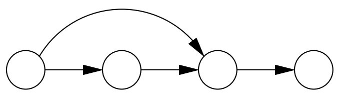
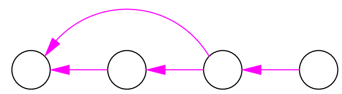
Solutions Long Quiz #3.1 (07-Apr): CSC-2259: Discrete Structures, Sp 2020

Your answers must be to the point. Total = 50; marks for each question is shown in [ ]. **There will be another Take-home Long Quiz #3.2. These two together with total score 100 will replace original Long Quiz #3.**

LastName: FirstName

1. For the (unlabeled) digraph G shown below, show the links of the reverse-digraph $r(G)$ using the nodes on the right (relative positions of the nodes remain the same). [5]

G: 

$r(G)$: 

Show the following for G. [3+3+3+3+3]

(a) Outdeg-sequence for G: $\langle 2, 1, 1, 0\rangle$

(b) Indeg-sequence for G: $\langle 2, 1, 1, 0\rangle$

(c) (Total) Degree-sequence: $\langle 3, 2, 2, 1\rangle$

(d) Shows #(paths) of lengths 2 and 3 (in that order): 3, 1

(e) DegreePair-sequence: $\langle (2,0), (1,2), (1,1), (1,0)\rangle$

Which of (a)-(e) above help to distinguish between G and $r(G)$? [2] (e)

How many labeled digraphs can you get from G if we label its nodes with $\{a, b, c, d\}$? [3] 24

2. If two unlabeled digraphs have the same degreePair-sequence, then what can you say about the following. [3+3+3]

(a) their outdeg-sequence will be the same.

(b) their indeg-sequence will be the same.

(c) their (total) degree-sequence will be the same.

Is it true that if two unlabeled digraphs have different outdeg-sequence, then their degreePair-sequence will be different? [3]

Yes.

Is it true that if two unlabeled digraphs have different indeg-sequence, then their degreePair-sequence will be different? [3]

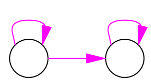
Yes.

Consider the three equality-tests of unlabeled (n, m)-digraphs based on: (i) outdeg-sequence, (ii) indeg-sequence, and (iii) degreePair-sequence. Which of these is the most powerful? [3]

(iii)

3. Give a clear argument that the maximum #(links) in an anti-symmetric relation on n items is $n(n+1)/2$. Also, verify the count $n(n+1)/2$ for $n = 2$ by showing the structure of anti-symmetric relation(s) on X with maximum #(links) when $|X| = 2$. [5+2]

(a) We select all n loops (i.e., links of the form (x, x)).
(b) For each of $n(n-1)/2$ distinct node-pairs $\{x, y\}$, we select exactly one of the links (x, y) and (y, x).
(c) Thus, #(links) = $n + n(n-1)/2 = n(n+1)/2$.



Here, #(links) = 3 = 2×3/2.