Math2090, Section 1
Quiz 3, Total 10 points

Last name: First name:

Please justify your answer.

(1) Use Gauss elimination to determine the solution set of the system:

$$\begin{aligned} 2x_1 - 6x_2 + 2x_3 &= 10 \\ x_1 - 6x_2 - 2x_3 &= 2 \\ 2x_1 - 9x_2 - x_3 &= 7. \end{aligned}$$

$$\left[\begin{array}{ccc|c} 2 & -6 & 2 & 10 \\ 1 & -6 & -2 & 2 \\ 2 & -9 & -1 & 7 \end{array}\right] \xrightarrow{A_{21}(-1)} \left[\begin{array}{ccc|c} 1 & 0 & 4 & 8 \\ 1 & -6 & -2 & 2 \\ 2 & -9 & -1 & 7 \end{array}\right] \xrightarrow[A_{13}(-2)]{A_{12}(-1)} \left[\begin{array}{ccc|c} 1 & 0 & 4 & 8 \\ 0 & -6 & -6 & -6 \\ 0 & -9 & -9 & -9 \end{array}\right]$$

$$\xrightarrow[M_3(\frac{-1}{9})]{M_2(\frac{-1}{6})} \left[\begin{array}{ccc|c} 1 & 0 & 4 & 8 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{array}\right] \xrightarrow{A_{23}(-1)} \left[\begin{array}{ccc|c} 1 & 0 & 4 & 8 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Let $\boxed{x_3 = t}$ then $x_2 + t = 1 \Rightarrow \boxed{x_2 = 1 - t}$

$x_1 + 0 \cdot x_2 + 4x_3 = 8 \Rightarrow x_1 + 4t = 8 \Rightarrow \boxed{x_1 = 8 - 4t}$

(2) Compute A^{-1} for $A = \begin{pmatrix} 1 & 0 & -2 \\ 2 & -1 & -4 \\ -1 & 0 & 1 \end{pmatrix}$. Check your answer.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 2 & -1 & -4 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[A_{13}(1)]{A_{12}(-2)} \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & -1 & 0 & -2 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[M_3(-1)]{M_2(-1)} \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & -1 \end{array}\right] \xrightarrow{M_{31}(2)} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & -1 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} -1 & 0 & -2 \\ 2 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}.$$

To check your answer, you have to show that $AA^{-1} = A^{-1}A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

$$AA^{-1} = \begin{bmatrix} 1 & 0 & -2 \\ 2 & -1 & -4 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 & -2 \\ 2 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ and $$A^{-1}A = \begin{bmatrix} -1 & 0 & -2 \\ 2 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -2 \\ 2 & -1 & -4 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$