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Short Quiz #4 (20-Feb): CSC-2259: Discrete Structures, Sp 2020
Your answers must be to the point. Total = 20; marks for each question is shown in [ ].

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1. Suppose H and W are m-subsets of an n-set. Complete the following equations. [2+2+2+2]

(a) #(H) = ..m.. × C(n,m) and #((H, W)-pairs) = .. 2^{2m} × C(n,m)^2 ..

(b) #((H, W)-pairs with $H = W$) = .. 2^m × C(n, m)

(c) #((H, W)-pairs with $H \subseteq W$) = .. 2^{n-m} × C(n,m)

(d) Verify your answers in (c) by giving all (H, W)-pairs for the n-set $\{a, b, c, d\}$ and $m = 1$.

? {a,a} {b,b} {c,c} {d,d}
{∅,a} {∅,b} {∅,c} {∅,d}

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2. Give average #(iterations in testing $H = W$ in the code given in the class) when H, W are binary arrays of length $n \geq 1$. Also, give the value of the average for $n = 10$ in simplified form. [2+1]

$2(1-(\frac{1}{2})^n)$ (or $n=10$, $2(1-(\frac{1}{2})^{10}) = 2(1-\frac{1}{1024}) = 2(\frac{1023}{1024})$)

3. Complete the equation below and verify the equation for $x = -1$ and $n = 5$. [2+1]

$1 + x + x^2 + \cdots + x^n =$ ..$\frac{1-x^{n+1}}{1-x}$.., if $x \neq 1$.

$1+(-1)+(-1)^2+(-1)^3+(-1)^4+(-1)^5 = 0$: $\frac{1-(-1)^{5+1}}{1-5} = \frac{1-(-1)^6}{-4} = \frac{1-1}{-4} = \frac{0}{-4} = 0$ ✓

$= \frac{1023}{512}$

4. Give the details to show $1 + 2x + 3x^2 + \cdots + nx^{n-1} = (1 - x^n)/(1 - x)^2 - nx^n/(1 - x)$, when $x \neq 1$. [4]

$S = 1 + 2x + 3x^2 + \ldots + nx^{n-1}$

$x \cdot S = x + 2x^2 + 3x^3 + \ldots + nx^n$

$S - xS = 1 + x + x^2 + x^3 + \ldots + x^{n-1} - nx^n$

$S(1-x) = \frac{1-x^n}{1-x} - nx^n$

$S = \frac{1-x^n}{(1-x)^2} - \frac{nx^n}{(1-x)}$

5. Suppose we replace the if-statement in the code we considered for testing $H = W$ by "`if (1 == H[i] + W[i]) return(false);`". Will the new code still work? If not explain, with some example H and W; otherwise, explain the new code wil be less efficient. [2]

The new code will still work.
It is less efficient because it contains 1 comparison and 1 addition in the if statement, while before it only had 1 comparison and 0 addition