Math2090, Section 3
Quiz 4, Total 10 points

Last name: First name:

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(1) Compute the determinant the following matrix and decide whether the matrix is invertible.

$$\begin{pmatrix} 1 & -1 & 2 & 0 \\ 0 & 2 & 0 & 4 \\ -1 & 1 & 3 & 2 \\ 2 & 0 & 1 & 0 \end{pmatrix}$$

$$\text{Det} = 1 . \begin{vmatrix} 2 & 0 & 4 \\ 1 & 3 & 2 \\ 0 & 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & 4 \\ -1 & 3 & 2 \\ 2 & 1 & 0 \end{vmatrix} + 2 \begin{vmatrix} 0 & 2 & 4 \\ -1 & 1 & 2 \\ 2 & 0 & 0 \end{vmatrix} + 0$$

$$= 2 \begin{vmatrix} 3 & 2 \\ 1 & 0 \end{vmatrix} - 0 + 4 \begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} + 4 \begin{vmatrix} -1 & 3 \\ 2 & 1 \end{vmatrix} + 2 \left[-2 \begin{vmatrix} -1 & 2 \\ 2 & 0 \end{vmatrix} + 4 \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix} \right]$$

$$= 2(-2) + 4 + 4(-1-6) + 2[-2(-4) + 4(-2)]$$

$$= -4 + 4 - 28 + 2[+8 - 8]$$

$$= -28.$$

The matrix is invertible because the determinant is not zero.

(2) Determine the eigenvalues of the matrix $A = \begin{pmatrix} 0 & 6 \\ 1 & 1 \end{pmatrix}$. That is, determine the scalars λ such that $\det(A - \lambda I) = 0$.

$$A - \lambda I = \begin{pmatrix} 0 & 6 \\ 1 & 1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -\lambda & 6 \\ 1 & 1-\lambda \end{pmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} -\lambda & 6 \\ 1 & 1-\lambda \end{vmatrix} = -\lambda(1-\lambda) - 6$$

$$= \lambda^2 - \lambda - 6$$

$$\det(A - \lambda I) = 0 \Rightarrow \lambda^2 - \lambda - 6 = 0$$

$$\Rightarrow (\lambda + 2)(\lambda - 3) = 0$$

$$\lambda = 3, -2.$$