

A(a)i.  $O(n^6)$  ✓

A(a)ii.  $O(n)$  ✓

A(b)i. This claim is true.

A. Let  $f$  and  $g$  be functions from  $\mathbb{Z}^+ \rightarrow \mathbb{R}^+$  that is positive real-valued functions on the domain of positive integers.

$$f(n) \in \Theta(g(n)) \Leftrightarrow \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = c, 0 < c < \infty$$

B Let  $f(n) = \ln \sqrt{n^3 + 2n}$  and  $g(n) = \ln n$ .

( I want to show that  $\lim_{n \rightarrow \infty} \frac{\ln \sqrt{n^3 + 2n}}{\ln n} = c, 0 < c < \infty$

$$\begin{aligned} D. \lim_{n \rightarrow \infty} \frac{\ln \sqrt{n^3 + 2n}}{\ln n} &= \lim_{n \rightarrow \infty} \frac{\frac{1}{2} \ln(n^3 + 2n)}{\ln n} = \lim_{n \rightarrow \infty} \frac{\frac{1}{2} (\ln(n^3 + 2n))'}{(\ln n)'} \\ &= \lim_{n \rightarrow \infty} \frac{\frac{1}{2} (3n^2 + 2)}{1/n} = \lim_{n \rightarrow \infty} \frac{1}{2} \cdot \frac{3n^2 + 2}{n} = \lim_{n \rightarrow \infty} \frac{1}{2} \cdot \frac{(3n^2 + 2)'}{n'} \\ &= \lim_{n \rightarrow \infty} \frac{1}{2} \cdot \frac{6n + 2}{1} = \lim_{n \rightarrow \infty} \frac{1}{2} \cdot (6n + 2) \\ &= \lim_{n \rightarrow \infty} \frac{1}{2} \cdot 3 = 3/2 \end{aligned}$$

For  $c = \frac{3}{2}$ ,  $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = c, 0 < c < \infty$ .

Therefore,  $\ln \sqrt{n^3 + 2n} \in \Theta(\ln n)$  ✓



A(b)ii. This claim is false.

Nice!!

A. Let  $f$  and  $g$  be functions from  $\mathbb{Z}^+ \rightarrow \mathbb{R}^+$  that is, positive real-valued functions on the domain of positive integers.

$$f(n) \in O(g(n)) \Leftrightarrow \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = c, 0 \leq c < \infty$$

B. Let  $f(n) = \sqrt{9^{n+2}}$  and  $g(n) = 2^n$ .

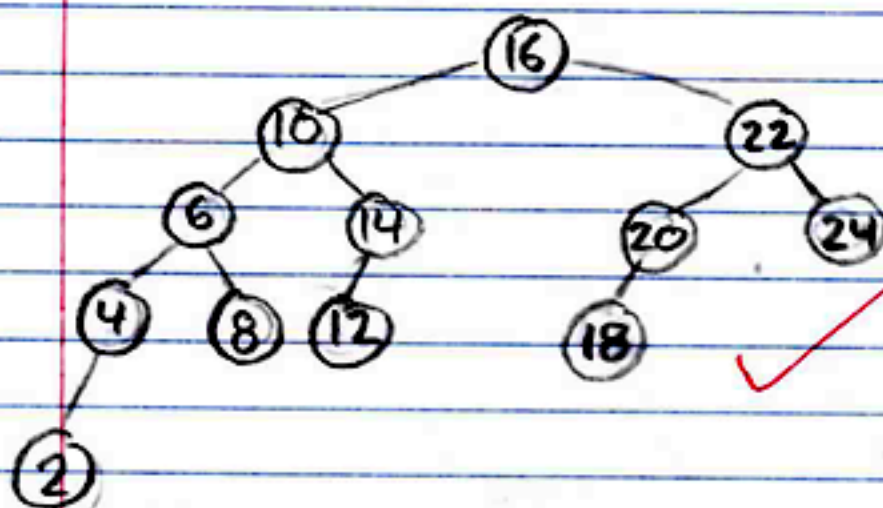
C. I want to show that  $\lim_{n \rightarrow \infty} \frac{\sqrt{9^{n+2}}}{2^n} = c, 0 \leq c < \infty$  is impossible.

$$\begin{aligned} D. \lim_{n \rightarrow \infty} \frac{\sqrt{9^{n+2}}}{2^n} &= \lim_{n \rightarrow \infty} \frac{\sqrt{9^{n+2}}}{2^n} = \lim_{n \rightarrow \infty} \frac{9 \sqrt{9^n}}{2^n} = \lim_{n \rightarrow \infty} \frac{9 \cdot (9^n)^{1/2}}{2^n} \\ &= \lim_{n \rightarrow \infty} \frac{9 \cdot (3^{2n})^{1/2}}{2^n} = \lim_{n \rightarrow \infty} \frac{9 \cdot 3^n}{2^n} = \lim_{n \rightarrow \infty} 9 \cdot \left(\frac{3}{2}\right)^n \\ &= 9 \cdot \left(\frac{3}{2}\right)^\infty = \infty \end{aligned}$$

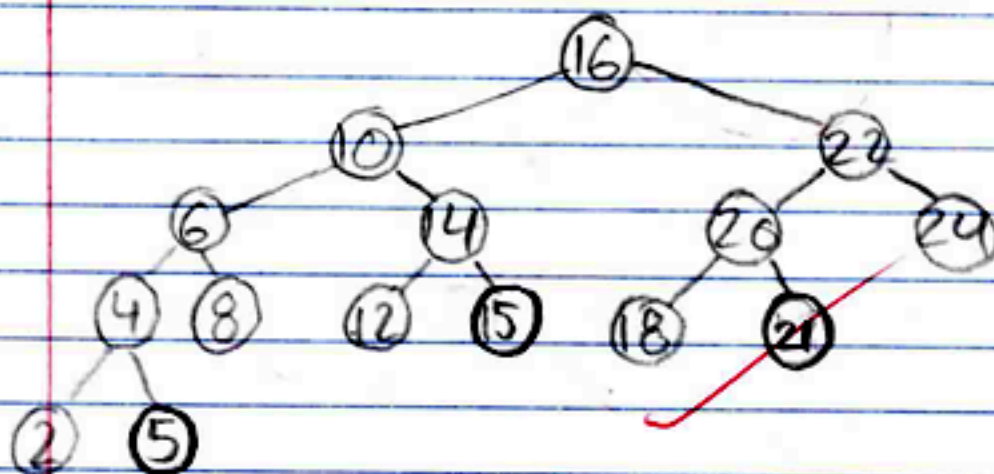
For  $c = \infty, 0 \leq c < \infty$  is false.

Therefore,  $\sqrt{9^{n+2}} \notin O(2^n)$ .

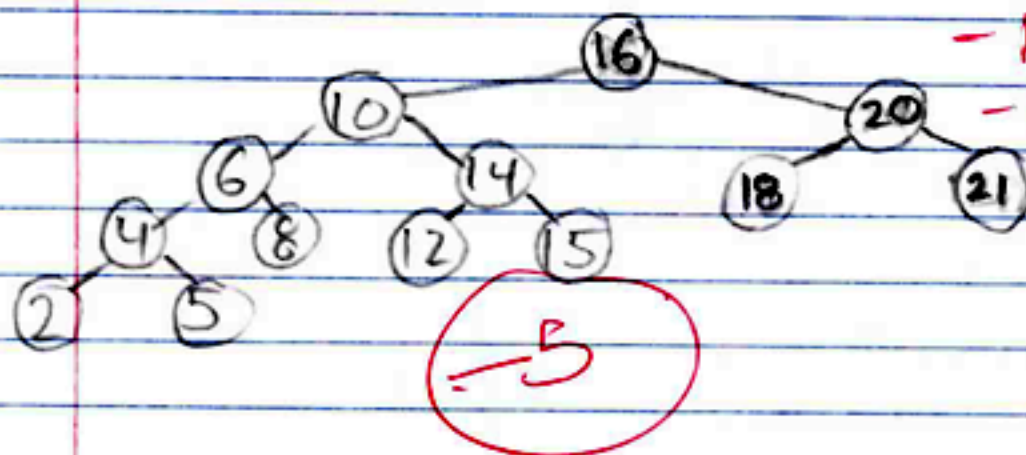
B(a)



B(b)



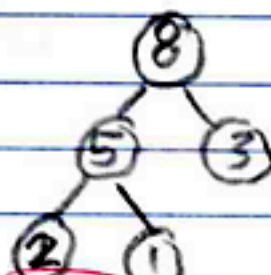
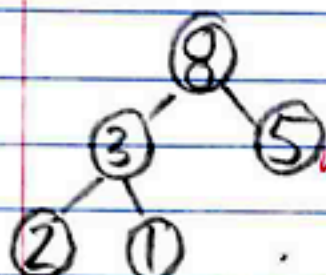
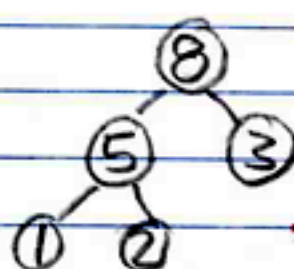
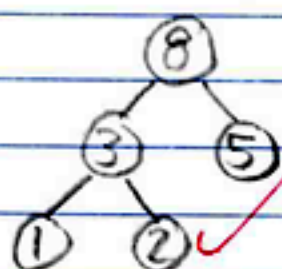
B(c)



- Not balanced  
- Needs a left rotation.



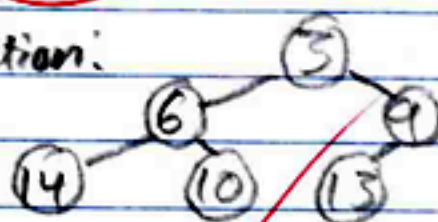
C.(a)



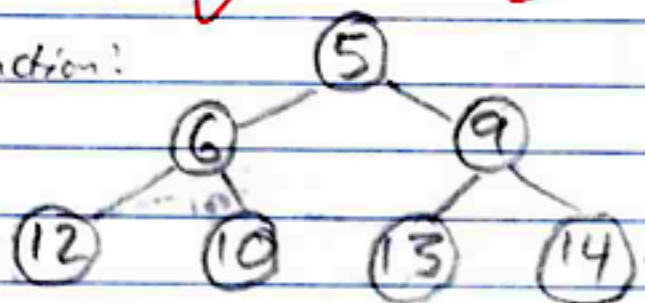
four more

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C.(b) after 6th instruction:



after 9th instruction:



D. (a)

fruit	quant.	profit	profit/quant.
apple	2	3	$\frac{3}{2} = 1.5$ (best)
avocado	5	6	$\frac{6}{5} = 1.2$
cherry	4	5	$\frac{5}{4} = 1.25$ (3rd best)
strawb.	3	4	$\frac{4}{3} = 1.33...$ (2nd best)

The farmer should take 2 tons of apples, 3 tons of strawberries, and 3 tons of cherries to maximize profit.

$$\begin{aligned}
 \text{max total profit: } & 2\left(\frac{3}{2}\right) + 3\left(\frac{4}{3}\right) + 3\left(\frac{5}{4}\right) \\
 & = 3 + 4 + \frac{15}{4} \\
 & = 7 + 3.75 \\
 & = \boxed{10.75}
 \end{aligned}$$

D. (b) \$10

D. (c) He must select the avocados and strawberries in order to maximize profit.

We filled in Table 3 left to right, top to bottom

For any given  $V[i, j]$  if there was not enough weight capacity to add the fruit of the row,  $V[i, j]$  would just equal the cell above, or  $V[i-1, j]$ . If there is enough weight capacity, you take the maximum of the upper cell,  $V[i-1, j]$ , and the sum of adding the new fruit to the max of leftover capacity or  $V[i-1, j-w_i] + V_i$ .