MATHEMATICS 2090
Final Examination Practice Problems

Print name ______________________________
Last name First name

1. Let $A = \begin{pmatrix} 1 & 4 & -1 & 3 \\ 2 & 9 & -1 & 7 \\ 2 & 8 & -2 & 6 \end{pmatrix}$.
   a) Compute the rank and nullity of A.
   b) Find a basis for the kernel of the linear map $T : \mathbb{R}^4 \to \mathbb{R}^3$ which is given by $T(\underline{v}) = A\underline{v}$.

2. Solve the linear system

$$\begin{aligned} -2x_1 + 4x_2 + x_3 &= -5 \\ 3x_1 - 2x_2 - x_3 &= 2 \\ 4x_1 - 3x_2 + 2x_3 &= 1 \end{aligned}$$

3. Let $A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$.

a) Compute its determinant. Is A invertible? If yes, please compute its inverse; if not, please justify.
b) Compute the characteristic polynomial, eigenvalues and eigenspaces of the matrix.
c) Is A defective or non-defective?
d) Determine the general solution of the differential equation system $\underline{x}' = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix} \underline{x}$.

4. Solve the initial value problem $y'' + 6y' + 13y = 0, \quad y(0) = 0, \quad y'(0) = 1$, using a method of your choice.

5. Use the variation-of-parameter technique to find the general solution to the order-1 differential equation system $\underline{x}' = \begin{pmatrix} -1 & 1 \\ 3 & 1 \end{pmatrix} \underline{x} + \begin{pmatrix} 20e^{3t} \\ 12e^{t} \end{pmatrix}$.

6. a) Are $\underline{x_1} = \begin{pmatrix} t\sin(t) \\ \cos(t) \end{pmatrix}$, $\underline{x_2} = \begin{pmatrix} -t\cos(t) \\ \sin(t) \end{pmatrix}$ linearly independent? Please justify your answer.

b) Are they both solutions of the linear system $\underline{x}' = \begin{pmatrix} 1/t & t \\ -1/t & 0 \end{pmatrix} \underline{x}$? Please justify your answer.

7. Solve the initial value problem $y'' - 4y = 12e^{2t}, \quad y(0) = 2, y'(0) = 3$, using a method of your choice.

8. a) Compute the Laplace transform $L\left[e^{2t}\sin(3t) - t/2 + 4e^{t}\right](s)$

b) Compute the inverse Laplace transform $L^{-1}\left[\frac{2s}{s^2+2s+2} + \frac{12}{(s-2)^4}\right](t)$.

9. Use Laplace transform to solve the initial value problem $y' - 2y = u_2(t)e^{t-2}, \quad y(0) = 2$ where $u_2(t)$ is the unit step function.