

Practice Questions for Apr 26, 2019

1. Consider a spinning wheel with 6 equal parts, one part labeled R , two parts labeled B , and three parts labeled W so that $P(R) = 1/6$, $P(B) = 2/6$, and $P(W) = 3/6$. Let our experiment consist of 3 turns of the wheel. Thus, the sample space has 27 points of the form $c_1c_2c_3$, where c_i is the color (R , B , or W) observed in the i^{th} turn. Let $X = \#(R \text{ or } B)$ and $Y = \#(B \text{ or } W)$; clearly, $0 \leq X, Y \leq 3$.

- (a) The table below shows the elementary events for some combinations of values for X and Y ("—" indicates this combinations not possible). Fill in the entries marked "?". (Each elementary event should appear exactly once.)

Elementary events for different combination of values of X and Y .					
	$Y=0$	$Y=1$	$Y=2$	$Y=3$	#(elem. events in row)
$X=0$	—	—	—	WWW	1
$X=1$	—	—	?	?	?
$X=2$	—	RRW, RWR, WRR	?	?	?
$X=3$	RRR	RRB, RBR, BRR	?	?	?
#(elem. events in column)	1	6	?	?	

- (b) Compute $E(X)$; show the details. Do the same for $E(Y)$ and $E(XY)$.
 (c) Verify whether $E(XY) = E(X)E(Y)$ or not.
2. Is there anything wrong in the following calculations; give the correct form. (Here, X and Y have nothing to do with those in Problem 1.)

$$\begin{aligned}
 E((X+Y)^2) &= E(X^2 + 2XY + Y^2) = E(X^2) + 2E(XY) + E(Y^2) \\
 E^2(X+Y) &= [E(X) + E(Y)]^2 = E^2(X) + 2E(X)E(Y) + E^2(Y) \\
 \text{Var}(X+Y) &= E((X+Y)^2) - E^2(X+Y) = E(X^2) - E^2(X) + E(Y^2) - E^2(Y) + 2[E(XY) - E(X)E(Y)] \\
 &= \text{Var}(X) + \text{Var}(Y) + 2[E(XY) - E(X)E(Y)]
 \end{aligned}$$

3. Consider the pair of integers (x, y) , $0 \leq x, y \leq 4$, as inputs to the following three codes to compute maximum of x and y .

(i) $\text{max} = x;$	(ii) $\text{max} = x;$	(iii) $\text{if } (x < y)$
$\text{if } (\text{max} < y)$	$\text{if } (\text{max} \leq y)$	$\text{max} = y;$
$\text{max} = y;$	$\text{max} = y;$	$\text{else max} = x;$

Assume that each point (x, y) has the probability $1/25$. Complete the two tables below, where $X = \#(\text{Comparisons})$ and $Y = \#(\text{Assignments})$. (We group the points (x, y) into the categories " $x < y$ ", " $x = y$ ", and " $x > y$ " because X and Y depend only on the latter for each code.)

Event	Probability	Code (i)		Code (ii)		Code (iii)	
		X	Y	X	Y	X	Y
$x < y$							
$x = y$							
$x > y$							

Code	Expected Values						Variances		
	X	Y	$X+Y$	X^2	Y^2	$(X+Y)^2$	X	Y	$X+Y$
(i)									
(ii)									
(iii)									

4. Describe the experiment, its outcomes (Sample Spaces), and their probabilities in relation to $E(X)$ and $\text{Var}(X)$ in Problem 3.