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20

Short Quiz #2 (28-Jan): CSC-2259: Discrete Structures, Sp 2020

Your answers must be to the point. Total = 20; marks for each question is shown in [].

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1. The following code is a slight variant of the one given in the class for computing all $C(n, m)$, $0 \leq m \leq n$ for a fixed n using a recursive formula for $C(n, m)$.

```
1. int[] c = new int[n+1]; //assume n has a value assigned
2. c[0] = 1; //c[m] = C(n, m); c[1], c[2], ... are computed below
3. for (int m = 1; m <= n; m++)
4.     c[m] = c[m-1] * (n-m+1) / m;
```

- (a) Give the number of iterations of the for-loop. [1]

n iterations ✓

- (b) Give #arithmetic and assignment operation in all iterations of line 4). [2]

$6n$ ✓

- (c) Let $n = 6$. Give a value of m to show in detail why changing line 4 to $c[m] = (n-m+1)/m * c[m-1]$ does not work. [2]

If $m = 2$, $c[2] = (5)/2 * c[1] = 2 * 6 = 12$, which is incorrect. The order causes rounding between $5/2$ which there should not be rounding.

2. Complete the following sentences/equations; use the notation $C(\cdot, \cdot)$ whenever possible. [1+1+1+2+3]

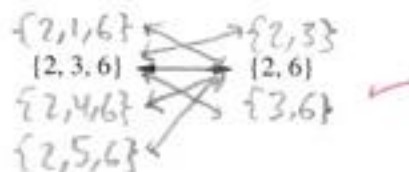
- (a) #(m -subsets of $\{1, 2, \dots, n\}) = C(n, m)$ ✓

- (b) #lines connecting each m -subset to the $(m-1)$ -subsets in the way we considered in the class) = m ✓

- (c) #lines connecting each $(m-1)$ -subset to the m -subsets in the way we considered in the class) = $n-m+1$ ✓

- (d) Therefore, $m \cdot C(n, m) = (n-m+1) \cdot C(n, m-1)$ ✓

- (e) For $n = 6$ and $m = 4$, complete the diagram below by showing all 2-subsets to which the 3-subset $\{2, 3, 6\}$ is linked and also all 3-subsets that are linked to $\{2, 6\}$.



3. Consider the code below to compute top and bottom so that $C(n, m) = \text{top}/\text{bottom}$ for a fixed n and m ($1 \leq m \leq n$).

```
1. int top = n, bottom = m; //assume n and m have values assigned
2. for (int i = 1; i < m; i++)
3. {   top *= n - i;   //top = top * (n-i)
4.     bottom *= i; //successive values: m*1, m*1*2, m*1*2*3, ...
5. }
```

Complete the following sentences/equations. [1+1+1+2+2]

- (a) #(assignments and arithmetic operations in lines 3-4 per iteration) = 5 ✓

- (b) #(assignments and arithmetic operations in lines 3-4 for all iterations) = $5(m-1)$ ✓

- (c) Final value of bottom is $1*2*3*\dots*m$; final value of top is $(n)*(n-1)*(n-2)*\dots*(n-m+1)$ ✓

- (d) If we replace "for (int i = 1; i < m; i++)" above by "for (int i = 2; i <= m; i++)", then both "top *= n - i" and "bottom *= i" have to be modified. Give the modified form of "top *= n - i".

$\frac{n(m+1)}{2}$

- (e) The sum $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$ ✓

top *= $n-i+1$ ✓