44 अवकलन
(DIFFERENTIATION)

महत्त्वपूर्ण सूत्र

1. यदि $y = x^n$ तो $\frac{dy}{dx} = nx^n - 1$
2. यदि $y = \text{Sin}x$, तो $\frac{dy}{dx} = \text{Cos}x$
3. यदि $y = \tan x$, तो $\frac{dy}{dx} = \text{Sec}^2 x$
4. यदि $y = \text{Cos}x$, तो $\frac{dy}{dx} = -\text{Sin}x$
5. यदि $y = \text{Cot}x$, तो $\frac{dy}{dx} = -\text{Cosec}^2 x$
6. यदि $y = \text{Sec}x$, तो $\frac{dy}{dx} = \sec x.\tan x$
7. यदि $y = \text{Cosec}x$ तो $\frac{dy}{dx} = -\text{Cosec}x.\text{Cot}x$
8. यदि $y = \text{Sin}^{-1}x$, तो $\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}$
9. यदि $y = \text{Cos}^{-1}x$ तो $\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$
10. यदि $y = \tan^{-1}x$ तो $\frac{dy}{dx} = \frac{1}{1+x^2}$
11. यदि $y = \text{Cot}^{-1}x$, तो $\frac{dy}{dx} = -\frac{1}{1+x^2}$
12. यदि $y = \text{Sec}^{-1}x$, तो $\frac{dy}{dx} = \frac{1}{x\sqrt{x^2-1}}$
13. यदि $y = e^x$ तो $\frac{dy}{dx} = e^x$
14. यदि $y = a^x$ तो $\frac{dy}{dx} = a^x \log a$
15. यदि $y = \log e^x$ तो $\frac{dy}{dx} = \frac{1}{x}$
16. यदि $y = \log a^x$ तो $\frac{dy}{dx} = \frac{1}{x}\log a^e$
17. यदि $y = u \pm v$ तो $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$
18. यदि $y = uv$, तो $\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$
19. यदि $y = \frac{u}{v}$, तो $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
20. यदि $y = a$ तो $\frac{dy}{dx} = 0$

अवकलन पर आधारित प्रश्न

1. $\sqrt{x}$ का प्रथम सिद्धांत से अवकलन गुणांक निकालें ?

Speedy Solution :-

मान लिया कि $y = \sqrt{x}$ ...(i)

अब यदि x तथा y में संवादी वृद्धि dx तथा dy हो, तो

$y + dy = \sqrt{x+dx}$

समीकरण (ii) में से (i) को घटाने पर

$dy = \sqrt{x+dx} - \sqrt{x}$

$$\frac{dy}{dx} = \frac{\sqrt{x+dx} - \sqrt{x}}{dx} \times \frac{\sqrt{x+dx} + \sqrt{x}}{\sqrt{x+dx} + \sqrt{x}}$$

$$= \frac{x+dx-x}{dx\left[x+dx+\sqrt{x}\right]} = \frac{1}{\sqrt{x+dx}+\sqrt{x}} = \frac{1}{\sqrt{x}+\sqrt{x}}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

2. यदि $y = \tan\left(\text{Sin}^{-1}x\right)$ तो $\frac{dy}{dx}$ का मान निकाले ?

Speedy Solution :-

$y = \tan\left(\text{Sin}^{-1}x\right)$

$$\frac{dy}{dx} = \frac{d\tan\left(\text{Sin}^{-1}x\right)}{d\left(\text{Sin}^{-1}x\right)} \times \frac{d\left(\text{Sin}^{-1}x\right)}{dx}$$

$$\frac{dy}{dx} = \text{Sec}^2\left(\text{Sin}^{-1}x\right) \times \frac{1}{\sqrt{1-x^2}} = \frac{\text{Sec}^2\left(\text{Sin}^{-1}x\right)}{\sqrt{1-x^2}}$$

3. यदि $y = \tan^{-1}(\text{Sin}x)$ तो $\frac{dy}{dx}$ का मान निकाले ?

Speedy Solution :-

$y = \tan^{-1}(\text{Sin}x)$

$$\frac{dy}{dx} = \frac{d\tan^{-1}(\text{Sin}x)}{d\text{Sin}x} \times \frac{d\text{Sin}x}{dx} = \frac{1}{1+\text{Sin}^2 x} \times \cos x = \frac{\text{Cos}x}{1+\text{Sin}^2 x}$$

4. $8x^3$ का differential co-efficient निम्नलिखित में से कौन है ?

Speedy Solution :-

$$\frac{d}{dx}\left(8x^3\right) = 8\frac{d}{dx}x^3 = 8.3x^2 = 24x^2$$

5. $6\sqrt{x}$ का अवकल गुणांक x के सापेक्ष क्या होगा ?

Speedy Solution :-

$$\frac{d}{dx}\left(6\sqrt{x}\right) = 6\frac{d}{dx}x^{\frac{1}{2}} = 6.\frac{1}{2}x^{\frac{-1}{2}} = \frac{3}{\sqrt{x}}$$

6. $\text{Sin}4x$ का अवकल गुणांक x के सापेक्ष क्या होगा ?

Speedy Solution :-

$$\frac{d}{dx}(\text{Sin}4x) = \frac{d}{d(4x)}(\text{Sin}4x) \times \frac{d(4x)}{dx} = 4\,\text{Cos}4x$$

7. $\log(3x+4)$ का d.c., x के सापेक्ष होगा -

Speedy Solution :-

$$\frac{d}{dx}\{\log(3x+4)\}$$

$$= \frac{d}{d(3x+4)}\left\{\log(3x+4).\frac{d(3x+4)}{dx}\right\} = \frac{1}{3x+4} \times 3 = \frac{3}{3x+4}$$

8. यदि $y = \log_e x^3$ तो $\frac{dy}{dx} = ?$

Speedy Solution :-

$$y = \log_e x^3 = 3\log_e x \qquad [\log \text{के नियम से}]$$

$$\therefore \frac{dy}{dx} = 3.\frac{1}{x} = \frac{3}{x}$$

9. यदि $y = \sqrt{\frac{1+\text{Cos}2x}{1-\text{Cos}2x}}$ तो $\frac{dy}{dx} = ?$

Speedy Solution :-

$$y = \sqrt{\frac{1+\cos 2x}{1-\cos 2x}} = \sqrt{\frac{2\text{Cos}^2 x}{2\text{Sin}^2 x}} = \frac{\text{Cos}x}{\text{Sin}x}$$

$$\therefore y = \text{Cot}x \qquad \therefore \frac{dy}{dx} = -\text{Cosec}^2 x$$

10. यदि $u = 3t^4 - 5t^3 - 18$ तो $\frac{du}{dt} = ?$ जब $t = 1$

Speedy Solution :-

$$\frac{du}{dt} = \frac{d}{dx}(3t^4 - 5t^3 - 18) = 3\times 4t^3 - 5\times 3t^2 - 0 = 12t^3 - 15t^2$$

यदि $t = 1$ तो

$$\frac{du}{dt} = 12(1)^2 - 15(1)^2 = 12 - 15 = -3$$

11. यदि $y = \tan^{-1}\left\{\frac{\text{Cos}x - \text{Sin}x}{\text{Cos}x + \text{Sin}x}\right\}$ तो $\frac{dy}{dx} = ?$

Speedy Solution :-

$$y = \tan^{-1}\left\{\frac{\text{Cos}x - \text{Sin}x}{\text{Cos}x + \text{Sin}x}\right\} = \tan^{-1}\left\{\frac{1-\tan x}{1+\tan x}\right\}$$

$$= \tan^{-1}\left\{\tan\left(\frac{\pi}{4} - x\right)\right\} = \frac{\pi}{4} - x$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{4}\right) - \frac{dx}{dx} = 0 - 1 = -1$$

12. $y = \tan^{-1}\left\{\frac{2x}{1-x^2}\right\}$ तो $\frac{dy}{dx} = ?$

Speedy Solution :-

$$y = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1}x$$

$$\therefore \frac{dy}{dx} = 2\frac{d}{dx}(\tan^{-1}x) = \frac{2}{1+x^2}$$

13. यदि $x^2 + y^2 = 8$ तो $\frac{dy}{dx} = ?$

Speedy Solution :-

$$x^2 + y^2 = 8 \qquad \therefore \frac{dx^2}{dx} + \frac{dy^2}{dx} = \frac{d}{dx}8$$

$$\Rightarrow 2x + 2y\frac{dy}{dx} = 0 \qquad \Rightarrow \frac{dy}{dx} = \frac{-x}{y}$$

14. $y = \sqrt{\text{Sin}x + \sqrt{\text{Sin}x + \sqrt{\text{Sin}x + \ldots\ldots \text{to}\,\infty}}}$ तो $\frac{dy}{dx} = ?$

Speedy Solution :-

$$y = \sqrt{\text{Sin}x + \sqrt{\text{Sin}x + \sqrt{\text{Sin}x + \ldots\ldots \text{to}\,\infty}}}$$

दोनों तरफ वर्ग करने पर

$$y^2 = \text{Sin}x + \sqrt{\text{Sin}x + \sqrt{\text{Sin}x + \ldots\ldots \text{to}\,\infty}}$$

$$\Rightarrow y^2 = \text{Sin}x + y$$

दोनों तरफ x के सापेक्ष differentiation करने पर

$$2y\frac{dy}{dx} = \text{Cos}x + \frac{dy}{dx}$$

$$\Rightarrow (2y-1)\frac{dy}{dx} = \text{Cos}x \qquad \Rightarrow \frac{dy}{dx} = \frac{\text{Cos}x}{2y-1}$$

15. $\tan x$ का अवकल गुणांक $\text{Sin}x$ के सापेक्ष होगा -

Speedy Solution :-

माना कि $s = \tan x$ $\qquad \therefore \frac{ds}{dx} = \text{Sec}^2 x$

तथा $t = \text{Sin}x$ $\qquad \therefore \frac{dt}{dx} = \text{Cos}x$

$\therefore \tan x$ का अवकल गुणांक $\text{Sin}x$ के सापेक्ष

$$= \frac{ds}{dt} = \frac{\frac{ds}{dx}}{\frac{dt}{dx}} = \frac{\text{Sec}^2 x}{\text{Cos}x} = \text{Sec}^3 x$$

16. यदि $x = a\text{Cos}\theta$ तथा $y = a\text{Sin}\theta$ तो $\frac{dy}{dx} = ?$

Speedy Solution :-

$x = a\text{Cos}\theta$ $\qquad \therefore \frac{dx}{d\theta} = -a\text{Sin}\theta$

$y = a\text{Sin}\theta$ $\qquad \therefore \frac{dy}{d\theta} = a\text{Cos}\theta$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\text{Cos}\theta}{-a\text{Sin}\theta} = -\text{Cot}\theta$$

17. यदि $s = 3t^2 + 2t + 5$ तो $\frac{d^2s}{dt^2} = ?$

Speedy Solution :-

$s = 3t^2 + 2t + 5 \qquad \therefore \frac{ds}{dt} = 6t + 2$

$\therefore \frac{d^2s}{dt^2} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d}{dt}(6t+2) = 6$

18. यदि $x = a(\theta - \text{Sin}\theta), y = a(1 - \text{Cos}\theta)$ तो $\frac{dy}{dx}$ निकालें ?

Speedy Solution :-

$\because x = a(\theta - \text{Sin}\theta) \qquad \therefore \frac{dx}{d\theta} = a(1 - \text{Cos}\theta)$

$y = a(1 - \text{Cos}\theta) \qquad \therefore \frac{dy}{d\theta} = a\text{Sin}\theta$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\text{Sin}\theta}{a(1-\text{Cos}\theta)} = \frac{2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} = \cot\frac{\theta}{2}$

19. यदि $y = \tan^{-1}\sqrt{\frac{1-\text{Cos}x}{1+\text{Cos}x}}$ तो $\frac{dy}{dx}$ का मान निकालें ?

Speedy Solution :-

$y = \tan^{-1}\sqrt{\frac{1-\text{Cos}x}{1+\text{Cos}x}} = \tan^{-1}\sqrt{\tan^2\frac{x}{2}} = \tan^{-1}.\tan\frac{x}{2} = \frac{x}{2}$

$y = \frac{x}{2} \qquad \therefore \frac{dy}{dx} = \frac{1}{2}$

20. यदि $y = \sin(\cot x)$ तो $\frac{dy}{dx}$ ज्ञात करें ?

Speedy Solution :-

$\because y = \sin(\cot x)$

$\therefore \frac{dy}{dx} = \frac{d}{dx}[\sin(\cot x)]$

$\frac{d}{d\cot x}[\sin(\cot x)] \times \frac{d}{dx}(\cot x)$

$= \cos(\cot x)(-\text{cosec}^2 x) = -\text{cosec}^2 x\cos(\cot x)$

21. यदि $y = a^x$ तो $\frac{dx}{dx}$ ज्ञात करें ?

Speedy Solution :-

जहाँ $y = ax = e^{\log a^x} = e^{x\log a}$

$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(e^{x\log a}\right) = \frac{d}{dx}\left(e^{x\log a}\right) \times \frac{d}{dx}(x.\log a)$

$= e^{x\log a}.\log a = \log a$

22. यदि $y = \log(3x+4)$ तो $\frac{dy}{dx}$ ज्ञात करें ?

Speedy Solution :-

$\because y = \log(3x+4)$

$\therefore \frac{dy}{dx} = \frac{d}{dx}[\log(3x+4)] = \frac{d[\log(3x+4)]}{d(3x+4)} \times \frac{d}{dx}(3x+4)$

$= \frac{1}{3x+4} \times 3 = \frac{3}{3x+4}$

23. यदि $y = \sqrt{\frac{1+\cos 2x}{1-\cos 2x}}$ तो $\frac{dy}{dx}$ ज्ञात करें ?

Speedy Solution :-

यहाँ $y = \sqrt{\frac{1+\cos 2x}{1-\cos 2x}} = \sqrt{\frac{2\cos^2 x}{2\sin^2 x}} = \cot x$

$\therefore \frac{dy}{dx} = \frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

24. यदि $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ तो $\frac{dy}{dx}$ ज्ञात करें ?

Speedy Solution :-

$y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$x = \tan\theta$ रखने पर $\frac{1-x^2}{1+x^2} = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$

$\therefore y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \cos^{-1}(\cos 2\theta) = 2\theta = 2.\tan^{-1}x$

$\therefore \frac{dy}{dx} = 2\frac{d}{dx}(\tan^{-1}x) = 2.\frac{1}{1+x^2} = \frac{2}{1+x^2}$

25. यदि $x + y = \sin(xy)$ तो $\frac{dy}{dx}$ ज्ञात करें ?

Speedy Solution :-

$x + y = \sin xy$

$\therefore \frac{d(x+y)}{dx} = \frac{d}{dx}(\sin xy) = \frac{d}{dx}(\sin xy) \times \frac{d}{dx}(xy)$

$\Rightarrow \frac{d(x)}{dx} + \frac{dy}{dx} = \cos(xy) \times \left(x\frac{dy}{dx} + 1.y\right)$

$\Rightarrow 1 + \frac{dy}{dx} = x\cos(xy)\frac{dy}{dx} = y\cos xy$

$\Rightarrow (1 - x\cos(xy))\frac{dy}{dx} = y\cos(xy) - 1$

$\therefore \frac{dy}{dx} = \frac{y\cos(xy)-1}{1-x\cos(xy)}$

PREVIOUS YEAR'S RRB'S QUESTIONS

1. $y=\tan^{-1}\sqrt{x}$ तो $\frac{dy}{dx}$ का मान निकालें ?

(A) $\frac{1}{2\sqrt{x}(1+x)}$ (B) $\frac{1}{(1+x)}$

(C) $\frac{1}{2(1+x)}$ (D) $\frac{1}{2x(1+x)}$

(RRB कोलकता J.E., 2000)

Speedy Solution : (A)

$$\frac{dy}{dx}=\frac{d\tan^{-1}\sqrt{x}}{d\sqrt{x}}\cdot\frac{d\sqrt{x}}{dx}=\frac{1}{1+(\sqrt{x})^2}\cdot\frac{1}{2\sqrt{x}}=\frac{1}{2\sqrt{x}(1+x)}$$

2. यदि $y=\sqrt{x+\sqrt{x+\sqrt{x+\sqrt{x+....}}}}$ to ∞ तो $\frac{dy}{dx}$ निकालें?

(A) $\frac{1}{y-1}$ (B) $\frac{y}{y-1}$ (C) $\frac{1}{2y-1}$ (D) $\frac{y}{2y-1}$

(RRB गोरखपुर E.S.M.- III, 2003)

Speedy Solution : (C)

$y=\sqrt{x+y}$ (क्योंकि दाएँ पक्ष में प्रथम पद छोड़ने पर भी y के ही बराबर रहता है, चूँकि पदों की संख्या अनंत है।)

$\Rightarrow y^2=x+y$

x के सापेक्ष अवकलन करने पर $2y\frac{dy}{dx}=1+\frac{dy}{dx}$

$\Rightarrow (2y-1)\frac{dy}{dx}=1$ $\therefore \frac{dy}{dx}=\frac{1}{2y-1}$

3. $y=\text{Cos}(2x+y)$ तो $\frac{dy}{dx}$ का मान निकालें ?

(A) $\frac{2\text{Sin}(2x+y)}{1+\text{Sin}(2x+y)}$ (B) $\frac{\text{Sin}(2x+y)}{1+\text{Sin}(2x+y)}$

(C) $\frac{\text{Sin}(2x+y)}{(2x+y)}$ (D) $\frac{(2x+y)}{\text{Sin}(2x+y)}$

(RRB अजमेर Laba Asst.., 1999)

Speedy Solution : (A)

$y=\text{Cos}(2x+y)$

दोनों पक्षों का x के सापेक्ष अवकलन करने पर

$$\frac{dy}{dx}=\frac{d\text{Cos}(2x+y)}{d(2x+y)}\times\frac{d(2x+y)}{dx}=-\text{Sin}^2(2x+y).\left(2+\frac{dy}{dx}\right)$$

$$\therefore \frac{dy}{dx}=\frac{2\text{Sin}(2x+y)}{1+\text{Sin}(2x+y)}$$

4. यदि $y=\text{Sin}x$ तो $\frac{d^2y}{dx^2}$ का मान होगा –

(A) $\text{Sin}x$ (B) $-\text{Sec}x$ (C) $-\text{Cos}x$ (D) $-\text{Sin}x$

(RRB अजमेर T.A., 2003)

Speedy Solution : (D)

$y=\text{Sin}x$

$$\therefore \frac{dy}{dx}=\text{Cos}x \quad \therefore \frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dx}\text{Cos}x=-\text{Sin}x$$

5. यदि $y=x^3+5x^2+7$ तो $x=0$ पर $\frac{d^2y}{dx^2}$ का मान होगा–

(A) 6 (B) 7 (C) 10 (D) 5

(RRB चेन्नई Diesel Driver, 2001)

Speedy Solution : (C)

$y=x^3+5x^2+7$ $\therefore \frac{dy}{dx}=3x^2+10x$

पुनः अवकलन करने पर $\frac{d^2y}{dx^2}=6x+10$

$\therefore$ जब $x=0$ तब $\frac{d^2y}{dx^2}=10$

6. $\text{Sin}x$ का अवकल गुणांक $\text{Cos}x$ के सापेक्ष होगा –

(A) $-\tan x$ (B) $-\text{Cot}x$ (C) $-\text{Sin}x$ (D) $\text{Sec}x$

(RRB बंगलोर A.S.M., 2004)

Speedy Solution : (B)

माना कि $s=\text{Sin}x$ तथा $t=\text{Cos}x$

$$\therefore \frac{ds}{dx}=\text{Cos}x \text{ तथा } \frac{dt}{dx}=-\text{Sin}x$$

$$\therefore \frac{d(\text{Sin}x)}{d(\text{Cos}x)}=\frac{ds}{dt}=\frac{\frac{ds}{dx}}{\frac{dt}{dx}}=\frac{\text{Cos}x}{-\text{Sin}x}=-\text{Cot}x$$

7. यदि $y=x^3+5x^2-7x-8$ तो $x=0$ पर $\frac{d^2y}{dx^2}$ का मान ज्ञात करें –

(A) 10 (B) 16 (C) 0 (D) –4

(RRB चंडीगढ़ E.S.M., 2004)

Speedy Solution : (A)

यहाँ $y=x^3+5x^2-7x-8$

$$\therefore \frac{dy}{dx}=\frac{d}{dx}\left(x^3+5x^2-7x-8\right)=3x^2+10x-7=3x^2+10x-7$$

$$\therefore \frac{d^2.y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dx}\left(3x^2+10x-7\right)=6x+10-0=6x+10$$

$\therefore x=0$ पर $\frac{dy^2}{dx^2}=6\times0+10=10$