Math2090, Section 1
Quiz 10, Total 11 points

Last name: First name:

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1. Find $L[f(t)](s)$:
   a) $f(t) = e^{4t}\sin(3t)$

$$L[f(t)] = L[\sin(3t)](s-4)$$
$$L[\sin(3t)] = \frac{3}{s^2+9} \quad \therefore \quad \boxed{L[f(t)] = \frac{3}{(s-4)^2+9}}$$

   b) $f(t) = (t-2)u_2(t)$

$$L[f(t)] = e^{-2s} L[t] = \boxed{\frac{e^{-2s}}{s^2}}$$

2. Use Laplace transform to solve the initial valued problem $y'' + y = u_1(t)$, $y(0) = 1$, $y'(0) = 0$.

$$L[y'' + y] = L[u_1(t)] = e^{-s} \cdot \frac{1}{s}$$
$$\Rightarrow s^2 L[y] - s y(0) - y'(0) + L[y] = \frac{e^{-s}}{s}$$
$$\Rightarrow (s^2+1) L[y] - s = \frac{e^{-s}}{s}$$
$$\Rightarrow (s^2+1) L[y] = \frac{e^{-s}}{s} + s$$
$$\Rightarrow L[y] = \frac{e^{-s}}{s(s^2+1)} + \frac{s}{s^2+1}$$
$$\Rightarrow y = L^{-1}\left[\frac{e^{-s}}{s(s^2+1)}\right] + \cos(t)$$

$$\frac{1}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1} = \frac{s^2A + A + Bs^2 + Cs}{s(s^2+1)}$$
$$= \frac{(A+B)s^2 + Cs + A}{s(s^2+1)}$$

$$\therefore \quad 1 = A, \quad C = 0, \quad A+B = 0 \Rightarrow B = -1$$

$$\therefore \quad \frac{1}{s(s^2+1)} = \frac{1}{s} - \frac{s}{s^2+1} \Rightarrow L^{-1}\left[\frac{1}{s(s^2+1)}\right] = 1 - \cos(t)$$

$$\Rightarrow L^{-1}\left[\frac{e^{-s}}{s(s^2+1)}\right] = u_1(t)(1 - \cos(t-1))$$

$$\therefore \quad y = \boxed{u_1(t)(1 - \cos(t-1)) + \cos(t)}$$