MATHEMATICS 2090
Practice Problems

(1) Using Gaussian elimination to determine the solution set to the lienar systerm

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 1 \\ 3x_1 + 5x_2 + x_3 &= 3 \\ 2x_1 + 6x_2 + 7x_3 &= 1 \end{aligned}$$

(2) Let $A = \begin{bmatrix} 1 & -2 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 3 \end{bmatrix}$.

a) Compute its determinant.
b) Is A invertible? Please justify your answer.
c). If A is invertible, please compute its inverse A^{-1} and compute $A \cdot A^{-1}$.

(3) Compute the eigenvalues of the matrix

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

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(4) Show that $\mathbf{v}_1 = (-1, 3, 2)$, $\mathbf{v}_2 = (1, -2, 1)$, $\mathbf{v}_3 = (2, 1, 1)$ span $\mathbb{R}^3$, and express $\mathbf{v} = (0, 1, -1)$ as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.

(5) Compute the Wronskian of the functions $\{x, \sin x, \cos x\}$.
Are they linearly dependent? Please justify.

(6) Compute the dimension of the linear span of $\left\{ \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} \right\}$.

Are $\left\{ \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} \right\}$ linearly dependent or independent?

(7) Does $\{1 + x, 2 + x + x^2, x - x^2\}$ form a basis for the vector space of polynomials in variable x with degree at most 2? Please justify your answer.

(8) For $A = \begin{pmatrix} 2 & -1 & 4 & 0 \\ 1 & 4 & 0 & 5 \end{pmatrix}$.

a) Please determine its null space.
b) Write down a basis for the null space of A.
c) Determine the dimension of for the null space of A.

(9) Does the following subset of $M_2(\mathbb{R})$ form a subspace? Please justify your answer.

(a) $S = \left\{ \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \in M_2(\mathbb{R}) \mid a_{11} + a_{22} = 0. \right\}$
(b) $S = \left\{ \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \in M_2(\mathbb{R}) \mid a_{11} + a_{22} = 1. \right\}$