94%

Kha Le
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CSC 3102

85%
+3
88%

1(a) $\{f_2\}, \{f_3, f_5\}, \{f_1, f_4\}$

$\xrightarrow{\hspace{6cm}}$
increasing asymptotic time complexity

1(b) $\sqrt{3n^2+5} \in \Theta(n\lg n)$

This claim is false.

Proof by Limits

A.) Definition: Let f and g be functions from $Z^+ \to R^+$.
$f(n) \in \Theta(g(n)) \iff \lim_{n\to\infty} \frac{f(n)}{g(n)} = K, \quad K \neq 0, \infty$

B.) Let $f(n) = \sqrt{3n^2+5}$ and $g(n) = n\lg n$

C.) I want to show that $\lim_{n\to\infty} \frac{\sqrt{3n^2+5}}{n\lg n} \neq K, \quad K \neq 0, \infty$

D.) Proof: $\lim_{n\to\infty} \frac{\sqrt{3n^2+5}}{n\lg n} = \lim_{n\to\infty} \frac{\sqrt{3n^2+5}}{\sqrt{n^2}} \cdot \frac{1}{\lg n} = \lim_{n\to\infty} \sqrt{\frac{3n^2+5}{n^2}} \cdot \frac{1}{\lg n}$

$= \lim_{n\to\infty} \sqrt{3+5/n^2} \cdot \frac{1}{\lg n}$

$= \left(\sqrt{3+\lim_{n\to\infty}(5/n^2)}\right) \cdot \lim_{n\to\infty} \frac{1}{\lg n} = \left(\sqrt{3+\frac{5}{\infty}}\right)\left(\frac{1}{\lg \infty}\right)$

$= (\sqrt{3+0})(0) = \sqrt{3}\cdot 0 = 0$

E.) Conclusion: Since $f(n) \in \Theta(g(n)) \iff \lim_{n\to\infty} \frac{f(n)}{g(n)} = K, K \neq 0, \infty$ and $\lim_{n\to\infty} \frac{\sqrt{3n^2+5}}{n\lg n} = 0$, $\sqrt{3n^2+5} \notin \Theta(n\lg n)$.

1(c) pt. 1 $\lg(n^2+3n) \in O(\lg n)$

This claim is true.

Proof by Limits

A.) Definition: Let f and g be functions from $Z^+ \to R^+$.
$f(n) \in O(g(n)) \iff \lim_{n\to\infty} \frac{f(n)}{g(n)} = K, \quad K \in [0, \infty)$

B.) Let $f(n) = \lg(n^2+3n)$ and $g(n) = \lg n$

C.) I want to show that $\lim_{n\to\infty} \frac{\lg(n^2+3n)}{\lg n} = K, \quad K \in [0,\infty)$

D.) Proof: $\lim_{n\to\infty} \frac{\lg(n^2+3n)}{\lg(n)} = \frac{\infty}{\infty}$, Apply L'Hôpital's Rule...

$= \lim_{n\to\infty} \frac{\frac{d}{dn}(\lg(n^2+3n))}{\frac{d}{dn}(\lg(n))} = \lim_{n\to\infty} \frac{[(1)/(n^2+3n)](2n+3)}{1/n}$

$= \lim_{n\to\infty} \frac{2n^2+3n}{n^2+3n} = \frac{\infty}{\infty}$, Apply L'Hôpital's Rule...

$= \lim_{n\to\infty} \frac{\frac{d}{dn}(2n^2+3n)}{\frac{d}{dn}(n^2+3n)} = \lim_{n\to\infty} \frac{4n+3}{2n+3} = \frac{\infty}{\infty}$, Apply L'Hôpital's Rule...

$= \lim_{n\to\infty} \frac{\frac{d}{dn}(4n+3)}{\frac{d}{dn}(2n+3)} = \lim_{n\to\infty} \frac{4}{2} = \lim_{n\to\infty} 2 = \boxed{2}$

E.) Conclusion: $\lim_{n\to\infty} \frac{\lg(n^2+3n)}{\lg n} = 2$, $2 \in [0,\infty)$, therefore $\lg(n^2+3n) \in O(\lg n)$

Kha Le
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1(c) $\lg(n^2+3n) \in O(\lg n)$ ← This claim is true.

pt. 11

proof by definition

A.) Definition: Let $f$ and $g$ be functions from $Z^+ \to R^+$.
$f(n) \in O(g(n))$ iff $\exists c \in R^+$ and $\exists n_0 \in Z^+ \ni f(n) \le cg(n), \forall n \ge n_0$

B) Let $f(n) = \lg(n^2+3n)$ and $g(n) = \lg n$

C.) I want to find a $c \in R^+$ and $n_0 \in Z^+$ such that
$\lg(n^2+3n) \le c\lg n, \forall n \ge n_0$

D.) Proof: i. $\lg(n^2+3n) \le \lg(n^2+3n),\ n \ge 1$ (reflexive property)
$\lg(n^2+3n) \le \lg(n(n+3)),\ n \ge 1$
$\boxed{\lg(n^2+3n) \le \lg(n) + \lg(n+3),\ n \ge 1}$ } (log rules)

ii. $n+3 \le n^2,\ n \ge 3$
$\lg(n+3) \le \lg(n^2),\ n \ge 3$
$\lg(n+3) \le \lg(n) + \lg(n),\ n \ge 3$ (log rule)
$\boxed{0 \le \lg(n) + \lg(n) - \lg(n+3)},\ n \ge 3$ (rearrange inequality)

iii. Combining results from i and ii, we get
$\lg(n^2+3n) \le \lg(n) + \lg(n) + \lg(n) + \lg(n+3) - \lg(n+3),\ n \ge 3$
$\boxed{\lg(n^2+3n) \le 3\lg(n)}$
$c = 3,\ n_0 = 3$

Could be simpler

$\lg n \le \lg n$;
$\lg(n+3) \le 2\lg n$
$\lg(n^2+3n) \le 3\lg$
$n \ge 3$

E.) For $c = 3$ and $n_0 = 3$, $f(n) \le cg(n), \forall n \ge n_0$

Conclusion: Therefore $\lg(n^2+3n) \in O(\lg n)$

2.(a) $T(n) = n+1+T(n-1),\ n \ge 2 \quad T(2) = 3$

$T(n) = n+1+(n-1)+1+T(n-2) = 2n+2-1+T(n-2)$

$T(n) = 2n+2-1+n-2+1+T(n-3) = 3n+3-(1+2)+T(n-3)$

$T(n) = 3n+3-(1+2)+n-3+1+T(n-4) = 4n+4-(1+2+3)+T(n-4)$

$T(n) = kn+k-(1+2+3+\ldots+k-1)+T(n-k) = kn+k-\frac{(k-1)(k)}{2}+T(n-k)$

$n-k=2,\ k=n-2,\ T(n) = (n-2)(n)+(n-2)-\frac{(n-3)(n-2)}{2}+T(2)$

$T(n) = n^2-2n+n-2-\frac{n^2-5n+6}{2}+3$

$\boxed{T(n) = \frac{1}{2}n^2 + \frac{3}{2}n - 2}$

$-3$

$\frac{(n+1)(n+2)}{2} - 3$

2.(b) $T(2)=3,\ T(3)=7,\ T(4)=12,\ T(5)=18,\ T(6)=25$

2.(c) quadratic polynomial time

Kha Le
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3(a) $\alpha(n) = \sum_{i=1}^{n}[(i+1)^2 - i^2] = \sum_{i=1}^{n}(i+1)^2 - \sum_{i=1}^{n}(i)^2$

$= (2^2+3^2+4^2+\ldots+n^2+(n+1)^2) - (1^2+2^2+3^2+\ldots+(n-1)^2+n^2)$

$= -1^2 + (2^2-2^2)+(3^2-3^2)+\ldots+((n-1)^2-(n-1)^2)+(n^2-n^2)+(n+1)^2$

$= -1^2+0+0+0+\ldots+0+(n+1)^2 = (n+1)^2 - 1^2 = n^2+2n+1-1$

$\boxed{\alpha(n) = n^2+2n}$

3(b) $\sum_{i=1}^{n}[(i+1)^2-i^2] = \sum_{i=1}^{n}(i^2+2i+1-i^2) = \sum_{i=1}^{n}(2i+1) = \boxed{2\sum_{i=1}^{n} i + \sum_{i=1}^{n} 1}$

3(c) $2\sum_{i=1}^{n} i + \sum_{i=1}^{n} 1 = n^2+2n$

$2\sum_{i=1}^{n} i + \underbrace{(1+1+1+\ldots+1)}_{n \text{ terms}} = n^2+2n$

$2\sum_{i=1}^{n} i + n = n^2+2n \rightarrow 2\sum_{i=1}^{n} i = n^2+n \rightarrow \boxed{\sum_{i=1}^{n} i = \frac{n^2+n}{2} = \beta(n)}$

proof by limits

claim: $\beta(n) \in \Theta(n^2)$

A.) Defn: Let f and g be functions from $Z^+ \rightarrow R^+$.
$f(n) \in \Theta(g(n)) \Leftrightarrow \lim_{n\to\infty} \frac{f(n)}{g(n)} = K,\ K \neq 0, \infty$

B.) Let $f(n) = \frac{n^2+n}{2} = \beta(n)$ and $g(n) = n^2$

C.) I want to show that $\lim_{n\to\infty} \frac{(n^2+n)/2}{n^2} = K,\ K\neq 0,\infty$

D.) Proof: $\lim_{n\to\infty} \frac{(n^2+n)}{n^2}\cdot\frac{1}{2}$

$= \lim_{n\to\infty} \frac{1}{2}(1+\frac{1}{n}) = \frac{1}{2}(1+\lim_{n\to\infty}\frac{1}{n}) = \frac{1}{2}(1+\frac{1}{\infty}) = \frac{1}{2}(1+0) = \frac{1}{2}(1)$

$= \frac{1}{2}$

formal proof is not required here

E.) Conclusion: Since $f(n) \in \Theta(g(n)) \Leftrightarrow \lim_{n\to\infty}\frac{f(n)}{g(n)} = K,\ K\neq 0,\infty$ and $\lim_{n\to\infty} \frac{(n^2+n)/2}{n^2} = \frac{1}{2}$, $\beta(n) = \frac{n^2+n}{2} \in \Theta(n^2)$

4(a) claim: $\sum_{i=1}^{n} \lg i \in O(n\lg n)$. This claim is true.

A.) Definition: Let f and g be functions from $Z^+ \rightarrow R^+$.
$f(n) \in O(g(n))$ iff $\exists\, c \in R^+$ and $\exists\, n_0 \in Z^+ \ni f(n) \leq c\,g(n), \forall n \geq n_0$.

B.) Let $f(n) = \sum_{i=1}^{n} \lg i$ and let $g(n) = n\lg n$

C.) I want to ~~show that~~ find a $c \in R^+$ and $n_0 \in Z^+$ such that $\sum_{i=1}^{n} \lg i \leq c\,n\lg n, \forall n \geq n_0$

D.) Proof: i. $1\cdot 2\cdot 3\cdot\ldots\cdot n \leq n\cdot n\cdot n\cdot n\cdot\ldots\cdot n$, $n\geq 1$ since $1\leq n, 2\leq n, \ldots n-1\leq n$

ii. $\lg(1\cdot 2\cdot 3\cdot\ldots\cdot n) \leq \lg(n\cdot n\cdot n\cdot\ldots n)$, $n \geq 1$

iii. $\lg(1)+\lg(2)+\lg(3)+\ldots+\lg(n) \leq \lg(n^n)$, $n\geq 1$ } log rules

iv. $\sum_{i=1}^{n} \lg i \leq n\lg n$, $n \geq 1$, $c=1, n_0=1$

E.) Conclusion: For $c=1, n_0=1$, $f(n)\leq c\,g(n), \forall n\geq n_0$, therefore $\sum_{i=1}^{n} \lg i \in O(n\lg n)$

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4(b) $\sum_{i=1}^{n} \lg i \in \Omega(n\lg n)$

This claim is true.

A.) Definition: Let f and g be functions from $Z^+ \to R^+$.
Then $f(n) \in \Omega(g(n))$ iff $\exists c \in R^+$ and $\exists n_0 \in Z^+ \ni$
$f(n) \geq cg(n), \forall n \geq n_0$.

B.) Let $f(n) = \sum_{i=1}^{n} \lg i$ and $g(n) = n\lg n$.

C.) I want to find a $c \in R^+$ and $n_0 \in Z^+$ such that $\sum_{i=1}^{n} \lg i \geq c\sum_{i=\frac{n}{2}+1}^{n} \lg \frac{n}{2}, \forall n \geq n_0$ in order to prove $\sum_{i=1}^{n} \lg i \in \Omega(n\lg n)$

D.)

i. $\lg(1)+\lg(2)+\ldots+\lg(n) \geq \lg(1)+\lg(2)+\ldots+\lg(n)$, $n \geq 1$

ii. $\lg(1)+\lg(2)+\ldots+\lg(n) \geq \lg(\frac{n}{2})+\lg(\frac{n}{2}+1)+\ldots+\lg(n)$, $n \geq 1$

iii. $\underbrace{\lg(\frac{n}{2})+\lg(\frac{n}{2}+1)+\ldots+\lg(n)}_{n/2 \text{ terms}} \geq \underbrace{\lg(\frac{n}{2})+\lg(\frac{n}{2})+\ldots+\lg(\frac{n}{2})}_{n/2 \text{ terms}}$, $n \geq 1$

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Fuzzy math

iv. Combining the inequalities in ii and iii, we get
$\lg(1)+\lg(2)+\ldots+\lg(n) \geq \underbrace{\lg(\frac{n}{2})+\lg(\frac{n}{2})+\ldots+\lg(\frac{n}{2})}_{n/2 \text{ terms}}$, $n \geq 1$

$$\sum_{i=1}^{n} \lg i \geq \sum_{i=\frac{n}{2}+1}^{n} \lg \frac{n}{2}, n \geq 1$$

$c=1; n_0=1$

E.) Conclusion: For $c=1$ and $n_0=1$, $\sum_{i=1}^{n} \lg i \geq c\sum_{i=\frac{n}{2}+1}^{n} \lg \frac{n}{2}, \forall n \geq n_0$.
Therefore, $\sum_{i=1}^{n} \lg i \in \Omega(\sum_{i=\frac{n}{2}+1}^{n} \lg \frac{n}{2})$. However, $\Omega(\sum_{i=\frac{n}{2}+1}^{n} \lg \frac{n}{2}) = \Omega(\frac{n}{2}\lg\frac{n}{2})$ has the same time complexity as $\Omega(n\lg n)$.
Therefore, $\sum_{i=1}^{n} \lg i \in \Omega(n\lg n)$.

4(c) $f(n) \in \Theta(g(n))$ iff $\exists c \in R^+$ and $\exists n_0 \in Z^+ \ni$
$c_1 g(n) \leq f(n) \leq c_2 g(n), \forall n \geq n_0$

4.(a) proves $\sum_{i=1}^{n} \lg i \in O(n\lg n)$, implying $\sum_{i=1}^{n} \lg i \leq n\lg n$, $\forall n \geq n_0$, $c_2=1, n_0=1$

4(b) proves $\sum_{i=1}^{n} \lg i \in \Omega(n\lg n)$, implying $\alpha n\lg n \leq \sum_{i=1}^{n} \lg i$, $\forall n \geq n_0$, $c_1=\alpha, n_0=1$

Therefore, $\alpha n\lg n \leq \sum_{i=1}^{n} \lg i \leq n\lg n, \forall n \geq n_0, c_1=\alpha, c_2=1, n_0=1$.

Thus, $\sum_{i=1}^{n} \lg i \in \Theta(n\lg n)$