Sample 6e Steps

Equations

$$m_1 \frac{d^2 x_1}{dt^2} = -k_1 x_1 + k_2 (x_2 - x_1)$$

$$m_2 \frac{d^2 x_2}{dt^2} = -k_2 (x_2 - x_1) + k_3 (x_3 - x_2)$$

$$m_3 \frac{d^2 x_3}{dt^2} = -k_3 (x_3 - x_2) - k_4 x_3$$

Steps

1) Rearrange the right sides of the equations by collecting the terms that multiply x_1, x_2 and x_3:

$$m_1 \frac{d^2 x_1}{dt^2} = -(k_1 + k_2) x_1 + k_2 x_2$$

$$m_2 \frac{d^2 x_2}{dt^2} = k_2 x_1 - (k_2 + k_3) x_2 + k_3 x_3$$

$$m_3 \frac{d^2 x_3}{dt^2} = k_3 x_2 - (k_3 + k_4) x_3$$

2) Divide both sides of the equations by the masses:

$$\frac{d^2 x_1}{dt^2} = \frac{1}{m_1} \left(-(k_1 + k_2) x_1 + k_2 x_2 \right)$$

$$\frac{d^2 x_2}{dt^2} = \frac{1}{m_2} \left(k_2 x_1 - (k_2 + k_3) x_2 + k_3 x_3 \right)$$

$$\frac{d^2 x_3}{dt^2} = \frac{1}{m_3} \left(k_3 x_2 - (k_3 + k_4) x_3 \right)$$

3) Define the matrix A that contains the coefficients of x_1, x_2 and x_3 in the equations. The first column of the matrix A contains the coefficients of x_1, the second column contains the coefficients of x_2, etc. The first equation gives the first row of the matrix A, the second equation gives the second row, etc. If x_1, x_2 or x_3 does not appear in an equation, its coefficient is 0 on the row of the array.

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A = [ -(k1+k2)/m1          k2/m1              0
          k2/m2    -(k2+k3)/m2          k3/m2
             0          k3/m3    -(k3+k4)/m3   ]
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