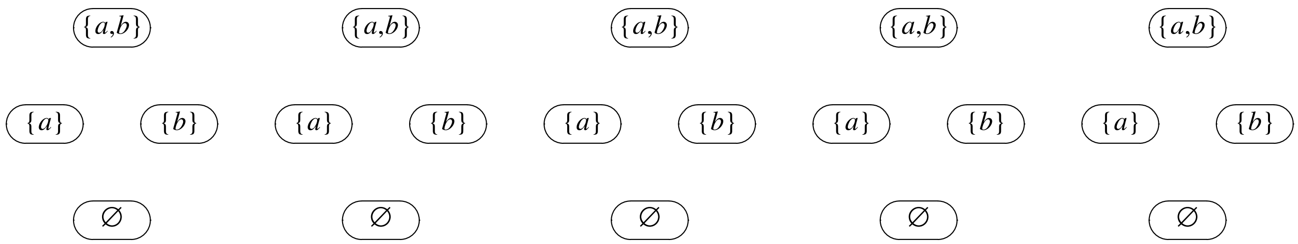
Practice Questions for Mar 19, 2019

1. Consider the "disjoint" relationship between subsets of $S = \{a,\ b\}$. We write $R_{disjoint}(x,\ y)$ or $(x,\ y) \in R_{disjoint}$ when subsets x is disjoint from subsett y; we draw all such links $(x,\ y)$ in the digraph representation for $R_{disjoint}$. Fill-in the links in the diagram in (a) below for the relation $R_{disjoint}$.



(a) $R_{disjoint}$. (b) $R_{disjoint\text{-}or\text{-}equal}$. (c) R_{subset}. (d) $R_{propersubset}$. (e) $R_{subset\text{-}and\text{-}disjoint}$.

Likewise, fill-in the links in the diagram in (b) above for the relation "disjoint or equal", fill-in the links in the diagram in (c) above for the relation "subset", and fill-in the links in the diagram in (d) above for the relation "proper subset", and fill-in the links in the diagram in (e) above for the relation "subset and disjoint".

2. Indicate which of the relations in Q1 that are symmetric. Also, for all non-symmetric relations, mark 'X' every link in the relation that causes the relation to be non-symmetric.

3. Recall the formula $\sum_d C(n, d)C((n^2 - n)/2, (m - d)/2)$, summed over all $0 \le d \le n$ such that $m - d$ is divisible by 2, for counting the number of symmetric relations on an n-set having size m, i.e., having m links.
   - (a) What does the symbol d represents in the above formulas and why we have the restriction $0 \le d \le n$?
   - (b) Why do we have the factor $C(n, d)$ in the above formula?
   - (c) Why do we have the restriction $m - d$ is divisible by 2 and what does $(m - d)/2$ represent?
   - (d) What does $(n^2 - n)/2$ correspond to in the above formula.
   - (e) Why do we have the factor $C((n^2 - n)/2, (m - d)/2)$ in the above formula.

4. Complement of a relation R on S is a relation R^c on S defined as follow:

$$(x,\ y) \text{ is in } R^c \text{ if and only if } (x,\ y) \text{ is not in } R.$$

Show the diagram of $R_{not\text{-}disjoint}$ corresponding to $R_{disjoint}$ in Q1. Is it true that R^c is symmetric when R is symmetric? How about R^c is non-symmetric when R is non-symmetric?

5 Apply the formula in Q3 to determine $S_{n,m}$ = #($n \times n$ symmetric relations with m links) for $n = 3$ and $m = 0, 1, 2, \cdots, 9$. Make a table like the one shown below for computations for $S_{3,m}$'s.

m	d	$C(3,\ d)$	$C(3,\ (m-d)/2)$	#(3×3 Symm. rels. with m links)
...	...			

Finally, verify the formula $2^{n(n+1)/2}$ for the total number of symmetric relations on an n-set.