Math2090, Section 1
Quiz 1. Total 10 points

~~Last name:~~ First name:

Please justify your answer.

(1) Find a solution to the initial value problem $y' = y^2 \cos(x)$, $y(0) = 1$. (5 pt)

$$\frac{dy}{dx} = y^2 \cos x$$

$$dy = y^2 \cos x \, dx$$

$$\frac{1}{y^2} dy = \cos x \, dx$$

$$\therefore \int \frac{1}{y^2} dy = \int \cos x \, dx$$

$$-\frac{1}{y} = \sin x + C \quad \dots (*)$$

Now, we find the value of c that makes $y(0) = 1$

$$-\frac{1}{1} = \sin 0 + C$$

$$\boxed{-1 = C}$$

$$\therefore \frac{-1}{y} = \sin x - 1 .$$

(2) Find the general solution of $e^{2x}\frac{dy}{dx} + e^{2x}y = 1$. 5 pts

1) write the equation in the form $\frac{dy}{dx} + P(x)\, y = Q(x)$.

$$e^{2x}\frac{dy}{dx} + e^{2x} y = 1 \Rightarrow \frac{e^{2x}}{e^{2x}}\frac{dy}{dx} + \frac{e^{2x} y}{e^{2x}} = \frac{1}{e^{2x}}$$

$$\therefore \frac{dy}{dx} + y = \frac{1}{e^{2x}} \Rightarrow \frac{dy}{dx} + \underbrace{y}_{P(x)=1} = \underbrace{e^{-2x}}_{Q(x)}$$

2) Find the integrating factor.

$$I = e^{\int P(x)} = e^{\int 1\, dx} = e^{x}$$

" don't write it as e^{x+C} "

3) Compute $\int Q(x)\, e^{\int P(x)}$.

$$\int Q(x)\, e^{\int P(x)} = \int e^{-2x} \cdot e^{x} = \int e^{-x} = -e^{-x} + C$$

4) The General Solution is $y = \frac{1}{I}\int Q(x)\, e^{\int P(x)}$

$$\therefore y = \frac{1}{e^x}\left[-e^{-x} + C\right] = e^{-x}\left[-e^{-x} + C\right] = -e^{-2x} + C e^{-x}.$$

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