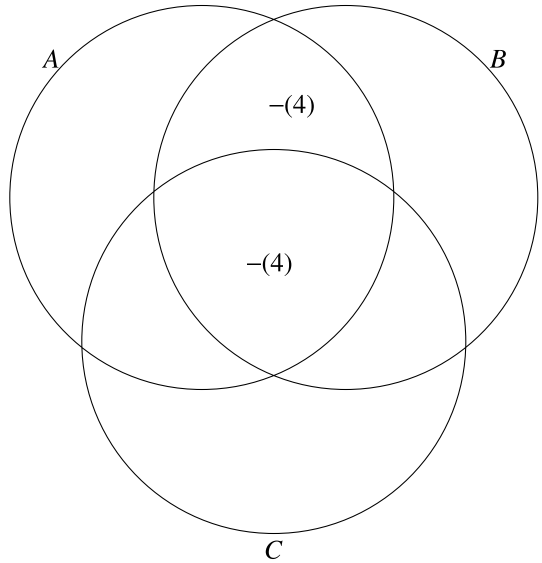

1. Consider the +ve and −ve terms below, numbered 1 to 7 terms.

$+|A|$ (term #1)	$-|A\cap B|$ (term #4)	$+|A\cap B\cap C|$ (term #7)
$+|B|$ (term #2)	$-|A\cap C|$ (term #5)	
$+|C|$ (term #3)	$-|B\cap C|$ (term #6)	

   In the above Venn Diagram, we have marked all areas that are accounted by $|A\cap B|$ as "−(4)", where "−" emphasizes that it is a −ve term and "(4)" indicates that it corresponds to term #4.

   (a) Fill the diagram by adding similar marks "+(i)" and "−(j)" etc. corresponding to the other 6 terms.

   (b) Show that the number of +ve marks for each part of $A\cup B\cup C$ (and only those parts) exceeds the number of −ve marks by exactly 1.

   (c) What does this prove?

2. Consider a program P, which given the inputs b = #(big fruits in the basket X), s = #(sour fruits in X), and f = #(all fruits in X). computes M = maximum number of fruits that could be both big and sour, and also computes m = minimum number of fruits that have to be both big and sour,

   (i) How do you know that P is no good if it outputs $M = 8$ for input $(b, s, f) = (6, 7, 14)$?

   (ii) Now suppose the program is modified and it outputs $M = 6$ for the above input but its other output $m = 2$. What can you say about the modified program and why?

   (iii) Give the code for computing M (= at most ...) and m (= at least ...) from inputs b, s, and f.

   (iv) If we just want to compute M, what should be the inputs? What if we just want to compute m?

3. Let B = subset of big fruits in the basket X, S = subset of sour fruits in X, $b = |B|$, $s = |S|$, and f = #(all fruits in X). Then, there are at most $\min(b, s)$ big and sour fruits in X, and at least $\max(0, b + s - f)$ big and sour fruits.

   (i) Let R = subset of ripe fruits in X and $r = |R|$. Complete the sentence below; use a formula in terms of b, s, etc.

   There are at most . . . . many fruits that are big, sour, and ripe in X.

   (ii) Suppose $b = 6$, $s = 7$, $r = 4$, and $f = 10$. Give a Venn diagram to verify your answer in (a).

   (iii) For the case in (ii), what can you say about at least how many fruits are big, sour, and ripe? Give a Venn Diagram to illustrate your answer.

   (iv) How would that answer change if $f = 9$ and $f = 8$?

   (v) Find the condition(s) for the answer 0 for the minimum number of fruits that have to be big, sour, and ripe. (Hint: If any of $|B\cap S|$, $|B\cap R|$, and $|S\cap R|$ equal 0, then clearly $|B\cap S\cap R| = 0$; there are still other ways that $|B\cap S\cap R|$ may be 0.)

4. Let S = A set of things, H = subset of things in S that I have, and W = subset of things in S that I want.

The sentence "I have every thing I want (which has the same meaning as "I have what I want") can be expressed in set notation as $H \supseteq W$ (or, equivalently, $H^c \subseteq W^c$ or $W \subseteq H$).

Express each of the following sentences in set notation; avoid complement as much as possible.

I have something(s) that I want.
I have nothing that I want.
I have every thing that I don't want.
I have nothing that I don't want.
I have something(s) that I don't want.
I have something(s) that I want and I have something(s) don't want.
I don't have everything that I want.
I don't have something(s) that I want.
I don't have anything that I want.