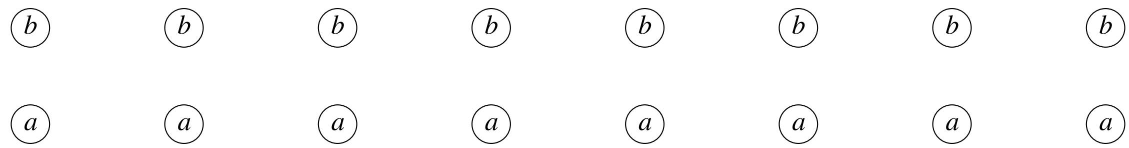
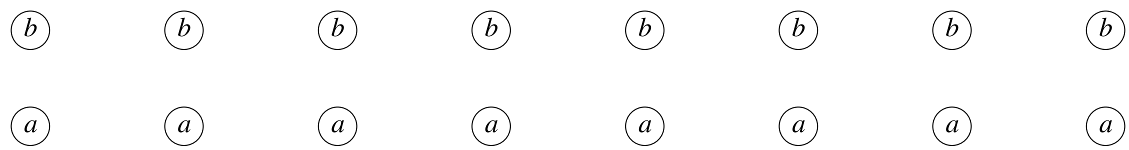
Practice Questions for Mar 14, 2019

1. We have shown there are 2^{n^2} (binary) relations on an n-set. Draw the diagrams for all 16 relations on the 2-set $\{a, b\}$ by filling the links in (r.1)-(r.16) below. As usual, follow some systematic rules to avoid duplicate diagrams; for example, first show all diagrams with no links, then those with just 1 link, then those with just 2 links, and so on.



(r.1) (r.2) (r.3) (r.4) (r.5) (r.6) (r.7) (r.8)



(r.9) (r.10) (r.11) (r.12) (r.13) (r.14) (r.15) (r.16)

2. Cross out all relations in Q1 that are not symmetric.
3. Correct all errors in the argument below in counting #(symmetric relations on an n-set).
   (a) For the $n \times n$ matrix $R[i][j]$, $0 \leq i,\ j < n$, of a symmetric relation, we can choose the diagonal items and the items above the diagonal arbitrarily (0 or 1).
   (b) The number of diagonal items and the items above the diagnal are $n + (n-1) + (n-2) + \cdots + 2 + 1 = n(n-1)/2$.
   (c) Thus, #(symmetric relations on an n-set) $= 2^{n(n+1)/2} = 2^{n(n+1)/2}$.
4. What does this say about #(non-symmteric relations on an n-set)
5. Give the number of symmetric relations on a 2-set. How does this match with your answers in Q1-Q2?
6. Explain why the statement below

   #(relations on an n-set) > #(symmetric relations on an n-set)

   is not true. Then, give two best possible correct versions of the statement. How will you prove them?
7. Fill in the links below for a largest size (in terms of #(links)) symmetric and a largest non-symmetric relation on $S = \{a, b, c\}$. Do the same for a smallest symmetric and a smallest non-symmetric relation. Which of these are unique? For the non-unique ones, give the number of possible alternatives.

(i) A largest symmetric relation. (ii) A largest non-symmetric relation. (iii) A smallest symmetric relation. (iv) A smallest non-symmetric relation.

8. How many ways can you fill the positions marked '?' in $A = \begin{bmatrix} 1 & 0 & 1 \\ ? & 0 & 0 \\ ? & ? & ? \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ ? & 0 & ? \\ ? & ? & ? \end{bmatrix}$. to get a symmetric relation?

   How about to get a non-symmetric reltaion?
9. Count the number of relations on an n-set having m links. Verify your answer for $n = 2$ based on your answer in Q1. Repeat the above for symmetric relations.