Math2090, Section 1
Quiz 10, Total 10 points

Last name: First name:

1. Find $L[f(t)](s)$ where $f(t) = 2e^{3t} + 4\cos(3t)$, 4 points

$$L[2e^{3t}] = \frac{2}{s-3} \quad \text{and} \quad L[4\cos 3t] = \frac{4s}{s^2+9}$$

$$\therefore \quad L[f(t)](s) = \frac{2}{s-3} + \frac{4s}{s^2+9}$$

2. **Use Laplace transform** to solve $y'' + y' - 2y = 0, y(0) = 1, y'(0) = 4$. 6 points

$$L[y''+y'-2y] = 0 \Rightarrow (s^2Y(s) - sy(0) - y'(0)) + sY(s) - y(0) - 2Y(s) = 0$$

$$(s^2+s-2)Y(s) - s - 4 - 1 = 0 \Rightarrow Y(s) = \frac{s+5}{s^2+s-2} = \frac{s+5}{(s-1)(s+2)}$$

we will find A, B such that $\frac{s+5}{(s-1)(s+2)} = \frac{A}{s-1} + \frac{B}{s+2}$

$$s+5 = A(s+2) + B(s-1)$$

If $s=1$ then $6 = 3A + 0\cdot B \Rightarrow \boxed{A=2}$

If $s=-2$ then $3 = 0\cdot A - 3B \Rightarrow \boxed{B=-1}$

$$\therefore \quad Y(s) = \frac{2}{s-1} + \frac{-1}{s+2}$$

$$\therefore \quad y(t) = L^{-1}\left[\frac{2}{s-1} + \frac{-1}{s+2}\right] = L^{-1}\left[\frac{2}{s-1}\right] + L^{-1}\left[\frac{-1}{s+2}\right]$$

$$= 2e^{t} - e^{-2t}.$$