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CSC 2259 master guide (→ to midterm)

Divider 1

- discrete vs non-discrete (continuous)
- syntax tree
- most efficient min code:

```
if (x<y) min = x;
   else min = y;
```
} → 1 test, 1 assignment

- pascal's triangle: $(x+y)^n \to$ binomial theorem

row 0: 1
row 1: 1 1
row 2: 1 2 1
row 3: 1 3 3 1

- definition of choose

$C(5,2) = \binom{5}{2} = \frac{5 \cdot 4}{2 \cdot 1}$ $C(n,m) = \frac{n(n-1)(n-2)\dots(n-m+1)}{m(m-1)(m-2)\dots(1)}$

Divider 2

- $\{x_1, x_2, \dots, x_m\}$ ← $\{x_1, x_2, \dots, x_{m-1}\}$

$\binom{n}{m}$ $\{x_1, x_2, \dots, x_{m-2}, x_m\}$

$\vdots$

$\{x_1, x_3, \dots, x_m\}$

$\{x_2, x_3, \dots, x_m\}$ } m lines

$\{x_1, x_2, \dots, x_{m-1}\}$ $\{x_1, x_2, \dots, x_{m-1}, x_m\}$

$\binom{n}{m-1}$ $\{x_1, x_2, \dots, x_{m-1}, x_{m+1}\}$

$\vdots$

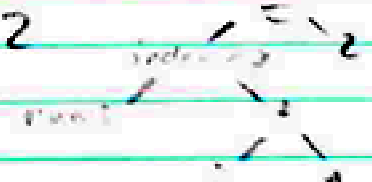
$\{x_1, x_2, \dots, x_{m-1}, x_{n-1}\}$

$\{x_1, x_2, \dots, x_{m-1}, x_n\}$ } n-m+1 sets

total # lines $= m \cdot C(n,m) = (n-m+1) \cdot C(n, m-1)$

$C(n,m) = C(n,m-1) \cdot \frac{n-m+1}{m}$

- nums[i+1] = 2



- structures - how different pieces relate to each other
- $C(n,m)$ = # of ways of choosing m items out of n items
- symmetric property: $\binom{n}{m} = \binom{n}{n-m}$
- $\sum_{k=0}^{n} \binom{n}{k} = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n} = 2^n$
- example of recursive proof

  $\{a,b,c,d,e\} = n$

  $\{a,b\}$ $\{b,c\}$ $\{c,d\}$ $\{d,e\}$ $\qquad$ $\{a\}$ $\{d\}$

  $\{a,c\}$ $\{b,d\}$ $\{c,e\}$ $\qquad$ $\{b\}$ $\{e\}$

  $\{a,d\}$ $\{b,e\}$ $\qquad$ $\{c\}$

  $\{a,e\}$

  (10) $\qquad$ (5)

  connecting these two sets: $2 * 10 = 4 * 5$
- non recursive proof (via recursive)

$$C(n,1) = C(n,0) * \frac{n-1+1}{1} = C(n,0) * n = n$$

$$C(n,2) = C(n,1) * \frac{n-2+1}{2} = n * \frac{n-1}{2}$$

$$C(n,3) = C(n,2) * \frac{n-3+1}{3} = \frac{n(n-1)(n-2)}{3(2)1}$$

$$C(n,m) = C(n,m-1) * \frac{n-m+1}{m}$$

$$= C(n,m-2) * \frac{n-m+1+1}{m-1} * \frac{n-m+1}{m}$$

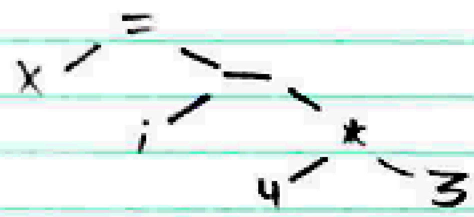
$$= C(n,m-3) * \frac{n-m+3}{m-2} * \frac{n-m+2}{m-1} * \frac{n-m+1}{m}$$

$$\vdots$$

$$= C(n,0) * \frac{n}{1} * \frac{n-1}{2} * \dots * \frac{n-m+3}{m-2} * \frac{n-m+2}{m-1} * \frac{n-m+1}{m}$$

$$= \frac{n(n-1)(n-2)\dots(n-m+1)}{m(m-1)(m-2)\dots(1)}$$

- $x = i - 4 * 3$



Divider 3

- for recursive proof.
  - #(m-subsets) connect #((m-1)-subsets)
- recurse code

```
int[] C = new int[n+1];
C[0] = 1;
C[1] = n;
for (int m=2; m≤n; m++)
    C[m] = C[m-1] · (n-m+1)/m;
```

$\#(\times/\div) = 2$, $\#(+/-) = 3$ → $\#(\text{arith}) = 5$

$\#(\text{ass.}) = 1$

$\#(\text{iterat.}) = n-1$

$\text{total} = (n-1)(6)$

- dont use curly brackets when unnecessary
- non-recurse code

```
for (int m=2; m≤n; m++)
    int top = n, bottom = m;
    for (int i = 2; i<m; i++)
    {   top *= (n-i+1);
        bottom *= (m-i+1);
    }
    C[m] = top / bottom;
```

$\#(\text{ope.}/\text{it. in } i\text{-loop}) = 8$

$\#(\text{it. of } i\text{-loop}) = m-1$

total for m: $(n-1)(8(n-1)+4)$

total for i-loop $8(n-1)$

mistake ↑

$8(2-1)+4+8(3-1)+4+ \dots + 8(n-1)+4$

$4(n-1)+8(1+2+3+\dots+n-1)$

$= 4(n-1)+8\left(\frac{(n-1)(n)}{2}\right) = 4(n-1)+4(n-1)(n)$

$= 4(n-1)(1+n) = 4(n^2-1)$ $\boxed{4(n^2-1) \text{ vs } 6(n-1)}$

- non-recursive uses much more computation

Divider 4

- $\cup$ - union, $\cap$ - intersection, $A \cap B = \emptyset$ means disjoint
- X^c - complement, $|X|$ - size/cardinality, $\bar{X} = X^c$
- $|X| = m$, $|X^c| = n-m$, $\{a,c\}^c = \{a,b,c\}$
- X and X^c has a 1-1 and onto relationship
- pascal triangle property = $C(n,m) = C(n-1,m-1) + C(n-1,m)$

  ex: $C(5,2) = C(4,1) + C(4,2)$

- {a,b,c} — B   idea of
  {a,b,d}
  {a,b,e}
  {a,c,d}
  {a,c,e}
  {a,d,e} — A
  {b,c,d}
  {b,c,e}
  {b,d,e}
  {c,d,e}

$C(n,m) = C(n-1, m-1) + C(n-1, m)$
with
$C(5,3) = C(4,2) + C(4,3)$

- efficient $C(n,m)$ code

```
int top = n, bot = m;
for (int i = 1; i < m, i++)
{  top *= n - i;
   bot *= i;
}
```

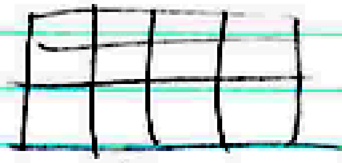
- disprove $C[m] = (n-m+1)/m * C[m-1]$
  $C(6,2)$ ✓
- for $n=6$, $m=4$, show links given these 2:

{2,1,6}	{2,3}
{2,3,6}	{2,6}
{2,4,6}	{3,6}
{2,5,6}	

- Divider 5
- box problem. box has 10 fruits. 7 sour. 6 big.
  - types of statements e.g. "at most 6 are sour & big", "3 are not sour"
- line problem   max # intersection points $C(n,2)$

- any 3 mutually intersecting lines form a △
- if there are actually 2 lines then an intersection point
- why can't $C(6,3)$ triangles? 1 points not joined by any lines
  2 3≥ collinear points

- grid problem
  - mississippi method
  - leunda's casework
    #(ways with 0 turns) ... and so on



this is a "4 by 5" grid according to leunda

- binary strings
  - $00001^c = 11110$
  - basically another $C(n,m)$ type deal

Divider 6

- binary string (cont.)
  - binary strings of m ones length n = $C(n,m)$
  - all binary strings of size $n = \sum_{k=0}^{n} \binom{n}{k} = 2^n$
  - proving $\sum \binom{n}{k} = 2^n$ via binary strings
- structural property
  - ex: - recursive def of $\binom{n}{m}$ — $\sum_{0}^{n} \binom{n}{k} = 2^n$
    - symmetry prop of $\binom{n}{m}$
    - pascal
  - not: - non-recurse def of $\binom{n}{m}$

proving $\sum \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots = 2^{n-1}$ using $C(n,m) = C(n-1,m-1) + C(n-1,m)$

- ? refer to 02/04/26 notes, ☆
  some bullshit about adding odd terms
- $\sum \binom{n}{k}$ for odds $= 2^{n-1}$
  $\sum \binom{n}{k}$ for even $= 2^{n-1}$ $\Big\} 2^n$
- grid prob (cont.)
  - # ways to use 4R-moves in 7-moves $C(7,4)$
  - # paths from $(0,0)$ to $(n-1, m-1)$ is $C(m-1+n-1, m-1)$
- another odd term BS

Divider 7

- 

X_4: items not in A and not in B
neither in A nor B

- $S = \{a \ldots g\}$, $A = \{a \ldots d\}$

#(subsets of A^c) $= 2^3 = 8$

#(subsets of S but not of A) $= 2^7 - 2^4 = 112$

or the $2^4(2^3-1)$ idea

$\updownarrow$ $\binom{4}{0}(2^3-1)$

$\left.\begin{matrix} \binom{4}{0}(2^3-1) \\ \binom{4}{1}(2^3-1) \\ \vdots \\ \binom{4}{4}(2^3-1) \end{matrix}\right\} 2^4(2^3-1)$

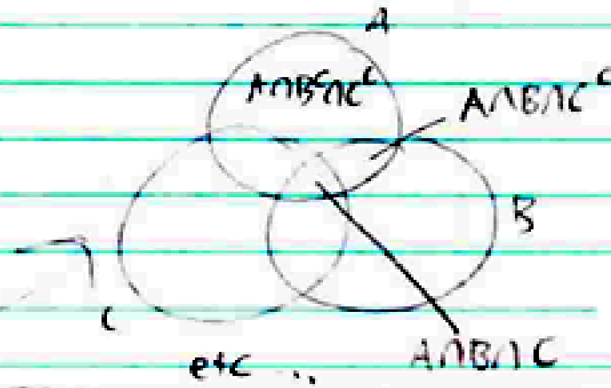
- demorgan's law

$(A \cup B)^c = A^c \cap B^c$

$(A \cap B)^c = A^c \cup B^c$

- clique problem

	x_1	x_2	x_3	x_4
$A \setminus B$	Y	N	N	N
all A and B	Y	Y	N	N
$B \setminus A$	N	Y	N	N



- this thing

Divider 8

- max # fruit in basket that are big and sour $= \min(b, s)$

min # fruit in basket that are big and sour $= \max(0, b - f + s)$

- $m \cdot C(n, m)$
- $0 \cdot \binom{n}{0} + 1 \cdot \binom{n}{1} + 2 \cdot \binom{n}{2} + \cdots + n \cdot \binom{n}{n} = n \cdot 2^{n-1}$

$=$ #(items in S) $\cdot$ #(subsets of S containing a given item)

$= (n) \cdot 2^{n-1}$

-
```
int[] c = new int[n+1];
c[0] = 1;
for (int m=1; m<=n; m++)
    c[m] = c[m-1]*(n-m+1)/m;
```

Divider 9

I have something that I want $H \cap W \neq \emptyset$

I have everything that I want $H \supseteq W$

I have nothing that I want. $H \cap W = \emptyset$

$W^c \subseteq H^c$

$|H| = |W|$ vs $H = W$

$|H|=0$, $\#(W's)=16$
$|H|=1$, $\#(W's)=8$
$|H|=2$, $\#(W's)=4$
$|H|=3$, $\#(W's)=2$
$|H|=4$, $\#(W's)=1$

$$\sum_{m=0}^{n} C(n,m)x^{n-m}y^{m}$$

$\#$(ways of choosing H,W) such $H=W$

H and W such that $H \subseteq W$

$$\sum_{m=0}^{n} \underbrace{C(n,m)}_{H \text{ of size } m} \cdot \underbrace{2^{n-m}}_{W \text{ can have remaining}} = (2+1)^{m} = 3^{n}$$

- $H[i] > W[i] \Leftrightarrow$ efficient
- for (int i=0; i<n; i++)

```
for (int i=0; i<n; i++)
    if (H[i] > W[i])
        return(false);
return(true);
```

Divider (C)

Suppose there are 4 things.

ways choose H & W such $H=W$?

such $H \subseteq W$ knowing $|H|$?

such $H \subseteq W$

$\#$ ($W \supseteq H$ where $|H|=m$)
$= C(n,m) \cdot 2^{n-m}$

$\#((H,W)$-pairs such that $H \subseteq W) = 3^{n}$

$\frac{\text{all cases}}{\#(\text{cases})}$ $\frac{\text{sums}}{\text{iteration}}$ $\frac{\text{all oranges}}{\#\text{boxs}}$

- code to test $H \subseteq W$ test iterations
- proof for $1.2.4^{n-1}+2.2^{2}.4^{n-2}+\ldots+n.2^{n}+n.2^{n}$ (for $H=W$ first)
- $1.4^{n-1}+2.3\,4^{n-2}+3.3^{2}.4^{n-3}+\ldots+n.3^{n-1}+n.3^{n}/4^{n} = 4(1-(\frac{3}{4}))^{n}$

Divide H

$H \Delta W = W \Delta H = (H \setminus W) \cup (W \setminus H)$