Practice Questions for Apr 02, 2019

1. Define the following properties of a relation. For each of these properties, complete the code below for testing whether a relation-matrix R satisfies that property or not (make sure your code is efficient).

   (a) Anti-symmetry.

   //code to test anti-symmetry

```
1. for (int i=0; i<R.length; i++)
2.     for (int j=i+1; j<R.length; j++)

3.         if ( ...................... )
4.            return(false); //not anti-symmetric
5. return(true); //anti-symmetric
```

   Now, give an example of 4×4 matrix R for which the code returns value false after three iterations of line 3 and give the number of such matrices.

   (b) Transitivity.

   //code to test transitivity;

```
1. for (int i=0; i<R.length; i++)
2.     for (int j=0; j<R.length; j++)
3.         if ((i != j) && (R[i][j] == 1))
4.            for (int k=0; k<R.length; k++)

5.                if ( ................................................. )
6.                   return(false); //not transitive
7. return(true); //transitive
```

   Now, give an example of 4×4 matric R for which the code returns value false after 1st iterations of line 5 and give the number of such matrices.

2. Partial order and linear (total) order.

   (a) State the three properties of a partial order.

   (b) Show all different *types* of Hasse-diagram of partial orders on 3 items (avoid labeling the nodes; draw Hasse-diagrams properly, with the links going upwards as we have done in the lectures); also, next to each of them show the number of possible partial orders having Hasse diagram of that *type*.

   (c) State the specal property that distinguishes a linear order from a partial order.

   (d) Give the number of linear orders on an n-set. Compare it with the number of symmteric relations, the number of relfexive relations, and the number of relations that are both reflexive and symmetric (to find who is bigger than whom and for wahtvalues of n).

   (e) Suppose R is the matrix of a partial order. Give the code to test whether R is a linear order or not.

3. Is there a flaw in the argument below to count anti-symmtric relations on an n-set? If so, correct it.

   (a) A relation matrix R can be anti-symmetric if and only if for some $i{\neq}j$ we have $R[i][j] + R[j][i] \neq 2$.

   (b) Thus, for each pair of off-diagonal items $R[i][j]$ and $R[j][i]$ there are three possible values (0,0), (1,0), and (1,1). The value (1,1) is not allowed.

   (c) The diagonal items can be any thing.

   (d) This give 2^n possibilities for the diagonal items and $3^{(n^2-n)/2}$ possibilities for off-diagonal items.

   (e) This gives #($n{\times}n$ anti-symmetric relation matrices) = $2^n 3^{(n^2-n)/2}$.

   Verify whether the above formula is correct or not by drawing diagrams for different types 2×2 anti-symmetric relations and counting the possible relations of each type.