1. सरलीकरण के लिए VBODMAS नियम का पालन करें –

जहाँ   V = Viniculum (Bar)   = (–)

B = Bracket $= \left[\{()\}\right]$

O = of   = ×

D = Division   = ÷

M = Multiplication   = ×

A = Addition   = +

S = Substraction   = –

अर्थात् किसी व्यंजक के सरलीकरण के लिए उपर्युक्त नियम से सर्वप्रथम BAR '(–)', छोटा कोष्ठक '( )', मध्यम कोष्ठक '{}', एवं बड़ा कोष्ठक '[ ]', का 'x', भाग '÷', गुणा 'x', जोड़ '+', और अन्त में घटाव '-' की क्रिया की जाती है।

सरलीकरण के लिए बीजगणितीय सूत्र

1. $a^2 - b^2 = (a+b)(a-b)$

2. $(a+b)^2 = a^2 + 2ab + b^2$

3. $(a-b)^2 = a^2 - 2ab + b^2$

4. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

5. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

6. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

7. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

8. $a^3 + b^3 + c^3 - 3abc = (a+b+c)\begin{pmatrix} a^2 + b^2 + c^2 \\ -ab - bc - ca \end{pmatrix}$

9. $a^2 + b^2 + c^2 - ab - bc - ca$
$= \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$

10. $(a-1)(a^3 + a^2 + a + 1) = x^4 - 1$

Speedy Solution के लिए याद रखें

1. $\frac{a^2 - b^2}{a+b} = a - b$

2. $\frac{a^2 - b^2}{a-b} = a + b$

3. $(a+b)^2 + (a-b)^2 = 2\left(a^2 + b^2\right)$

4. $(a+b)^2 - (a-b)^2 = 4ab$

5. $\frac{a^3 + b^3}{a^2 - ab + b^2} = a + b$

6. $\frac{a^3 - b^3}{a^2 + ab + b^2} = a - b$

7. $\frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - bc - ca} = a + b + c$

8. यदि $a + b + c = 0$ हो, तो $a^3 + b^3 + c^3 - 3abc = 0$

9. $a^3 + b^3 + c^3 = 3abc$

महत्त्वपूर्ण तथ्य और सूत्र पर आधारित प्रश्न

1. $7\frac{1}{2} - \left[2\frac{1}{4} + \left\{1\frac{1}{4} - \frac{1}{2}\left(1\frac{1}{2} - \frac{1}{3} - \frac{1}{6}\right)\right\}\right]$ का मान निकालें ?

Speedy Solution :–

$7\frac{1}{2} - \left[2\frac{1}{4} + \left\{1\frac{1}{4} - \frac{1}{2}\left(1\frac{1}{2} - \frac{1}{3} - \frac{1}{6}\right)\right\}\right]$

$= \frac{15}{2} - \left[\frac{9}{4} + \left\{\frac{5}{4} - \frac{1}{2}\left(\frac{3}{2} - \frac{1}{3} - \frac{1}{6}\right)\right\}\right]$

$= \frac{15}{2} - \left[\frac{9}{4} + \left\{\frac{5}{4} - \frac{1}{2} \times 1\right\}\right] = \frac{15}{2} - \left[\frac{9}{4} + \left\{\frac{3}{4}\right\}\right]$

$= \frac{15}{2} - \left[\frac{9}{4} \times \frac{4}{3}\right] = \frac{15}{2} - 3 = \frac{9}{2} = 4\frac{1}{2}$

2. $\frac{(0.5)^3 + (0.6)^3}{(0.5)^2 - (0.3) + (0.6)^2}$ का मान निकालें ?

Speedy Solution :–

यदि $0.5 = a$ तथा $0.6 = b$ हो तो

व्यंजक $\frac{(0.5)^3 + (0.6)^3}{(0.5)^2 - (0.3) + (0.6)^2} = \frac{a^3 + b^3}{a^2 - ab + b^2} = a + b$

∴ अभीष्ट मान $= 0.5 + 0.6 = 1.1$

3. $\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{6}\right)\ldots\ldots\left(1 - \frac{1}{n}\right)$ का मान बतायें ?

Speedy Solution :-

$$\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\left(1-\frac{1}{6}\right)\cdots\left(1-\frac{1}{n}\right)$$

$$=\frac{3}{4}\times\frac{4}{5}\times\frac{5}{6}\cdots\frac{n-1}{n}=\frac{3}{n}$$

4. $\left(2-\frac{1}{3}\right)\left(2-\frac{3}{5}\right)\left(2-\frac{5}{7}\right)\cdots\left(2-\frac{997}{999}\right)$ का मान निकालें ?

Speedy Solution :-

$$\left(2-\frac{1}{3}\right)\left(2-\frac{3}{5}\right)\left(2-\frac{5}{7}\right)\cdots\left(2-\frac{997}{999}\right)$$

$$=\frac{5}{3}\times\frac{7}{5}\times\frac{9}{7}\times\cdots\frac{1001}{3}=\frac{1001}{3}$$

5. $1+\frac{1}{1\times2}+\frac{1}{1\times2\times4}+\frac{1}{1\times2\times4\times8}+\frac{1}{1\times2\times4\times8\times16}$ का मान निकालें ?

Speedy Solution :-

$$1+\frac{1}{1\times2}+\frac{1}{1\times2\times4}+\frac{1}{1\times2\times4\times8}+\frac{1}{1\times2\times4\times8\times16}$$

$$=1+\frac{4\times8\times16+8\times16+16+1}{1\times2\times4\times8\times16}$$

$$=1+\frac{512+128+16+1}{1024}=1+\frac{657}{1024}=1.6416$$

6. $\left(1\frac{3}{5}-\frac{2}{3}\div\frac{12}{13}+\frac{7}{5}\times\frac{1}{3}\right)$ का मान निकालें ?

Speedy Solution :-

$$\because 1\frac{3}{5}-\frac{2}{3}\div\frac{12}{13}+\frac{7}{5}\times\frac{1}{3}=\frac{8}{5}-\frac{2}{3}\times\frac{13}{12}+\frac{7}{5}\times\frac{1}{3}$$

$$=\frac{8}{5}-\frac{13}{18}+\frac{7}{15}=\frac{144-65+42}{90}=\frac{121}{90}=1\frac{31}{90}$$

7. $\dfrac{17\times13\times11\times3+17\times17\times17+11\times11\times11+13\times13\times13}{17\times17+11\times11+13\times13-17\times11-13\times11-17\times13}$ का मान निकालें ?

Speedy Solution :-

यदि $17=a$, $13=b$ तथा $11=c$ हो, तो

$$\frac{a^3+b^3+c^3+3abc}{a^2+b^2+c^2-ab-bc-ca}=a+b+c$$

∴ अभीष्ट मान $=17+13+11=41$

8. यदि $\frac{a}{b}=\frac{7}{8}$ हो, तो $\left(\frac{14}{13}-\frac{2b-a}{2b+a}\right)$ का मान निकालें ?

Speedy Solution :-

$a=7$ तथा $b=8$ रखने पर,

$$\left(\frac{14}{13}-\frac{2b-a}{2b+a}\right)=\frac{14}{23}-\frac{2\times8-7}{2\times8+7}=\frac{14}{23}-\frac{9}{23}=\frac{5}{23}$$

9. $\dfrac{1}{3-\dfrac{1}{2-\dfrac{1}{5}}}$ का मान निकालें ?

Speedy Solution :-

$$\frac{1}{3-\dfrac{1}{2-\dfrac{1}{5}}}=\frac{1}{3-\dfrac{1}{\dfrac{9}{5}}}=\frac{1}{3-\dfrac{5}{9}}=\frac{9}{22}$$

10. $\dfrac{2+2\times2}{2\div2\times2}\div\dfrac{\dfrac{1}{2}+\dfrac{1}{2}\text{ का }\dfrac{1}{2}}{\dfrac{1}{2}+\dfrac{1}{2}\text{ का }\dfrac{1}{2}}$ का मान निकालें ?

Speedy Solution :-

$$\frac{2+2\times2}{2\div2\times2}\div\frac{\dfrac{1}{2}+\dfrac{1}{2}\text{ का }\dfrac{1}{2}}{\dfrac{1}{2}+\dfrac{1}{2}\text{ का }\dfrac{1}{2}}=\frac{2+4}{1\times2}\div\frac{\dfrac{1}{2}+\dfrac{1}{4}}{\dfrac{1}{2}+\dfrac{1}{4}}$$

$$=\frac{6}{2}\div\frac{\dfrac{1}{2}\times\dfrac{4}{1}}{\dfrac{3}{4}}=3\div\frac{2}{3}=3+\frac{8}{3}=3\times\frac{3}{8}=\frac{9}{8}=1\frac{1}{8}$$

11. $5-\left[\frac{3}{4}+\left\{2\frac{1}{2}-\left(0.5+\overline{\frac{1}{6}-\frac{1}{7}}\right)\right\}\right]$ का मान बतायें ?

Speedy Solution :-

$$5-\left[\frac{3}{4}+\left\{2\frac{1}{2}-\left(0.5+\overline{\frac{1}{6}-\frac{1}{7}}\right)\right\}\right]$$

$$=5-\left[\frac{3}{4}+\left\{\frac{5}{2}-\left(\frac{1}{2}+\frac{1}{42}\right)\right\}\right]$$

$$=5-\left[\frac{3}{4}+1\frac{41}{42}\right]=5-\frac{229}{84}=2\frac{23}{84}$$

12. $\dfrac{5.32\times56+5.32\times44}{(7.66)^2-(2.34)^2}$ का मान क्या होगा ?

Speedy Solution :-

$$\frac{5.32\times56+5.32\times44}{(7.66)^2-(2.34)^2}=\frac{5.32(56+44)}{(7.66+2.34)(7.66-2.34)}$$

$$=\frac{5.32\times100}{10\times5.32}=10$$

[19]

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1. हल करें –

$$\frac{(0.87)^3 + (0.13)^3}{(0.87)^2 + (0.13)^2 - (0.87)(0.13)}$$

(A) 0.13   (B) 0.74   (C) 0.87   (D) 1

(RRB बंगलोर T.C., 1997)

Speedy Solution : (D)

माना 0.87 = a तथा 0.13 = b

$$\because \frac{a^3 + b^3}{a^2 + b^2 - ab} = \frac{(a+b)(a^2 + b^2 - ab)}{(a^2 + b^2 - ab)} = a + b$$

$$= 0.87 + 0.13 = 1$$

2. $\left(\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \frac{1}{5 \times 6}\right)$ का योगफल है-

(A) $\frac{2}{13}$ (B) $\frac{3}{13}$ (c) $\frac{5}{12}$ (D) $\frac{5}{6}$

(RRB गोरखपुर T.C., 1998)

Speedy Solution : (D)

$$\left(\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \frac{1}{5 \times 6}\right)$$

$$= \left(\frac{360 + 120 + 60 + 36 + 24}{1 \times 2 \times 3 \times 4 \times 5 \times 6}\right) = \frac{600}{720} = \frac{5}{6}$$

3. $4 - \dfrac{5}{1 + \dfrac{1}{3 + \dfrac{1}{2 + \dfrac{1}{4}}}}$ का मान है –

(A) $\frac{40}{31}$ (B) $\frac{4}{9}$ (C) $\frac{1}{8}$ (D) $\frac{31}{40}$

(RRB मुम्बई C.C., 1999)

Speedy Solution : (C)

$$\therefore 4 - \frac{5}{1 + \dfrac{1}{3 + \dfrac{1}{2 + \frac{1}{4}}}} = 4 - \frac{5}{1 + \dfrac{1}{3 + \dfrac{1}{\frac{9}{4}}}}$$

$$= 4 - \frac{5}{1 + \dfrac{1}{3 + \frac{4}{9}}} = 4 - \frac{5}{1 + \dfrac{1}{\frac{31}{9}}} = 4 - \frac{5}{1 + \frac{9}{31}}$$

$$= 4 - \frac{5}{\frac{40}{30}} = 4 - \frac{5 \times 31}{40} = \frac{160 - 155}{40} = \frac{5}{40} = \frac{1}{8}$$

Note : इस प्रकार के प्रश्नों को नीचे से हल करते जाए।

4. $\dfrac{0.216 + 0.064}{0.36 + 0.16 - 0.24}$ किसके बराबर है

(A) 0.64   (B) 1.0   (C) 0.6   (D) 0.4

(RRB महेन्द्रघाट Goods Guard.1998)

Speedy Solution : (B)

$$\frac{0.216 + 0.064}{0.36 + 0.16 - 0.24} = \frac{(0.6)^3 + (0.4)^3}{(0.6)^2 + (0.4)^2 - 0.6 \times 0.4}$$

माना कि 0.6 = a तथा 0.4 = b

$$\therefore \frac{a^3 + b^3}{a^2 + b^2 - ab} = a + b = 0.6 + 0.4 = 1$$

5. व्यंजक $3 + \left[(8-5) + \left\{(4-2) + \left(2 + \frac{8}{13}\right)\right\}\right]$ का मान है-

(A) $\frac{17}{13}$ (B) $\frac{138}{13}$ (C) $\frac{13}{17}$ (D) $\frac{13}{68}$

(RRB गुवाहाटी ASM. 2001)

Speedy Solution : (B)

$$3 + \left[(8-5) + \left\{(4-2) + \left(2 + \frac{8}{13}\right)\right\}\right]$$

$$= 3 + \left[3 + \left\{2 + \left(\frac{26 + 8}{13}\right)\right\}\right]$$

$$= 3 + \left[3 + \left\{2 + \frac{34}{13}\right\}\right]$$

$$= 3 + \left[3 + \frac{60}{13}\right]$$

$$= 3 + \left[\frac{39 + 60}{13}\right] = 3 + \frac{99}{13} = \frac{39 + 99}{13} = \frac{138}{13}$$

6. यदि $\dfrac{a}{3} = \dfrac{b}{4} = \dfrac{c}{7} = \dfrac{(7a - 4b + 3c)}{P}$, तो P का मान होगा –

(A) 16   (B) 58   (C) 26   (D) 28

(RRB गोरखपुर, ASM. 2001)

Speedy Solution : (C)

प्रश्नानुसार,

$$\frac{a}{3} = \frac{b}{4} = \frac{c}{7} = \frac{(7a - 4b + 3c)}{P},$$

माना प्रत्येक पद = K है

$$\frac{a}{3} = \frac{b}{4} = \frac{c}{7} = \frac{(7a - 4b + 3c)}{P} = K$$

$\Rightarrow a = 3k,\quad b = 4k,\quad c = 7k \quad$ तथा $\quad 7a - 4b + 3c = Pk$

$\Rightarrow 7 \times 3k - 4 \times 4k + 3 \times 7k = Pk$

$\Rightarrow 21k - 16k + 21k = Pk \qquad \Rightarrow k(21 - 16 + 21) = Pk$

$\therefore P = 42 - 16 = 26$

7. $3.5 \times 3.5 + 2 \times 6.5 \times 3.5 + 6.5 \times 6.5$ का मान होगा –

(A) 10   (B) 100   (C) 200   (D) 1000

Speedy Solution : (B)

माना $3.5 = a$ तथा $6.5 = b$

$\therefore a^2 + 2 \times ab + b^2 = (a + b)^2 = (3.5 + 6.5)^2 = 100$

8. $\dfrac{4.359 \times 4.359 - 1.641 \times 1641}{4.359 - 1.641} = ?$

(A) 6.3   (B) 6   (C) 3.2   (D) 4.6

Speedy Solution : (B)

माना $4.359 = a$ तथा $1.641 = b$

$\therefore \dfrac{a^2 - b^2}{a - b} = a + b$

$= 4.359 + 1.641 = 6$

9. यदि $\dfrac{a}{3} = \dfrac{b}{4} = \dfrac{3}{7}$ तो $\dfrac{a + b + c}{c}$ का मान क्या होगा –

(A) 2   (B) 7   (C) $\dfrac{1}{2}$ (D) $\dfrac{1}{7}$

Speedy Solution : (A)

माना $\dfrac{a}{3} = \dfrac{b}{4} = \dfrac{c}{7} = K$

$\Rightarrow a = 3k,\quad b = 4k,\quad c = 7k$

$\therefore \dfrac{a + b + c}{c} = \dfrac{3k + 4k + 7k}{7k} = \dfrac{14k}{7k} = 2$

10. $15\dfrac{1}{4} + \dfrac{2}{3} \times 7\dfrac{3}{4} - 6\dfrac{2}{3}$ का मान होगा –

(A) $15\dfrac{35}{36}$ (B) $13\dfrac{3}{4}$ (C) $12\dfrac{3}{4}$ (D) $10\dfrac{3}{4}$

Speedy Solution : (B)

$15\dfrac{1}{4} + \dfrac{2}{3} \times 7\dfrac{3}{4} - 6\dfrac{2}{3}$

$= \dfrac{61}{4} + \dfrac{2}{3} \times \dfrac{31}{4} - \dfrac{20}{3} = \dfrac{61}{4} + \dfrac{31}{6} - \dfrac{20}{3} = 13\dfrac{3}{4}$

11. यदि $a = 16$ तथा $b = 5$ है, तो $\dfrac{a^2 + b^2 + ab}{a^3 - b^3}$ का मान बराबर है –

(A) $\dfrac{1}{19}$ (B) $\dfrac{1}{11}$ (C) $\dfrac{121}{3971}$ (D) $\dfrac{17}{191}$

Speedy Solution : (B)

$\dfrac{a^2 + b^2 + ab}{a^3 - b^3} = \dfrac{1}{a - b} = \dfrac{1}{16 - 5} = \dfrac{1}{11}$

12. सरल करें –

$\dfrac{2(x - 1)}{5} + \dfrac{3(x - 3)}{4} + \dfrac{6(x - 2)}{10} = ?$

(A) $\dfrac{35x + 77}{20}$ (B) $\dfrac{35x - 77}{10}$ (C) $\dfrac{35x - 77}{20}$ (D) $\dfrac{35x - 77}{12}$

Speedy Solution : (C)

$\dfrac{2(x - 1)}{5} + \dfrac{3(x - 3)}{4} + \dfrac{6(x - 2)}{10}$

$= \dfrac{2x - 2}{5} + \dfrac{3x - 9}{4} + \dfrac{6x - 12}{10}$

$= \dfrac{8x - 8 + 15x - 45 + 12x - 24}{20} = \dfrac{35x - 77}{20}$

13. $\left(\dfrac{1}{6} + \dfrac{1}{12} + \dfrac{1}{20} + \dfrac{1}{30} + \dfrac{1}{42} + \dfrac{1}{56} \right) +$

$\left(\dfrac{1 \times 3 \times 6 + 2 \times 6 \times 12 + 3 \times 9 \times 19 + \dots}{1 \times 4 \times 7 + 2 \times 8 \times 14 + 3 \times 12 \times 21 + \dots} \right)$ का मान है –

(A) $\dfrac{86}{57}$ (B) $\dfrac{50}{75}$ (C) $\dfrac{57}{56}$ (D) इनमें कोई नहीं

Speedy Solution : (C)

$\left(\dfrac{1}{6} + \dfrac{1}{12} + \dfrac{1}{20} + \dfrac{1}{30} + \dfrac{1}{42} + \dfrac{1}{56} \right) +$

$\left(\dfrac{1 \times 3 \times 6 + 2 \times 6 \times 12 + 3 \times 9 \times 19 + \dots}{1 \times 4 \times 7 + 2 \times 8 \times 14 + 3 \times 12 \times 21 + \dots} \right)$

$= \left(\dfrac{1}{2} - \dfrac{1}{3} + \dfrac{1}{3} - \dfrac{1}{4} + \dfrac{1}{4} - \dfrac{1}{5} + \dfrac{1}{5} - \dfrac{1}{6} + \dfrac{1}{6} - \dfrac{1}{7} + \dfrac{1}{7} - \dfrac{1}{8} \right)$

$+ \dfrac{18 + 144 + 486 + \dots}{28 + 224 + 756 + \dots}$

$= \dfrac{1}{2} - \dfrac{1}{8} + \dfrac{18}{28} \left(\dfrac{1 + 8 + 27 + \dots}{1 + 8 + 27 + \dots} \right)$

$= \dfrac{4 - 1}{8} + \dfrac{18}{28} = \dfrac{3}{8} + \dfrac{18}{28} = \dfrac{57}{56}$

14. $\left(1+\dfrac{1}{x+1}\right)\left(1-\dfrac{1}{x+2}\right)\left(1+\dfrac{1}{x+3}\right)\left(1+\dfrac{1}{x+4}\right)$ का मान होगा –

(A) $1+\dfrac{1}{x+5}$ (B) $\dfrac{x+1}{x+5}$ (C) $\dfrac{x+5}{x+1}$ (D) इनमें कोई नहीं

Speedy Solution : (C)

$=\dfrac{x+2}{x+1}\times\dfrac{x+3}{x+2}\times\dfrac{x+4}{x+3}\times\dfrac{x+5}{x+4}=\dfrac{x+5}{x+1}$

15. सरलीकृत करने पर, गुणनफल

$\left(2-\dfrac{1}{3}\right)\left(2-\dfrac{3}{5}\right)\left(2-\dfrac{5}{7}\right)\dots\left(2-\dfrac{997}{999}\right)$ बराबर है –

(A) $\dfrac{5}{999}$ (B) $\dfrac{1001}{999}$ (C) $\dfrac{1001}{3}$ (D) इनमें कोई नहीं

Speedy Solution : (C)

$=\left(\dfrac{6-1}{3}\right)\left(\dfrac{10-3}{5}\right)\left(\dfrac{14-5}{9}\right)\dots\left(\dfrac{1998-997}{999}\right)$

$=\dfrac{5}{3}\times\dfrac{7}{5}\times\dfrac{9}{7}\times\dots\times\dfrac{1001}{99}=\dfrac{1001}{3}$

16. यदि $\dfrac{x}{y}=\dfrac{6}{5}$ तो $\dfrac{x^2+y^2}{x^2-y^2}$ का मान है –

(A) $\dfrac{36}{25}$ (B) $\dfrac{25}{36}$ (C) $\dfrac{61}{11}$ (D) $\dfrac{11}{61}$

Speedy Solution : (C)

$a=6$ तथा $y=5$ लेने पर

$\dfrac{x^2+y^2}{x^2-y^2}=\dfrac{(6)^2+(5)^2}{(6)^2-(5)^2}=\dfrac{36+25}{36-25}=\dfrac{61}{11}$

17. $\dfrac{8.73\times8.73\times8.73+4.27\times4.27\times4.27}{8.73\times8.73-8.73\times4.27+4.27\times4.27}=?$

(A) 11 (B) 12 (C) 13 (D) इनमें कोई नहीं

Speedy Solution : (C)

माना $8.73=a$ तथा $4.27=b$

$\therefore\ \dfrac{a^3+b^3}{a^2-ab+b^2}=a+b$

$=8.73+4.27=13$

18. $\sqrt{x^{-1}y}\times\sqrt{y^{-1}z}\times\sqrt{z^{-1}x}=?$

(A) 1 (B) xyz (C) $\sqrt{xyz}$ (D) $(xyz)^2$

Speedy Solution : (A)

$\sqrt{x^{-1}y}\times\sqrt{y^{-1}z}\times\sqrt{z^{-1}x}$

$=\left(x^{-1}y\right)^{1/2}\times\left(y^{-1}z\right)^{1/2}\times\left(z^{-1}x\right)^{1/2}$

$=x^{-1/2}\,y^{1/2}\times y^{-1/2}\,z^{1/2}\times z^{-1/2}\,x^{1/2}$

$=\dfrac{x^{1/2}\times y^{1/2}\times z^{1/2}}{x^{1/2}\times y^{1/2}\times z^{1/2}}=1$

19. यदि $\dfrac{x}{2y}=\dfrac{3}{2}$, तो $\dfrac{2x+y}{x-2y}$ का मान क्या है –

(A) $\dfrac{1}{7}$ (B) 7 (C) 7.1 (D) 6.8

Speedy Solution : (B)

$\dfrac{x}{2y}=\dfrac{3}{2}\quad\therefore\ \dfrac{x}{y}=\dfrac{3\times2}{2}=\dfrac{3}{1}$

$x=3$ तथा $y=1$ रखने पर,

$\therefore\ \dfrac{2x+y}{x-2y}=\dfrac{2\times3+1}{3-2\times1}=\dfrac{6+1}{3-2}=\dfrac{7}{1}=7$

20. यदि $\dfrac{a}{b}=\dfrac{3}{4}$ और $8a+5b=22$ हो, तो 'a' का मान होगा –

(A) 2 (B) $\dfrac{2}{3}$ (C) $\dfrac{5}{8}$ (D) $\dfrac{3}{2}$

Speedy Solution : (D)

$\dfrac{a}{b}=\dfrac{3}{4}$

$\therefore\ b=\dfrac{4a}{3}$

$\Rightarrow 8a+5\times\dfrac{4a}{3}=22$

$\Rightarrow \dfrac{24a+20a}{3}=22$

$\Rightarrow 44a=22\times3$

$\therefore\ a=\dfrac{22\times3}{44}=\dfrac{3}{2}$

21. यदि $x+\dfrac{1}{x}=3$ हो, तो $x^3+\dfrac{1}{x^3}=?$

(A) 9 (B) 18 (C) 27 (D) 6

$$\therefore \left(x+\frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3x\frac{1}{x}\left(x+\frac{1}{x}\right)$$

$$\Rightarrow (3)^3 = \left(x^3 + \frac{1}{x^3}\right) + 3\times 3 \quad \Rightarrow 27 = \left(x^3 + \frac{1}{x^3}\right) + 9$$

$$\therefore x^3 + \frac{1}{x^3} = 27 - 9 = 18$$

22. यदि $a+b+c=0$, तो $\dfrac{a^2+b^2+c^2}{c^2-ab}$ का मान ज्ञात करें-

(A) 0   (B) 1   (C) 2   (D) −2

(RRB राँची ASM, 2002)

Speedy Solution : (C)

$$\because a+b+c=0$$

$$\Rightarrow a+b=-c \qquad \Rightarrow (a+b)^2 = c^2$$

$$\Rightarrow a^2+b^2+2ab = c^2 \qquad \Rightarrow a^2+b^2 = c^2 - 2ab$$

$$\therefore \frac{a^2+b^2+c^2}{c^2-ab} = \frac{c^2-2ab+c^2}{c^2-ab} = \frac{2(c^2-ab)}{(c^2-ab)} = 2$$

23. $\dfrac{x+y+z}{x^{-1}y^{-1}+y^{-1}z^{-1}+z^{-1}x^{-1}}$ का मान है -

(A) $\dfrac{1}{xyz}$ (B) $\dfrac{x}{xy}+\dfrac{x}{yz}+\dfrac{x}{zx}$

(C) xyz (D) इनमें से कोई नहीं

RRB कोलकाता G.G., 2002

Speedy Solution : (C)

$$\frac{x+y+z}{x^{-1}y^{-1}+y^{-1}z^{-1}+z^{-1}x^{-1}}$$

$$= \frac{x+y+z}{\frac{1}{xy}+\frac{1}{yz}+\frac{1}{zx}} = \frac{x+y+z}{\frac{z+x+y}{xyz}}$$

$$= \frac{(x+y+z).xyz}{x+y+z} = xyz$$

24. यदि $x^4+\dfrac{1}{x^4}=727$ हो, तो $\left(x^3-\dfrac{1}{x^3}\right)$ का मान ज्ञात करें -

(A) 125   (B) 140   (C) 155   (D) 170

(RRB कोलकाता/भुवनेश्वर T.C., 2003)

Speedy Solution : (B)

$$x^4 + \frac{1}{x^4} = 727$$

$$\Rightarrow (x^2)^2 + \left(\frac{1}{x^2}\right)^2 = 727 \quad \Rightarrow \left(x^2+\frac{1}{x^2}\right)^2 - 2x^2.\frac{1}{x^2} = 727$$

$$\Rightarrow \left(x^2+\frac{1}{x^2}\right)^2 = 727 + 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = \sqrt{729} = 27 \qquad \dots (i)$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^2 + 2.x.\frac{1}{x} = 27 \quad \Rightarrow \left(x-\frac{1}{x}\right)^2 = 27-2$$

$$\Rightarrow x - \frac{1}{x} = \sqrt{25} = 5$$

अब, $x^3 - \dfrac{1}{x^3} = \left(x-\dfrac{1}{x}\right)\left(x^2 + x.\dfrac{1}{x} + \dfrac{1}{x^2}\right)$

$$= 5 \times \left(x^2 + \frac{1}{x^2} + 1\right) \qquad [(i) \text{ से}]$$

$$= 5 \times (27+1) = 5 \times 28 = 140$$

25. यदि $a-b=3$ और $a^2+b^2=29$, तो ab का मान ज्ञात कीजिए -

(A) 10   (B) 12   (C) 15   (D) 18

| R मुम्बई/ भोपाल गंगमैन, 2003 |

Speedy Solution : (A)

$$\because a-b=3$$

$$\Rightarrow (a-b)^2 = 9 \qquad \Rightarrow a^2+b^2-2ab = 9$$

$$\Rightarrow 29 - 2ab = 9 \qquad \Rightarrow 2ab = 29-9$$

$$\therefore ab = \frac{20}{2} = 10$$

26. यदि $\dfrac{a-x}{a} + \dfrac{2a-x}{2a} = \dfrac{3a-x}{3a}$ हो, तो x का मान है -

(A) 0   (B) $\dfrac{1}{3a}$ (C) $\dfrac{5a}{6}$ (D) $\dfrac{6a}{7}$

(RRB सिकन्दराबाद गंगमैन, 2003)

Speedy Solution : (D)

$$\frac{a-x}{a} + \frac{2a-x}{2a} = \frac{3a-x}{3a}$$

$$\Rightarrow \frac{2a-2x+2a-x}{2a} = \frac{3a-x}{3a} \quad \Rightarrow 3(4a-3x) = 2(3a-x)$$

$$\Rightarrow 12a-9x = 6a-2x \qquad \Rightarrow 6a = 7x$$

$$\therefore x = \frac{6a}{7}$$

[23]