Sample 6a Steps

Convert the second order equation for x:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 32\sin(3t)\cos(5t)$$

into two first order equations by doing steps a-e below:

a) Rearrange the second order equation to get the term containing the second derivative by itself on the left:

$$m\frac{d^2x}{dt^2} = 32\sin(3t)\cos(5t) - c\frac{dx}{dt} - kx$$

b) Define the first derivative of x to be the velocity v of the mass:

$$\frac{dx}{dt} = v$$

c) Substitute this into the second derivative of x. When this substitution is done, the second derivative of x becomes the first derivative of v:

$$\frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d}{dt}(v) = \frac{dv}{dt}$$

d) Then substitute this into the term that contains the second derivative of x in the equation in part a, and also substitute the equation in part b into the term that contains the the first derivative of x. When these two substitutions are done, the second order equation in part a becomes the following first order equation:

$$m\frac{dv}{dt} = 32\sin(3t)\cos(5t) - cv - kx$$

e) Then in order to get the first derivative by itself on the left in the above equation, divide both sides of the equation by m:

$$\frac{dv}{dt} = \frac{1}{m}\left(32\sin(3t)\cos(5t) - cv - kx\right)$$

Steps a-e above have converted the second order equation for x in the statement of the problem into the two first order equations given in steps b and e, where the equation in step b is the first order equation for x and the equation in step e is the first order equation for for v.