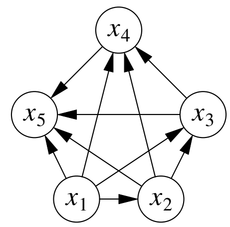
Practice Questions for Feb 21, 2019

1. Consider the following digraph, where we have a link (x_i, x_j) for each $1 \le i < j \le n$ = #(nodes) = 5. We have seen such a digraph before when we let x_i represent the integer i and the links represent "<" relationship between integers. Because there is no cycle in this digraph, such digraphs are called *acyclic* digraph (in short, DAG = directed acyclic graph). Also, because it has the maximum number of links possible for a DAG, it is called a *complete* DAG.



   (a) Give all details of computing the number of (acyclic) x_1x_5-paths.

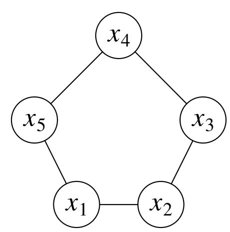
   (b) For general $n \ge 2$, give #(x_1x_n-paths).

2. If G is the complete (undirected) *graph* on n nodes $\{x_i: 1 \le i \le n\}$ and $n \ge 2$, then give a brief argument to show that the number of acyclic x_1x_n-paths is

$$1 + P(n-2, 1) + P(n-2, 2) + \cdots + P(n-2, n-2)$$

   where $P(n, m)$ = #(permutations of m items chosen from a set of n items).

3. Add as many links as possible to the graph below to get a planar graph. The resulting graph is called *maximal* planar graph on 5 nodes (maximal because you cannot add any more links and still keep the graph planar.)



   Give the number of maximal planar graphs on 5 nodes