1. Binary Strings
   (a) Show all binary strings of length 4, grouping them according to #(ones) = 0, 1, 2, 3, and 4.
   (b) Relate the #(binary strings) in each group in Problem (a) with the numbers $C(4, m)$ for different m.
   (c) Assume that the successive bits from left to right in the binary strings of length 4 correspond to the items a, b, c, and d of the set $X = \{a, b, c, d\}$. Show the subset of X related to each binary string of length 4 in Problem (a).
   (d) How does the result in (c) help to show $C(4, 3)$ = #(binary string with 3 ones) in Problem (b)?
   (e) Why do we have #(binary strings of length 4) $= 16 = C(4, 0) + C(4, 1) + \cdots + C(4, 4)$?
   (f) What does generlization of (e) give for arbitrary $n \geq 1$? (For $n = 0$, the only binary string of length 0 is the empty-string and thus $C(n, 0) = 1$ = #(binary strings of length 0).)
   (g) Show a matching (1-1 and onto relationship) between binary strings of length 4 with one 1 and binary strings of length 4 with one 0.
   (h) How does this help to show $C(4, 1) = C(4, 3)$?
   (i) What is the connection between the notions of "complement of a binary string" and "the complemt of a subset"?.IP (j) How does (i) help to show that $C(n, m) = C(n, n - m)$, the symmetry property of Binomial numbers?

2. How can you use the formula $C(n, m) = \dfrac{n(n-1)(n-2)\cdots(n-m+1)}{m(m-1)(m-2)\cdots 2.\ 1}$ to show $C(n, m) = C(n, n - m)$? (Because one of m and $n - m$ is going to be $\leq n/2$, we can assume $m \leq n/2$. By the way, why one of m and $n - m$ is $\leq n/2$?)

3. Application $m.\ C(n, m) = (n - m + 1).\ C(n, m - 1)$ for $1 \leq m \leq n$.
   (a) Show the equation obtained by replacing m with $n - m + 1$ throughout the above formula. (Note that $1 \leq n - m + 1 \leq n$ if $1 \leq m \leq n$.)
   (b) Now assume that $C(n, m - 1) = C(n, n - (m - 1))$, i.e., $C(n, m - 1) = C(n, n - m + 1)$. Then, what can you obtain from the result in (a)?

4. Suppose we are given three facts: (1) a box has 10 fruits, (2) 6 of those fruits are big, and (3) 7 of those fruits are sour.‘
   (i) List at least 8 new valid conclusions; state each conclusion in the strongest form. (The statement "3 fruits are not sour" is stronger, i.e., more informative than each of the statements "at least 3 fruits are not sour" and "at most 3 fruits are not sour"; the last one is stronger than "at most 4 fruits are not sour". However, the conclusion "at most 2 fruits are not sour" is wrong/invalid as is "at least 4 fruits are not sour". One can also have conclusions of the form "At least ... many fruits are sour or not big".)

      (a)

      (b)

      (c)

      (d)

      (e)

      (f)

      (g)

      (h)

      ...

      ...

   (ii) Replace now 10, 6, and 7 in the facts (1)-(3) by f, b, and s; also assume $0 < b, s < f$. Restate your answers in (i) in terms f, b and s. (Hint: You may need to use maximum and minimum of two or more of b, s, f, $f - b$, $f - s$, etc.)