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MidTerm(05-Mar): CSC-2259: Discrete Structures, Sp 2020
Your answers must be to the point. Total = 20; marks for each question is shown in [ ].

LastName: Le ; FirstName Kha

1. Complete the sentence below. [2+2+2]
   We study Discrete Structure so that:
   (a) we can determine efficiency of a program by ..counting.......... #(tests and assignments) in the program's executions for different inputs, and
   (b) we can write better, i.e., clean and ..more.. ....efficient....... programs.

2. Put a circle around the best (most efficient and most logical) code among the ones below. [2]

```
max = x;                    max = x;                     max = x;
if (x < y) max = y;         if (x <= y) max = y;         if (max < y) max = y;
```

   Give the best code to compute max of x and y. [2]

```
if (x<y) max = y;
else max=x;
```

3. Give the details to show $S = 1 + 2x + 3x^2 + \cdots + nx^{n-1} = (1 - x^n)/(1 - x)^2 - nx^n/(1 - x)$, when $x \neq 1$. [8]

$$xS = x + 2x^2 + 3x^3 + \cdots + nx^n$$
$$S - xS = 1 + x + x^2 + x^3 + \cdots + x^{n-1} - nx^n$$
$$S(1-x) = \frac{1-x^n}{1-x} - nx^n$$
$$S = \frac{1-x^n}{(1-x)^2} - \frac{nx^n}{1-x}$$

4. Consider the code below to test $H = W$, where H and W are binary arrays of length n.

```
for (int i = 0; i < n; i++)
    if (H[i] != W[i]) return(false);
return(true);
```

   Complete the following sentences. [2+2+2+2]
   (a) #((H, W)-pairs giving true-return) = ...2^n....
   (b) #((H, W)-pairs giving false-return) = ...$4^n - 2^n$....
   (c) #((H, W)-pairs that give false-return value in mth iteration, $1 \leq m \leq n$) = ...$2^m \cdot 4^{n-m}$....
   (d) The average #(iterations for all (H, W)-pairs) in simplified form = ...$2(1-(\frac{1}{2})^n)$....

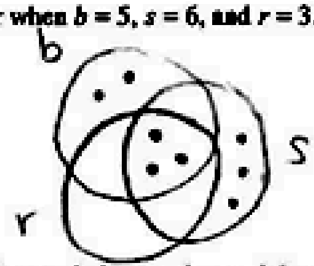
5. Complete the following sentences. [(2+2)+2+2+2]
   (a) #((H, W)-pairs such that $H \subseteq W$) = $\sum_{m=0}^{n} C(n, m)$.$* 2^{n-m}$.... = ...3^n...., by Binomial Theorem.
   (b) #((H, W)-pairs such that $H \cap W = \varnothing$) = ...3^n....
   (c) For a fixed H of size m, $0 \leq m \leq n$, #(W disjoint from H) = ...$C(\cancel{k}\, n, m) * \boxed{2^{n-m}}$
   (d) #((H, W)-pairs such that $H \cap W = \varnothing$ and $|H| = m$) = ...$C(n, m) * 2^{n-m}$....

-1

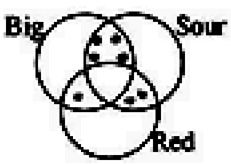
6. Give the formula for maximum #(big, sour, and ripe fruits), when we have b = #(big fruits), s = #(sour fruits), and r = #(ripe fruits). [4]

min(b,s,r)

Give a Venn diagram to explain the above answer when $b = 5$, $s = 6$, and $r = 3$. [3]



Now assume #(fruits) = 7. Complete the Venn-diagram below to show minimum #(big, sour, and ripe fruits) = 0. [3]



7. Complete the sentences below describing the given situations; keep the sentences short and clear. [2+2+2]

(a) $H = W$: I have ..exactly what I want.........................

(b) $|H| = |W|$: I have ..the same number of things as the number of things I want.

−2 (c) $H \subseteq W$: I have ..what I want..................................

nothing that I don't want.

Express the following situations using set notation (avoid the use of complement as much as possible). [2+2+2]

(d) I don't have something(s) that I want: ..$H \cap W = \emptyset$.....
(= I have nothing that I want.)

(e) I have every thing that I want: ..$H \supseteq W$..............

(f) I have every thing that I do not want: ..$H \supseteq W^c$...........

8. Complete the sentences below. Assume $S = \{1, 2, 3, \cdots, n\}$. [12]

(a) $C(n, m)$ = #(m..-subsets of S) = #(binary strings of length...n... containing exactly m ones).

(b) $C(n, m) = C(n,$ $n-m$$)$ because the m-subsets of S can be matched with $n-m$-subsets of S via complements

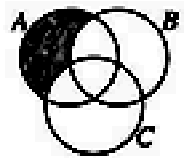
−1 (c) $C(n, 0) + C(n, 1) + \cdots + C(n, n)$ = #(all subsets of S) = #(all binary strings of length n...) = ..2^n...

(d) $m.C(n, m)$ = Total size of all m-subsets of S.

(e) $0.C(n, 0) + 1.C(n, 1) + 2.C(n, 2) + 3.C(n, 3) + \cdots + n.C(n, n)$ = #(items in S)·#(subsets of S containing a given item) = $n2^{n-1}$ because each item belongs to 2^{n-1} subsets.

size of all subsets

9. In the Venn-diagram below, shade the relevant area for the set of all items that belong to *A* and not to any of *B* and *C*. On the rightside, give the size of that subset in terms of $|A|$, ⋯, $|A \cap B|$, ⋯, and $|A \cap B \cap C|$. [2+4]

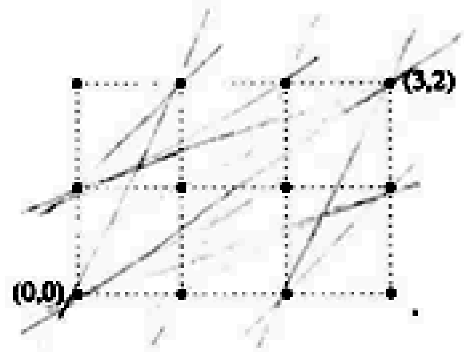


$|A| - |A \cap B| - |A \cap C| + |A \cap B \cap C|$ ✓

Now give the formula for the size of the set of all items that belong to exactly one of *A*, *B*, and *C*. [4]

$|A| + |B| + |C| - 2|A \cap C| - 2|A \cap B| - 2|B \cap C| + 3|A \cap B \cap C|$ ✓

10. Show clearly one line with each positive slope that contains exactly 2 of the grid points in the 3×4 grid below. Also, show the number of such lines for each slope. (We are not talking here about line segments, but about the whole straight-lines extending to infinity.) [5+5]



#(slope 1): 2
#(slope 2): 3
#(slope 1/2): 4
✓ #(slope 2/3): 1
#(slope 1/3): 2

11. Given integer $n \geq 0$, the code below efficiently computes $c[m] = C(n, m)$, $0 \leq m \leq n$ using a recursive formula for $C(n, m)$.

```
1.   int[] c = new int[n+1];
2.   c[0] = 1;
3.   for (int m = 1; m <= n; m++)
4.       c[m] = c[m-1]*(n-m+1)/m;
```

(a) Give #(arithmetic and assignment operation in all iterations of line 4). [5]

6n ✓

(b) Explain why rewriting line 4 as `c[m] = (n-m+1)/m*c[m-1]` does not work. [5]

If (n-m+1) is not divisible by m, (n-m+1)/m will round to an integer, causing c[m] to equal an incorrect value. ✓

12. BONUS. Assume we have *n* straight lines $L_1, L_2, \cdots, L_n$ which give the maximum number of points of intersections. Complete the following sentences/equations. [2+2+2+2+2]

(a) Then any three of those lines form a triangle because exactly . . 2 . . . lines go through a point of intersection and there are no . parallel . . lines. ✓

+8

(b) The #(ways choosing 3 lines out of $L_1, L_2, \cdots, L_n$) = . C(n,3) . ✓

(c) Thus, #(triangles formed by $L_1, L_2, \cdots, L_n$) = . C(n,3).

(d) For n = 18, there will be . . . . 816 . . . . triangles. ✓ $C(18,3) = \frac{18 \cdot 17 \cdot 16}{3 \cdot 2 \cdot 1} = 3 \cdot 17 \cdot 16 = 816$

(e) If the lines L_i do not give the maximum number of intersection points, then #(triangles formed by $L_1, L_2, \cdots, L_n$) is

.indeterminate ✗ < C(n,3)