Math2090, Section 1
Quiz 2, Total 10 points

Last name: First name:

Please justify your answer.

(1) (a) Show that the differential equation $\underbrace{(y-x^2)}_{M}dx + \underbrace{(x+2y)}_{N}dy = 0$ is exact. 2 pts
(b) Solve the differential equation. 4 pts

$M_y = 1$, $N_x = 1$. Since $M_y = N_x$ then the differential equation is exact.

$\phi = \int M dx = \int y - x^2 dx = yx - \frac{1}{3}x^3 + \psi(y)$

$\phi_y = N \Rightarrow x + \psi'(y) = x + 2y$

$\Rightarrow \psi'(y) = 2y \Rightarrow \psi(y) = y^2 \Rightarrow \phi(x,y) = yx - \frac{1}{3}x^3 + y^2$

$\therefore$ the general solution is $yx - \frac{1}{3}x^3 + y^2 = C$.

(2) Let $A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & -1 & -4 \end{bmatrix}_{2\times 3}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 0 & -1 \end{bmatrix}_{3\times 2}$. 4 pts

(a) Compute A^T.
(b) Compute $A + B$ if it is well-defined.
(c) Compute AB if it is well-defined.
(d) Compute BA if it is well-defined.

a) $A^T = \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -2 & -4 \end{bmatrix}$

b) we cann't compute $A+B$ because A, B have different dimension.
[dimension of A is 2×3, dimension of B is 3×2]

c) $AB = \begin{bmatrix} 1\cdot 1 + 0\cdot 2 + (-2)\cdot(0) & 1\cdot 0 + 0\cdot(-1) + (-2)(-1) \\ 2\cdot 1 + (-1)\cdot(2) + (-4)\cdot 0 & 2\cdot 0 + (-1)\cdot(-1) + (-4)\cdot(-1) \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 5 \end{bmatrix}$

d) $BA = \begin{bmatrix} 1\cdot 1 + 0\cdot 2 & 1\cdot 0 + (0)\cdot(-1) & 1\cdot(-2) + 0\cdot(-4) \\ 2\cdot 1 + (-1)\cdot 2 & 2\cdot 0 + (-1)(-1) & 2\cdot(-2) + (-1)(-4) \\ 0\cdot 1 + (-1)\cdot 2 & 0\cdot 2 + (-1)(-1) & 0(-2) + (-1)(-4) \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$