) = 7$.
3. Find the general solution to the differential equation $(D^2 - D - 2)y = 10 \sin x$.
4. Find the general solution for $y'' + 4y' + 4y = e^{-2x} \ln x$.
5. Find the general solution of the system of differential equations: $\mathbf{x}' = A \cdot \mathbf{x}$ where $A = \begin{pmatrix} 5 & -3 & -3 \\ 12 & -10 & -12 \\ -6 & 6 & 8 \end{pmatrix}$.
6. Find the general solution for $\mathbf{x}'(t) = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \mathbf{x}(t) + \begin{pmatrix} 0 \\ 4e^t \end{pmatrix}$.

1) Eigenvalues $\lambda = 1, 2$

Eigenvectors for $\lambda = 1$: $\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

Eigenvectors for $\lambda = 2$: $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\therefore$ A is defective

2) $y = 2e^{4x} - xe^{4x}$

4) $y = C_1 e^{-2x} + C_2 e^{-2x} + e^{-2x} x^2 \left(\frac{1}{2} \ln x - \frac{3}{4}\right)$