Sample 11

Consider the following One-Dimensional Heat Equation for u(x,t) for $0 \le x \le 1$ and $0 \le t \le .2$:

$$\frac{\partial u}{\partial t} = a\frac{\partial^2 u}{\partial x^2} + f(x,t)$$

$$f(x,t) = 0$$

$$a = 1$$

with the following initial conditions:

u(x,0) = u0(x) = sin(πx)

and the following boundary conditions:

u(0,t) = gleft(t) = 0

u(1,t) = gright(t) = 0

Write a MATLAB program as follows:

1) Use the explicit full discretization scheme to calculate numerical values for the unknown u(x,t) for $0 < x < 1$ and $0 < t \le .2$. Divide the x interval [0, 1] into 12 equal subdivisions and the t interval [0, .2] into 96 equal subdivisions (there will be 13 equally spaced grid points in the x interval and 97 equally spaced grid points in the t interval). Use the variables L for the length of the x interval, T for the size of the t interval, nx and nt for the number of grid points in the x and t intervals, and hx and ht for the stepsizes in the x and t intervals. The main program will call a function named heat1 that solves the One-Dimensional Heat Equation for the unknown u and returns it to the main program. The first line of heat1 is:

   function u = heat1(f, u0, gleft, gright, a, nx, nt, L, T)

2) Plot u versus x and t for $0 \le x \le 1$ and $0 \le t \le .2$. u will be a surface in 3-dimensional space. Use the MATLAB function surf to plot u.

The graph should look like the one on the attached sheet.