

Update

$$S[i, j] > S[i, k] + S[k, j]$$

$k=0$

$$S = \begin{bmatrix} 0 & 1 & \infty & \infty \\ \infty & 0 & \infty & 2 \\ 4 & 7 & 0 & \infty \\ \infty & \infty & 3 & 0 \end{bmatrix}$$

$$\pi = \begin{bmatrix} 1 & 2 & \emptyset & \emptyset \\ \emptyset & 2 & \emptyset & 4 \\ 1 & 2 & 3 & \emptyset \\ \emptyset & \emptyset & 3 & 4 \end{bmatrix}$$

$k=1$

$$S[2, 3] \not> S[2, 1] + S[1, 3]$$

$$[3, 2] > [3, 1] + [1, 2]$$

$$[4, 2] \not> [4, 1] + [1, 2]$$

$$[2, 4] \not> [2, 1] + [1, 4]$$

$$[3, 4] \not> [3, 1] + [1, 4]$$

$$[4, 3] \not> [4, 1] + [1, 3]$$

$$S = \begin{bmatrix} 0 & 1 & \infty & \infty \\ \infty & 0 & \infty & 2 \\ 4 & 5 & 0 & \infty \\ \infty & \infty & 3 & 0 \end{bmatrix}$$

$$\pi = \begin{bmatrix} 1 & 2 & \emptyset & \emptyset \\ \emptyset & 2 & \emptyset & 4 \\ 1 & 1 & 3 & \emptyset \\ \emptyset & \emptyset & 3 & 4 \end{bmatrix}$$

$$K = 1$$

$$S = \begin{bmatrix} 0 & 1 & \infty & \infty \\ \infty & 0 & \infty & 2 \\ 4 & 5 & 0 & \infty \\ \infty & \infty & 3 & 0 \end{bmatrix}$$

$$\Pi = \begin{bmatrix} 1 & 2 & \emptyset & \emptyset \\ \emptyset & 2 & \emptyset & 4 \\ 1 & 1 & 3 & \emptyset \\ \emptyset & \emptyset & 3 & 4 \end{bmatrix}$$

$$K = 2$$

$$\begin{aligned} [1,3] & \neq [1,2] + [2,3] \\ [3,1] & \neq [3,2] + [2,1] \end{aligned}$$

$$\begin{aligned} [1,4] & > [1,2] + [2,4] \\ [3,4] & > [3,2] + [2,4] \end{aligned}$$

$$S = \begin{bmatrix} 0 & 1 & \infty & 3 \\ \infty & 0 & \infty & 2 \\ 4 & 5 & 0 & 4 \\ \infty & \infty & 3 & 0 \end{bmatrix}$$

$$\Pi = \begin{bmatrix} 1 & 2 & \emptyset & 2 \\ \emptyset & 2 & \emptyset & 4 \\ 1 & 1 & 3 & 2 \\ \emptyset & \emptyset & 3 & 4 \end{bmatrix}$$

$$[4,1] \neq [4,2] + [2,1]$$

$$[4,3] \neq [4,2] + [2,3]$$

$k=2$

$$S = \begin{bmatrix} 0 & 1 & 2 & 3 \\ \infty & 0 & \infty & 2 \\ 4 & 5 & 0 & 5 \\ 2 & 2 & 3 & 0 \end{bmatrix}$$

$$\pi = \begin{bmatrix} 1 & 2 & \emptyset & 2 \\ \emptyset & 2 & \emptyset & 4 \\ 1 & 1 & 3 & 2 \\ \emptyset & \emptyset & 3 & 4 \end{bmatrix}$$

$k=3$

$$\begin{aligned} [1,2] &\neq [1,3] + [3,2] & [1,4] &\neq [1,3] + [3,4] \\ [2,1] &\neq [2,3] + [3,1] & [2,4] &\neq [2,3] + [3,4] \\ [4,1] &> [4,3] + [3,1] & [4,2] &> [4,3] + [3,2] \end{aligned}$$

$$S = \begin{bmatrix} 0 & 1 & \infty & 3 \\ \infty & 0 & \infty & 2 \\ 4 & 5 & 0 & 5 \\ 7 & 8 & 3 & 0 \end{bmatrix}$$

$$\pi = \begin{bmatrix} 1 & 2 & \emptyset & 2 \\ \emptyset & 2 & \emptyset & 4 \\ 1 & 1 & 3 & 2 \\ 3 & 3 & 3 & 4 \end{bmatrix}$$

$$K = 3$$

$$S = \begin{bmatrix} 0 & 1 & \underline{8} & 3 \\ \infty & 0 & \infty & 2 \\ 4 & \infty & 0 & 5 \\ 7 & \infty & 3 & 0 \end{bmatrix}$$

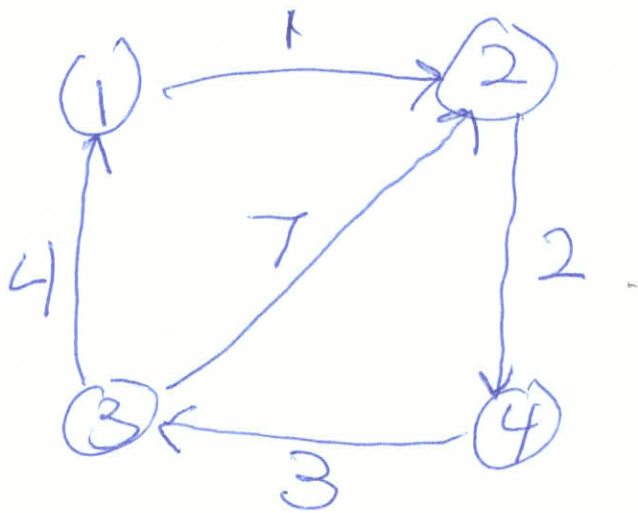
$$\Pi = \begin{bmatrix} 1 & 2 & \emptyset & 2 \\ \emptyset & 2 & \emptyset & 4 \\ 1 & 1 & 3 & 2 \\ 3 & 3 & 3 & 4 \end{bmatrix}$$

$$K = 4$$

$$\begin{aligned} [1, 2] &\neq [1, 4] + [4, 2] & [1, 3] &> [1, 4] + [4, 3] \\ [2, 1] &> [2, 4] + [4, 1] & [2, 3] &> [2, 4] + [4, 3] \\ [3, 1] &\neq [3, 4] + [4, 1] & [3, 2] &\neq [3, 4] + [4, 2] \end{aligned}$$

$$S = \begin{bmatrix} 0 & 1 & 6 & 3 \\ 9 & 0 & 5 & 2 \\ 4 & \infty & 0 & 5 \\ 7 & \infty & 3 & 0 \end{bmatrix}$$

$$\Pi = \begin{bmatrix} 1 & 2 & 4 & 2 \\ 4 & 2 & 4 & 4 \\ 1 & 1 & 3 & 2 \\ 3 & 3 & 3 & 4 \end{bmatrix}$$



$$S = \begin{bmatrix} 0 & 1 & 6 & 3 \\ 9 & 0 & 5 & 2 \\ 4 & 5 & 0 & 7 \\ 7 & 8 & 3 & 0 \end{bmatrix}$$

$$\Pi = \begin{bmatrix} 1 & 2 & 4 & 2 \\ 4 & 2 & 4 & 4 \\ 1 & 1 & 3 & 2 \\ 3 & 3 & 3 & 4 \end{bmatrix}$$

$$\begin{aligned} \Delta(1, 3) &= 6 \\ \Pi[1, 3] &= \Pi[1, 4] \rightarrow \Pi[4, 3] \\ &= 1 \xrightarrow{3} 4 \rightarrow \Pi[4, 3] \\ &= 1 \xrightarrow{3} 4 \xrightarrow{3} 3 \end{aligned}$$

$$\begin{aligned} \Delta(4, 1) &= 7 \\ \Pi[4, 1] &= \Pi[4, 3] \rightarrow \Pi[3, 1] \\ &= 4 \xrightarrow{3} \Pi[3, 1] \\ &= 4 \xrightarrow{3} 3 \xrightarrow{4} 1 \end{aligned}$$