Calculate the eigenvalues and eigenvectors of the 2x2 matrix A:

A = [7 -1
      6   2]

Answer

$$\begin{pmatrix} 7 & -1 \\ 6 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \lambda \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$7x_1 - x_2 = \lambda x_1$$
$$6x_1 + 2x_2 = \lambda x_2$$

$$(7-\lambda)x_1 - x_2 = 0$$
$$6x_1 + (2-\lambda)x_2 = 0$$

$$\begin{vmatrix} 7-\lambda & -1 \\ 6 & 2-\lambda \end{vmatrix} = 0$$

$$(7-\lambda)(2-\lambda) + 6 = 0$$
$$\lambda^2 - 9\lambda + 20 = 0$$
$$(\lambda - 4)(\lambda - 5) = 0$$
$$\lambda = 4, 5$$

$\lambda_1 = 4$ | $\lambda_2 = 5$

$(7-4)x_1 - x_2 = 0$ | $(7-5)x_1 - x_2 = 0$

$3x_1 - x_2 = 0$ | $2x_1 - x_2 = 0$

$x_2 = 3x_1$ | $x_2 = 2x_1$

$V_1 = \begin{pmatrix} x_1 \\ 3x_1 \end{pmatrix}$ | $V_2 = \begin{pmatrix} x_1 \\ 2x_1 \end{pmatrix}$

$V_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ | $V_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

eigenvalues

$$\lambda_1 = 4, \lambda_2 = 5$$

eigenvectors

$$V_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}, V_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$