Long Quiz #1 (05 Feb): CSC-2259: Discrete Structures, Sp 2019
Your answers must be to the point. Total = 100; marks for each question is shown in [ ].

LastName: FirstName

1. State one of the DeMorgan's laws and one of the Distributive laws for set operations. [2+3]

   (a) (b)

   Complete the equations below in terms of $|A|$, $|B|$, $|A \cap B|$, etc. [2+4+4]

   (a) $|A \cup B|$ = ...

   (b) $|A \cup B \cup C|$ = ...

   (c) max $|A \cup B \cup C|$ = ... and min $|A \cup B \cup C|$ = ...

2. For three sets A, B, and C, express the numbers in (a)-(b) in terms of $|A|$, $|B|$, $|C|$, $|A \cap B|$, $\cdots$, $|A \cap B \cap C|$. [3+5+2]

   (a) #(items in A and not in $B \cup C$) =

   (b) #(items in exactly 1 of A, B, and C) =

   (c) Draw a Venn-diagram and shade the area for items in exactly 1 of A, B, and C.

3. Complete the sentences below. [2+(4+2)+2+2+3]

   (i) The number of m-subsets of the n-set $\{x_1, x_2, \cdots, x_n\}$ is denoted by C(......., ...).

   (ii) For $m \geq 1$, the m-subsets of $\{x_1, x_2, \cdots, x_n\}$ are of two types: (a) does not contain x_n, and (b) contains x_n.

   For $n = 4$ and $m = 3$, show the m-subsets of type (a) and type (b) below; also show the $(m-1)$-subsets of the $(n-1)$-set $\{x_1, x_2, x_3\}$.

3-subsets of $\{x_1, \cdots, x_4\}$ not containing x_4 (type (a))	3-subsets of $\{x_1, \cdots, x_4\}$ containing x_4 (type (b))	2-subsets of $\{x_1, x_2, x_3\}$

   Draw lines from subsets in column 2 to those in column 3 to show a suitable relationship between them.

   (iii) For general $n \geq m \geq 1$, the number of m-subsets of type (a) is .... and that of type (b) is ....

   (iv) This gives the total number of m-subsets of an n-set = ...

   (v) Thus, ... = ...

4. Complete the statements equations below. [2+2+2+2]
   (a) The total number of subsets of an n-set = ...
   (b) $C(n, 0) + C(n, 1) + \cdots + C(n, n)$ = the total ....
   (c) Thus, 2^n = ...
   (d) The symmetry property of $C(n, m)$ means $C(n, m)$ = ...

5. Suppose Ω = Universe of discourse (the set of things under consideration), H = the set of things in Ω that I have, and W = the set of things in Ω that I want. Express each of the following using set notations (subset, union, intersection, complement, etc) unless indicated othrrwise. (Use of Venn-diagram's maybe helpful.) [2+2+2+2+2+2]
   (a) I have every thing I want:
   (b) I have nothing that I want:
   (c) I only have things that I want (i.e., I don't have any thing that I don't want):
   (d) I have nothing that I don't want:
   (e) Express $H \supseteq W^c$ in another way using set notations but without using complement.
   (f) Complete the sentence below to express in English the situation $H \supseteq W^c$ (without using set terminology).

       I have .....

6. Give a clean and efficient code for testing whether $H = W$ or not. Do the same for testing $H \subseteq W$. Assume that sets H and W are given as 0/1-arrays. [10+4]

7. Consider a given Ω and a subset H of Ω, where $0 \le m = |H| \le |\Omega| = n$. [2+2+2]
   (a) Give #(subsets W of Ω such that $H \subseteq W$).
   (b) Give #(subsets W' of Ω such that $H \cap W' = \emptyset$).
   (c) How are the sets W in (a) related to the sets W' in (b)?

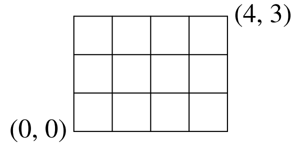
8. Suppose we have 10 straigtlines L_1, L_2, $\cdots$, L_{10}. [3+4+3]
   (a) Why is the maximum number of intersection points of these lines is $C(10, 2) = 45$?

   (b) Give two reasons why the maximum number of triangles formed by these lines must be less than $C(45, 3)$?
   (i)

   (ii)

   (c) Explain why the maximum number of triangles formed by these intersection points is 120?

9. Consider an $m \times n$ 2-dimensional grid of unit squares; shown below is such a grid for $m = 4$ and $n = 3$.



Answer the following questions. [4+2+(2+2)]
   (a) Give the number of rectangles of area 2 (i.e., 2×1 or 1×2) that can be formed using the unit squares for an $m \times n$ grid.

   (b) Give the number of paths from the left bottom corner (0, 0) to the top right corner (m, n) when we consider only east-moves and north-moves (but avoid west-moves and south-moves).

   (c) If we have an $m \times n \times p$ 3-dimensional grid, then give the number of paths from (0, 0, 0) to (m, n, p) when we only consider horizontal east-moves (E), horizontal north-moves (N), and vertical upward-moves (U).

   Give one such move-sequence for $m = 4$, $n = 3$, $p = 3$.