Practice Questions for Apr 23, 2019

1. Consider a spinning wheel with 6 equal parts, one part labeled R, two parts labeled B, and three parts labeled W so that $P(R) = 1/6$, $P(B) = 2/6$, and $P(W) = 3/6$. Let our experiment consist of 3 turns of the wheel. Thus, the sample space has 27 points of the form $c_1c_2c_3$, where c_i is the color (R, B, or W) observed in the ith turn. Let X = "#(R or B)" and Y = "#(B or W)"; clearly, $0 \leq X, Y \leq 3$.

   (a) The table below shows the elementary events for some combinations of values for X and Y ("−" indicates this combinations not possible). Fill in the entries marked '?'. (Each elementary event should appear exactly once.)

   Elementary events for different combination of values of X and Y.

	$Y=0$	$Y=1$	$Y=2$	$Y=3$	#(elem. events in row)
$X=0$	−	−	−	WWW	1
$X=1$	−	−	?	?	?
$X=2$	−	RRW, RWR, WRR	?	?	?
$X=3$	RRR	RRB, RBR, BRR	?	?	?
#(elem. events) in column	1	6	?	?	

   (b) Compute $E(X)$; show the details. Do the same for $E(Y)$ and $E(XY)$.

   (c) Verify whether $E(XY) = E(X)E(Y)$ or not.

2. Is there any thing wrong in the following calculations; give the correct form. (Here, X and Y have nothing to do with those in Prpblem 1.)

$$E((X+Y)^2) = E(X^2 + 2XY + Y^2) = E(X^2) + 2E(XY) + E(Y^2)$$
$$E^2(X+Y) = [E(X) + E(Y)]^2 = E^2(X) + 2E(X)E(Y) + E^2(Y)$$
$$Var(X+Y) = E((X+Y)^2) - E^2(X+Y) = E(X^2) - E^2(X) + E(Y^2) - E^2(Y) + 2[E(XY) - E(X)E(Y)]$$
$$= Var(X) + Var(Y) + 2[E(XY) - E(X)E(Y)]$$

3. Consider the pair of integers (x, y), $0 \leq x, y \leq 4$, as inputs to the following three codes to compute maximum of x and y.

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(i)  max = x;            (ii)  max = x;            (iii) if (x < y)
       if (max < y)              if (max <= y)              max = y;
          max = y;                  max = y;            else max = x;
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   Assume that each point (x, y) has the probability 1/25. Complete the two tables below, where X = #Comparisons) and Y = #(Assignments). (We group the points (x, y) into the categories "$x < y$", "$x = y$", and $x > y$" because X and Y depend only on the latter for each code.)

Event	Probability	Code (i)		Code (ii)		Code (iii)	
		X	Y	X	Y	X	Y
$x < y$							
$x = y$							
$x > y$							

Code	Expected Values						Variances		
	X	Y	$X+Y$	X^2	Y^2	$(X+Y)^2$	X	Y	$X+Y$
(i)									
(ii)									
(iii)									

4. Describe the experiment, its outcomes (Sample Spaces), and their probabilities in relation to $E(X)$ and $Var(X)$ in Problem 3.