Practice questions for Feb 19, 2020. (Work on Problem 1 before Problem 2 before Problem 3, etc.)

1. Consider the code below to test $H = W$, where H and W are binary arrays of length n.

```
for (int i = 0; i < n; i++)
    if (H[i] != W[i]) return(false);
return(true);
```

   Complete the following sentences and give a justifications for your answer:

   (a) #((H, W)-pairs that give false-return value in mth iteration, $1 \le m \le n$) = ...

   (b) #((H, W)-pairs giving false-return) = ...

   (c) #((H, W)-pairs giving true-return) = ...

   (d) For $n = 4$, the average #(itertions for all (H, W)-pairs) = ...

   (e) For $n = 5$, the average #(itertions for all (H, W)-pairs) = ...

   (f) For $n = 6$, the average #(itertions for all (H, W)-pairs) = ...

   (g) For general $n \ge 1$, the average #(itertions for all (H, W)-pairs) = ...

2. Give an alternate if-condition for the code in Problem 1 that would also correctly test $H = W$.

3. Express the set theoretic relationship between H and W, when we think of $H$and W representing subsets of an n-set, that is tested by the following code.

```
for (int i = 0; i < n; i++)
    if (H[i] != W[i]) return(false);
    else return(true);
```

4. Complete the following sentences.

   (a) For a given H of size m, $0 \le m \le n$, #(W disjoint from H) = ...

   (b) #((H, W)-pairs such that $H \cap W = \emptyset$ and $|H| = m$) = ...

   (c) #((H, W)-pairs such that $H \cap W = \emptyset$) = ...

   (d) A matching of (H, W)-pairs with $H \cap W = \emptyset$ and (H, W')-pairs with $H \subseteq W'$ can be obtained as follows . . . .
   For example, the pair ($\{a, b\}$, $\{c\}$) of 1st kind can be matched with the pair . . . . of 2nd kind.

   (e) How many (H, W)-pairs would give true return-value for the code in Problem 3?

5. What is wrong in the following way of counting (H, W) pairs with $H \cap W$ non-empty? (Here, H and W are subsets of an n-set.)

   (a) For a given m-subset X, $1 \le m \le n$, #((H, W)-pairs with $H \cap W = X$) = 3^{n-m}.

   (b) #((H, W)-pairs with $|H \cap W| = m$) = $C(n, m)3^{n-m}$.

   (c) #((H, W)-pairs with $|H \cap W| \ge 1$) = $\sum_{m=1}^{n} C(n, m)3^{n-m} = 4^n - 3^n$.