96

Long Quiz #1 (06-Feb): CSC-2259: Discrete Structures, Sp 2020
Your answers must be to the point. Total = 100; marks for each question is shown in [ ].

LastName: Le

FirstName: Kha

1. Complete the sentences below. Assume $S = \{1, 2, 3, \cdots, n\}$. [12]

   (a) $C(n, m)$ = #(m-subsets of S) = #(binary strings of size n containing exactly m ones).

   (b) $C(n, m) = C(n, n-m)$ because the m-subsets of S can be matched with n-m -subsets of S via complements

   (c) $C(n, 0) + C(n, 1) + \cdots + C(n, n)$ = #(all subsets of S) = #(all binary strings of size n) = 2^n

   (11)

   (d) $m . C(n, m)$ = Total size of all ~~even~~ m -subsets of S.

   (e) $0 . C(n, 0) + 1 . C(n, 1) + 2 . C(n, 2) + 3 . C(n, 3) + \cdots + n . C(n, n)$ = $n . 2^{n-1}$

   Compute the value of the sum on leftside of (e) by showing below all relevant subsets and their sizes for $n = 4$. List the subsets systematically by sise (**no points otherwise**). Also, show the the value of the sum. [6+2]

   (8)

C(4,0)	C(4,1)	C(4,2)	C(4,3)	C(4,4)
∅	1	1,2	1,2,3	1,2,3,4
	2	1,3	1,2,4	
	3	1,4	1,3,4	
	4	2,3	2,3,4	
		2,4		
		3,4		

   Sum = 0(1)+1(4)+2(6)+3(4)+4(1) = 32

   Sum = 32

   Complete the sentences below. [4]

   (g) The item 4 in S is counted in the size of 2^3 many subsets (give the number as a power of 2) in your list of subsets above. They are the subsets of S containing item 4

   (3)

   For each item in $S = \{1, 2, 3, 4\}$, each item is counted in the size of the same number of subset.

   Thus, the sum in (e) equals #(~~odd sets~~ items in S).#(subsets of S containing a given item) = $n . 2^{n-1}$.

2. Give a detailed efficient code to compute all $C(n, m)$ for a fixed $n \geq 1$. Use an array `c[]` to store the values of $C(n, m)$, $0 \leq m \leq n$. [10]

   (8+2)

```
int[] c = new int[n+1];
c[0] = 1;
for (int m = 1; m <= n; m++)
    c[m] = c[m-1] * (n-m+1)/m;
```

   Also, give #(arithmatic and assignment operations in the body) per loop-iteration and for all iterations. [3+3]

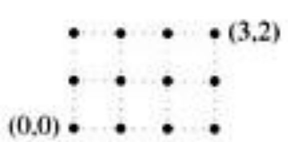
   (6)

   per loop-iteration: 6   for all iterations: 6n

3. Consider the 4×3 grid-points shown below, where one can only go to right (R) and to north (N) from a grid point. A step here means going from one grid point to the next grid point to its right or to its north.

   (a) Show all paths in a systematic order from (0,0) to (3,2) (**no points otherwise**) as strings of R and N on the rightside of the figure below. [10]

   10



   Paths from (0,0) to (3,2):

   RRRNN, RRNRN, RRNNR, RNRRN, RNRNR, RNNRR, NRRRN, NRRNR, NRNRR, NNRRR

2 (b) Consider the general case, where c = #(columns) and r = #(rows). Show #(paths from (0,0) to $(c-1, r-1)$) in the form of $C(?, ?)$. [2]

$C(c+r-2, c-1)$

2 (c) Verify your answer in (b) in the case of considered in (a). [2]

$C(4+3-2, 4-1) = C(5,3) = 10$

(d) Explain clearly where the following arguments went wrong. [4]

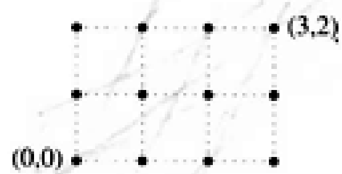
> (a) Any two points chosen out of the 12 points in the above grid give a line.
> (b) There are $C(12, 2) = 55$ ways of choosing two points.
> (c) Thus, there are 55 lines each containing exactly 2 grid points.

3 (c) is incorrect. The line may not contain exactly 2 grid points. For instance, a vertical line at (0,0) contains (0,1) and (0,2), which is a total of 3 grid points. That means some lines were overcounted.

(e) Find out the number of lines that contain exactly 2 grid points. Show one line with each possible positive slopes. (Hint: group lines together with the same slope and count them). [10]

#(vertical slope): none
#(slope 0): none
#(slope 1): 2
#(slope 2): 3
#(slope $\frac{1}{2}$): 2 ← 4
#(slope $\frac{1}{3}$): 2
#(slope $\frac{2}{3}$): 1

9



#(positive slope): 2+3+2+2+1 = $\boxed{10}$

#(lines contain exactly 2 grid points) = 2 × #(positive slope) = $\boxed{20}$

4. Consider $n \geq 1$ *distinct* straight lines $L_1, L_2, \cdots, L_n$. Let P_{ij} be the interesection point of L_i and L_j, if they are not parallel. Suppose k = #(intersection points of L_1 to L_n>

3 (a) For $n = 4$, show all possible ways we can get exactly $k = 0$ intersection points (describe in English if possible; otherwise draw diagrams).

Only when all lines are parallel

3 (b) Repeat (a) for $k = 1$.

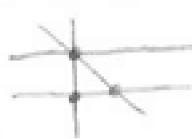
Only when all lines intersect at the same point.

3 (c) Repeat (a) for $k = 2$.

There is no possible way

3 (d) Repeat (a) for $k = 3$.

way #1: 3 lines are parallel and 1 line intersects all of them

way #2: 

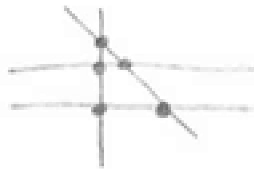
4

3 (e) Repeat (a) for $k = 4$.

way #1:  way #2: 

(f) Repeat (a) for $k = 5$.

3



(g) Repeat (a) for $k = 6$.

3

Only when none of the lines are parallel and all intersection points contain only 2 lines.

Q5. general

maximum # of fruit in basket that are big and sour $= \min(b, s)$

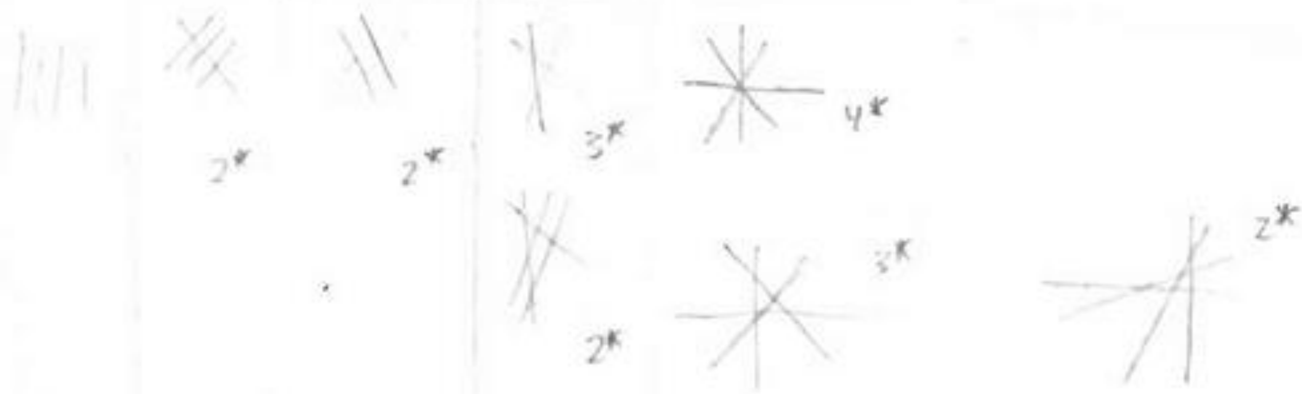
minimum # of fruit in basket that are big and sour $= \max(0, b - f + s)$

11

when $b=6, s=10$ and $f=15$

max # of big and sour $= \min(6, 10) = \boxed{6}$

min # of big and sour $= \max(0, 6-15+10) = \max(0, 1) = \boxed{1}$

parallel	4	3|1	2|2	2|1|1	1|1|1|1	
cases:						
intersections	0	3	4	3 5	1 4	6