Sample 10 Steps

Poisson Equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -2\pi^2 \sin(\pi x)\sin(\pi y)$$

Before writing the function poisson, obtain the equation to be iterated by doing the following:

1) **Approximate the second order partial derivatives in the Poisson Equation by the 3-point second order central difference formula, using the point with indices i,j as the central point:**

$$\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2} + \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{h^2} = f_{i,j}$$

where $u_{i,j} = u(x_i, y_j)$, $f_{i,j} = f(x_i, y_j)$ where f(x,y) is the function on the right of the equals sign in the Poisson Equation, and h is the stepsize in both the x and y intervals.

2) **Solve the equation for u_{ij} :**

$$u_{i,j} = \frac{1}{4}\left(u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1} - h^2 f_{i,j}\right)$$

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% function poisson
function u = poisson(f, g, L, n, accuracy)
h = L/(n-1);
u = zeros(n,n);
for(i = 1:n)
   u(i,1) = g( (i-1)*h, 0 );
   u(i,n) = g( (i-1)*h, L );
end
for(j = 1:n)
   u(1,j) = g( 0, (j-1)*h );
   u(n,j) = g( L, (j-1)*h );
end
max_diff = 1;
while(max_diff >= accuracy)
   max_diff = 0;
   for(i = 2:n-1)
      for(j = 2:n-1)
         uij_old = u(i,j);
         u(i,j) = 1/4*( u(i-1,j) + u(i+1,j) + u(i,j-1) + u(i,j+1) ...
                        - h^2 * f( (i-1)*h, (j-1)*h) );
         diff = abs(u(i,j) - uij_old);
         if(diff > max_diff)
             max_diff = diff;
         end
      end
   end
end
```