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CSC 02/04/20

long quiz thursday, if you finish early, you can leave early

binary strings $\quad$ matching (1-1, onto) $\quad$ subset, subset^c, bin.str., bin.str.^{c}

$\{a,b,c,d,e\}$

01001 $\quad$ subset: $\{b,e\}$

\# (strings of length n with m ones) $= C(n,m) =$ \#(m-subsets of an n-set)

binary

\# (all binary strings of length n) $= C(n,0)+C(n,1)+\cdots+C(n,n) = 2^n$

$00\cdots0 \qquad 11\cdots1$

$C(n,0)+C(n,1)+\cdots+C(n,n)=2^n$, proved via bin.str.

structural property of PASCAL's (binomial) numbers in row n

$n+1$ number in row n

$C(n,m) = \frac{n(n-1)\cdots(n-m+1)}{m(m-1)\cdots(1)}$ $\quad$ not a structural property

$C(n,m) = C(n,m-1)\cdot\frac{n-m+1}{m}$ $\quad$ property of multiple items, so it is a structural property.

$C(n,m) = C(n,n-m)$, a structural property

$C(n,m) > C(n,m-1)$ if $\frac{n-m+1}{m} > 1$, i.e. $n-m+1 > m$

i.e. $n+1 > 2m$

what happens when $m = \frac{n+1}{2}$: $\quad$ i.e. $\frac{n+1}{2} > m$

$C(n,m) = C(n,m-1)$

$n=5, m=3$ $\quad$ if n is even, no two consecutive terms equal

when n is odd, there you go

$m < \frac{n+1}{2} \Longleftrightarrow C(n,m) > C(n,m-1)$

" $>$ " " " $<$

" $=$ " " " $=$

adding odd terms gets you something

$C(m-1+n-1, m-1) = C(m-1+n-1, n-1)$

(paths from (0,0) to (n-1, m-1))

$1+4+6+4+1 = 2^4$ $\quad$ $1+6+1 = 2^3$

$1+5+10+10+5+1 = 2^5$ $\quad$ $1+10+5 = 2^4$

$1+6+15+20+15+6+1 = 2^6$ $\quad$ $1+15+15+1 = 2^5$

$1+7+21+35+35+21+7+1 = 2^7$ $\quad$ $1+21+35+64 = 2^6$

$C(n,0) + C(n,2) + \ldots + = 2^{n-1}$

$\boxed{\text{structural property}}$

general statement

illustration

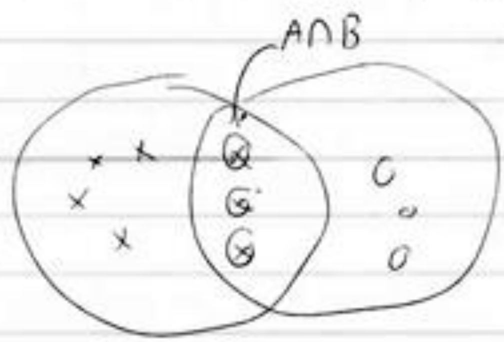
$C(n,1) + C(n,3) + \ldots + = 2^{n-1}$

$C(n,0) + C(n,1) + \ldots + C(n,n) = 2^{n-1} + 2^{n-1} = 2^n$

$2^{n-1} = C(n-1,0) + C(n-1,1) + \ldots + C(n-1,n-1)$

$C(n,m) = C(n-1,m-1) + C(n-1,m)$

$C(n,1) = C(n-1,0) + C(n-1,1)$	$C(n,1) + C(n,3) + \ldots$
$C(n,3) = C(n-1,2) + C(n-1,3)$	$= C(n-1,0) + C(n-1,1) + C(n-1,2) + \ldots$
$C(n,5) = C(n-1,4) + C(n-1,5)$	$= 2^{n-1}$



RRRRNNN
RRRNRNN
RRRNNRN
RRRNNNR
RRNRRNN
RRNRNRN
RRNRNNR
RRNNRRN
RRNNRNR
RRNNNRR

$|A| = 7 \quad |B| = 6 \quad |A \cap B| = 3$

$10 = |A \cup B| \quad |A| + |B| - |A \cap B| = |A \cup B|$

A and B disjoint, $A \cap B = \emptyset$ $\quad$ $|A \cap B| = 0$

$|A \cup B| = |A| + |B|$

= # (ways to use 4 R-moves in 7-moves)

$= C(7,4) = C(7,3) = 35$