MA 555 - Numerical Analysis

Homework # 1
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1. Minimize $I(a,b) = \int_{-1}^{1}(x^2+ax+b)^2dx$. Graph the function and locate its maximum value over the interval [-1,1].

In order to minimize this function we take the partial derivatives of I with respect to a and b and set them equal to 0. We end up with a linear system with two equations and two unknowns. Starting with a...

$$\frac{\partial I}{\partial a} = 2\int_{-1}^{1}(x^2+ax+b)(x)dx = 0 \tag{1}$$

By solving this integral in Mathematica we get the result

$$0 = \frac{-2a}{3} \tag{2}$$

and conclude that $a = 0$. Now to b...

$$\frac{\partial I}{\partial b} = 2\int_{-1}^{1}(x^2+ax+b)dx = 0 \tag{3}$$

Again solving in Mathematica we see that

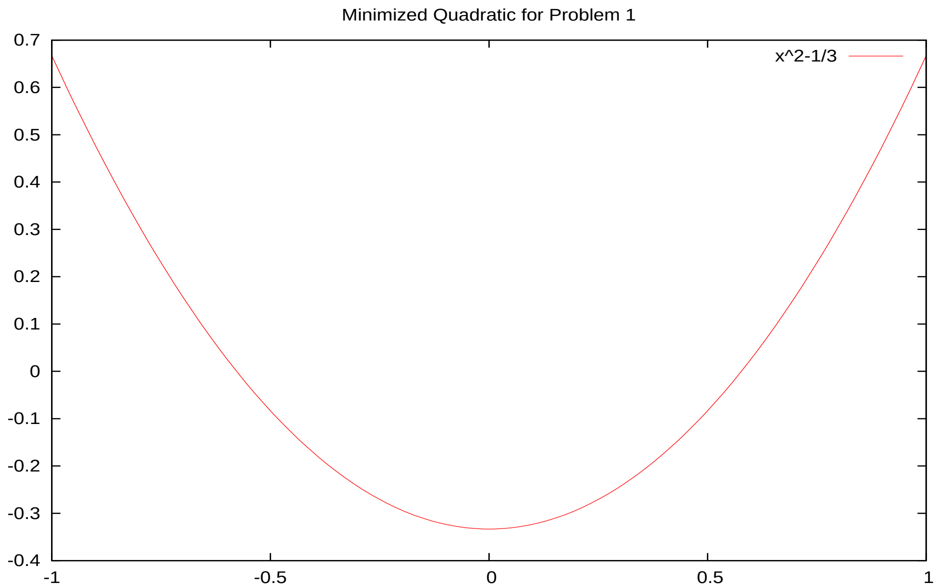
$$0 = \frac{2}{3} + 2b \tag{4}$$

$$b = -\frac{1}{3} \tag{5}$$

Therefore, our minimized function is

$$I_{min}(a,b) = \int_{-1}^{1}(x^2 - \frac{1}{3})^2dx \tag{6}$$

Graphing the quadratic function $x^2 - \frac{1}{3}$ yields the following figure



from which it is clear that the maximum value is $\frac{2}{3}$, occuring at $|x| = 1$.

2. Minimize a similar function $I(c, d)$ using the previous result as a weight function.

$$\frac{\partial I}{\partial c} = 2\int_{-1}^{1} (x^2 - \frac{1}{3})^2 (x^2 + cx + d)(x) dx = 0 \tag{7}$$

Solving this integral in Mathematica gives the result

$$0 = \frac{88c}{945} \tag{8}$$

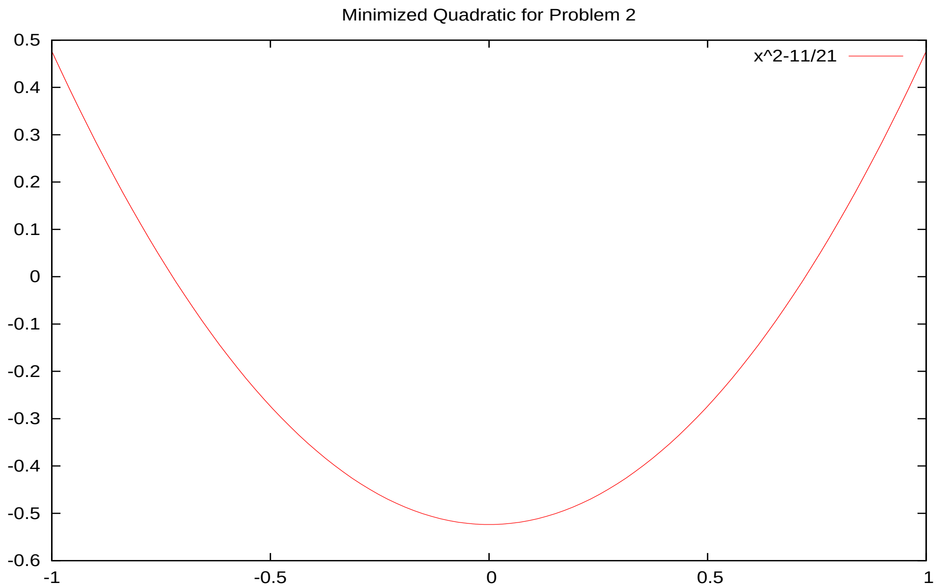
From which we conclude $c = 0$. Moving on to d...

$$\frac{\partial I}{\partial d} = 2\int_{-1}^{1} (x^2 - \frac{1}{3})^2 (x^2 + cx + d) dx = 0 \tag{9}$$

Solving this integral in Mathematica gives the result

$$0 = \frac{44}{945} + \frac{84d}{945} \tag{10}$$

Solving for d shows that $d = -\frac{11}{21}$. Intuition (and Professor Fried) tells us that the best value we can get is $-frac12$, so this is pretty good. Graphing the quadratic function $x^2 - \frac{11}{21}$ yeilds the figure



The max clearly occurs at $|x| = 1$, and solving the equation for $x = \pm 1$ yeilds tha max of $\frac{10}{21}$.

3. Minimize the function $I(b) = \int_{-1}^{1}(x^3 - bx)^2 dx$.

$$\frac{\partial I}{\partial b} = 2\int_{-1}^{1}(x^3 - bx)(x)dx = 0 \tag{11}$$

Solving this integral yields

$$0 = \frac{2}{5} - \frac{2b}{3} \tag{12}$$

From which we see that $b = \frac{3}{5}$ and

$$I_{min}(b) = \int_{-1}^{1}(x^3 - \frac{3}{5})^2 dx \tag{13}$$

4. Calculate $x = \frac{1.23678-1.23456}{1.23555-1.23444}$ using 4,5, and 6 digit arithmetic.

Here is a copy and paste from my terminal. I used Python and its round function.

```
>>> for n in range (4,7):
...     x=round(1.23678-1.23456,n)/round(1.23555-1.23444,n)
...     print str(x) + ", " + str(n)
...
2.0, 4
2.0, 5
2.0, 6
```

This seemed strange at first, but it makes sense if you actually do it out by hand. The 4^{th}, 5^{th}, and 6^{th} always have the same difference during subtraction.