Math2090, Section 1
Quiz 6, Total 10 points

Last name: First name:

(1) Determine whether the following are linear transformations. Please justify if your answer is NO. 3 pt

a). $T : C^2(I) \to C^0(I)$ defined by $T(y) = y'' + 3y' - xy$.

1) $T(z+y) = (z+y)'' + 3(z+y)' - x(z+y) = z'' + y'' + 3z' + 3y' - xz - xy = (z''+3z'-xz)+(y''+3y'-xy)$
$= T(z) + T(y)$.

2) $T(ky) = (ky)'' + 3(ky)' - x(ky) = ky'' + k(3y)' - k(xy) = k\,T(y)$. Therefore T is a linear transformation.

b). $T : P_2(\mathbb{R}) \to \mathbb{R}$ defined by $T(a + bx + cx^2) = a + b + c + 2$.

No. Ex: $T(1+x^2) = 1+1+2 = 4$

but $T(2(1+x^2)) \neq 2\,T(1+x^2)$

because $T(2(1+x^2)) = T(2+2x^2) = 2+2+2 = 6 \neq 2 \cdot 4 = 8$.

(2) (a) Determine the matrix of the transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined by 2 pt

$$T(x_1, x_2, x_3) = (x_1 - x_2 + 3x_3, x_3 + x_1).$$

(b) Find the kernel and the range of T by describing $\mathrm{Ker}(T)$ and $\mathrm{Rng}(T)$ as sets, and compute their corresponding dimensions. 5 pt

a) The standard basis for $\mathbb{R}^3$ $e_1 = (1,0,0)$, $e_2 = (0,1,0)$, $e_3 = (0,0,1)$

$T(e_1) = (1,1)$, $T(e_2) = (-1,0)$, $T(e_3) = (3,1)$

$\therefore$ The matrix of T is $[T(e_1), T(e_2), T(e_3)] = \begin{bmatrix} 1 & -1 & 3 \\ 1 & 0 & 1 \end{bmatrix}$.

b).

$\mathrm{Ker}\,T = \{(x_1,x_2,x_3) : T(x_1,x_2,x_3) = (0,0,0)\}$

$x_1 - x_2 + 3x_3 = 0$ and $x_1 + x_3 = 0$. $x_1 = -x_3$ [redacted] let $x_3 = t$ then $x_1 = -t$

and $x_2 = 2t$

$\therefore$ $\mathrm{Ker}\,T = \{(-t, 2t, t) : t \in \mathbb{R}\} = \{t(-1,2,1) : t \in \mathbb{R}\}$

$\dim \mathrm{Ker}\,T = 1$.

$= \mathrm{span}\{(-1,2,1)\}$.

$\mathrm{Rang}(T) = \{T(x_1,x_2,x_3) : (x_1,x_2,x_3) \in \mathbb{R}^3\}$

$= \{x_1(1,1) + x_2(-1,0) + x_3(3,1)$ where $x_1, x_2, x_3 \in \mathbb{R}\}$

$= \mathrm{span}\{(1,1), (-1,0), (3,1)\} = \mathrm{span}\{(-1,0), (3,1)\}$ because $(1,1) = 2(-1,0) + (3,1)$

$\therefore$ $\dim \mathrm{Rang}(T) = 2$. because $(-1,0)$ and $(3,1)$ are linear independent