

Make sure you understand the following argument, and you should be able to give similar arguments for other combination of at-most/at-least, combined with and/or and big/not-big and sour/not-sour.

Let B = subset of big fruits, S = subset of sour fruits, $b = |B|$, and $s = |S|$.
Let $\neg S$ denote the complement of S (i.e., subset of fruits that are not sour),
 $B \cap \neg S$ denote the intersection of B and $\neg S$ corresponding to the fruits that are big and non-sour, and $B \cup \neg S$ denote the union of B and $\neg S$ corresponding to the fruits that are big or non-sour.

Maximizing $B \cap \neg S$ means maximizing overlap of B and $\neg S$. Because $|B| = b$ and $|\neg S| = f - s$, maximum overlap is of size $\min(b, f - s)$. Clearly, it cannot be more than either of b and $f - s$ and hence cannot be more than $\min(b, f - s)$.

For $f = 10$, $b = 6$, and $s = 7$, we get $\max |B \cap \neg S| = \min(6, 10 - 7) = 3$. If we had $s = 2$, then $\max |B \cap \neg S|$ would be $\min(6, 10 - 2) = 6$. Indeed, we could fit all 6 big fruits within 8 non-sour fruits.

In conclusion, there are at most $\min(b, f - s)$ fruits that are big and not sour.

What is the minimum size of $B \cap \neg S$? This requires keeping B as much as possible inside S , that is maximizing $B \cap S$. From above, this is $\min(b, s)$. Note that by subtracting the size of $B \cap S$ from that of B we get the size of $B \cap \neg S$. Thus, there are at least $b - \min(b, s) = b + \max(-b, -s) = \max(b - b, b - s) = \max(0, b - s)$ fruits that are big and not sour.

For $f = 10$, $b = 6$, and $s = 7$, we have at least $\max(0, 6 - 7) = 0$ fruits that are big and not sour. Because we could fit all big fruits within the sour fruits, in this case there are no fruits that are big and not sour. If s were 2, then there would be at least $\max(0, 6 - 2) = 4$ fruits that are big and not sour. Indeed, we could have at most 2 fruits that are big and sour and hence at least 4 big and non-sour fruits.

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