CSC 3102 Midterm Exam

Section (1)

February 27, 2019

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- This is a four-part exam.
- Blue book is required. Fill in the information on the cover of your blue book and on the exam sheet.
- Answer exercise D.(b) on the exam paper and all other exercises in your blue book.
- Show the steps and/or calculations used to arrive at the answers in A.(b), D.(a) and D.(c).
- As indicated in the exam guidelines, you were allowed to staple up to eight pages of crib notes at the back of your blue book before the start of the exam. You may refer to those notes during the exam.
- No digital or electronic device is allowed during the exam.
- Turn in the exam and your blue book before you leave.

DURATION: 80 Minutes

Table 1: Distribution of Points

PART	WORTH	SCORE
A	$x_1 = 25$	
B	$x_2 = 25$	
C	$x_3 = 25$	
D	$x_4 = 25$	
Total	$\sum_{i=1}^{4} x_i$	85
Exam Score	100	/100

Good

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

Exercises

Instruction: Read each question carefully before providing an answer.

A. In A.(a) each function is a growth rate function $f_i : \mathbb{Z}^+ \to \mathbb{R}^+$; that is, each f_i is a positive real-valued function defined on the domain of positive integers. For the assertions in A.(b), $n \in \mathbb{Z}^+$. Base 2 logarithm is denoted by lg.

(a) For each function, give the most-restrictive asymptotic upper bound $\mathrm{O}(n^k)$, where $k \geq 0$ is an integer, or indicate no such bound exists.

i. $f_1(n) = (n^2+2)^3$ [2.5 points] ii. $f_2(n) = \lg(n^3+1)$ [2.5 points]

(b) Prove or disprove the assertion using limits and possibly L'Hôpital's rule. Give a detailed formal proof that includes the equivalence between the asymptotic notation in the assertion and a limit involving the functions.

i. $\ln\sqrt{n^3+2n} \in \Theta(\ln n)$ [10 points]

ii. $\sqrt{9^{n+2}} \in \mathrm{O}(2^n)$ [10 points]

B. For each exercise below, draw the binary tree as indicated.

(a) Draw a **Fibonacci** tree of height 4. Write the integers $2, 4, 6, \cdots, 2n$, where n is the size of the tree, in the nodes of the tree so that it is a search tree. [8 points]

(b) Draw the **full** binary search tree formed by adding the fewest number of nodes possible to the tree drawn in B.(a) so that its height remains the same. Also, write odd integers in the new nodes added. [8 points]

(c) Given that the tree drawn in B.(b) is an **AVL** tree, draw the tree formed by successively deleting its largest and second largest entries. Use in-order successor replacement strategy, if applicable. [9 points]

C. Draw the binary heap showing the links, nodes and numeric entries.

(a) Draw all possible **max-heaps**, each of which has entries 1, 2, 3, 5 and 8. [15 points]

(b) Suppose h is an initially empty binary **min-heap** of integers. Draw h after the sixth instruction is executed and again after the ninth instruction is executed in Listing 1. [10 points]

Listing 1: A Sequence of Instructions on a Binary Min-Heap

```
1 h.insert(6) ;   4 h.insert(10);   7 h.insert(5);
2 h.insert(14);   5 h.insert(3);    8 h.insert(12);
3 h.insert(13);   6 h.insert(9);    9 h.remove();
```

D. Suppose a farmer grows four kinds of fruits as shown in Table 2. The farmer's truck can take at most 8 tons of fruit to the farmer's market which is several miles away. He can make only one trip on market day.

Table 2: Quantity of Fruit on Hand

Fruit	Quantity in Tons	Profit in $1000
Apple	2	$3
Avocado	5	$6
Cherry	4	$5
Strawberry	3	$4

(a) Which fruits and what fraction of each should the farmer take to the market to maximize his total profit? What is the maximum total profit? [10 points]

(b) Suppose the fruits are indivisible, what is the maximum total profit for the fruits that he can take to the market? Trace the action of the dynamic programming algorithm used to obtain the answer by filling the empty cells in Table 3 with the relevant profits. [10 points]

Table 3: Hand-Trace of the Dynamic Programming Algorithm

	Capacity								
Grade (weight,cost)	0	1	2	3	4	5	6	7	8
NONE (0,$0)	0	0	0	0	0	0	0	0	0
Apple (2,$3)	0	0	3	3	3	3	3	3	3
Avocado (5,$6)	0	0	3	3	3	6	6	9	9
Cherry (4,$5)	0	0	3	3	5	6	8	9	9
Strawberry (3,$4)	0	0	3	4	5	7	8	9	10

(c) What kind of fruits must he select to maximize profit? Give a precise explanation on how the answer was determined using the completed Table 3. Use the 2D-array-like notation $V[i, j]$ to refer to entries of the unshaded region of the table, where $V[0, 0]$ is the top-left entry of the unshaded region. [5 points]