Sample 10

Consider the following Poisson partial differential equation with Dirichlet boundary conditions:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -2\pi^2 \sin(\pi x)\sin(\pi y) \qquad 0 < x,y < 1$$

$$u(x,y) = 0 \qquad x = 0,1 \text{ or } y = 0,1$$

Write a MATLAB program as follows:

1) Use the 5-point scheme to calculate numerical values for the unknown u for 0 < x < 1 and 0 < y < 1 . Divide both the x interval [0, 1] and the y interval [0, 1] into 16 equal subdivisions (there will be 17 equally spaced grid points in both the x and y directions). Use 1e-8 as the accuracy factor. The main program will call a function named poisson that solves the Poisson equation for the unknown u (u is a two dimensional array) and returns it to the main program.

2) Plot u versus x and y for $0 \le x \le 1$ and $0 \le y \le 1$. u will be a surface in 3-dimensional space. Use the MATLAB function surf to plot u.

The graph should look like the one on the attached sheet.