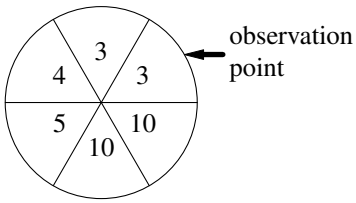
Practice Questions for Apr 09, 2019

1. Consider a spinning wheel with 6 equal parts (see below), marked as 3, 3, 4, 5, 10, and 10. We make an experiment consisting of 3 spinings and note down the numbers observed; we write the result as a triplet $<s_1, s_2, s_3>$, where s_i = the observed value in the ith spin.



Is there any error in the following ansawers? If so, find the correct answers.

(a) What is the probability that the triplet $<s_1, s_2, s_3>$ forms the sides of a right-angled triangle?

Answer. The sides have to be {3, 4, 5} and this may arise from any of the triplets <3, 4, 5>, <3, 5, 4>, etc. There are 6 of them, each with probability (2/6)(1/6)(1/6) = 1/108. This gives P(right-angled triangle) = 6/108 = 1/18.

(b) What is the probability that the triplet $<s_1, s_2, s_3>$ forms the sides of an equilateral triangle?

Answer. The output triplet must be of the form $<s, s, s>$, with s = 3, 4, 5, and 10. For example, P(equilateral triangle with sides of length 3) = $(2/6)^3$. Thus, P(equilateral triangle) = $(2/6)^3 + (1/6)^3$ = 18/216 = 1/12.

(c) What is the probability that the triplet $<s_1, s_2, s_3>$ does not form the sides of a triangle?

Answer. The triplet has to be of the following forms and their rearrangements (i.e., rearranging s_1, s_2, and s_3 in other ways):

Form (has ≥1 of 3)	#(triplets)	Probability of each triplet	Form (has no 3, has ≥1 of 4)	#(triplets)	Probability of each triplet
<3, 3, 10>	3	(2/6)(2/6)(2/6) = 8/216	<4, 4, 10>	3	(1/6)(1/6)(2/6) = 2/216
<3, 4, 10>	6	(2/6)(1/6)(2/6) = 4/216	<4, 5, 10>	6	(1/6)(1/6)(2/6) = 2/216
<3, 5, 10>	6	(2/6)(1/6)(2/6) = 4/216			

Form (has no 3 or 4)	#(triplets)	Probability of each triplet
<5, 5, 10>	3	(1/6)(1/6)(2/6) = 2/216

P(the lengths $<s_1, s_2, s_3>$ do not form a triangle) = (3×8 + 6×4 + 6×4 + 3×2 + 6×2 + 3×2)/216 = 96/216 = 4/9.

(d) What is P(the lengths $<s_1, s_2, s_3>$ form a triangle)?

Answer. 1 – P(the lengths $<s_1, s_2, s_3>$ do not form a triangle) = 1 – 4/9 = 5/9.

2. For the experiment in Problem 1, do the following.
   (a) Compute P(isosceles triangle) using a table like that in (c) above.
   (b) Compute P(scalene triangle) = P(triangle) – P(equilateral triangle) – P(isosceles triangle).

3. Suppose the wheel consists of two 3's, one 4, and one 10. Compute the various probabilities as in Problems 1 and 2.