

Should be able to answer questions like the following.

1. Let B = subset of big fruits in the box, $b = |B|$, S = subset of sour fruits in the box, $s = |S|$, and $f = |\text{all fruits in the box}|$. Then, $B \cup S$, where \cup means union, is the subset fruits that are big or sour. It can be as much as $b + s$ but not bigger than f .

Thus, $\max |B \cup S| = \min(b+s, f)$. That is, there are at most $\min(b+s, f)$ many fruits that are big or sour. For $b=6$, $s=7$, and $f=10$, there are at most $\min(6+7, 10) = 10$ fruits that are big or sour. If we had $f = 15$, then there would have at most $\min(6+7, 15) = 13$ fruits that are big or sour.

What is the maximum number of fruits that are big or not sour, i.e. $\max |B \cup -S|$. This is at most $\min(b+f-s, f)$, which is obtained by simply replacing s by $f-s$ for $|-S| = \#(\text{not-sour fruits})$ in the formula obtained earlier. For $b=6$, $s=7$, and $f=10$, there are at most $\min(6+10-7, 10) = \min(9, 10) = 9$ big or not sour fruits. Note that if we let $s = 10-7 = 3$, then there would be at most $\min(6+10-3, 10) = 10$ big or not sour fruits (here "not sour" plays the role of "sour" in the very first case.)

What about $\min |B \cup S|$? If $b \geq s$, then we can make sour-fruits a subset of big-fruits and $B \cup S = B$ and $\min |B \cup S| = |B| = b$. Similarly, if $b \leq s$, then $|B \cup S| = |S| = s$. Thus, we can say $|B \cup S|$ is at least $\max(b, s)$, i.e., there is at least $\max(b, s)$ many big or sour fruits.

How about $\min |B \cup -S|$? It is $\max(b, f-s)$. Replacing S by $-S$ means replacing s by $f-s$ in the formula $\max(b, s)$. Thus, there is at least $\max(b, f-s)$ fruits that are big or not sour.

2. You should be able to argue why there is exactly $C(n,3)$ triangles formed if we have maximum number of points of intersection of n lines. In that case, no 3 lines share a common point and no 2 lines are parallel. These are the exact two conditions needed for three lines to form a triangle. Thus, any 3 lines from the n lines would now form a triangle and there are $C(n, 3)$ ways of choosing 3 lines out of n lines. If the lines do not form maximum number of intersection points, then we do not get $C(n,3)$ triangles. For example, If all n lines are parallel, then no triangle is formed. Thus, for any n lines, we can say there will be at most $C(n, 3)$ triangles formed. If we have exactly $C(n, 3)$ triangles, then can we say that there are exactly $C(n, 2)$ intersection points, i.e., the maximum number of intersection points? If so, then given any intersection point P_{ij} = the intersection point of lines L_i and L_j how many triangles would be there with P_{ij} as one of its vertex?

3. You should be able to find how many lines have exactly 2 of the grid-points in a small grid like 4×3 grid. (Hint: group the lines based on their slopes and count the lines for each different slope, assuming that each line contains exactly 2 of the grid points.)