Kha Le

1. (a) For every $x \neq y$ in X, exactly one of (x,y) or (y,x) is in R.

(b) 

$\#(\text{linear orders on } X) = 3! = 6$

(c) 

$\#(\text{strict linear orders on } X) = 3! = 6$

(d) $\{1,2,4,8\}$

(e) - $1+2+3+\dots+n = \boxed{\frac{n(n+1)}{2}}$

- true
- in R, each row should have a distinct number of 1's from $\{1,2,\dots,n\}$ if it is a linear order

(f) This argument assumes that each of the $n(n-1)/2$ pairs $(R[i][j], R[j][i])$ are valid. Linear orders must be transitive, so you cannot have something like $R[1][2]=1$, $R[2][3]=1$, $R[1][3]=0$

2. (a) $\{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

(b) $\{HHH, HHT, HTH, THH\}$

(c) $\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} = \boxed{\frac{4}{27}} = Prob(HHT)$

(d) $\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} + 3\left(\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3}\right) = \frac{8}{27} + \frac{12}{27} = \boxed{\frac{20}{27}}$

(e) $Prob(E \cup E') = Prob(E) + Prob(E') - Prob(E \cap E')$

(f) $E_1^c =$ "at most 1 heads"
$Prob(E_1^c) = 1 - Prob(E_1) = 1 - \frac{20}{27} = \boxed{\frac{7}{27}}$

(g) (g.1) $\{(3,3,3), (3,3,8), (3,3,4), (3,3,5)\}$

(g.2) $Prob(E_{1,2}) = \frac{1}{3} \cdot \frac{1}{3} \cdot \left(\frac{1}{3} + \frac{1}{3} + \frac{1}{6} + \frac{1}{6}\right) = \frac{1}{9}(1) = \boxed{\frac{1}{9}}$

(g.3) $Prob(E_{1,2} \cup E_{2,3}) = Prob(E_{1,2}) + Prob(E_{2,3}) - Prob(E_{1,2} \cap E_{2,3})$
$= 1/9 + 1/9 - \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$
$= 2/9 - 1/27$
$= \boxed{5/27}$