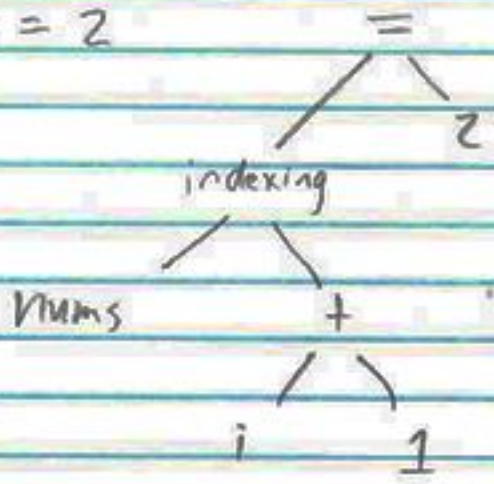
CS 2259 1/21/20

nums[i+1] = 2



structures - how different pieces relate to each other

pascal triangle - structure
- relation of #'s

# of ways of choosing m items out of n items
or
{1, 2, 3, ... n}

row 0 . . . . . 1 — C(n,m)

1 1 — 0 ≤ m ≤ n

1 2 1

1 3 3 1

row 4 . . . . . 1 4 6 4 1

$\binom{4}{0}$ $\binom{4}{1}$ $\binom{4}{2}$ $\binom{4}{3}$ $\binom{4}{4}$

symmetric property

C(4,1) = C(4,3)

C(4,0) = C(4,4)

C(n,m) = C(n,n-m)

$C(n,0) + C(n,1) + C(n,2) + \dots + C(n,n) = 2^n = \sum_{k=0}^{n} \binom{n}{k}$

proof for choose property (structural)

$C(n,m) = C(n,m-1) * \frac{n-m+1}{m}$

$\binom{n}{2}$ = # (sets $\{x,y\}$) = $\binom{n}{2}$ out of $\{1,2,3,\dots,n\}$

$\binom{n}{1} = \binom{n}{0} * \frac{n-1+1}{1} = \binom{n}{0} * n = n$

(sets $\{z\}$ out of $\{1,2,3,\dots,n\}$) = $\binom{n}{1}$

$\binom{n}{2} = \binom{n}{1} * \frac{n-2+1}{2} = n * \frac{n-1}{2}$

(lines coming out of $\{x,y\}$) = 2

$\binom{n}{3} = \binom{n}{2} * \frac{n-3+1}{3} = \frac{n(n-1)(n-2)}{3(2)}$

lines coming out of $\binom{n-1}{2}$ = $\binom{n-1}{2}$

$\binom{n}{m} = \binom{n}{m-1} * \frac{n-m+1}{m}$

$\binom{n}{k} = \binom{n}{n-k}$ $\binom{n}{k} = \binom{n}{k-1} \times \frac{(n-k+1)}{k}$

$\{1,2,3,\dots,n\}$ — $\{1,2,\dots,n-1\}$, $\{1,2,\dots,n-2,n\}$, $\{1,2,\dots,n-3,n-1,n\}$, $\{2,3,\dots,n\}$

$= \binom{n}{m-2} * \frac{n-m+3}{m-2} * \frac{n-m+2}{m-1} * \frac{n-m+1}{m}$

$= \binom{n}{0} * \frac{n}{1} * \frac{n-1}{2} * \cdots * \frac{n-m+3}{m-2} * \frac{n-m+2}{m-1} * \frac{n-m+1}{m}$

$= \frac{n(n-1)(n-2)\cdots(n-m+1)}{m(m-1)(m-2)\cdots(1)}$

$\binom{n}{m}$ = # (ways of choosing m items out of n things)

$n=5, m=2$

a, b, c, d, e

order not important

set $\{b,a\} = \{a,b\}$ $\{b,c\}$ $\{c,d\}$ $\{d,e\}$

$\{a,c\}$ $\{b,d\}$ $\{c,e\}$

$\{a,d\}$ $\{b,e\}$

$\{a,e\}$

you can list and $\binom{5}{2} = \frac{5\times4}{2\times1} = 10$

$\rightarrow$! has to be systematic

(lines coming out of $\{x,y\}$) = 2

(lines coming out of $\{z\}$) = 4

$\{a\}$ $\{b\}$ $\{c\}$ $\{d\}$ $\{e\}$

$\binom{5}{1}$

total # (lines) $= 2 * \#(\text{sets } \{x,y\})$

$2\times\binom{5}{2} = 4 * \#(\text{sets } \{z\})$

$4\times\binom{5}{2} = 4 * \#(\text{sets } \{z\})$

$\binom{5}{2} = \frac{4\times\binom{5}{1}}{2}$