Sample 12

Consider the following One-Dimensional Wave Equation for u(x,t) for $0 \le x \le 2\pi$ and $0 \le t \le 6$:

$$\frac{\partial^2 u}{\partial t^2} = a\frac{\partial^2 u}{\partial x^2} + f(x,t)$$

$$f(x,t) = 2e^{-\frac{t}{2}}\sin\left(\frac{x}{2}\right)$$

$$a = \frac{1}{4\pi^2}$$

with the following initial conditions at t=0:

u(x,0) = u0(x) = sin(x/2),

v(x,0) = v0(x) = -sin(x/2)

and the following boundary conditions:

u(0,t) = gleft(t) = sin(π/6*t)

u(2π,t) = gright(t) = sin(π/12*t)

Write a MATLAB program as follows:

1) Use the explicit full discretization scheme to calculate numerical values for the unknown u(x,t) for $0 < x < 2\pi$ and $0 < t \le 6$. Divide the x interval [0, 2π] into 20 equal subdivisions and the t interval [0, 6] into 30 equal subdivisions (there will be 21 equally spaced grid points in the x interval and 31 equally spaced grid points in the t interval). Use the variables L and T for the lengths of the x and t intervals, nx and nt for the number of grid points in the x and t intervals, and hx and ht for the stepsizes in the x and t intervals.

   The main program will call a function named wave1 that solves the One-Dimensional Wave Equation for the unknown u and returns it to the main program. The first line of wave1 is:

   ```
   function u = wave1(f,u0,v0,gleft,gright,a,nx,nt,L,T)
   ```

2) Plot u versus x and t for $0 \le x \le 2\pi$ and $0 \le t \le 6$. u will be a surface in 3-dimensional space. Use the MATLAB function surf to plot u.

The graph should look like the one on the attached sheet.