CN 530 Neural and Computational Models of Vision

Simulation Assignment 3

The theme of this assignment is *boundary-gated diffusion*. The problem statement is *deliberately* "open-ended," (not to say vague), as befits an exploration of a currently debated topic in the field, about which no consensus among experts exists.

Note carefully: The simulations asked for are all "1-D," so computational load should not be excessive. You are actively encouraged to consult the course TF concerning the relative merits of simulation speed vs. ease of development for such options as (1) your own C code vs. (2) MatLab. You can use the same software you used for Assignment 2 to generate FCS inputs and (with minor modification) BCS inputs, so implementation of the diffusion equations represents the major novelty. The requirements of your simulations are:

Item 1: Replication of simple filling-in simulation.
Replicate two of the simplest G & T 1988 simulations, specifically those whose results are plotted in Figures 14a and 14b. By "replicate" is meant "solve for equilibrium" the pertinent equations. *I will leave it up to you* whether to employ exactly the published parameters, simulation resolution, and so forth, or whether to use other parameters and resolutions that give qualitatively similar results. **Note 1:** You should solve all stages other than the diffusion stage at equilibrium; that is, do *not* numerically integrate those equations. **Note 2:** If you have access to software for solving the diffusion stage at equilibrium -- effectively requiring the inversion of a large, banded matrix to solve simultaneous algebraic equations -- you may use it. **Note 3:** If you choose to numerically integrate the diffusion equations, *consult the teaching fellow before proceeding, and see Note 5 in this paragraph*. If the time you have devoted to this step of Item 1 exceeds four hours, consult the teaching fellow. You are also welcome to approach the teaching fellow, in any case, *before* this "milestone" is achieved! Note 3 is included as a disclaimer against allegations of "injury" on the part of anyone experiencing difficulties. **Note 4:** You *do not have to* numerically integrate the diffusion equation, as reasonable results can be obtained in the present case by a relaxation approach: In the first iteration, solve for the "equilibrium" of each node based on the inputs -- from Levels 2, 5, and 6 -- to that node. After all nodes have been solved for, repeat the procedure with "new" input values for Level 6 nodes, based on the results of the first step, etc. In other words, solve Eq. 25 from Grossberg and Todorović (1988) repeatedly for each S_{ij}, using the S_{pq} values from the previous time step for each iteration. This calculation should be carried out until the maximum change across all S_{ij}, nodes falls below some

criterion in a given iteration. **Note 5: If you have a particular interest in performing a numerical solution of ordinary differential equations, either because you want the practice or because you want to observe the temporal dynamics of the boundary-gated diffusion process – that is, to see what the state of the system looks like at times before equilibrium – then contact me before proceeding. Experience with solutions of "stiff" systems of equations is a plus for this option.** We can negotiate some substitutions in the assignment statement, whereby you skip some of the parts in this assignment statement, in favor of a greater exploration of the system's temporal aspects. **Note 6:** Some of you may see narrow peaks in your simulation output, sticking up from each side of a "plateau," in the form of a "Batman" profile; others of you may not. It is not the case that either of these outcomes is correct and the other incorrect. Do try, however, to understand what aspects of parameter choices or simulation technique may be responsible for what you observe.

Item 2: Brightness ramps

Try to demonstrate *some* combination of inputs and parameters that yields a "noticeable" brightness gradient at equilibrium in the shape of a *ramp,* i.e. a sloping line. If the sloping line also curves a bit, that's okay. That is, the resulting plot is *not* composed entirely of horizontal plateaus and sharp steps between plateaus, but contains some smooth drop from a high to a low level over space. **Note:** The combination of inputs and parameters required to do this with the G & T, 1988 formulation *may* seem "implausible" in order to achieve this end in a way that is visible to the naked eye in your plots. If you find that you cannot accomplish this in a reasonable time (i.e. two hours of trying, assuming that you *did* get part 1 to work), then simply "write up" an explanation of *why* this task is tricky with the G & T, 1988 system.

Item 3: Border-Ownership (*thanks to Oliver Layton who patiently contributed to this item***)**

The discovery of cells that are sensitive to the direction of edge ownership may have implications for how we bridge the perceptual process of figure-ground segregation and early vision physiology. Although border-ownership cells (B cells) have very small receptive fields, they exhibit modulation due to global context (e.g. respond only if the figure is to the left instead of the right, even though the local edge in the receptive field appears identical). In this item, you will explore a 1D version of the model proposed by Craft et al. (2007).

1. Begin with a simple bar input that has a generous width compared to the total size of your 1D presentation. Use the BCS code you wrote (Item 1) to generate the complex cell responses to this stimulus.
2. Grouping cells (G cells) in the model of Craft et al. (2007) have 2D 'ring-like' kernels that geometrically resemble the upper half of a torus (i.e. the top half of a sliced bagel).

First, think about how a 1D version of the kernel would appear and then implement it (e.g. a rectified inverted Mexican hat).

3. Decompose the G cell kernel into 'fragments'. In other words, produce two additional kernels that are the same size as the G cell kernel that each contains only one of the 'lobes'.
4. Numerically integrate the following system of coupled ordinary differential equations:

$$\tau_B \frac{dB_{i,\rightarrow}}{dt} = -B_{i,\rightarrow} + [C_i - \rho(G_i * K_{\rightarrow})]^+$$

$$\tau_B \frac{dB_{i,\leftarrow}}{dt} = -B_{i,\leftarrow} + [C_i - \rho(G_i * K_{\leftarrow})]^+$$

$$\tau_G \frac{dG_i}{dt} = -G_i + \gamma\sqrt{(B_{i,\rightarrow} * K_{\rightarrow})(B_{i,\leftarrow} * K_{\leftarrow})}$$

The τ are the time constants, the arrows indicate the direction of border ownership of each B cell, C indicates the complex cell input at the same spatial location, ϱ and γ specify proportionality constants, $[]^+$ depicts rectification, the * operator denotes convolution, and the K are the G cell kernel 'fragments' you derived in 3. Choices for the parameter values are up to you.

5. Plot the G cell activity to show the cells respond to the interior of uniform regions. Adjust the parameters of your G kernel as necessary to show this behavior. What do you notice about the relationship between your kernel parameter choices and the location of the G cell responses?
6. Next consider the B cell activity by computing an analogue of the "vectorial modulation index" for the pair of B cells at each spatial location using the following equation:

$$v_i = \frac{B_{i,\leftarrow} - B_{i,\rightarrow}}{B_{i,\leftarrow} + B_{i,\rightarrow}}$$

A positive v at each position i indicates a rightward horizontal direction of ownership and conversely a negative value shows a leftward horizontal direction of ownership. Depending on the direction convention you used to determine the direction of each fragment, you may obtain a reversed mapping. You can set the sign by examining the situation in which the B cells point inward (or outward) at edge locations where the complex cells activity is maximum when presented with a bar stimulus. Plot the modulation indices (a vector plot is

not required) and make sure you indicate what the sign of v means at the edges. You may have symmetric modulation indices on either side of an edge depending on the properties of your kernel, which is fine.

7. Answer the following questions: In the G cell differential equation, why are we multiplying the B cells terms of opposite directions? How can this generalize in 2D? How do the B cells develop their ownership directional sensitivity? Can you present a stimulus for which the model has trouble representing the appropriate B cell direction? If so, include the stimulus, complex cell, G cell, and B cell modulation plots along with a description why the model fails in the particular case you tested. Finally, how does the representation used in this model differ from that of the BCS/FCS, and how does it compare?