Math2090, Section 1
Quiz 8, Total 10 points

Last name: First name:

1. Determine the general solution of differential equation $y'' - 6y' + 9y = 0$. 4 pt

The auxiliary polynomial $P(r) = r^2 - 6r + 9$

$r^2 - 6r + 9 = 0 \Rightarrow (r-3)^2 = 0 \Rightarrow r = 3$

$\therefore y_1(x) = c_1 e^{3x}$, $y_2(x) = c_2 x e^{3x}$

$\therefore$ The general solution is $y(x) = c_1 e^{3x} + c_2 x e^{3x}$.

2. Determine the general solution to the differential equation $(D-2)(D+3)y = 15e^{-3x}$. 6 pt

The auxiliary polynomial $P(r) = (r-2)(r+3)$

$\therefore y_1(x) = c_1 e^{2x}$, & $y_2(x) = c_2 e^{-3x}$

$\therefore y_c(x) = c_1 e^{2x} + c_2 e^{-3x}$.

$F(x) = 15e^{-3x}$. Now, $A(D) = D+3$ annihilate $15e^{-3x}$

$\therefore (D+3)(D-2)(D+3)y = 0$

$(r+3)(r-2)(r+3) = 0 \Rightarrow r = 2, r = -3, -3$

$\therefore y(x) = c_1 e^{2x} + c_2 e^{-3x} + A_0 x e^{-3x}$

Now, we find A_0.

Let $y_p(x) = A_0 x e^{-3x} \Rightarrow y_p'(x) = A_0 e^{-3x}(-3x+1)$

$y_p''(x) = A_0 e^{-3x}(9x-6)$

$A_0 e^{-3x}((9x-6) + (-3x+1) - 6x) = 15e^{-3x}$

$-5A_0 e^{-3x} = 15e^{-3x} \Rightarrow A_0 = -3$

$\therefore$ The general solution $y(x) = c_1 e^{2x} + c_2 e^{-3x} - 3x e^{-3x}$.