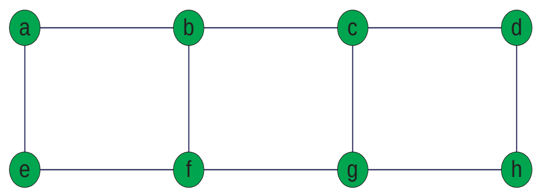
Homework Assignment №2

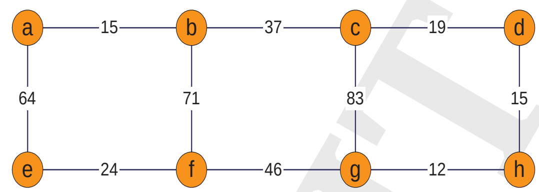
Dr. Duncan, CSC 3102, Louisiana State University

Due: 03/17/2020 at noon

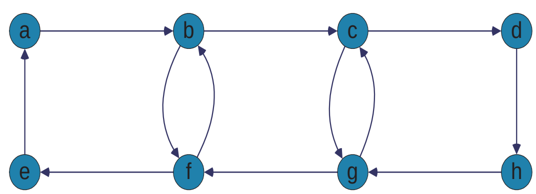
1. Consider the graphs below and answer the exercises that follow, assuming lexicographical order where applicable.



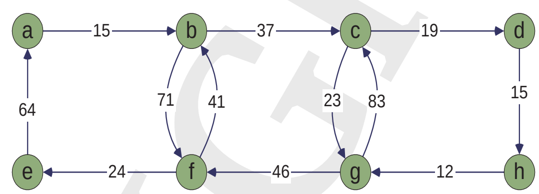
(a) An Undirected Graph



(b) A Weighted Graph



(c) A Digraph



(d) A Weighted Digraph

Figure 1: Illustration of various Graphs

   (a) Give the adjacency list representation for the graphs in Figures 1a - 1d. [12 points]

   (b) Give the adjacency matrix representation for the graphs in Figures 1a - 1d. [13 points]

2. When generating the traversal for each graph, assume that the neighbors of each vertex are explored in lexicographical order during the execution of the algorithm.

   (a) Give the breadth-first-search and the pre-order depth-first-search traversals for the graph in Figure 1a beginning at vertex *a*. [10 points]

   (b) Give the breadth-first-search and the pre-order depth-first-search traversals for the digraph in Figure1c beginning at vertex *a*. [15 points]

3. Consider the incidence matrix β for a digraph shown in Figure 2. Assume the matrix was constructed using the conventions discussed in class.

$$\beta = \begin{bmatrix} -6 & 4 & 0 & 0 & 0 & 0 & 0 \\ 6 & 0 & 5 & 8 & -9 & 0 & 0 \\ 0 & 0 & -5 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & -8 & 0 & -5 & 7 \\ 0 & -4 & 0 & 0 & 9 & 0 & -7 \end{bmatrix}$$

Figure 2: An Incidence Matrix for a Digraph

(a) Draw the digraph whose incidence matrix is given in Figure 2 using A, B, C, $\cdots$ to label the vertices. [5 points]

(b) Draw the transpose of the graph drawn in 3(a).[10 points]

Definition 1. *The **transpose**, also called an **converse**, of a digraph D is another digraph D' on the same set of vertices as D with all of the edges reversed compared to those in D; that is, edge $(u, v) \in E(D')$ if and only if edge $(v, u) \in D$.*

(c) Give the adjacency matrix of the transitive closure of the digraph in 3(b). [10 points]

Definition 2. *The **transitive closure** of a digraph D, denoted $C(D)$, is a reachability matrix of D with dimensions $|D| \times |D|$. The (i, j) entry of $C(D)$ is 1 if vertex j is reachable from vertex i; otherwise, it is 0.*

4. Assuming a simple connected graph, consider the definition below.

Definition 3. *A **bipartite** graph, also called a **bigraph**, is a set of graph vertices decomposed into two disjoint sets such that no two graph vertices within the same set are adjacent. A bipartite graph with m and n vertices in its partitions is denoted a $K_{m,n}$.*

(a) A bipartite graph that is maximal with respect to edges is said to be complete. Draw a complete $K_{7,4}$ bipartite graph. Arrange the vertices in each partition horizontally with the larger partition above the smaller one. [5 points]

(b) Draw a connected $K_{7,4}$ bipartite graph that is minimal with respect to edges. Arrange the vertices in each partition horizontally with the larger partition above the smaller one. [5 points]

(c) Give a formula for the number of edges $\|K_{m,n}\|_{\max}$ in a complete bipartite graph $K_{m,n}$, where $m \geq n > 0$. [5 points]

(d) Give a formula for the number of edges $\left\|K^{con}_{m,n}\right\|_{\min}$ in a connected bipartite graph $K_{m,n}$, where $m \geq n > 0$. [5 points]

(e) Give the maximum possible degree, in terms of m and n, in a connected bipartite graph $K_{m,n}$ that is minimal with respect to number of edges. Assume that $m \geq n > 0$. [5 points]