AN EFFICIENT CODE TO TEST $H \subseteq W$

Assume H is a binary-array of length n, where 1 in a position means the associated item is in H. Similarly for W.

```
for (int i = 0; i < n; i++)
    if (H[i] > W[i]) return(false);
return(true); //i = n
```

Counting #((H, V)-pairs that returns false after k loop-iterations).

We had shown in the class the cases $k = 1$ and 2 when $n = 4$. Shown below are those cases for general n.

- Case $k = 1$: #((H, V)-pairs) = $2^{n-1}. 2^{n-1} = 4^{n-1}$.

  This is because here ($H[0]$, $W[0]$) = (1, 0), i.e., $H[0] = 1$, $W[0] = 0$; also, all other ($H[i]$, $W[i]$) can be arbitrary, giving $2^{n-1}. 2^{n-1}$ choices.

- Case $k = 2$. #((H, V)-pairs) = $3. 2^{n-2}. 2^{n-2} = 3. 4^{n-2}$.

  (a) This is because here ($H[0]$, $W[0]$) can be anything but (1, 0), i.e., any of (0, 0), (0, 1), and (1, 1), which give 3 possibilities.

  (b) ($H(1)$, $W(1)$) = (1, 0).

  (c) The remaining ($H[i]$, $W[i]$) for $2 \le$ i <= n can be arbitrary, giving $2^{n-2}. 2^{n-2} = 4^{n-2}$ choices.

Practice Problems.

1. Assume $n = 4$. Determine the following:

   (a) #((H, W)-pairs that return false value after k-iterations) for $k = 3$ and 4.

   (b) #((H, W)-pairs that return true value after 4-iterations).

   (c) The average number of iterations.

2. Give all details to show that for the general case of $1 \le k \le n$ loop-iterations and returning false value, #((H, W)-pairs) = $3^{k-1}4^{n-k}$.

   Give all details to show that for the returning true value (after n loop-iteration), #((H, W)-pairs) = 3^n.

   Show that the sum of all the counts (for return value false and the return value true) is 4^n. (Hint: see the note below.)

Note

- For $n = 4$, the sum of all the counts (for return value false and the return value true) equals $(4^3 + 3. 4^2 + 3^2. 4 + 3^3) + 3^4 = 4^3 + 3. 4^2 + 3^2. 4 + 3^3. 4 = 4^3 + 3. 4^2 + 3^2. 4^2 = 4^3 + 3. 4^3 = 4^4$ = total number of all (H, V)-pairs where both H and W are binary-arrays of length 4