Derivation of the formula for $u_{i,k}$

One-Dimensional Heat Equation

$$\frac{\partial u}{\partial t} = a\frac{\partial^2 u}{\partial x^2} + f(x,t)$$

$$f(x,t) = 0$$

$$a = 1$$

Before writing the function heat1, obtain the equation to be used in the explicit scheme by doing the following:

1) **In the One-Dimensional Heat Equation, approximate the first order partial derivative by the 2-point first order forward difference formula and approximate the second order partial derivative by the 3-point second order central difference formula, using the point with indices i,k-1 as the central point:**

$$\frac{u_{i,k} - u_{i,k-1}}{h_t} = a\frac{u_{i-1,k-1} - 2u_{i,k-1} + u_{i+1,k-1}}{h_x^2} + f_{i,k-1}$$

where $u_{i,k}$ = u(i,k) , $f_{i,k-1}$ = f(x_i,t_{k-1}) , and h_x and h_t are the stepsizes in the x and t intervals.

2) **Solve the equation for $u_{i,k}$:**

$$u_{i,k} = \frac{ah_t}{h_x^2}\left(u_{i-1,k-1} - 2u_{i,k-1} + u_{i+1,k-1}\right) + h_t f_{i,k-1} + u_{i,k-1}$$