HOMEWORK ASSIGNMENT №1

Dr. Duncan, CSC 3102, Louisiana State University — Due: 2/4/2020 at noon

1. In 1.(a), group the functions in classes, such that if f_i and f_j belong to the same class, then $\Theta(f_i) = \Theta(f_j)$. In 1.(b) and 1.(c) formally prove or disprove the assertion using limits, possibly with L'Hôpital's rule. Also, if the assertion is true, prove that it is true directly from the definition of the asymptotic notation by deriving values for the relevant constants.

   (a) Consider each function $f_i : \mathbb{Z}^+ \to \mathbb{R}^+$: $f_1(n) = 3(n+1)^2$, $f_2(n) = 2^{100}$, $f_3(n) = 2^{\lg n}$, $f_4(n) = 3n^2 + 4n$ and $f_5(n) = \sqrt{7n^2 + 4n - 1}$. Enclose functions with the same asymptotic growth rate in a pair of braces, separated by commas. List the classes in order of increasing asymptotic time complexity. [5 points]

   (b) $\sqrt{3n^2+5} \in \Theta(\mathrm{n} \lg \mathrm{n})$ [10 points]

   (c) $\lg\left(n^2 + 3n\right) \in \mathrm{O}(\lg n)$ [10 points]

2. Give the closed-form solution of the recurrence relation where indicated using algebraic *unrolling* and answer the questions regarding the sequence that is denoted by the relation.

   (a) Solve $T(n) = n + 1 + T(n-1)$, $n \geq 2$, with the initial condition $T(2) = 3$. [15 points]

   (b) Give the first five terms, $T(2), T(3), \cdots, T(6)$, of the sequence denoted by the recurrence relation in 4(a). [5 points]

   (c) Give the most restrictive polynomial-time asymptotic upper bound for $T(n)$ if one exists or explain why no such bound exists. [5 points]

3. Answer the exercises below, showing work in each case.

   (a) Give the closed-form solution for telescoping series $\alpha(n) = \sum_{i=1}^{n} \left[(i+1)^2 - i^2\right]$, in terms of n, by writing the sigma notation as the difference of two series and then expanding and simplifying the difference. [10 points]

   (b) Expand the summand in the $\alpha(n)$ and write the series as an equivalent expression consisting of multiple sigma notations with simpler summands.[10 points]

   (c) Give a closed-form formula for $\beta(n) = \sum_{i=1}^{n} i$, in terms of n, by equating the expressions in 3.(a) and 3(b). Explain why $\beta(n) \in \Theta(n^2)$. [5 points]

4. Prove that $\sum_{i=1}^{n} \lg i \in \Theta(n \lg n)$ using the definition of the Θ-notation. Derive values for the constants c_1, c_2 and n_o in the definition.

   (a) First, prove that $\sum_{i=1}^{n} \lg i \in \mathrm{O}(n \lg n)$. [10 points]

   (b) Second, Prove that $\sum_{i=1}^{n} \lg i \in \Omega(n \lg n)$. [Hint: Compare $\sum_{i=1}^{n} \lg i$ and $\sum_{i=\frac{n}{2}+1}^{n} \lg \frac{n}{2}$, the series consisting of $\frac{n}{2}$ terms of $\lg \frac{n}{2}$.] [10 points]

   (c) Give a concluding statement based on results obtained in 4.(a) and 4.(b). [5 points]